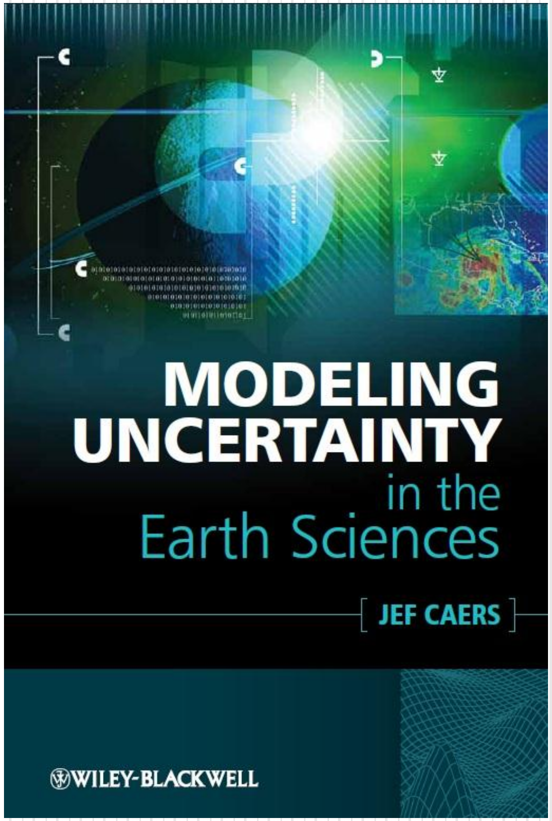
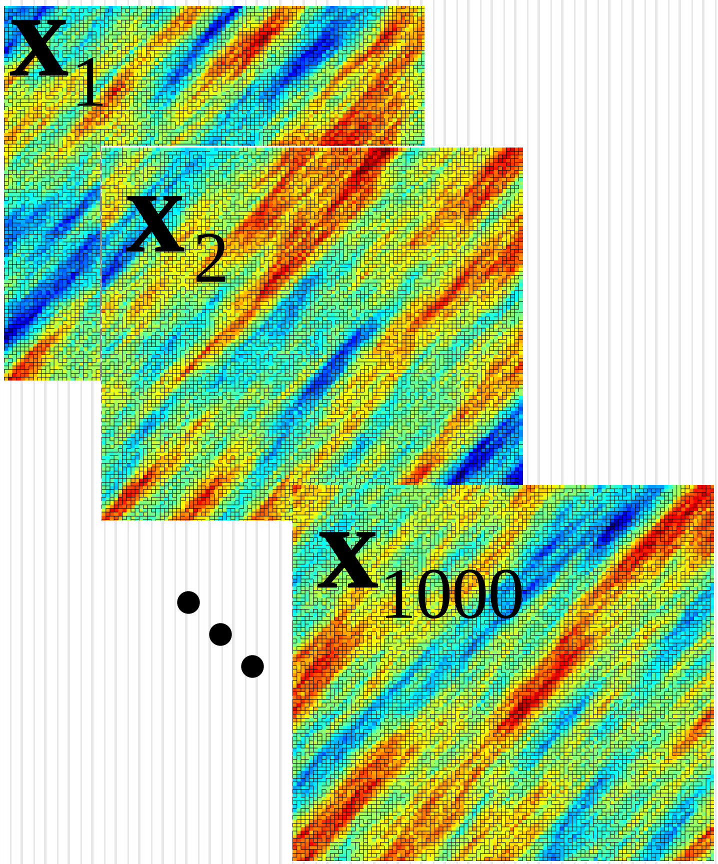
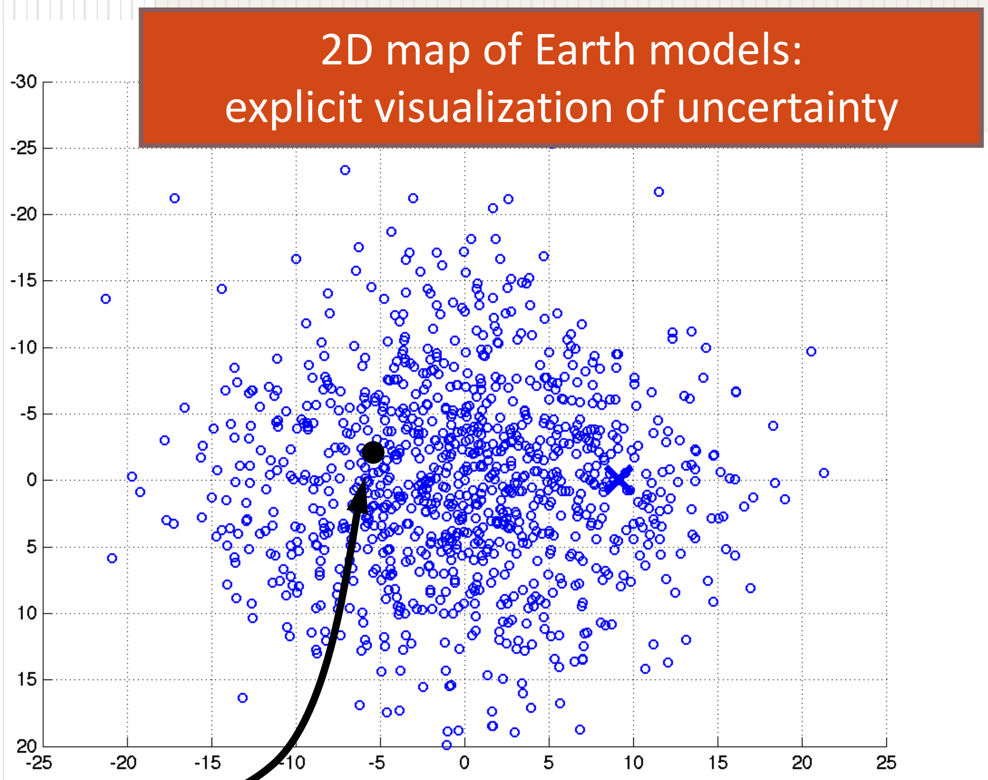
Modeling response uncertainty



Distances: multi-dimensional scaling

1000 SGSIM model





Calculate Euclidean distance

$$d_{ij} = \sqrt{(\mathbf{x}_i - \mathbf{x}_j)^T (\mathbf{x}_i - \mathbf{x}_j)}$$

MDS summary

matrix of all Earth models $X = \left[\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_\mathrm{L} \right]^T$

$$\Downarrow$$

Euclidean distance matrix A

$$\Downarrow$$

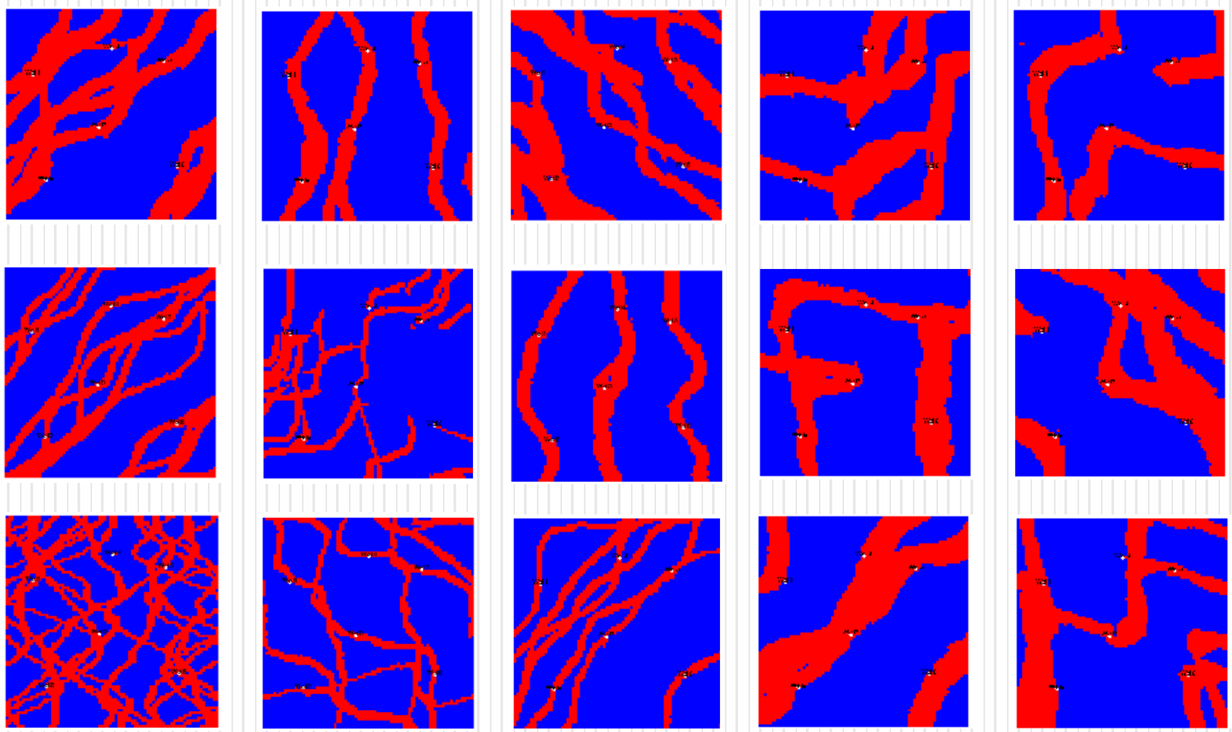
Dot-product $B = XX^T = V \Lambda V^T$

$$\Downarrow$$

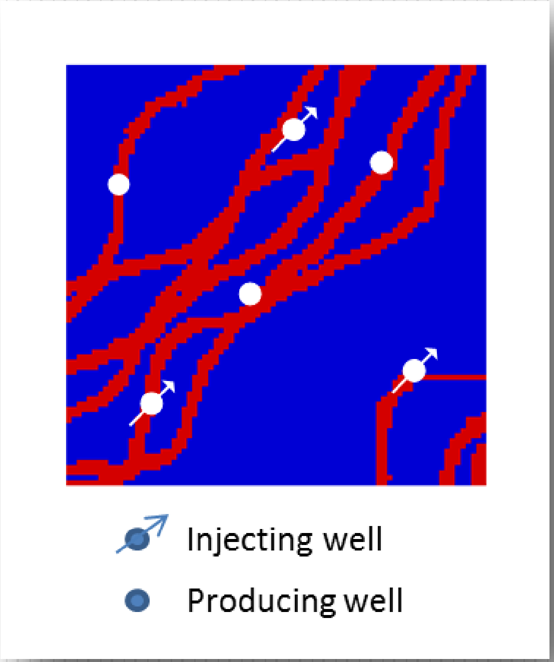
Reconstruction $X = V \Lambda^{1/2}$

Example: Earth models

Created 405 2D channel Earth models
with different proportion, direction, width



Example: response



Basic questions

What is the uncertainty in Oil production over time in terms of [P10, P90]?

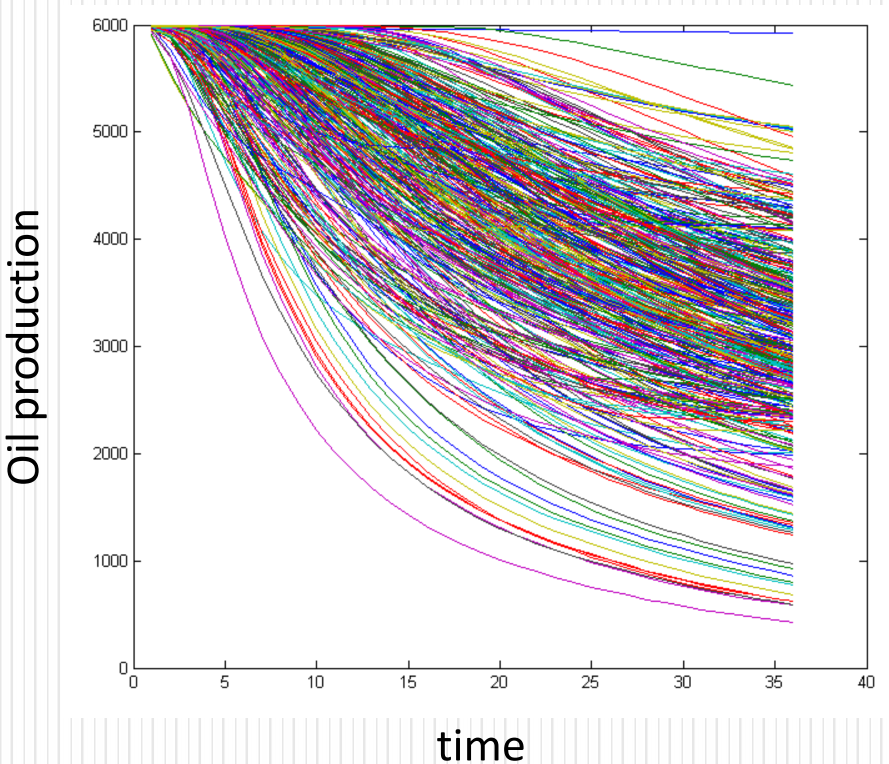
What are the most influencing factors ?

True response takes too long to evaluate on 405 models.

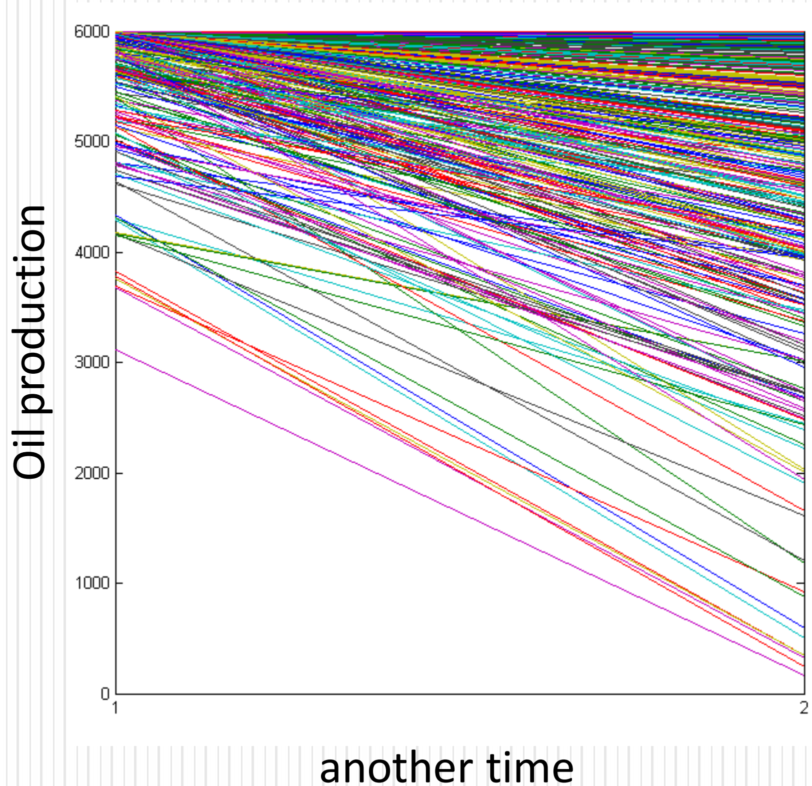
We have a proxy response

Responses

True Response



Proxy Response



Multi-dimensional scaling in MATLAB

- Calculating distances
  - `dvector = pdist(response)`
  - `d = squareform(dvector)`

- Basic command for MDS
  - `[Y e] = cmdscale(d)`

- Plotting
  - `scatter` for 2D plots
  - `scatter3` for 3D plots
  - `plot_MDS_response_value` for adding a color that corresponds to some (single) response value

Kernel transformation

distance

D	1	2	3	4
1	0	1	1	$\sqrt{2}$
2	1	0	$\sqrt{2}$	1
3	1	$\sqrt{2}$	0	1
4	$\sqrt{2}$	1	1	0

$\lambda_1 = \lambda_2 = 1$

$\lambda_3 = \lambda_4 = 0$

k= 1-exp(-d)

new distance

K	1	2	3	4
1	0	0.86	0.86	0.94
2	0.86	0	0.94	0.86
3	0.86	0.94	0	0.86
4	0.94	0.86	0.86	0

$\lambda_1 = \lambda_2 = 0.44$

$\boldsymbol{\lambda_3 = 0.31}$

$\lambda_4 = 0$

Kernel transformation

Transformation from one metric space to another does not require knowledge of φ, only knowledge of the dot-product $\varphi^{T}\varphi$

$$\varphi^{T}(\mathbf{x})\varphi(\mathbf{y}) = k(\mathbf{x},\mathbf{y})$$

example

Euclidean distance obtained with MDS

$$k(\mathbf{x},\mathbf{y}) = \exp\left(-\frac{\sqrt{(\mathbf{x}-\mathbf{y})^{T}(\mathbf{x}-\mathbf{y})}}{\sigma}\right)$$

<u>Role of the Kernel</u>

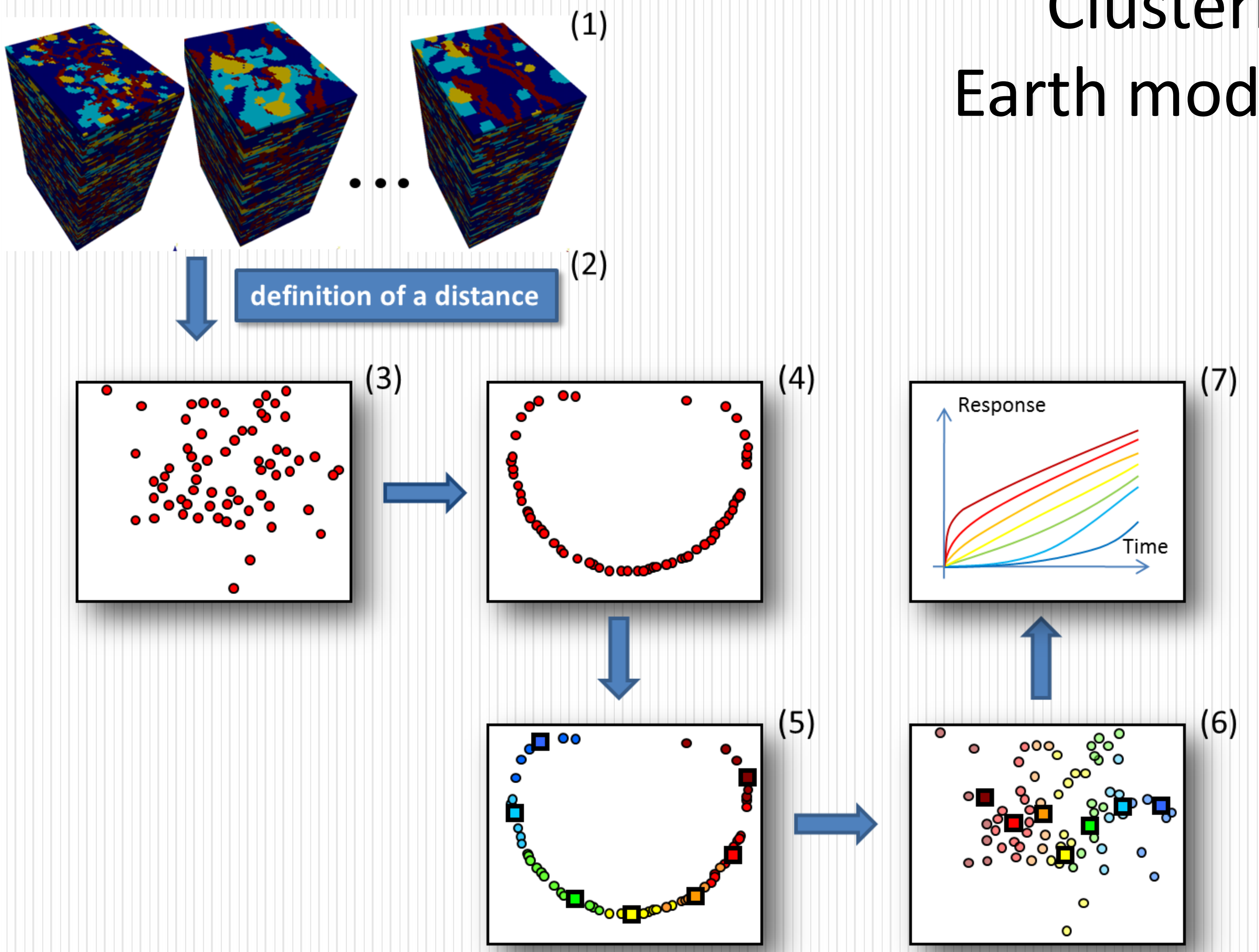
Increase dimension, seperability and linearity

Command

- Radial basis kernel
- Input = coordinates after projection with MDS
- Output = Kernel matrix K

- `K=rbf_kernel(coord)`

Clustering Earth models



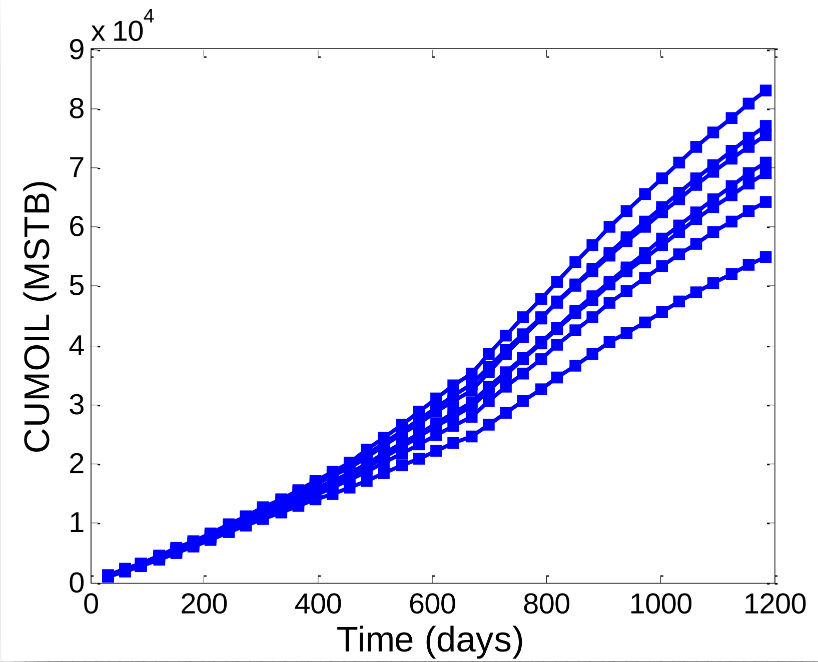
Script

- `kmedoids.m`

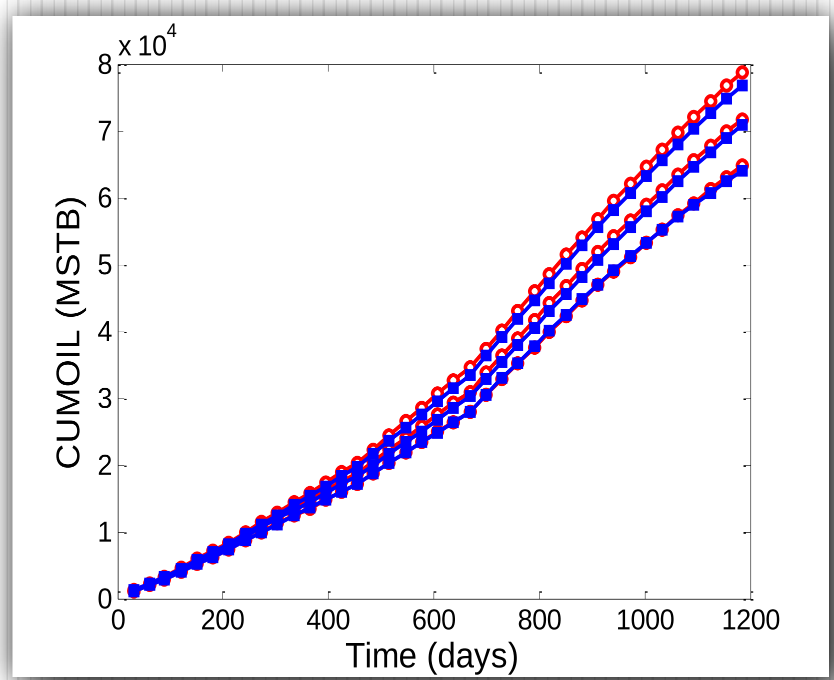
- Input (in script)
  - Number of clusters
  - Number of iterations

Response calculation

Response of 7 selected Earth models



Calculated P10, P50 and P90



Command

- `Quantiles_Calculation`

- Input
  - Responses
  - Vector containing percentages corresponding to the quantiles you want to calculate, e.g. [0.10 0.50 0.90]

- Output: quantiles of the responses

Command

- `Sensitivity_Analysis_Classical`

- Input
  - Parameter values (experimental design indicators)
  - Responses belong to those parameter values
  - Parameter names
  - Flag variable to indicate whether you want to study interaction

- Output
  - Effect estimates for all parameters