TIME-AVERAGE EQUATION REVISITED

JACK DVORKIN AND AMOS NUR
DEPARTMENT OF GEOPHYSICS, STANFORD UNIVERSITY, STANFORD, CA 94305-2215

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INTRODUCTION

Expressions that relate velocity to porosity and to pore-fluid compressibility are among the most important deliverables of rock physics. Such relations are often used as additional controls for inferring porosity from well logs, as well as in-situ indicators of pore fluid type. The oldest and most popular is the Wyllie et al. (1956) equation:

$$\tau_P = \tau_S + \tau_F, \tag{1}$$

where τ_P is the measured travel time of a P-wave, τ_S is the travel time expected in the solid-phase material, and τ_F is that expected in the pore fluid. It follows from equation (1) that

$$\frac{1}{V_p} = \frac{1-\varphi}{V_{pS}} + \frac{\varphi}{V_{pF}}, \tag{2}$$

where φ is porosity, V_p is the measured P-wave velocity, and V_{pS} and V_{pF} are the P-wave velocities in the solid and in the pore-fluid phases, respectively.

Formulas (1) and (2) present a simple and convenient, but deceptive form of summarizing extensive experimental data. Indeed, there is no physical reason for the total travel time of a wave in a two-component composite to be the sum of the travel times in the individual components (unless the two components are arranged in layers normal to the direction of propagation, and the wavelength is small as compared to the thickness of an individual layer).

At the same time, a host of first-principle-based effective medium models exists that provide relations among velocity, porosity, and pore-fluid compressibility. Such relations explicitly take into account the internal structure of rock. Examples of effective medium models are: Hudson (1990), for cracked rocks; Kuster and Toksöz (1974), for low-porosity rocks; Berryman (1980), for low- to medium-porosity rocks; Digby (1981), Walton (1987), and Dvorkin et al. (1994) for high-porosity granular rocks. Reviews of such theories are given by Zimmerman (1991) and Wang and Nur (1992).

Is it necessary to abandon the non-physical time average equation, or equally simple traditional empirical relations, in favor of rigorous physics-oriented models?

In this short note we address this question by applying traditional velocity-porosity transformations to three sandstone datasets. In addition to the Wyllie et al. (1956) equation (WGG) we consider another popular porosity-to-velocity transformation of Raymer et al. (1980):

$$V_p = (1-\varphi)^2 V_{pS} + \varphi V_{pF}, \quad \varphi < 0.37, \tag{3}$$

as well as the relation between rock density (ρ) and velocity of Gardner et al. (1974):

$$\rho = 0.23 V_p^{\,0.25}. \tag{4}$$

In equation (4) velocity is in ft/s, and density is in g/cm^3. The latter two transformations are referred to as RHG and GGG, respectively.

Rock density can be related to the density of the solid phase (ρ_S) and to the density of the fluid (ρ_F) as

$$\rho = (1-\varphi)\rho_S + \varphi\rho_F. \tag{5}$$

By combining equations (4) and (5) we arrive at the following velocity-to-porosity transformation (GGG):

$$V_p = [\frac{(1-\varphi)\rho_S + \varphi\rho_F}{0.23}]^4. \tag{6}$$

EXAMPLES

In order to assess the accuracy of equations (2), (3), and (6), we apply them to three sandstone datasets. The velocity measurements have been conducted in the laboratory on small plugs at ultrasonic frequencies. The first dataset (Han, 1986) includes about 70 consolidated sandstone samples with porosity between 0.05 and 0.3, and volumetric clay content between zero and 0.5. The second dataset includes high-porosity slightly-cemented rocks from the Oseberg field in the North Sea (Strandenes, 1991). In these rocks cement is either quartz or clay. The third dataset is from the Troll field in the North Sea (Blangy, 1992). It includes high-porosity unconsolidated and uncemented samples where the dominant minerals are quartz, feldspar, and mica.

The experimental equations under consideration are intended to be applied to water-saturated rocks at high effective pressures. Accordingly, the effective pressure for the first two datasets is 40 MPa, and it is 30 MPa for the third dataset. Ultrasonic measurements of P-wave velocities in saturated rocks often overestimate the seismic- and sonic-frequency velocities expected in situ (e.g., Han, 1986). This is why in order to obtain V_p in a saturated rock, we use the corresponding high-frequency P- and S-wave velocities measured in the dry rock, and then apply Gassmann's (1951) transformation. By so doing we assume that velocities in dry homogeneous rock samples are frequency-independent (e.g., Bourbié et al., 1987). The pore fluid is water with V_{pF} = 1.5 km/s and ρ_F = 1 g/cm^3. These saturated-rock velocities are referred to as "true" velocities.

In cases where the solid phase contains more that one (n) mineral components, we calculate the effective elastic moduli of the solid phase using Hill's (1952) average:

$$M_S = 0.5(M_V + M_R),$$
$$M_V = \sum_{i=1}^{n} X_i M_i,\ M_R = (\sum_{i=1}^{n} \frac{X_i}{M_i})^{-1}, \tag{7}$$

where M is either bulk or shear modulus, M_i is the modulus of the i-th component, and X_i is its volumetric concentration in the solid phase. In this case the density of the solid phase (ρ_S) is calculated from the densities of the components (ρ_i) as

$$\rho_S = \sum_{i=1}^{n} X_i \rho_i. \tag{8}$$

Finally, the P-wave velocity in the solid phase is

$$V_{pS} = \sqrt{(K_S + \frac{4}{3}G_S)/\rho_S}, \tag{9}$$

where K_S and G_S are the bulk and shear moduli of the solid phase, respectively.

The bulk modulus, shear modulus, and density of quartz are chosen as 38 GPa, 44 GPa, and 2.65 g/cm^3, respectively (Carmichael, 1990). The resulting velocity is 6.038 km/s. The value recommended by Wyllie et al. (1958) is 5.948 km/s.

The bulk modulus, shear modulus, and density of clay are chosen as 21 GPa, 7 GPa, and 2.58 g/cm^3, respectively. The resulting velocity is 3.41 km/s. These clay moduli are calculated from the velocity values derived by extrapolating the experimental linear velocity-porosity-clay relations obtained by Tosaya (1982) for clay-bearing Gulf sandstones. The density was calculated from the measured density of samples with high clay content, and known porosity and mineralogy.

Clean sandstones

First we select relatively clay-free samples (volumetric clay content not exceeding 0.03), and apply equations (2), (3), and (6) to calculate the velocity. Some of the Troll samples have large amounts of feldspar and mica. Still we find that the theoretical velocity calculated using such mineral mixtures are very close to those calculated using only quartz. Therefore, in our calculations we assume that for the selected samples (including all the Troll samples) the solid phase is pure quartz.

The results are given in Figure 1. The RHG equation accurately predicts low-frequency saturated-rock velocity in consolidated sandstones and in cemented high-porosity rocks. The accuracy of the WGG equation is not as good. The GGG predictions significantly differ from the true values. Neither of the three equations can adequately predict velocity in the high-porosity uncemented Troll sands.

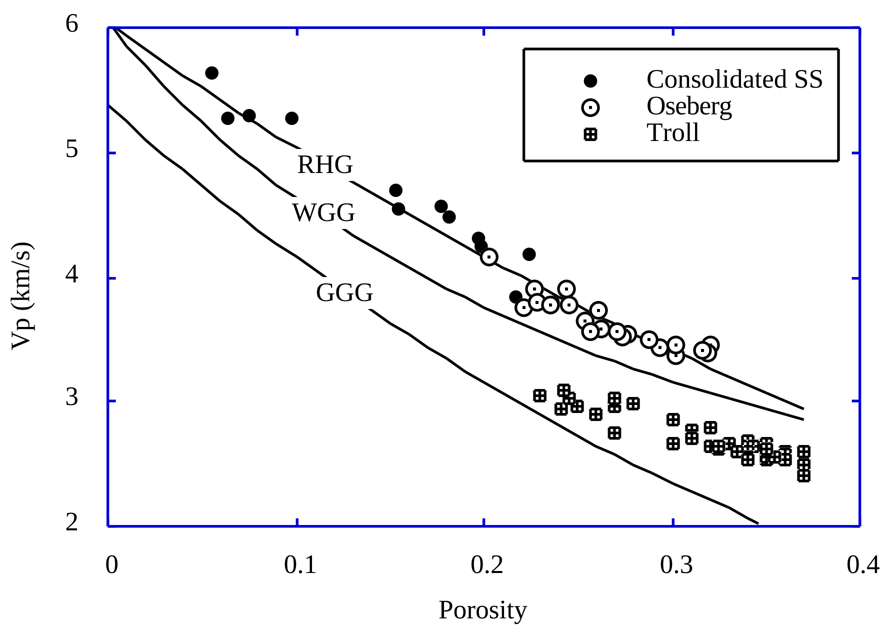

Figure 1. Velocity versus porosity in clay-free sandstones. The symbols represent the three datasets. The solid lines are theoretical predictions.

Consolidated sandstones

We plot the velocity calculated from equations (2) and (3) versus the true velocity for all consolidated sandstone samples. The RHG predictions (Figure 2a) appear to be very accurate -- the relative error which is calculated as

$$(V_p^{\text{THEORETICAL}} - V_p^{\text{TRUE}}) / V_p^{\text{TRUE}}$$

rarely exceeds 5% (Figure 3a). The WGG transformation is less accurate and tends to underestimate the true values (Figures 2b and 3b). The GGG predictions (not shown) strongly overestimate velocity in the samples with clay. They result in large relative errors that may exceed 25%.

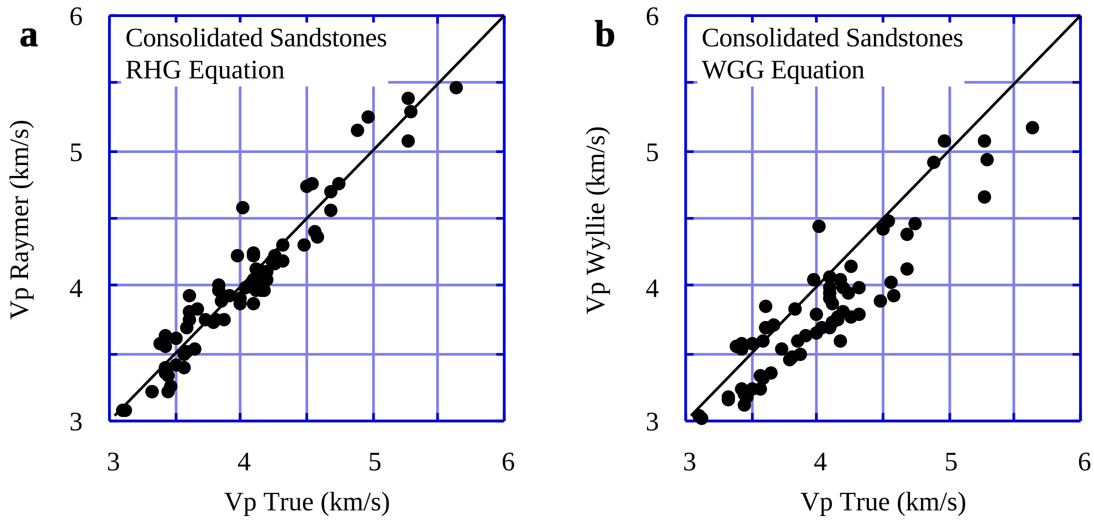

Figure 2. Calculated velocity versus true velocity in consolidated sandstones. On the solid line, the predicted velocity equals the true velocity. a. The RHG equation. b. The WGG equation.

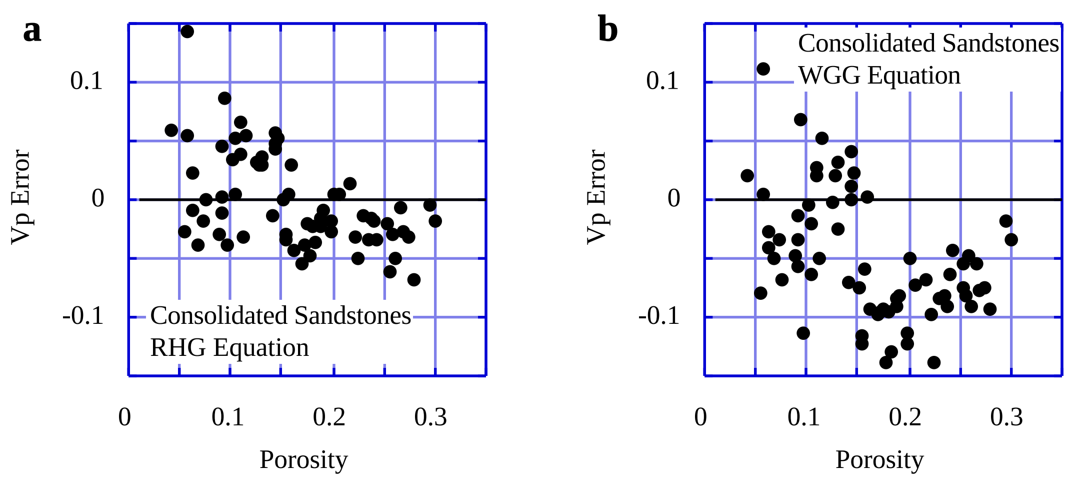

Figure 3. Consolidated sandstones. Relative errors of the RHG and WGG equations versus porosity.

High-porosity cemented sandstones

The RHG and WGG model predictions are plotted versus the true velocity in Figure 4. Again, the RHG predictions are very accurate: the relative error is rarely larger than 5% (Figure 5a). The WGG predictions are less accurate and tend to underestimate the true velocity (Figures 4b and 5b). However, the WGG prediction relative error can be decreased by the appropriate choice of the solid-phase velocity. For example, if for quartz we use V_{pS} = 7.3 km/s instead of 6.038 km/s, the WGG predictions for clean samples become as accurate as the RHG predictions (Figure 6). The success of this manipulation emphasizes the role of V_{pS} in the time-average equation as an adjustable parameter.

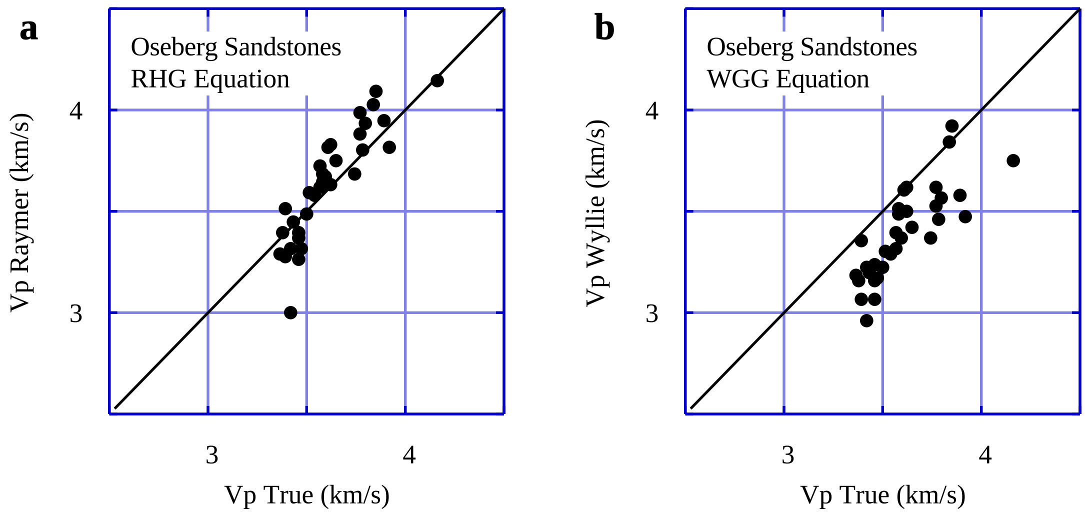

Figure 4. Calculated velocity versus true velocity in the Oseberg sandstones. On the solid line, the predicted velocity equals the true velocity. a. The RHG equation. b. The WGG equation.

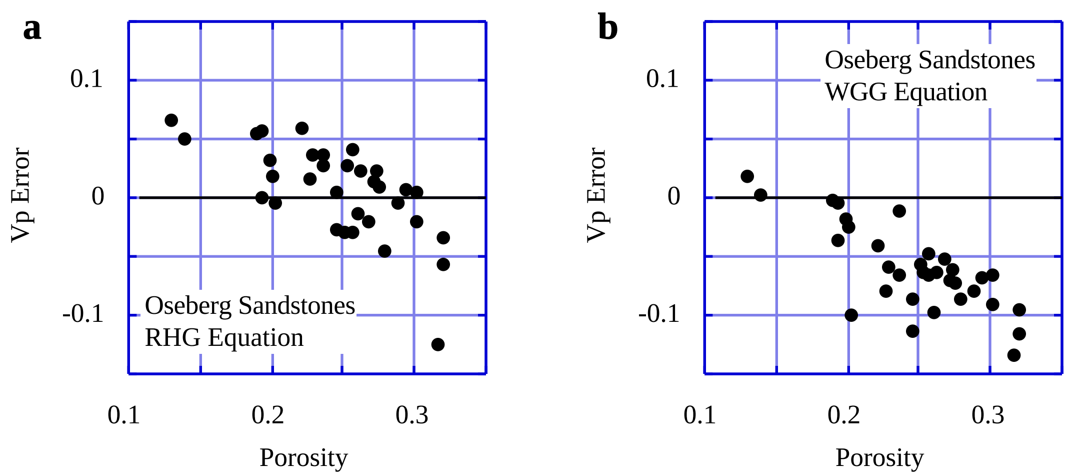

Figure 5. The Oseberg sandstones. Relative errors of the RHG and WGG equations versus porosity.

High-porosity uncemented sandstones

Figure 7 is a velocity-porosity plot for the Troll samples at 30 MPa effective pressure. Clearly, none of the traditional velocity-porosity transformations provides an adequate approximation for the true velocity. As in the previous example, the WGG and RHG curves can be lowered and placed close to the datapoints by assuming that the velocity in pure quartz is 4.5 - 4.7 km/s. However, such a manipulation is hardly acceptable.

DISCUSSION

The above examples show that the Raymer et al. (1980) velocity-porosity transform, with a reasonably chosen solid-phase velocity, can be reliably used for cemented sandstones in the porosity interval from zero to 0.35. The Wyllie et al. (1956) time-average equation is less accurate, but still can be used to evaluate porosity from sonic logs in such sandstones. This result is in line with the original conclusion of Wyllie et al. (1958) and Gardner et al. (1974). However, none of the traditional transforms should be used for uncemented rocks. An accurate deterministic model is needed in the latter case. An example is the model of Dvorkin and Nur (1996). It allows one to accurately estimate velocity in uncemented rocks and relate it not only to porosity, but also to the effective pressure (Figure 8).

Are rigorous models needed for cemented sandstones? Yes, if the goal is not merely to relate porosity to velocity, but to **diagnose** the rock. For example, if velocity **and** porosity are known (which is often the case in modern well logging), it may be possible to estimate crack aspect ratios by using the aforementioned effective medium models for low- and medium-porosity sandstones. Such estimates may in turn lead to evaluating stress-sensitivity and permeability of sandstones.

High-porosity sandstones can be diagnosed, for example, by using the effective medium cementation and Hertz-Mindlin theories (Dvorkin and Nur, 1996). By mapping the available velocity and porosity datapoints onto the velocity-porosity plane (Figure 9a), and assessing their proximity to a model-predicted (V_p, φ) trajectory, one can determine whether the rock is uncemented or cemented. In the latter case, it may be even possible to predict the cement type (Figure 9b).

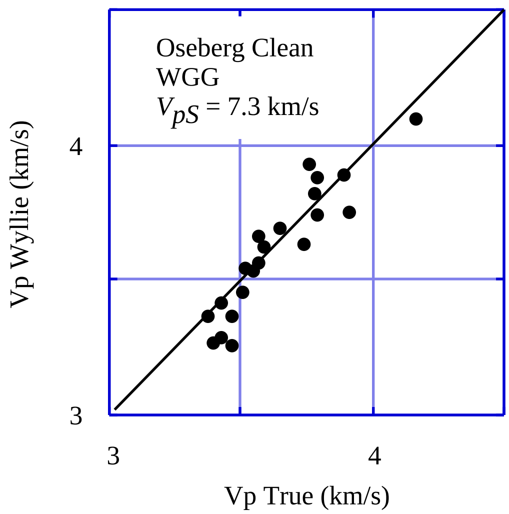

Figure 6. Calculated "manipulated" velocity versus true velocity in the clean Oseberg sandstones.

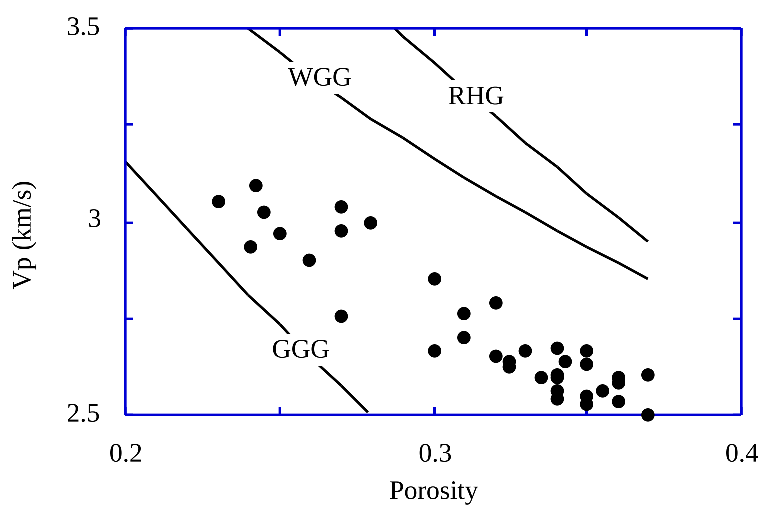

Figure 7. True velocity versus porosity in the Troll samples. Solid lines are the model predictions.

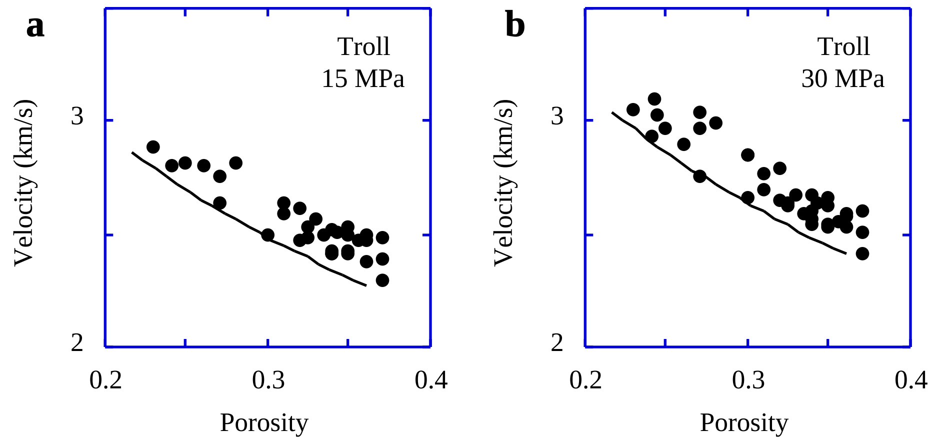

Figure 8. True velocity versus porosity in the Troll samples. Solid lines are the Dvorkin and Nur (1996) model predictions. a. 15 MPa effective pressure. b. 30 MPa effective pressure.

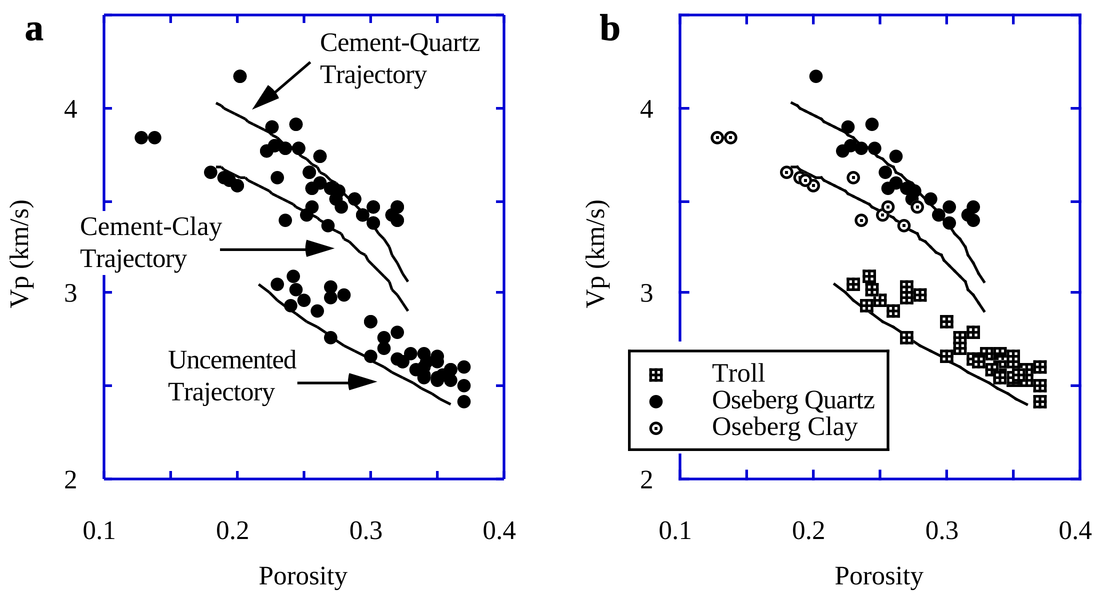

Figure 9. Diagnosing high-porosity sandstones in the (V_p, φ) plane. a. Mixed Troll and Oseberg datapoints mapped together with the theoretical trajectories for uncemented rocks, cemented rocks with quartz cement, and cemented rocks with clay cement. We speculate that the internal structure of the rock is that predicted by the effective-medium model near whose trajectory the datapoint falls. b. Confirmation of the diagnostic.

CONCLUSION

- The Raymer et al. (1980) equation which is an improved version of the Wyllie et al. (1956) time-average equation can be reliably used to relate P-wave velocity to porosity in cemented saturated sandstones in a wide porosity range.
- These empirical equations should not be used to relate velocity to porosity in unconsolidated uncemented rocks.
- Rigorous effective medium relations between porosity and velocity are indispensable when the goal is to **diagnose** the rock, i.e. to derive its internal grain-scale structure from velocity **and** porosity data.

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