Earthquake Ruptures with Strongly Rate-Weakening Friction and Off-Fault Plasticity: 2. Nonplanar Faults

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Abstract

Observations demonstrate that faults are fractal surfaces, with deviations from planarity at all scales. Self-similar surfaces have local slope perturbations of order $10^{-3}$ to $10^{-2}$ that are independent of scale. We study dynamic rupture propagation on such faults using two-dimensional plane strain models, which feature strongly rate-weakening fault friction and off-fault plasticity. The latter is necessary to bound otherwise unreasonably large stress perturbations in the vicinity of bends, and furthermore prevents fault opening. A consequence of strongly rate-weakening friction is the existence of a critical background stress level at which self-sustaining rupture propagation (in the form of self-healing slip-pulses) is just barely possible. Around this level, at which natural faults are expected to operate, ruptures become extremely sensitive to fault roughness and exhibit substantial fluctuations in rupture velocity. Under a wide range of conditions, the fluctuations are roughly correlated with the local fault slope; self-similarity of the surface thus implies that the fluctuations occur with equal amplitude at all scales. These accelerations and decelerations excite waves of all wavelengths, resulting in ground acceleration spectra that are flat at high frequency, consistent with observed strong motion records.
Introduction

Natural fault surfaces exhibit slight deviations from planarity at all scales (Brown and Scholz, 1985; Power and Tullis, 1988, 1991, 1995; Lee and Bruhn, 1996; Renard et al., 2006; Sagy et al., 2007; Candela et al., 2009; Resor and Meer, 2009; Kaven and Pollard, 2010). Slip on such faults perturbs the local stress field, leading to inelastic deformation of the off-fault material and increasing the resistance to slip (Chester and Chester, 2000; Dieterich and Smith, 2009). Roughness-induced stress perturbations will alter rupture propagation, both by introducing heterogeneities in the slip distribution and by causing rapid accelerations and decelerations of the rupture front. These processes generate high frequency seismic waves (Haskell, 1964; Madariaga, 1977; Boore and Joyner, 1978; Spudich and Frazer, 1984).

In this work, we explore how fault roughness influences the spontaneous propagation of shear ruptures using two-dimensional plane strain models. We use strongly rate-weakening fault friction, as supported by many recent laboratory experiments (Tsutsumi and Shimamoto, 1997; Tullis and Goldsby, 2003a,b; Prakash and Yuan, 2004; Hirose and Shimamoto, 2005; Beeler et al., 2008), and account for off-fault inelastic deformation in the framework of continuum plasticity (Andrews, 2005; Duan and Day, 2008; Templeton and Rice, 2008; Viesca et al., 2008). The latter prevents unreasonably large stress concentrations around abrupt bends in the fault and, at least for the particular elastic-plastic model we use, completely eliminates fault opening. This study is the second of a two-part series. The first part (Earthquake Ruptures with Strongly Rate-Weakening Friction and Off-Fault Plasticity: 1. Planar Faults; Eric M. Dunham, David Belanger, Lin Cong, and Jeremy E. Kozdon, submitted to BSSA), hereinafter referred to as Part 1, investigates rupture dynamics with plasticity and strongly rate-weakening friction laws on flat faults. The present study extends this to nonplanar faults.

Our focus is entirely on roughness at scales larger than the maximum slip distance in a single event. Of course, roughness exists at scales less than the slip distance as well, but that particular limit is presently inaccessible to us, at least in direct computational simulations of rupture propagation along entire faults. Despite this approximation, our models account for fault roughness over about three orders of magnitude in scale. While our results can be presented in nondimensional form, we can also view our simulations as representing earthquakes on faults of $\sim$100 km length, with roughness down to $\sim$100 m (and the numerical grid spacing is yet another order of magnitude
smaller than this, to ensure proper resolution). Perturbations to the rupture at these shortest wavelengths excite waves at frequencies up to ∼10 Hz. The high frequency band between ∼1–10 Hz is of particular interest to earthquake engineers, as the fundamental frequency of most man-made structures (with notable exceptions like tall buildings) lies within it.

Studies of strong motion accelerograms indicate a loss of coherence at high frequencies (Housner, 1947). Indeed, as discussed by Hanks (1979); McGuire and Hanks (1980); Hanks and McGuire (1981), the portion of far-field acceleration records corresponding to direct shear wave arrivals are, to a first approximation, band-limited white noise. While scattering and local site effects certainly contribute to high frequency ground motion, the observations do suggest that the source process itself becomes random at short wavelengths, and that fluctuations in slip and rupture velocity assume a stochastic character. This idea has formed the basis of a wide variety of ground motion and earthquake source models.

In the most direct approach, synthetic acceleration time series are generated as (non-stationary) stochastic signals (e.g., Housner, 1955; Iyengar and Iyengar, 1969; Boore, 1983). Others have developed stochastic source models. Many such approaches are kinematic in nature, and either impose a fractal distribution of slip heterogeneity or build a composite source model from a fractal distribution of small events (e.g., Andrews, 1980, 1981; Papageorgiou and Aki, 1983; Herrero and Bernard, 1994; Zeng et al., 1994). Such models are widely used to generate synthetic broadband seismograms (e.g., Saikia and Somerville, 1997; Hartzell et al., 1999; Pitarka et al., 2000; Mai and Beroza, 2003). A review of these and other models is given by Boore (2003). A related approach is taken in stochastic dynamic models, in which heterogeneity is added to the initial stress distribution or friction law parameters on flat faults (e.g., Andrews, 1980, 1981; Oglesby and Day, 2002; Guatteri et al., 2003; Ripperger et al., 2007). In many of these works is the suggestion that the imposed heterogeneities are to be regarded as a proxy for unmodeled geometrical complexities; as Andrews (1980) states in his pioneering study, “Variation of sliding friction on a plane is intended to represent the effect of random geometric irregularity of the fault surface.”

In the present work, we directly account for geometric irregularities and the associated set of physical processes that are likely to be active during coseismic slip on rough surfaces. Our models suggest how the random properties of natural fault surfaces can, under the right conditions, might provide an explanation of the observed levels and character of high frequency
ground motion (though a direct comparison with data is limited by the two-dimensional nature of our simulations). As such, our hope is to eventually use this type of model to explore the transition between coherent low frequency ground motion and random high frequency ground motion, which remains poorly understood despite its importance in the generation of synthetic broadband seismograms for seismic hazard calculations.

**Field and Laboratory Measurements of Fault Roughness**

Fault surface roughness can be quantified in a variety of ways (e.g., Russ, 1994; Candela et al., 2009). Consider a one-dimensional profile of the form

\[ y = h(x) \]  

(1)

and assume that it has zero mean,

\[ \int_{-\infty}^{\infty} h(x) dx = 0. \]  

(2)

One of the simplest quantities that can be determined is the root-mean-square (rms) roughness. For a profile measured over length \( L \) (and that, consequently, includes all wavelengths of roughness, \( \lambda \), less than \( L \)), this is defined as

\[ h_{\text{rms}}(\lambda < L) = \sqrt{\frac{1}{L} \int_{-L/2}^{L/2} h(x)^2 dx}. \]  

(3)

As revealed in the original studies by Brown and Scholz (1985); Aviles et al. (1987); Okubo and Aki (1987); Power and Tullis (1988, 1991, 1995); Lee and Bruhn (1996), the rms roughness is not an intrinsic property of the fault surface but instead depends on the profile length \( L \). This is a well-known property of fractal surfaces (e.g., Russ, 1994). The early studies concluded that natural fault surfaces are self-similar fractals; that is, they are statistically identical when viewed at different scales. An example is shown in Figure 1. More precisely, if the profile is decomposed into Fourier modes of various wavelengths \( \lambda \), then the amplitude of the deviations from planarity at scale \( \lambda \) are proportional to \( \lambda \). The largest fluctuations in \( h(x) \) are those with the longest \( \lambda \), such that

\[ h_{\text{rms}}(\lambda < L) = \alpha L, \]  

(4)
for a constant $\alpha$ known as the amplitude-to-wavelength ratio of the roughness. Stated another way, fluctuations in the slope of the fault surface,

$$m(x) = dh(x)/dx,$$

are of the same amplitude (of order $\alpha$) at all scales.

Measurements indicate that faults have larger $\alpha$ in the direction perpendicular to slip than in the slip-parallel direction. Values of $\alpha = 10^{-2}$ are typical for the slip-perpendicular direction, while values of $\alpha$ between $10^{-3}$ and $10^{-2}$ characterize the slip-parallel direction; the smaller end of this range is more representative of mature faults Power and Tullis (1991). Sagy and Brodsky (2009) have suggested that as faults mature, the wear processes that accompany slip decrease $\alpha$. It should be noted, however, that measurements of $\alpha = 10^{-3}$ for mature faults quantify the roughness of single fault strands; many fault zones contain multiple strands or segments, in some cases anastamosing, and taken as a whole the combination of structures that accommodate displacement in an earthquake might be more accurately described by a larger value of $\alpha$. For example, Chester et al. (2004) find that while the prominent fracture surface of the Punchbowl fault, which likely accommodated the majority of $\sim$44 km relative displacement, has $\alpha \approx 10^{-3}$, the boundary between the ultracataclastic fault core and the surrounding material has $\alpha \approx 10^{-1}$. This issue emerges also at the scale of fault systems, and explains the substantial differences in inferred $\alpha$ for the San Andreas fault in the studies of Okubo and Aki (1987) and Aviles et al. (1987).

Recent studies by Renard et al. (2006); Sagy et al. (2007); Sagy and Brodsky (2009); Candela et al. (2009) have provided high resolution data on fault surface roughness at scales of $\sim 10 \mu$m to $\sim 100$ m. The lower end of this range is covered by laboratory profilometers and the upper end by LiDAR scans. Analysis of this data, particularly by Candela et al. (2009), indicates that the surfaces are self-affine rather than self-similar. Self-affine surfaces have $h_{\text{rms}}(\lambda < L) \propto L^\zeta$, where $\zeta < 1$ is the Hurst exponent, such that the surfaces have larger amplitude-to-wavelength ratios at smaller scales (or, equivalently, appear smoother at larger scales). Values of $\zeta$ in the direction of slip are between 0.5 and 0.8. Similarly high resolution data has not yet been compiled at scales above $\sim 100$ m. Because of this, we have chosen in this work to model faults as self-similar surfaces.

A more precise mathematical description of rough surfaces in terms of power spectral density functions is given in the appendix Mathematical De-
Figure 1: Band-limited self-similar fault profile, $y = h(x)$, with amplitude-to-wavelength ratio $\alpha = 10^{-2}$, shown (a) to scale, and (b) exaggerated in the $y$-direction to emphasize fluctuations. Synthetic seismograms are calculated at station marked with triangle in (a). The maximum principal stress, $S_{\text{max}}$, is inclined at angle $\Psi$ to $y = 0$. Fault strength drops over a distance of $\sim R_0$; $R_0 = 300$ m is used in dimensional scales. Roughness is present at wavelengths $\lambda \geq \lambda_{\text{min}} = 1.25R_0 = 375$ m in this example.
Consequences of Fault Roughness

Several authors have investigated how deviations from planarity influence fault mechanics. Saucier et al. (1992) studied stress perturbations induced by slip on a frictionless nonplanar fault; they suggested that fault curvature might help explain the unexpected orientation of the principal stresses around the San Andreas Fault measured at the Cajon Pass borehole. Jayakumaran and Keer (1994) developed a methodology for calculating stresses around arbitrarily shaped frictional cracks, with particular emphasis on the influence of normal stress perturbations on fault strength. Using a boundary perturbation approach, Chester and Chester (2000) demonstrated that stress perturbations due to slip a frictional sinusoidal fault are most pronounced at the shortest wavelengths, and increase with cumulative slip (at least assuming elastic off-fault behavior). They furthermore made quantitative predictions of the location and extent of regions in which stresses are expected to activate inelastic deformation, according to a Mohr–Coulomb failure criterion. This boundary perturbation analysis is reviewed and extended to arbitrary profiles in the appendix First-Order Effects of Fault Nonplanarity.

More recently, Dieterich and Smith (2009) studied rough faults with the boundary element method. They found that roughness causes departures from the expected linear scaling of fault slip with fault length, if one assumes linear elastic response of the off-fault material. They then postulate that relaxation of the roughness-induced stress perturbations occurs via brittle failure processes (aftershocks and secondary faulting) in the surrounding material. They demonstrate how these relaxation processes restore the linear slip-length scaling and are roughly consistent with the decay of aftershock seismicity with distance from the fault.

Even minor amounts of roughness can induce large stress perturbations during slip. Unlike the rms roughness, which is determined by the longest wavelengths, the stress perturbations are dominated by the shortest wavelengths. As derived in the appendix First-Order Effects of Fault Nonplanarity, the rms normal stress perturbation arising from roughness at wavelengths $\lambda$ exceeding a minimum, $\lambda_{\text{min}}$, on a self-similar fault having slip $\Delta$ is
\[ \sigma_{rms}(\lambda > \lambda_{\text{min}}) = 2\pi^2 \alpha \frac{G}{1 - \nu} \frac{\Delta}{\lambda_{\text{min}}}. \]  

This expression contains several factors. It acquires its dimensions from the elastic stiffness, \( G/(1 - \nu) \), where \( G \) is the shear modulus and \( \nu \) is Poisson’s ratio, and is directly proportional to the amplitude-to-wavelength ratio of roughness, \( \alpha \). Furthermore, it depends on the ratio of \( \Delta \) to \( \lambda_{\text{min}} \), implying that the largest perturbations arise on faults that have large amounts of slip and short-wavelength roughness. Taking \( G = 30 \text{ GPa} \) and \( \nu = 1/4 \),

\[ \sigma_{rms}(\lambda > \lambda_{\text{min}}) \approx 80 \text{ MPa} \left( \frac{\alpha}{10^{-3}} \right) \left( \frac{\Delta}{1 \text{ m}} \right) \left( \frac{10 \text{ m}}{\lambda_{\text{min}}} \right). \]  

Such perturbations can easily become sufficiently large to induce fault opening (so long as the material response remains ideally elastic) and inelastic off-fault deformation. We can estimate when fault opening is expected by comparing the rms normal stress perturbations to the effective normal stress on a flat fault, \( -\sigma_{yy}^0 \). Opening is expected if

\[ \frac{\Delta}{\lambda_{\text{min}}} > \frac{-\sigma_{yy}^0 (1 - \nu)}{2\pi^2 \alpha G}. \]  

For a typical mid-seismogenic-depth value of \( -\sigma_{yy}^0 = 100 \text{ MPa} \), opening is predicted to occur when \( \Delta/\lambda_{\text{min}} \sim 0.1 \) for \( \alpha = 10^{-3} \) and when \( \Delta/\lambda_{\text{min}} \sim 0.01 \) for \( \alpha = 10^{-2} \). In order to prevent opening when all wavelengths down to the scale of slip are taken into account, then \( \alpha < 10^{-4} \).

The above estimates clearly indicate the importance of fault roughness, but are almost certainly overpredictions due to the assumed elastic behavior of the off-fault material. Natural faults are surrounded by damage zones of high fracture density. These regions are the signature of inelastic deformation processes and it is reasonable to expect that during additional slip events, their response will involve further inelastic deformation. In this work we model the off-fault material as an elastic-plastic solid with a shear yield strength that depends on the mean stress, as understood by Mohr–Coulomb theory. Such material descriptions are well known to describe the behavior of rock (e.g., Jaeger et al., 2007).
Model Description

We study rupture propagation on nonplanar faults, accounting for strongly rate-weakening friction on the fault and inelastic deformation of the material surrounding the fault, using two-dimensional plane-strain models. The medium is assumed to be homogeneous and infinite in extent. The fault is the curve $y = h(x)$, which deviates only slightly from $y = 0$. Band-limited self-similar profiles having amplitude-to-wavelength ratio $\alpha$ are generated using a Fourier transform method to obtain the desired power spectrum (e.g., Russ, 1994; Sahimi, 1998). We ensure accuracy of our numerical models by restricting the range of roughness wavelengths to those between $20\Delta x$, where $\Delta x$ is the grid spacing, and the full extent of the spatial domain. (For some problems, we also verify the accuracy of the numerical solution by further decreasing $\Delta x$ by a factor of two or four.) We consider only a single realization of the random surface, as shown in Figure 1, and reserve for later work investigations into variability of our results.

A complete description of the friction law and plasticity model is given in Part 1, but we summarize the relevant details here. The coefficient of friction obeys a rate-and-state law that features the direct effect and evolution, over slip $L$, toward a strongly velocity-weakening steady state friction coefficient. Such laws give rise to self-healing slip pulses when the background shear stress level, $\tau^b$, is sufficiently low (specifically, around a critical value $\tau^{pulse}$) (Zheng and Rice, 1998). For the parameters used in this work, $\tau^{pulse} = 0.2429\sigma^0$, where $\sigma^0$ is the initial normal stress acting on the fault (taken to be positive in compression). We restrict our attention in this work to ruptures in this mode. The off-fault material is described by an elastic-viscoplastic Drucker–Prager rheology without cohesion.

Parameters are identical to those in Part 1. The S-wave speed is $c_s$ and the shear modulus is $G$. The extent of the state-evolution region at the rupture front, over which the strength drop occurs, is characterized by a length scale $R_0$. The characteristic dynamic stress drop is $\Delta \tau$. We present both nondimensional and dimensional results; the latter use $c_s = 3.464$ km/s, $G = 32.04$ GPa, $\Delta \tau = 7.1195$ MPa, and $R_0 = 300$ m. We use a uniform initial stress field, which results in spatially heterogeneous initial shear and normal stresses when resolved on the nonplanar fault surface. An important parameter in determining the location of plastic deformation is the angle, $\Psi$, between the maximum compressive principal stress and the plane $y = 0$ (see Figure 1a). We also assume that the material above and below the fault is
initially moving, in only the $x$-direction, at a constant rate. Again, when resolved on the fault, the initial slip rate is heterogeneous, and the initial state variable is chosen to be consistent with these initial conditions.

**Ruptures on Rough Faults**

Having established the basic phenomenology of ruptures on fault faults with strongly rate-weakening friction and off-fault plasticity in Part 1, we now consider the influence of fault roughness. In no simulations do we observe any fault opening. This would require, if the fault were constrained against opening, tensile stress states, which are ruled out by our choice of a cohesionless yield surface. We note that opening, over large portions of the fault ($\sim10R_0 = 3$ km or larger in some cases), was a common feature in earlier simulations we conducted assuming linear elastic off-fault response.

Increasing $\alpha$ alters the proximity of the rupture to the critical conditions at which self-sustaining propagation is just barely possible. Figure 2 shows slip profiles for $\Psi = 50^\circ$ as a function of the amplitude-to-wavelength ratio of roughness, $\alpha$. The background stress, $\tau^b$, is increased with $\alpha$ to ensure propagation near the critical conditions. As $\alpha$ and $\tau^b$ are increased in this manner, the fluctuations in slip increase and the rupture velocity decreases. Note, however, that the average amount of slip remains roughly constant despite the increase in $\tau^b$.

In Figures 3 and 4, we focus on two examples, both having $\alpha = 10^{-2}$ but different values of $\Psi$ ($20^\circ$ and $50^\circ$). At lower values of $\Psi$, for which yielding on the extensional side is not so strongly favored, the stress perturbations due to roughness cause plastic flow on both sides of the fault. For higher values of $\Psi$, almost all of the plastic strain occurs on the extensional side of the fault, as it would have for a flat fault, but stress perturbations due to nonplanarity modulate the amount of plastic strain. There are thus fluctuations in the rate of energy dissipation in the material surrounding the rupture front, leading to enhanced variability in the rupture propagation speed.

Figure 4 shows the local rupture velocity, $v_r(x)$, for these two cases (calculated as the inverse of the spatial derivative of rupture front arrival time, defined as the time at which slip first reaches the state evolution distance). As can be seen, when yielding occurs on just one side of the fault (e.g., as for $\Psi = 50^\circ$), there is a clear relationship between rupture speed and the local fault slope. At reasonably high rupture speeds, the plane ahead of the
Figure 2: Profiles of slip for \( \Psi = 50^\circ \), illustrating the increase in slip heterogeneity as the amplitude-to-wavelength ratio of roughness, \( \alpha \), is increased. For all cases, the background stress, \( \tau_b \), is just slightly above that required for self-sustaining propagation; while this value of \( \tau_b \) increases with \( \alpha \), the average amount of slip remains roughly constant.

rupture front that is most favorably oriented for frictional slip is inclined at an angle with respect to the current propagation direction (e.g., Poliakov et al., 2002; Rice et al., 2005); that angle depends on both rupture velocity and \( \Psi \). Ruptures would thus branch onto a more favorably oriented fault, if one existed. For the nonplanar faults we study, this effect (for sufficiently large \( \Psi \)) causes the rupture to accelerate when the fault bends into the extensional quadrant and to decelerate when the fault bends in the opposite direction, leading to the pronounced anticorrelation of \( v_r(x) \) and \( m(x) \) for \( \Psi = 50^\circ \) in Figure 4.

**High Frequency Ground Motion**

The irregular rupture propagation observed in our rough fault models (e.g., Figure 4) leads to the generation of high frequency seismic waves. Our focus here is on strong ground motion in the near-source region, that is, the region within about one source dimension from the fault (which is the extent of our computational domain). Studies of far-field strong motion records by Hanks and McGuire (1981) suggest that acceleration time series from direct shear-wave arrivals are, to a reasonable level of approximation, band-limited white noise; that is, the power spectral density of appropriately windowed
Figure 3: (a) Profiles of slip for $\Psi = 20^\circ$ and $50^\circ$, both at $\alpha = 10^{-2}$ (at time intervals of $15.4849 R_0/c_s = 1.3411$ s). Snapshots of plastic strain field for (b) $\Psi = 20^\circ$ and (c) $50^\circ$ at time $t = 193.5607 R_0/c_s = 16.7633$ s.

Figure 4: Relation between rupture velocity, $v_r(x)$, and slope of fault profile, $m(x)$, for $\Psi = 20^\circ$ and $50^\circ$ with $\alpha = 10^{-2}$. 
far-field acceleration records is independent of frequency over a range of frequencies between the corner frequency, \( f_0 \), of the earthquake and the highest frequency, \( f_{\text{max}} \), that remains undiminished by attenuation along the source-to-site path. Andrews (1981) has shown that the high frequency spectral properties in the near-field region are the same as those in the far-field. Of course, acceleration records are not stationary stochastic signals (Iyengar and Iyengar, 1969); there is a finite duration of shaking that is important when predicting intensity measures from random vibration theory (Boore, 1983). Further scrutiny reveals a shift from shorter to longer periods over the duration of the record, and there have been efforts to generate synthetic seismograms with the same property (Der Kiureghian and Crempien, 1989). This is of critical importance for performance-based earthquake engineering, as the fundamental frequency of a building evolves with progressive accumulation of damage. In this section, we demonstrate how these observational characteristics emerge in our models. We focus on ground motion at a station located at \((x, y) = (87.5, 5.6)R_0 = (26.25, 1.68) \text{ km}\); see Figure 1a.

We begin with faults whose profile contains only a single Fourier mode \( \lambda \), that is, \( h(x) = \alpha \lambda \sin \left( \frac{2\pi x}{\lambda} \right) \). In Figure 5 we use a space-time diagram to link ground motion to the fault profile. Even though the fault has a single characteristic wavelength, \( \lambda \), the period of the ground motion oscillations changes with time. This feature is simply explained in terms of the Doppler shift due to the motion of the rupture first toward and then away from the station. To quantify this, assume that the rupture propagates at an average velocity \( \bar{v}_r \), such that it repeatedly encounters the same fault topography at a time interval of \( \lambda/\bar{v}_r \). Assuming that the ground motion oscillations are primarily caused by radiating shear waves, and making the approximation that \( \lambda \) is much smaller than the source-to-station distance, the expected period of the ground motion oscillations, \( T \), is predicted to be

\[
T \approx \frac{\lambda}{\bar{v}_r} \left(1 - \frac{\bar{v}_r}{c_s} \cos \theta \right),
\]

where \( \theta \) is the angle between a line passing through the center of the fault (i.e., \( y = 0 \)) and a line connecting the station to the section of the fault from which waves recorded at the time of interest were emitted. Hence, \( \theta \) increases from roughly zero to \( \pi/2 \) to \( \pi \) as the rupture approaches, passes, and then recedes from the station. The period thus increases monotonically from \( (\lambda/\bar{v}_r)(1 - \bar{v}_r/c_s) \) to \( (\lambda/\bar{v}_r)(1 + \bar{v}_r/c_s) \) as the rupture passes. A close examination of Figure 5 supports this quantitative interpretation. While there are other
explanations for the changing frequency content of seismograms at farther distances from the fault (e.g., the arrival of longer-period surface waves after direct body wave arrivals), the Doppler effect is a likely explanation in the near-source region.

When roughness makes only small perturbations to the rupture process (as occurs when conditions are not too close to the critical ones), we find that while the period of ground motion oscillations is of order $\frac{\lambda}{c_s}$, the amplitude depends primarily on the amplitude-to-wavelength ratio $\alpha$ (and not the amplitude of the profile, $\alpha \lambda$). However, when conditions are close to critical ones, the response becomes highly nonlinear and such simple relations do not exist.

Next, we turn to ruptures on self-similar faults. Figure 6 shows synthetic seismograms and Fourier acceleration spectra at the same station for several values of $\alpha$ (in each case, the value of $\tau^b$ is just above the critical level). Also shown is the Lucerne Valley (LUC) record from the 1992 $M_w$ 7.3 Landers earthquake. The station was located about 2 km from the fault; the record has been rotated into fault-normal and fault-parallel components using the average strike of the fault ($336^\circ$). The Landers earthquake occurred on a set of roughly vertical strike-slip faults, and the LUC record is widely considered to be representative of near-source ground motions expected from large subshear ruptures.

The comparison of our synthetic seismograms and the LUC record is only meant to be qualitative, given the two-dimensional nature of our simulations. However, the main features are remarkably similar; these include the two-sided fault-normal velocity pulse (which occurs as the rupture passes), and the sustained shaking that precedes this starting with the hypocentral shearwave arrival (i.e., between 8.5 and 10 s in the LUC record and between 7.5 and 10.5 s in the synthetics) that could be interpreted as the shear waves radiated by the rupture as it propagates toward the station in an irregular manner. A snapshot of the entire wavefield is shown in Figure 7.

**Discussion**

We have investigated the role of fault roughness in the earthquake rupture process, using numerical simulations that include both strongly rate-weakening fault friction and off-fault plasticity. Accounting for inelastic deformation near the fault is essential to prevent large-scale fault opening.
Figure 5: Space-time diagram of rupture on a sinusoidal fault $h(x) = \alpha \lambda \sin \left(2\pi x / \lambda \right)$ having wavelength $\lambda = 1.5$ km and $\alpha = 10^{-2}$. The black curve is the fault-normal acceleration, $a_y(t)$, at the station shown in Figure 1a. The large acceleration spike at $t \approx 7.5$ s is the hypocentral shear wave arrival (which is unreasonably large because of the abrupt artificial nucleation process). The blue curve is the rupture front arrival time, $t_r(x)$; the rupture speed has average value $\bar{v}_r$ and fluctuations with period $\lambda / \bar{v}_r$. High frequency waves are emitted at this period; their trajectories through space-time to the station are shown as the red curves. A shear wave emitted from the fault at $(x, y) = (x, h(x))$ arrives at the station at $(x_s, y_s)$ at time $t = t_r(x) + \sqrt{(x_s - x)^2 + [y_s - h(x)]^2} / c_s$; this equation can be inverted for $x = x(t)$, the fault location causing ground motion at time $t$. Using this relation, the fault profile $y = h(x)$ is plotted as a function of time ($h(x(t))$, green curve), making it is possible to directly relate the fault profile to the acceleration record (see inset for more detail). The Doppler effect explains the increase in ground motion period when the rupture passes the station at $t \approx 10.5$ s.
Figure 6: (a) Synthetic velocity seismograms for several values of the amplitude-to-wavelength ratio $\alpha$. Hypocentral P- and S-wave arrivals are marked; the two-sided fault-normal pulse occurs when the rupture passes the station. (b) The Lucerne Valley (LUC) record from the 1992 $M_w$ 7.3 Landers earthquake (2 km from the fault). (c) Fourier amplitude spectra (FAS) of fault-normal acceleration corresponding to seismograms in (a) windowed between 8 and 20 s to avoid the overly sharp hypocentral S-wave arrival due to the artificial nucleation. The absence of roughness wavelengths below $\lambda_{\min}$ prevents the excitation of waves at frequencies greater than $\sim c_s/\lambda_{\min}$. (d) FAS of LUC record (fault-normal component).
Figure 7: Snapshots of the velocity field as the rupture passes the station (triangle) at which seismograms in Figure 6 are calculated: (a) fault-parallel ($v_x$); (b) fault-normal ($v_y$). The hypocentral shear wave is marked “hypo S.” Note that 1 m/s = 1.295 $c_s\Delta\tau/G$. 

\[ 1 \text{ m/s} = 1.295 \frac{c_s\Delta\tau}{G} \]
Roughness with amplitude-to-wavelength ratios between $10^{-3}$ and $10^{-2}$ leads to an irregular rupture process, even if the initial stress field is spatially uniform. This irregularity manifests itself in heterogeneous slip distributions and fluctuations in rupture velocity; both contribute to high frequency ground motion.

As expected, production of high frequency radiation increases with increasing levels of roughness (specifically, the amplitude-to-wavelength ratio $\alpha$). In addition, high frequency waves are most efficiently generated when the background stress is just barely larger than the critical level required for self-sustaining propagation; under these conditions, the response becomes quite nonlinear, and slight perturbations in the fault profile can cause substantial changes in the propagation process. Thus, two ruptures on the same fault may produce different amounts of high frequency radiation if there are differences in the background stress level.

Our simulations leave open several issues. The first is dimensionality. There are differences between two- and three-dimensional wave propagation in terms of both geometrical spreading and the extended duration of influence of disturbances in two dimensions (i.e., the “tail” behind wavefronts in the two-dimensional Green’s function). Additionally, we expect that accounting for surface roughness in the unmodeled dimension (z-direction) will increase irregularity of the ground motion; our plane strain simulations effectively force coherence of the rupture process in the z-direction.

Second, our models only account for fault roughness over a limited range of wavelengths. Our minimum modeled wavelengths are $\sim 100$ m, but with additional computational resources it will be possible to include wavelengths down to the scale of slip (at which point, the assumption of grid points collocated on either side of the fault breaks down and the numerical method would need to be altered). Additionally, given the larger amplitude of stress perturbations and the additional resistance to slip introduced at short wavelengths, we expect there may be substantial changes in the predicted minimum background stress level at which faults are capable of hosting ruptures.

Third, we have assumed, for simplicity, a spatially uniform initial stress field. Slip introduces heterogeneities in the stress field; these become the initial conditions for subsequent events, and most certainly will influence the rupture process. We have not attempted to simulate multiple events and interseismic deformation on a single fault, but this must be done in order to have self-consistent initial conditions.

Finally, we have investigated only one possible source of high frequency
ground motion, and it is undoubtedly the case that both scattering along the path and local site conditions play a role in nature. There is a clear need for more extensive numerical modeling to resolve all of these issues.

Data and Resources

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Mathematical Description of Rough Surfaces

Consider a fault profile \( y = h(x) \) with zero mean. For an infinitely long profile exhibiting nonplanarity over its entire extent, \( h(x) \) is not absolutely integrable and its Fourier transform, as usually defined, does not exist. To deal with this, we limit roughness to a finite-length section of the profile by
defining \( h(x; L) = h(x) \) for \( |x| \leq L/2 \) and zero elsewhere. Note that \( L \) is also used in this manuscript to denote the state evolution distance in the friction law, but the particular meaning should be clear from context.

The Fourier transform of this finite portion of the profile exists and is given by

\[
H(k; L) = \int_{-\infty}^{\infty} h(x; L)e^{-ikx} \, dx = \int_{-L/2}^{L/2} h(x)e^{-ikx} \, dx; \tag{10}
\]

the inverse transform is

\[
h(x; L) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(k; L)e^{ikx} \, dk. \tag{11}
\]

The power spectral density of the profile is

\[
P_h(k) = \lim_{L \to \infty} \frac{1}{L} |H(k; L)|^2. \tag{12}
\]

Alternatively, the power spectral density can be defined as the Fourier transform of the autocorrelation function, \( R_h(x) \); that is,

\[
P_h(k) = \int_{-\infty}^{\infty} R_h(x)e^{-ikx} \, dx \tag{13}
\]

with

\[
R_h(x) = \lim_{L \to \infty} \frac{1}{L} \int_{-L/2}^{L/2} a(\xi)a(\xi + x) \, d\xi. \tag{14}
\]

In terms of the power spectral density, the root-mean-square (rms) roughness between wavenumbers \( k_{\min} \) and \( k_{\max} \) is

\[
h_{rms}(k_{\min}, k_{\max}) = \sqrt{\int_{k_{\min}}^{k_{\max}} P_h(k) \, dk}. \tag{15}
\]

Now consider a power-law expression for the power spectral density: \( P_h(k) \propto k^{-\beta} \) with \( 1 < \beta < 3 \). More specifically, if we define

\[
P_h(k) = C_h k^{-\beta} = \frac{(\beta - 1)\alpha^2}{2\pi} \left( \frac{2\pi/\Lambda}{k} \right)^{\beta} \Lambda^3 \tag{16}
\]

then
The rms roughness is dominated by the longest wavelengths; this is evident if we set \( k_{\text{min}} = 2\pi/L \) and take \( k_{\text{max}} \to \infty \):

\[
h_{\text{rms}}(2\pi/L, \infty) = \alpha L, \tag{19}
\]

and we see that the rms roughness is proportional to the profile length \( L \). Such a profile is known as self-similar. In this case, the amplitude-to-wavelength ratio of the roughness, defined as \( h_{\text{rms}}(2\pi/L, \infty)/L \), is constant and equal to the dimensionless coefficient \( \alpha \). For other values of \( \beta \), there exists a length scale below which the fault appears rougher and above which the fault appears smoother. If we define this crossover wavelength as the value of \( L \) satisfying \( h_{\text{rms}}(2\pi/L, \infty) = L \), then \( L = \alpha^{2/(3-\beta)} \Lambda \).

## First-Order Effects of Fault Nonplanarity

In this appendix we derive the first-order effects of roughness on frictional slip for static elasticity; a similar analysis was conducted by Chester and Chester (2000). The problem is cast as a boundary perturbation problem. There are two approximations that are employed in this procedure, both of which involve an expansion in terms of a small parameter \( \epsilon \) that characterizes the amplitude of the slope perturbations of the fault surface (\( \alpha \) for a self-similar fault). We express the slope as \( m(x) = \epsilon \tilde{m}(x) \) and the profile as \( h(x) = \epsilon \tilde{h}(x) \), and assume that \( \epsilon \ll 1 \). Boundary conditions are naturally expressed in terms of fields defined in a local orthogonal coordinate system based on the unit normal and tangent to the irregular boundary. In transforming between the \( xy \)-coordinate system and the local tangential-normal coordinate system, we make use of the small angle approximation. For example, the slip \( \delta(x) \) at point \((x, y = h(x))\), given by
\begin{align*}
\delta(x) &= \left[ u_x(x, h(x)^+) - u_x(x, h(x)^-) \right] \cos \theta(x) \\
&\quad + \left[ u_y(x, h(x)^+) - u_y(x, h(x)^-) \right] \sin \theta(x) \quad (20)
\end{align*}

for \( \tan \theta(x) = m(x) \), is approximated as

\begin{align*}
\delta(x) &= u_x(x, h(x)^+) - u_x(x, h(x)^-) \\
&\quad + \epsilon \tilde{n}(x) \left[ u_y(x, h(x)^+) - u_y(x, h(x)^-) \right] + O(\epsilon^2) \\
&= u_x(x, h(x)^+) - u_x(x, h(x)^-) \\
&\quad + m(x) \left[ u_y(x, h(x)^+) - u_y(x, h(x)^-) \right] + O(\epsilon^2). \quad (21)
\end{align*}

Rather than applying boundary conditions on \( y = h(x) \), we can apply them on \( y = 0 \) with the aid of another approximation. A field \( g(x, y) \) that is to be evaluated on the fault surface is approximated by Taylor series expansion as

\begin{align*}
g(x, h(x)) &= g(x, 0) + \epsilon \tilde{h}(x) \frac{\partial g}{\partial y}(x, 0) + O(\epsilon^2) \\
&= g(x, 0) + h(x) \frac{\partial g}{\partial y}(x, 0) + O(\epsilon^2). \quad (22)
\end{align*}

Following this procedure, the slip \( \delta(x) \), opening \( \omega(x) \), shear stress \( \tau(x) \), and normal stress \( \sigma(x) \) on the fault, neglecting terms of \( O(\epsilon^2) \), are

\begin{align*}
\delta(x) &= u_x(x, 0^+) - u_x(x, 0^-) + m(x) \left[ u_y(x, 0^+) - u_y(x, 0^-) \right] \\
&\quad + h(x) \left[ \frac{\partial u_x(x, 0^+)}{\partial y} - \frac{\partial u_x(x, 0^-)}{\partial y} \right], \quad (23)
\end{align*}

\begin{align*}
\omega(x) &= u_y(x, 0^+) - u_y(x, 0^-) - m(x) \left[ u_x(x, 0^+) - u_x(x, 0^-) \right] \\
&\quad + h(x) \left[ \frac{\partial u_y(x, 0^+)}{\partial y} - \frac{\partial u_y(x, 0^-)}{\partial y} \right], \quad (24)
\end{align*}

\begin{align*}
\tau^\pm(x) &= \sigma_{xy}(x, 0^+) - m(x) \left[ \sigma_{xx}(x, 0^+) - \sigma_{yy}(x, 0^+) \right] \\
&\quad + h(x) \frac{\partial \sigma_{xy}(x, 0^+)}{\partial y}, \quad (25)
\end{align*}
\[ \sigma^\pm(x) = -\sigma_{yy}(x, 0^\pm) + 2m(x)\sigma_{xy}(x, 0^\pm) - h(x)\frac{\partial \sigma_{yy}(x, 0^\pm)}{\partial y}. \] (26)

The ± symbol indicates fields that are defined on top or bottom of the fault, i.e., \( y = h(x)^\pm \). Of course, we will later enforce continuity of the traction components of stress, so we can write \( \tau(x) \) in place of \( \tau^\pm(x) \) without any confusion; the same applies to \( \sigma(x) = \sigma^\pm(x) \).

The next step is to expand the solution in powers of \( \epsilon \):

\[ u_i(x, y, t) = u_i^{(0)}(x, y, t) + \epsilon u_i^{(1)}(x, y, t) + O(\epsilon^2) \] (27)
\[ \sigma_{ij}(x, y, t) = \sigma_{ij}^{(0)}(x, y, t) + \epsilon \sigma_{ij}^{(1)}(x, y, t) + O(\epsilon^2), \] (28)

and for notational convenience we write the first-order terms as

\[ \hat{u}_i(x, y, t) = \epsilon u_i^{(1)}(x, y, t) \] (29)
\[ \hat{\sigma}_{ij}(x, y, t) = \epsilon \sigma_{ij}^{(1)}(x, y, t). \] (30)

Now substitute the expanded solution into the expanded expressions for fields on the fault and drop terms of \( O(\epsilon^2) \):

\[ \delta(x) = u_x^{(0)}(x, 0^+) - u_x^{(0)}(x, 0^-) + \hat{u}_x(x, 0^+) - \hat{u}_x(x, 0^-) \] (31)
\[ + m(x) \left[ u_y^{(0)}(x, 0^+) - u_y^{(0)}(x, 0^-) \right] + h(x) \left[ \frac{\partial u_x^{(0)}(x, 0^+)}{\partial y} - \frac{\partial u_x^{(0)}(x, 0^-)}{\partial y} \right], \]
\[ \omega(x) = u_y^{(0)}(x, 0^+) - u_y^{(0)}(x, 0^-) + \hat{u}_y(x, 0^+) - \hat{u}_y(x, 0^-) \] (32)
\[ - m(x) \left[ u_x^{(0)}(x, 0^+) - u_x^{(0)}(x, 0^-) \right] + h(x) \left[ \frac{\partial u_y^{(0)}(x, 0^+)}{\partial y} - \frac{\partial u_y^{(0)}(x, 0^-)}{\partial y} \right], \]
\[ \tau^\pm(x) = \sigma_{xy}^{(0)}(x, 0^+) + \hat{\sigma}_{xy}(x, 0^+) - m(x) \left[ \sigma_{xx}^{(0)}(x, 0^+) - \sigma_{yy}^{(0)}(x, 0^+) \right] \] (33)
\[ + h(x)\frac{\partial \sigma_{xy}^{(0)}(x, 0^+)}{\partial y}, \]
\[ \sigma^\pm(x) = -\sigma_{yy}^{(0)}(x, 0^+) - \hat{\sigma}_{yy}(x, 0^+) + 2m(x)\sigma_{xy}^{(0)}(x, 0^+) \] (34)
\[ - h(x)\frac{\partial \sigma_{yy}^{(0)}(x, 0^+)}{\partial y}. \]
We will also need to the Fourier transform of the fields on the fault:

\[ U^\pm_i(k) = \int_{-\infty}^{\infty} u_i(x, 0^\pm)e^{-i k x} dx \]  
\[ \Sigma^\pm_{ij}(k) = \int_{-\infty}^{\infty} \sigma_{ij}(x, 0^\pm)e^{-i k x} dx. \]

Elasticity relates the transforms of stress and displacement (e.g., Geubelle and Rice, 1995)

\[ \Sigma^\pm_{xx}(k)/G = i k \frac{2(3-2\nu)}{3-4\nu} U^\pm_x(k) \pm |k| \frac{4\nu}{3-4\nu} U^\pm_y(k) \]  
\[ \Sigma^\pm_{xy}(k)/G = \mp |k| \frac{4(1-\nu)}{3-4\nu} U^\pm_x(k) + i k \frac{2(1-2\nu)}{3-4\nu} U^\pm_y(k) \]  
\[ \Sigma^\pm_{yy}(k)/G = -i k \frac{2(1-2\nu)}{3-4\nu} U^\pm_x(k) \mp |k| \frac{4(1-\nu)}{3-4\nu} U^\pm_y(k). \]

The Fourier coefficients are to be determined by the boundary conditions.

**Static Perturbation Problem**

Consider the problem of uniform slip \( \Delta \) at constant friction with no opening in a prestressed medium. Along with continuity of the traction components of stress, \( \tau^+(x) = \tau^-(x) \) and \( \sigma^+(x) = \sigma^-(x) \), the boundary conditions are

\[ \omega(x) = 0 \]  
\[ \tau(x) = f \sigma(x). \]

The solution for a flat fault (the zeroth-order solution) is

\[ u^{(0)}_x(x, y) = \frac{\Delta}{2} \text{sgn}(y) \]  
\[ u^{(0)}_y(x, y) = 0 \]  
\[ \sigma^{(0)}_{xx}(x, y) = \sigma^0_{xx} \]  
\[ \sigma^{(0)}_{xy}(x, y) = \sigma^0_{xy} = -f \sigma^0_{yy} \]  
\[ \sigma^{(0)}_{yy}(x, y) = \sigma^0_{yy}. \]
(Terms that are linear in $x$ and/or $y$ may be added to the displacements in order to be compatible with the prestress state; such terms, however, they will play no role in the analysis to follow and are hence ignored.) The fields on the fault, given in (31)–(34), simplify with this zeroth-order solution:

$$\delta(x) = \Delta + \hat{u}_x(x,0^+) - \hat{u}_x(x,0^-),$$

$$\omega(x) = \hat{u}_y(x,0^+) - \hat{u}_y(x,0^-) - m(x)\Delta,$$

$$\tau^\pm(x) = \sigma^0_{xy} + \hat{\sigma}_{xy}(x,0^\pm) - m(x)\left(\sigma^0_{xx} - \sigma^0_{yy}\right),$$

$$\sigma^\pm(x) = -\sigma^0_{yy} - \hat{\sigma}_{yy}(x,0^\pm) + 2m(x)\sigma^0_{xy}.\quad(47)$$

To satisfy the boundary conditions to first order in $\epsilon$, the following conditions must hold:

$$\hat{\sigma}_{xy}(x,0^+ - \hat{\sigma}_{xy}(x,0^-) = 0\quad(51)$$

$$\hat{\sigma}_{yy}(x,0^+ - \hat{\sigma}_{yy}(x,0^-) = 0\quad(52)$$

$$u_y(x,0^+) - u_y(x,0^-) = m(x)\Delta\quad(53)$$

$$\hat{\sigma}_{xy}(x,0) + f\hat{\sigma}_{yy}(x,0) = m(x)\left(\sigma^0_{xx} - \sigma^0_{yy} + 2f\sigma^0_{xy}\right)\quad(54)$$

These expressions, which involve fields from the zeroth-order solution, constitute the boundary conditions that determine the first-order solution. By Fourier transforming them and using the elasticity relations (37)–(39), we find

$$\hat{\Sigma}_{yy}(k) = -\frac{G\Delta}{2(1-\nu)}|k|M(k)\quad(55)$$

$$\hat{\Sigma}_{xy}(k) = -f\hat{\Sigma}_{yy}(k) + M(k)\left(\sigma^0_{xx} - \sigma^0_{yy} + 2f\sigma^0_{xy}\right)\quad(56)$$

$$\hat{\Sigma}_{xx}^\pm(k) = \frac{G\Delta}{2(1-\nu)}\left(\mp2fi\kappa - |k|\right)M(k)$$

$$\pm2|k|H(k)\left(\sigma^0_{xx} - \sigma^0_{yy} + 2f\sigma^0_{xy}\right).\quad(57)$$

The first terms, proportional to $\Delta$, are the most relevant. They represent the perturbations due to slip; the other terms are simply the result of projecting the uniform prestress field onto the nonplanar fault surface.

We can invert these transforms using the convolution theorem to obtain
\begin{align*}
\hat{\sigma}_{yy}(x,0) &= -\frac{G\Delta}{2(1-\nu)}\mathcal{H}[m'(x)] \quad (58) \\
\hat{\sigma}_{xy}(x,0) &= -f \hat{\sigma}_{yy}(x,0) + m(x) \left(\sigma_{xx}^0 - \sigma_{yy}^0 + 2f\sigma_{xy}^0\right) \quad (59) \\
\hat{\sigma}_{xx}(x,0^\pm) &= \frac{G\Delta}{2(1-\nu)} \left\{ \mp 2fm'(x) - \mathcal{H}[m'(x)] \right\} \\
&\quad \pm 2\mathcal{H}[m(x)] \left(\sigma_{xx}^0 - \sigma_{yy}^0 + 2f\sigma_{xy}^0\right) \quad (60)
\end{align*}

where the Hilbert transform of some function \( g(x) \) is
\[
\mathcal{H}[g(x)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\xi)}{x-\xi} \, d\xi. \quad (61)
\]

**Normal Stress Perturbations**

Next we relate the power spectral density of normal stress perturbations on the fault, \( P_\sigma(k) \), to that of the profile itself. Specifically, we focus in the normal stress perturbations due to slip, not those due to resolving the uniform prestress field on the nonplanar surface. Thus we take the transform of the normal stress perturbation to be the single term \( \Sigma(k;L) = \hat{\Sigma}_{yy}(k;L) \). Then,
\[
\begin{align*}
P_\sigma(k) &= \lim_{L \to \infty} \frac{1}{L} \left| \Sigma(k;L) \right|^2 \\
&= \lim_{L \to \infty} \frac{1}{L} \left| -\frac{iGk|k|}{2(1-\nu)} H(k;L) \Delta \right|^2 \\
&= \left[ \frac{Gk^2\Delta}{2(1-\nu)} \right]^2 \lim_{L \to \infty} \frac{1}{L} |H(k;L)|^2 \\
&= \left[ \frac{Gk^2\Delta}{2(1-\nu)} \right]^2 P_h(k). \quad (62)
\end{align*}
\]

For the power spectral density (16),
\[
P_\sigma(k) = \left[ \frac{G\Delta}{2(1-\nu)} \right]^2 C_h k^{4-\beta} \quad (63)
\]
which simplifies for a self-similar fault ($\beta = 3$) to

$$P_\sigma(k) = 2 \left( \frac{\pi \alpha G \Delta}{1 - \nu} \right)^2 k. \quad (64)$$

The rms normal stress perturbation from wavenumbers between $k_{\text{min}}$ and $k_{\text{max}}$ is

$$\sigma_{\text{rms}}(k_{\text{min}}, k_{\text{max}}) = \sqrt{\int_{k_{\text{min}}}^{k_{\text{max}}} P_\sigma(k) dk}. \quad (65)$$

Unlike the rms roughness, which is determined by the longest wavelengths, the rms normal stress perturbation is dominated by the shortest wavelengths. This is illustrated by considering wavenumbers $k_{\text{min}} = 0$ and $k_{\text{max}} = 2\pi/\lambda_{\text{min}}$ for the case of a self-similar fault:

$$\sigma_{\text{rms}}(0, 2\pi/\lambda_{\text{min}}) = 2\pi^2 \alpha \frac{G \Delta}{1 - \nu \lambda_{\text{min}}}. \quad (66)$$