Lecture Notes 4

Movement and Entrapment of Petroleum

Interfacial Tension

When two immiscible liquids (e.g., water and oil) are in contact, interfacial tension occurs at the contact surface between the two liquids. The interfacial tension is due to the difference in molecular attraction between similar molecules within each fluid versus molecular attraction between different molecules of the two fluids. Because of this difference in molecular attraction, the contact surface is generally curved. Strictly speaking, the interfacial tension is defined as the amount of work that must be performed to separate a unit area of contact surface between the two liquids (Bear, 1972, p. 442). However, we can loosely think of interfacial tension as the force that acts on the contact surface (Berg, 1975, p. 940). The interfacial tension between two liquids generally depends on temperature and pressure. For oil and water, the interfacial tension (γ) is about 25 dynes/cm at surface conditions. Increasing the temperature and pressure tends to increase interfacial tension. At depth, the interfacial tension between oil and water is about 30 dynes/cm (Berg, 1975).

Contact Angle and Wettability

When two liquids are in contact with a solid surface, the contact surface between the two liquids will be formed as shown in Figure 4-1(a). The angle θ is called the contact angle, and is measured through the denser liquid. When $\theta < 90^o$, the denser liquid is known as the wetting fluid. When $\theta > 90^o$, the denser liquid is called the nonwetting liquid. For oil and water in a sedimentary material, the water is usually (but not always) the wetting liquid. The typical arrangement of oil and water in the pore space between grains is illustrated in Figure 4-1(b).

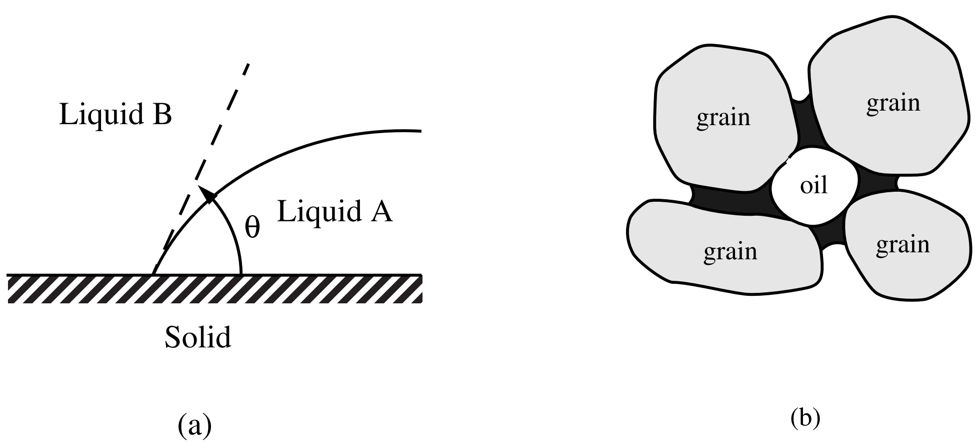

Figure 1. (a) Contact angle θ between two liquids resting on a solid. (b) Arrangement of oil and water in the pore space between grains

Capillary Pressure

The presence of interfacial tension at the contact surface between two immiscible liquids causes a pressure difference across the contact surface. This pressure difference is called the capillary pressure. The capillary pressure p_c between two liquids is linearly proportional to the interfacial tension γ between the liquids, and inversely proportional to the principal radii of curvature (r_1 and r_2) of the contact surface:

$$p_c = \gamma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{4-1}$$

If the surface is spherical (that is, that $r_1 = r_2$), then eq. (4-1) becomes

$$p_c = \frac{2\gamma}{r} \tag{4-2}$$

where r is the radius of curvature of the contact surface.

The sign of the capillary pressure does not have a standard convention. Bear (1972, p. 444) defines it as the pressure in the nonwetting fluid minus the pressure in the wetting fluid. Hubbert (1953, p. 1975) defines it as the pressure in the oil minus the pressure in the water (regardless of which is the wetting fluid). Here, we will merely note that for the common case of water-wet sedimentary material, the pressure is higher inside the oil (the nonwetting fluid), so the above two conventions are in agreement.

Oil Migration and Entrapment in the Absence of Flowing Ground Water

Because oil is less dense (lighter) than water, droplets of oil immersed in water will tend to rise vertically in the absence of flowing ground water. The net upward force on an oil droplet of volume V can be determined by application of Archimedes' principle. The downward force of gravity on the oil droplet is $\rho_{oil} gV$, where ρ_{oil} is the density of oil. The upward force on the oil droplet is equal to the weight of water displaced by the oil droplet, which is $\rho_w gV$, where ρ_w is the density of water. Therefore, the net upward force (buoyancy force) per unit volume of oil droplet is

$$F_b = (\rho_w - \rho_{oil})\, g \tag{4-3}$$

In the underground, the oil droplet must rise through the pore space between solid grains. Whether the buoyancy force is sufficiently large to push an oil droplet through the pore space is the key to petroleum entrapment. To analyze this entrapment mechanism (Berg, 1975, p. 941-944.) consider a spherical globule of oil at rest in a pore (Figure 4-2a). The radius of the globule is approximately equal to the pore radius r_p. According to eq. (4-2), the capillary pressure in the pore is

$$p_p = \frac{2\gamma}{r_p} \tag{4-4}$$

If buoyancy force is to push the oil globule through the pore throat between two grains, the globule will have to be distorted as shown in Figure 4-2b. If the throat radius is denoted by r_t, the capillary pressure at the throat is

$$p_t = \frac{2\gamma}{r_t} \tag{4-5}$$

Because the throat radius, r_t, is less than the pore radius, r_p, the capillary pressure is greater in the throat than in the pore. This creates a downward force on the oil globule. Noting the relation

$$\text{Force per unit volume} = \text{- pressure gradient,} \tag{4-6}$$

the downward force per unit volume of the oil globule is given by the capillary pressure gradient as

$$F_c = \frac{p_t - p_p}{H} = \frac{2\gamma}{H}\left(\frac{1}{r_t} - \frac{1}{r_p}\right) \tag{4-7}$$

where H is the vertical height of the globule. If the globule is to move past the pore throat, then the upward buoyancy force given by eq. (4-3) must be greater than the downward force given by eq. (4-7). Once the top of the globule moves past the pore throat (Figure 4-2c and d), the capillary pressure gradient is greatly reduced, and the rest of the globule can easily move through the pore throat. Therefore, the criterion for the oil globule to move through the pore throat is $F_b \geq F_c$, or

$$(\rho_w - \rho_{oil})\, g \geq \frac{2\gamma}{H}\left(\frac{1}{r_t} - \frac{1}{r_p}\right) \tag{4-8}$$

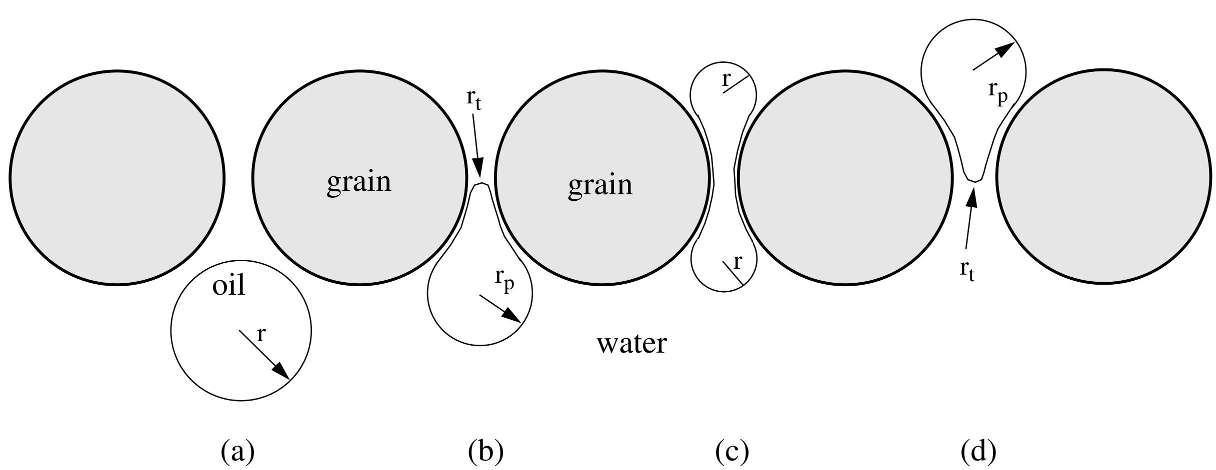

Figure 4-2. Movement of an oil droplet in water-wet clastic rock. (After Berg, 1975, Fig. 1)

Conversely, the upward movement of an oil globule or stringer may be impeded when the oil encounters a reduction in grain size (and therefore a reduction in pore throat radius.) If the buoyancy force is less than the capillary pressure gradient that would result as the oil moves through the throat of the finer-grained material, further upward movement will stop and the oil is "trapped." Note that this trapping mechanism does not require the presence of an "impermeable" layer.

For any given set of reservoir and fluid conditions (pore and throat diameter, fluid properties) the only value in eq. (4-8) that can change during oil migration is the vertical height H of the globule. The minimal value of H for which oil migration is possible is known as the critical height, H_c. According to eq. (4-8), this is given by

$$H_c = \frac{2\gamma}{(\rho_w - \rho_{oil})\, g}\left(\frac{1}{r_t} - \frac{1}{r_p}\right) \tag{4-9}$$

According to Berg (1975, p. 944), eq. (4-9) can be used to estimate the thickness of the oil trapped under a finer-grained stratum. In making this estimation, r_t is defined as the pore throat radius in the finer-grained stratum, and r_p is defined as the pore radius in the underlying coarser-grained stratum.

Oil Migration and Entrapment Under Hydrodynamic Conditions

The general theory for analysis of oil migration falls within the field of multiphase transport and is beyond the scope of this class. Here we follow the potential theory approach promoted by Hubbert (1953). In this approach, oil droplets or globules are assumed to be driven by hydrodynamic and buoyancy forces. Capillary effects are ignored except at contact between strata where there is a change in grain size (and therefore pore size).

We begin by reviewing Darcy's law:

$$\boldsymbol{q} = -K\nabla h \tag{4-10}$$

where $\boldsymbol{q}$ is the specific discharge, K is hydraulic conductivity, and h is hydraulic head. The hydraulic conductivity K is related to the permeability k by

$$K = \frac{\rho g k}{\mu} \tag{4-11}$$

where ρ is the fluid density, g is the acceleration of gravity, and μ is the dynamic viscosity of the fluid. For a fluid of constant density ρ, the hydraulic head at a point in the flow system can be written as

$$h = \frac{p}{\rho g} + z \tag{4-12}$$

where p is fluid pressure and z is the elevation of the point above the datum. Substituting eqs. (4-11) and (4-12) into (4-10) yields Darcy's law in terms of pressure and elevation:

$$\boldsymbol{q} = -\frac{k}{\mu}(\nabla p + \rho g \nabla z) \tag{4-13}$$

where ∇z is simply a unit vector in the upward vertical direction. It is common to define the gravitational acceleration vector as

$$\boldsymbol{g} = -g\nabla z \tag{4-14}$$

which points in the vertical downward direction. Then eq. (4-13) can be rewritten as

$$\boldsymbol{q} = \frac{\rho k}{\mu}\left(-\frac{\nabla \rho}{\rho} + \boldsymbol{g}\right) \tag{4-15}$$

The bracketed term on the right hand side of eq. (4-15) can now be interpreted as forces per unit mass of fluid. The quantity $-\nabla p/\rho$ is the force due to pressure gradient acting on a unit mass of fluid. The quantity $\boldsymbol{g}$ is the gravitational force acting on a unit mass of fluid. The sum of the two terms is the net force on a unit mass of fluid. Hubbert (1953) denotes this net force by

$$\boldsymbol{E} = -\frac{\nabla p}{\rho} + \boldsymbol{g} \tag{4-16}$$

These forces are illustrated in Figure 4-3.

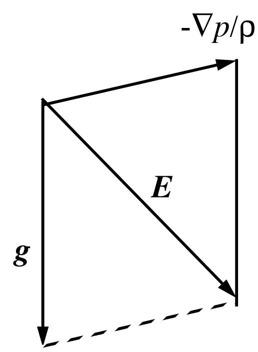

Figure 4-3. Forces acting on a unit mass of fluid.

For application to oil migration, we have to define hydraulic heads for water and oil separately. Thus, we write

$$h_w = \frac{p}{\rho_w g} + z \tag{4-17}$$

$$h_{oil} = \frac{p}{\rho_{oil} g} + \frac{p_c}{\rho_{oil} g} + z \tag{4-18}$$

where the subscripts *w* and *oil* denote water and oil, and p_c is the capillary pressure in the oil. However, for oil migration in permeable strata such as sandstone, the capillary pressure can generally be ignored. Then, the oil head can be written as

$$h_{oil} = \frac{p}{\rho_{oil} g} + z \tag{4-19}$$

To seek the relation between water and oil heads, we solve for p in eq. (4-17) and substituting the result into (4-19). This gives

$$h_{oil} = \frac{\rho_w}{\rho_{oil}} h_w - \frac{\rho_w - \rho_{oil}}{\rho_{oil}} z \tag{4-20}$$

Eq. (4-20) allows one to construct the oil head distribution from the water head distribution. In an isotropic medium, an oil droplet will move in the direction perpendicular to the lines of equal oil head, while a water particle will move in the direction perpendicular to lines of equal water head (Figure 4-4).

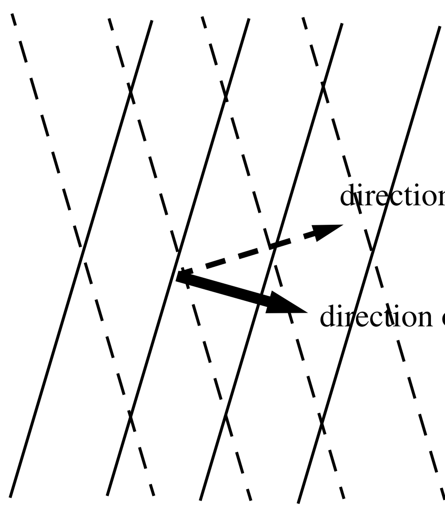

Figure 4-4. Lines of equal oil head (dotted), lines of equal water head (solid), and flow directions of oil and water.

To understand the difference in oil and water movement, consider the force per unit mass of water and oil. Writing eq. (4-16) separately for oil and for water, we have

$$\boldsymbol{E}_w = -\frac{\nabla p}{\rho_w} + \boldsymbol{g} \tag{4-21}$$

$$\boldsymbol{E}_{oil} = -\frac{\nabla p}{\rho_{oil}} + \boldsymbol{g} \tag{4-22}$$

The two forces $\boldsymbol{E}_w$ and $\boldsymbol{E}_{oil}$ and their components (pressure gradient and gravity) are shown in Figure 4-5. For both cases, the gravitational force $\boldsymbol{g}$ are the same. However, because ρ_{oil} is less than ρ_w, the pressure-gradient force $-\nabla p/\rho_{oil}$ on the oil is larger than the pressure-gradient force $-\nabla p/\rho_w$ on the water. Consequently, the direction of $\boldsymbol{E}_{oil}$ is tilted upward from the direction of $\boldsymbol{E}_w$. This upward tilt results from the buoyancy of the oil.

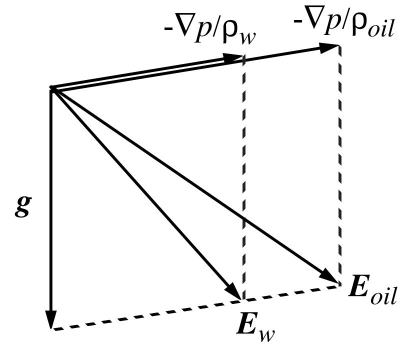

Figure 4-5. Relation between force per unit mass of water, $\boldsymbol{E}_w$, and force per unit mass of oil, $\boldsymbol{E}_{oil}$.

Once the oil head, h_{oil}, is determine, the average velocity of an oil droplet is

$$v_{oil} = -\frac{K_{oil}}{n}\nabla h_{oil} \tag{4-23}$$

where

$$K_{oil} = \frac{k\rho_{oil}g}{\mu_{oil}} \tag{4-24}$$

and n is porosity.

Slope of the Oil-Water Contact Beneath an Oil Trap

An interesting analysis perfromed by Hubbard (1953) is to derive the slope of the oil-water contact beneath an oil trap. Hubbert recognized that two requirements must be satisfied along an oil-water contact. First, because water cannot cross the oil-water contact, the contact must act like an impermeable boundary to water. In other words, the water must move parallel to the oil water contact. Second, if the oil is stagnant above the water, then the oil must have a uniform oil head. In other words, the oil-water contact must be a line of equal oil head.

The first requirement is straightforward to consider. According to Darcy's Law, water flows along the negative direction of the water head gradient. If we let $\boldsymbol{u}$ be vector of unit length directed along the direction of water movement (see Figure 4-6), then

$$\boldsymbol{u} = \begin{bmatrix} u_x \\ u_z \end{bmatrix} = -\frac{1}{J}\begin{bmatrix} \dfrac{\partial h_w}{\partial x} \\ \dfrac{\partial h_w}{\partial z} \end{bmatrix} \tag{4-25}$$

where

$$J = \left[\left(\frac{\partial h_w}{\partial x}\right)^2 + \left(\frac{\partial h_w}{\partial z}\right)^2\right]^{1/2} \tag{4-26}$$

is the magnitude of the water head gradient.

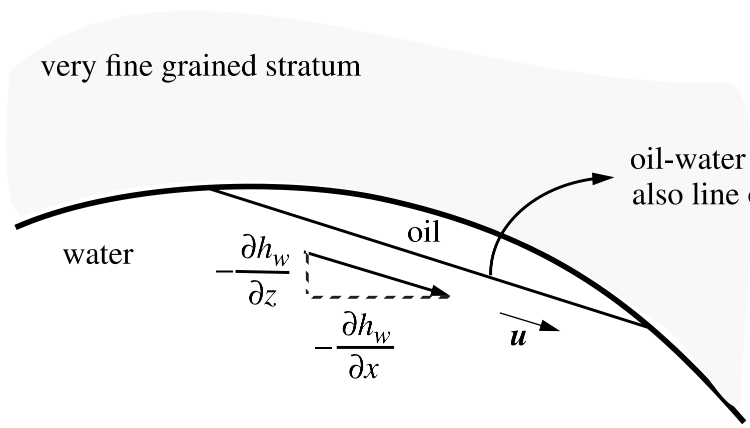

Figure 4-6. Entrapment of oil under very fine grained stratum. Larger arrow indicates water flow along oil-water contact. Smaller arrow indicates the unit vector $\boldsymbol{u}$ pointing along direction of water flow.

Next, we consider the second requirement. If the oil-water contact is a line of equal oil head, then h_{oil} must remain constant along the oil-water contact. Expressed in terms of directional derivative, this means that

$$\partial h_{oil}/\partial \boldsymbol{u} = \frac{\partial h_{oil}}{\partial x}u_x + \frac{\partial h_{oil}}{\partial z}u_z = 0 \tag{4-27}$$

Substituting eq (4-25) into (4-27) and multiplying through by -J, we have

$$\frac{\partial h_{oil}}{\partial x}\frac{\partial h_w}{\partial x} + \frac{\partial h_{oil}}{\partial z}\frac{\partial h_w}{\partial z} = 0 \tag{4-28}$$

Now substitute (4-20) into (4-28) and we obtain, after rearrangement:

$$\frac{\partial h_w}{\partial z} = \frac{\rho_w}{\rho_w - \rho_{oil}}\left[\left(\frac{\partial h_w}{\partial x}\right)^2 + \left(\frac{\partial h_w}{\partial z}\right)^2\right] \tag{4-29}$$

If we divide both sides of (4-29) by $\partial h_w/\partial x$, we obtain

$$\frac{\frac{\partial h_w}{\partial z}}{\frac{\partial h_w}{\partial x}} = \frac{\rho_w}{\rho_w - \rho_{oil}} \frac{\left[\left(\frac{\partial h_w}{\partial x}\right)^2 + \left(\frac{\partial h_w}{\partial z}\right)^2\right]}{\frac{\partial h_w}{\partial x}} \tag{4-30}$$

Note that the left hand side of (4-30) is simply the slope of the oil-water contact. Substituting (4-26) into (4-30) we have

$$\text{slope} = \frac{\rho_w}{\rho_w - \rho_{oil}} \frac{J^2}{\frac{\partial h_w}{\partial x}} \tag{4-31}$$

If the angle of the oil-water contact is denoted by θ_c (see Figure 4-7), and we note that $\partial h_w/\partial x = J\cos\theta$, then eq. (4-31) becomes

$$\text{slope} = \frac{\rho_w}{\rho_w - \rho_{oil}} \frac{J}{\cos(\theta_c)} \tag{4-32}$$

.

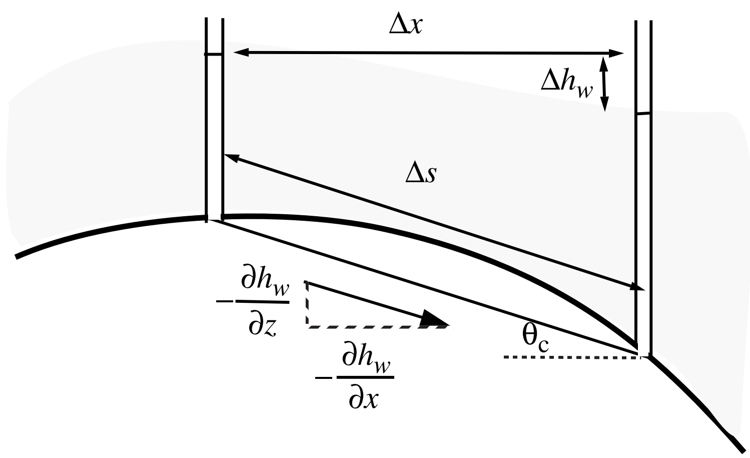

Figure 4-7.

The term $J/\cos\theta$ can be interpreted as follows (see Figure 4-7). J can be written as $\Delta h_w/\Delta s$, where Δs is an incremental distance along the flow line. The x-component of Δs is Δx, so that $\Delta x/\Delta s = \cos\theta$. Therefore $J\cos\theta$ can be written as $(\Delta h_w/\Delta s)/(\Delta x/\Delta s) = \Delta h_w/\Delta x$. Thus, eq. (4-32) is written as

$$\text{slope} = \frac{\rho_w}{\rho_w - \rho_{oil}} \frac{\Delta h_w}{\Delta x} \tag{4-33}$$

According to eq. (4-33), the slope of the oil-water contact is given by a constant $\rho_\omega/(\rho_\omega-\rho_{oil})$ times the slope of the potentiometric surface $\Delta h_w/\Delta x$. Because this constant is always greater than one, it can be thought of as an amplification factor. If the density of the oil is assumed to be 0.9 gm/cm, then the amplification factor is 10. This means that a slope of the potentiometric surface of 10 feet per mile would produce a oil-water contact slope of 100 feet per mile.

References

Bear, Jacob, 1972, Dynamics of Fluids in Porous Media, Elsevier, NY, 764 p.

Berg, R. R., 1975, Capillary pressures in stratigraphic traps, Bull. Amer. Assoc. Petrol. Geol., v. 59, no. 6, p. 939-956.

Hubbert, M. K., 1953, Entrapment of petroleum under hydrodynamic conditions, Bull. Amer. Assoc. Petrol. Geol., v. 37, no. 8, p. 1954-2026.