Lecture Notes 3

Steady-State Flow of Ground Water in Sedimentary Basins

Topography-Driven Flow

Among the several mechanisms that drive the flow of ground water in sedimentary basins, topography ranks as one of the most important driving forces. In a topography-driven flow system, ground water flows from high-elevation recharge areas (where hydraulic head is high) to low-elevation discharge areas (where hydraulic head is low). In this lecture, we consider topography-driven flow under steady-state conditions. Other flow driving mechanisms (such as compaction and density-driven flow) will be covered later in this course.

The classical studies of topography-driven flow are those of Hubbert (1940), Toth (1962, 1963), and Freeze and Witherspoon (1967). A summary of these studies is given by Freeze and Cherry (1979, Chapter 6). In many near-surface flow systems, the water table is a subdued replica of topography. Assuming that the flow system is bounded at the base by an impermeable boundary, the steady-state flow in a two-dimensional vertical section can be conceptualized as shown in Fig. 3-1.

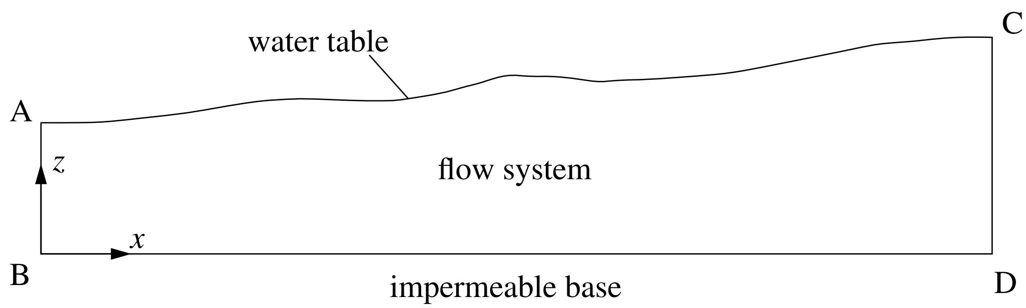

Fig. 3-1. Vertical section showing ground water flow system. AC is water table. AB and CD are ground-water divides. BC is impermeable base.

The mathematical problem of the ground-water flow system in Fig. 3-1 can be stated as follows. The governing equation is:

$$\frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right) = 0 \qquad \text{within the flow system bounded by ABCD} \qquad (3\text{-}1)$$

The boundary conditions are:

$$\frac{\partial h}{\partial x} = 0 \qquad \text{along the groundwater divides AB and CD.} \qquad (3\text{-}2)$$

$$\frac{\partial h}{\partial z} = 0 \qquad \text{along the impermeable base BC} \qquad (3\text{-}3)$$

$$h = z \qquad \text{along the water table AC} \qquad (3\text{-}4)$$

where z is the elevation of the water table.Initial condition is not needed for a steady-state flow problem. In general, the problem posed by (3-1) to (3-4) must be solved numerically. However, Toth (1962, 1963) has published analytical solutions for simple flow systems.

Analytical Solution for Simple Flow Systems

Toth's (1962, 1963) analytical solutions are based on three simplifying assumptions. First, the geologic material is assume homogeneous and isotropic with respect to hydraulic conductivity. Second, if the thickness of the flow system is large compared to the relief of the water table, then the top boundary of the flow system (AD) in Fig. 3-1 can be approximated by a horizontal line--however, the hydraulic head along the modified boundary is still equal to the elevation of the water table. Finally, the water table is assumed to have a simple shape. For example, if the water table has a uniform slope (Fig. 3-2), the solution is

$$h(x, z) = z_o + \frac{cs}{2} + \frac{4cs}{\pi^2} \sum_{m=0}^{\infty} \frac{\cos[(2m+1)\pi x/s] \cosh[(2m+1)\pi z/s]}{(2m+1)^2 \cosh[(2m+1)\pi z_o/s]} \qquad (3\text{-}5)$$

where $c = \tan\alpha$ is the slope of the water table, α being the angle between the water table and the horizontal line.

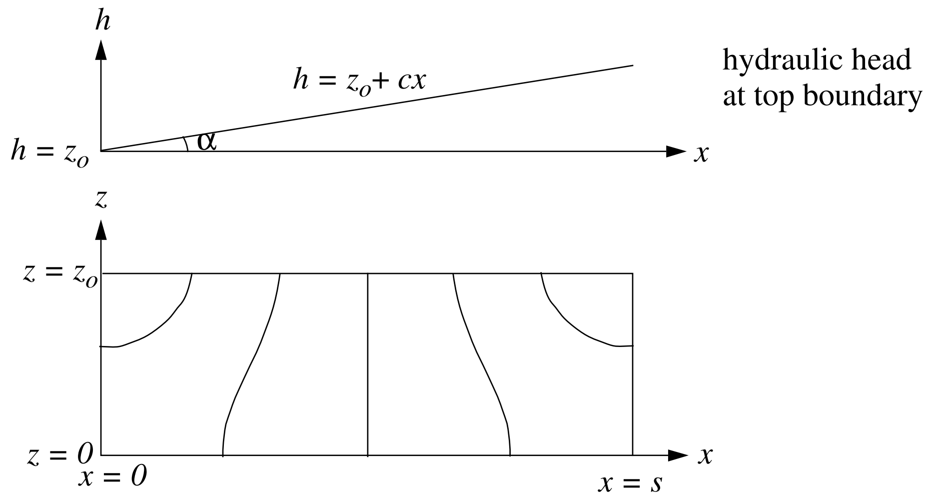

Figure 3-2. Vertical section showing a rectangular flow region with contours of equal hydraulic head (approximate). Hydraulic head varies linearly across the top boundary. All other boundaries are impervious to flow.

Numerical Solutions for Complex Topography

If the water table underlies a complex topography, the flow problem generally cannot be solved analytically. Instead, a numerical model is needed. Freeze and Witherspoon (1967) made a series of computer simulations for flow systems in various topographical settings. They found that topography with uniform gentle incline tends to produce a single, large scale flow system. By contrast, hummocky or hilly topography tends to produce many smaller scale (local) flow systems superimposed on the large-scale (regional) flow system. (See Figure 6-2 of Freeze and Cherry, 1979). Their conclusion is that complex topography can create complex systems of ground-water flow.

Effect of Heterogeneity

Heterogeneity in hydraulic conductivity can exert a strong influence on a ground water flow system. The presence of a high hydraulic conductivity zone can focus flow through the zone. A low hydraulic conductivity zone tends to deflect flow away from the zone. Hydraulic conductivity variation can also substantially perturb the distribution of hydraulic head, creating regions of “abnormal” or “anomalous” hydraulic head.

The presence of a highly conductive, basal aquifer beneath finer grained, less conductive sediments would tend to concentrate flow through the deeper part of the sedimentary basin, thus promoting regional flow (Fig. 3-3). A dramatic example of such a flow system is the White River flow system of Nevada, where the “basal aquifer” occurs in the Paleozoic carbonate rocks underlying a thick sedimentary sequence. According to Eakin (1966), the White River flow system extends in length about 380 km and to depths of greater then 1 km, and underlies more than 10 intermontane basins in east central Nevada.

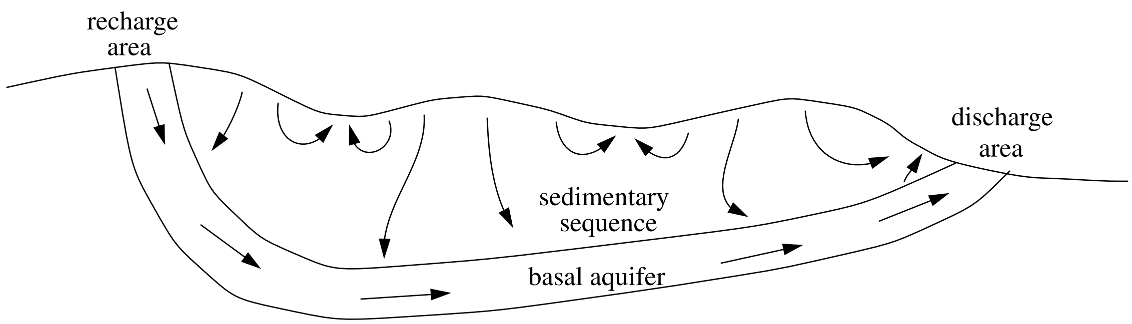

Fig. 3-3. Vertical section showing flow in basal aquifer (higher hydraulic conductivity) and overlying finer grained sedimentary sequence (lower hydraulic conductivity).

Belitz and Bredehoeft (1988) have analyzed the role of heterogeneity in causing anomalously low hydraulic head in the Denver basin. In parts of the Denver basin, hydraulic head in the Dakota sandstone is 600 - 900 m beneath land surface. Belitz and Bredehoeft provided two explanations for the anomalously low head. First, the Dakota sandstone is poorly connected to the high-elevation recharge area to the west (Front Range of the Rocky Mountains), but is directly connected to the low-elevation discharge area to the east (Nebraska and Kansas). Second, the transmissivity (hydraulic conductivity times thickness) of the Dakota sandstone is lower to the west and increases to the east. This is due to deeper burial and thinner sandstone to the west, and shallower burial and thicker sandstone to the east. Belitz and Bredehoeft (1988) illustrate their reasoning with two simple conceptual models.

Conceptual Model A illustrates the poor hydraulic connection between the Dakota sandstone and the recharge area. If we assume that the sandstone is cut off from the recharge area by faulting along the Front Range of the Rock Mountains, then recharge must enter the sandstone via the overlying Pierre Shale (Fig. 3-4a). To illustrate this flow system, Belitz and Bredehoeft used the “two-pipe” flow model as shown in Fig. 3-4b. The vertical pipe represents the Pierre shale, and the horizontal pipe represents the Dakota sandstone.

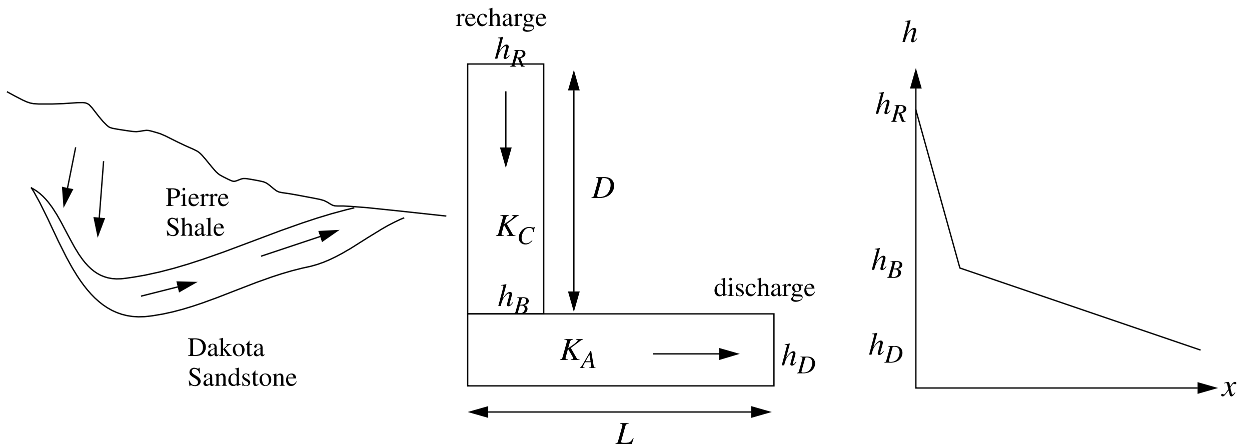

Figure 3-4. (a) Conceptual model of Denver Basin with Dakota sandstone cut off from recharge area. (b) “Two-pipe” model of flow in Denver Basin. (c) Distribution of hydraulic head.

Under steady state, flow into the vertical pipe must equal flow out of the horizontal pipe. Applying Darcy’s Law:

$$Q = -K\frac{dh}{L} \tag{3-6}$$

where Q is volumetric flow rate per unit area, K is hydraulic conductivity, dh is hydraulic head difference across the pipe, and L is length of pipe. Applying Darcy’s law to the continuity of flow in the two pipes, we can write:

$$K_C\frac{(h_R - h_B)}{D} = K_A\frac{(h_B - h_D)}{L} \tag{3-7}$$

where K_C is hydraulic conductivity of the Pierre Shale (confining unit), K_A is hydraulic conductivity of the Dakota sandstone (basal aquifer), h_R is hydraulic head in the recharge area, h_D is hydraulic head at the discharge area, and h_B is hydraulic head at the shale-sandstone contact. Eq. (3-7) can be rewritten as

$$\left(\frac{K_C}{K_A}\right)\left(\frac{L}{D}\right) = \frac{(h_B - h_D)}{(h_R - h_B)} \tag{3-8}$$

Assuming that h_R, h_D, L and D are fixed, then as K_C becomes progressively smaller than K_A, the ratio K_C/K_A also becomes progressively smaller. On the right hand side of (3-8), the numerator will become small, meaning that h_B is close to h_D. Consequently, the head throughout the Dakota sandstone will be close to the head in the discharge area.

In Conceptual Model B, the Dakota sandstone is directly connected to the recharge area, but the hydraulic conductivity increases from west to east. Recharge from the Pierre Shale is neglected. Figure 3-5 illustrates the (a) conceptual flow system, (b) simple model representation, and (c) hydraulic head distribution. Although the mechanism in Conceptual Model B is somewhat different than that of Conceptual Model A, the resultant distribution of hydraulic heads are similar: there is a large drop of hydraulic head on the western side of the flow system, causing the hydraulic head the remain depressed in the rest of the flow system. Through additional numerical simulation, Belitz and Bredehoeft concluded that the anomalously low hydraulic head in the Denver Basin is caused by combined effects of both conceptual models A and B.

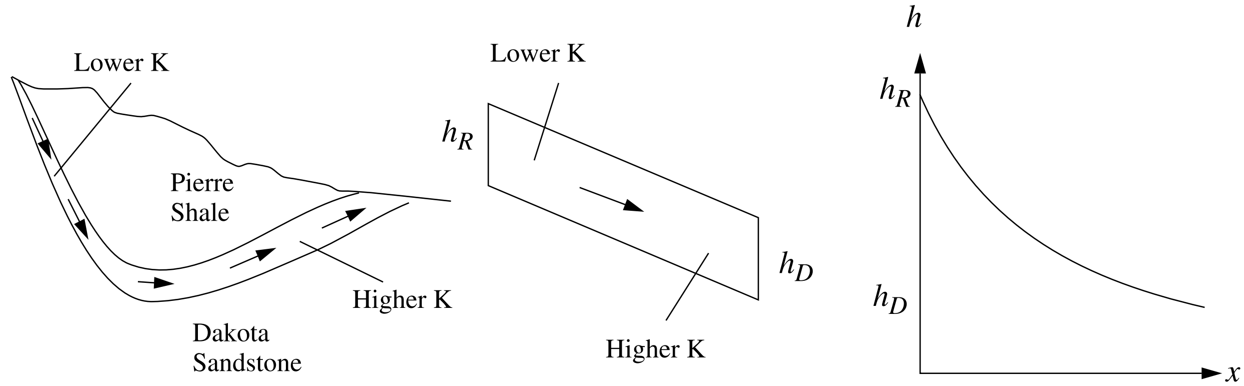

Figure 3-5. (a) Conceptual model of Denver Basin with eastward increasing hydraulic conductivity in Dakota sandstone. (b) Simple model of flow in Dakota Sandstone. (c) Distribution of hydraulic head.

Another interesting finding of the simulation study by Belitz and Bredehoeft is the volumetric flow of ground water through the Denver basin. Their model simulation suggested that about 1.8 m^3/s moves through the regional flow system. Although this flow rate is exceedingly small, it is found to be consistent with other investigations to simulate ground-water flow in large aquifer systems.

References

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