Viscous relaxation model for predicting least principal stress magnitudes in sedimentary rocks

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A B S T R A C T

We propose a new method for estimating stress magnitudes as a function of depth in sedimentary formations based on a laboratory constrained viscous rheology and steady tectonic loading. We apply this method to a well drilled in the Barnett shale in the Fort Worth Basin, Texas. Laboratory experiments show that shale gas reservoir rocks exhibit wide range of viscoplastic behavior mostly dominantly controlled by its composition. Stress relaxation in these formations is described by a simple power-law (in time) rheology. We demonstrate that a reasonable profile of the principal stress magnitudes can be obtained from geophysical logs by utilizing (1) the laboratory power-law constitutive law, (2) an estimate of the horizontal tectonic loading, and (3) the assumption that the ratio of principal stress differences ([S2 − S3]/[S1 − S3]) is relatively uniform with depth. Profiles of the principal stress magnitudes generated based on our proposed method for a vertical well in the Barnett shale generally agree with the occurrence of drilling-induced tensile fractures in the same well. Also, the predicted decrease in the least principal stress (fracture gradient) in the limestone formation underlying the Barnett shale appears to explain the downward propagation of hydraulic fractures observed in this region. This stress change is not captured by the extended Eaton (instantaneous loading) model even when incorporating formation anisotropy. We believe our approach is more consistent with the time-dependent processes associated with stress accumulation over the course of geological time and thus may provide a new method to predict vertical hydraulic fracture growth in targeted reservoirs.

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1. Introduction

Predicting variations of the magnitude of least principal stress within sedimentary basins has significant practical value in the petroleum industry. Newly created hydraulic fractures open when the pressure of the injected fluid exceeds the least principal stress magnitude. Knowledge of stress magnitude variation along a well path helps to identify intervals that will, or will not, act as barriers to optimize fracture containment within desired formations.

Direct measurements of the in-situ least principal stress magnitudes in subsurface reservoir formations rely on mini-frac and leak off tests. While these measurements can be conducted with reasonable accuracy, they only provide measurements of the in-situ stress magnitude at a specific depth and repeated measurements (e.g., above, below and within an interval to be hydraulically fractured) are extremely rare. Therefore, log-based algorithms for predicting stress magnitudes have been proposed which can create a continuous profile of the stress magnitude along a well after calibration against the limited direct measurements from mini-frac and leak off tests.

For over 40 years, the approach proposed by Eaton (1969) has been used and adapted by many researchers. The method assumes that the source of horizontal stress is the gravitational load of the overburden. When the vertical stress, Sv, from the overburden load is applied vertically to a formation with lateral confinement (uniaxial strain), the formation also experiences an increase in horizontal stress, Sh. Based on linearly elasticity, this relation between increase in vertical and horizontal stresses was proposed by Eaton to be

\[ S_h = \frac{\nu}{1-\nu}(S_v-P_p) + P_p \]  

(1)

where \( P_p \) is the pore pressure and \( \nu \) is the Poisson’s ratio of an isotropic elastic medium. Therefore, if the pore pressure gradient is known and profiles of \( S_v \) and \( \nu \) are determined from density and sonic logs, a profile of the minimum horizontal stress \( S_h \) can be obtained. However, despite the wide-spread use of this relation,
Eaton (1969) recognized from the outset that it was necessary to use an empirically-determined depth-dependent Poisson’s ratio that increases from 0.25 (at ~1000 ft) to 0.5 (at 20,000 ft) to fit available measurements of least principal stress in the Gulf of Mexico.

There are several fundamental shortcomings of the model described in Eq. (1). First, after deposition, the overburden load increases with time and burial as rocks undergo compaction and diagenesis which causes elastic properties to evolve throughout the process. Therefore, the use of present day elastic properties is not appropriate to compute the effects of processes that occur over geological time. Secondly, rock deformation involves plastic and time-dependent components not captured by linear elasticity and the assumption of instantaneously applied stress. Plastic deformation (e.g. compaction, fracturing, or faulting) do not contribute to the overall buildup of elastic stress but releases stress. Reservoir rocks can also exhibit time-dependent deformatonal behavior which can alter stress states through flow deformation over geological time depending on the degree to which it occurs (Sone and Zoback, 2014). These inelastic time-dependent effects cannot be ignored when considering processes leading up to the present day stress states. Finally, the overburden load is not the only source of horizontal stress, but tectonic processes also contribute to the horizontal stresses. While Eq. (1) predicts $S_h$ less than or equal to $S_v$, observations of strike-slip earthquakes ($S_{h\text{max}} > S_v > S_{h\text{min}}$) and reverse faulting earthquakes ($S_{h\text{max}} > S_{h\text{min}} > S_v$) provide clear evidence of horizontal stresses from sources other than gravitational loading.

To overcome some of these shortcomings, various modifications to Eq. (1) have been proposed in the past (we refer to these as the “extended Eaton models”) which also incorporate the effects of elastic anisotropy, poroelasticity, and tectonic loading (e.g. Amadei et al., 1987; Thiercelin and Plumb, 1994). Eq. (2) below is one such equation by Thiercelin and Plumb (1994), which assumes vertical transverse isotropy and horizontal tectonic loading

$$ S_{h\text{min}} - \alpha P_p = \frac{E_h}{E_0} \frac{\nu_h}{1-\nu_h} (S_v - \alpha P_p) + \frac{E_h}{1-\nu_h} (\epsilon_h - \nu_h \epsilon_h) $$

$$ S_{h\text{max}} - \alpha P_p = \frac{E_h}{E_0} \frac{\nu_h}{1-\nu_h} (S_v - \alpha P_p) + \frac{E_h}{1-\nu_h} (\epsilon_h - \nu_h \epsilon_h) $$

(2)

here $E_h$ and $\nu_h$ are the vertical and horizontal Young's moduli, respectively; $\nu_v$ and $\nu_h$ are the vertical and horizontal Poisson's ratio, respectively; $\alpha$ is the Biot's coefficient; $\epsilon_h$ and $\epsilon_v$ are the tectonic strains in the direction parallel to the maximum and minimum horizontal principal stresses ($S_{h\text{max}}$ and $S_{h\text{min}}$). In Eq. (2), the first term is the horizontal stress due to gravitational loading ($S_h$) similar to Eq. (1) except in the anisotropic form. The second term describes the horizontal stresses arising from horizontal tectonic strains with constant strain boundary condition in the horizontal direction and constant stress boundary condition in the vertical direction.

The addition of the horizontal tectonic loading particularly allows the presence of more compressive environments ($S_h > S_v$) which was not realizable in the original equation, and also appeared to explain the bed-to-bed variation in horizontal stress magnitudes observed in some sand-shale sequences in North America (Kry and Groenst, 1983; Hickman et al., 1985; Evans et al., 1989a). In these studies, in-situ horizontal stress magnitudes measured in boreholes were observed to be lower in the shale/siltstone layers than in the adjacent sandstone units. By comparing field stress data in the Appalachian Plateau with core mechanical properties (Evans et al., 1989b) and geophysical log data (Plumb et al., 1991), it was concluded that variation in horizontal stress correlates with the Young’s modulus of the formation better than with the Poisson’s ratio. These studies suggested that horizontal stress magnitudes are lower in the compliant shale layers compared to the adjacent stiff sandstones because tectonic horizontal stress caused by a given amount of tectonic strain is proportional to the elastic stiffness.

However, while some local examples of horizontal stress variation may be consistent with these elastic models, other observations are not. For instance in the Paris basin, minimum horizontal stress magnitudes are found to be larger within the clay-rich Bure argillite where the elastic modulus is smaller compared to the adjacent limestone (Gunzburger and Cornet, 2007). Also hydraulic fractures created in the Barnett shale are sometimes observed to extend downwards into the underlying limestone formations (Fisher and Warpinski, 2011) although these limestone formations are stiffer compared to the Barnett shale formation and according to extended Eaton models, they should act as fracture barriers due to the higher stress (see below).

Several studies have also incorporated viscoelastic rheology with detail consideration of the basin history (Prats, 1981; Warpinski, 1989), but these calculations were based on arbitrary viscoelastic constitutive laws not constrained through laboratory experiments but convenient for numerical calculations. Thus models based on more realistic rock behavior justified through laboratory experiments are needed.

In this paper, we propose a new method for estimating stress magnitude variations in shale gas reservoirs. The method superimposes the effect of tectonic loading and viscous stress relaxation, which is based on laboratory-observed creep behavior of a diverse suite of shale gas reservoir rock (Sone and Zoback, 2014). We first examine geophysical log data from a vertical well in Barnett shale, Fort Worth basin, Texas, to obtain information about the in-situ stress variation in shale gas reservoirs. Then laboratory results from creep experiments using core samples from several shale gas reservoirs are discussed to explore the possible relation between viscous formation properties and spatial variation in horizontal stress differences. We then finally attempt to calculate the profile of minimum horizontal principal stress based on the viscous formation properties. Results from our method are compared with those obtained from commonly used extended Eaton models.

2. In-situ stress in a vertical well from Barnett shale, Fort Worth Basin, Texas

2.1. Local geology in the studied well

We studied a set of geophysical and electrical image log that was obtained from a vertical well drilled in the northeastern region (Newark East Field) of the Fort Worth basin, Texas, to study the stress state around the Barnett shale formation and its relation to petrophysical properties. At this location, the Barnett shale is over 1000 ft (300 m) in thickness, divided into two units (upper and lower Barnett) by the Forestburg limestone, and overlies the Viola Limestone Formation (Fig. 1). We find the upper Barnett shale to be the quartz/clay rich unit that extends from about 7695–7840 ft in this well overlying the Forestburg limestone below. The lower extent of the Forestburg limestone is unclear, but we believe that it extends to about 8000 ft where rapid inter-laying of high- and low-resistivity units starts to appear in the electrical image log. This rapidly inter-layered section which occurs at the top of the lower Barnett shale is referred as the “limestone” (Pollastro et al., 2007). The mineral compositions in this interval obtained through analysis of elemental capture spectroscopy range from almost completely carbonate-dominated (> 90%) rocks, to quartz, feldspar, mica-
and clay-dominated shales with no carbonates (Fig. 1). We also see the presence of a high gamma ray layer at the base of the lower Barnett, referred as the “basal hot shale”, which can be tracked throughout most of the Fort Worth Basin (Pollastro et al., 2007). High gamma ray values in the basal hot shale are attributed to the abundance of organic- and phosphate-rich zones that are inter-bedded with laminated siliceous mudstone (Tian, 2010).

2.2. Observation of natural fractures and wellbore failures

2.2.1. Fracture distribution and orientation

A high quality image log data was available in the studied well which allowed us to clearly identify the presence of natural fractures and also wellbore failures needed for in-situ stress analysis. We found no breakouts, but there are drilling-induced tensile fractures (DITFs) present in the wellbore images (Fig. 1c) indicative of a moderately compressive environment (Moos and Zoback, 1990).
common and more vertical DITFs are observed compared to the overlying Barnett shale formation.

Fig. 3 summarizes the orientations at which the DITFs are observed and the strike of the natural fractures. Except for the Marble Falls limestone, DITFs are observed throughout the image log and their azimuth appears to be consistent (Fig. 1). From the Rose diagram plot (Fig. 3a), we see that the azimuths of DITFs are scattered around N20°E and N200°E, indicating that the direction of $S_{\text{max}}$ is in the NNE/SSW direction. This $S_{\text{max}}$ direction is consistent, but slightly more northerly, with the $S_{\text{max}}$ direction found from other studies in the Barnett shale (Gale et al., 2007; Vermylen, 2011) and seen in limited data in the World Stress Map data in northeast Texas (Heidbach et al. (2008)), which all show NE/SW to ENE/WSW directions.

Most of the natural fractures occur in the upper Barnett shale and the basal hot shale. These fractures have strikes around N50°W and dip 80°–85° to the NE. Natural fractures were also found in the Forestburg limestone and in the Bend group formation, but their orientations are not as consistent as in the basal hot shale fractures (Fig. 3b). These observations agree fairly well with natural fractures observed in cores reported by Gale et al. (2007; Vermylen, 2011) and seen in limited data in the World Stress Map data in northeast Texas (Heidbach et al. (2008)), which all show NE/SW to ENE/WSW directions.

2.2.2. Occurrence of DITFs and lithology

Although many DITFs are observed especially in the lower Barnett shale, they do not occur within all intervals at the small scale, but rather disappear and re-appear along the wellbore. Fig. 4 shows an example of such observations. Here DITFs disappear as the well enters a high-gamma ray section, but re-appears as the well re-enters a lower-gamma ray section. To check the consistency of this trend, we plot two histograms of gamma ray intensities of where DITFs do and do not exist. Fig. 5 shows data for the Lime Wash section. The black and gray histograms represent gamma ray intensities of the formation where DITFs are present and absent, respectively. The difference in the peak position of the histograms suggests that gamma ray intensity is, on average, higher for formations where DITFs do not occur. Although there does not seem to be a clear threshold value, DITFs generally do not occur above 70 GAPI. Because high-gamma ray intensity indicate either high clay content and/or high organic content in the formation, a lithological control on the patterns of DITF occurrence is implied such that clay- and organic-rich layers do not yield DITFs.

2.3. Constraining the in-situ stress state

2.3.1. Far-field stress

Given the occurrence of DITFs and absence of breakoutst, we constrained the in-situ stress state at reservoir depth based on the Kirsch equations for stress concentration around circular cavities and other constraints on principal stress magnitudes, rock properties, and drilling parameters, following methods described in Zoback (2007). The vertical stress, $S_v$, was estimated by integrating the density log data, and the minimum horizontal stress, $S_{\text{min}}$, was estimated from observations of instantaneous shut-in pressures (ISIPs) during a hydraulic fracturing operation in a nearby well. Other environmental and rock parameters were taken from the literature, log data, or assumed values. These are
Fig. 3. Orientations of DITFs and natural fractures: (a) Rose diagram of DITF azimuth and strike of natural fractures and (b) stereonet diagram of strike vs. dip of natural fractures.

summarized in Table 1. Values are determined for 8500 ft depth where applicable.

Fig. 6 shows the stress polygon representation (see Moos and Zoback (1990) or Zoback (2007) for an explanation of the polygon) of the allowable stress states calculated based on the parameters in Table 1. Presence of DITFs forces the stress state to be on the left side of the $T_0 = 0$ contour, and absence of breakouts forces the stress state to be below the $C_0 = 150$ MPa contour. Given $S_{\text{hmin}} = 44-47$ MPa and $S_{\text{p}} = 65$ MPa at 8500 ft, the magnitude of $S_{\text{hmax}}$ is constrained to be between $S_{\text{hmax}} = 64-85.5$ MPa. As a result, the allowable stress state mostly lies in the strike-slip faulting regime, but close to the boundary with the normal faulting regime. Note that the misalignment between the borehole axis and the vertical principal stress is ignored here because it was found that the effect of a misalignment of up to 15° does not influence the results greatly for this particular case (Sone, 2012).

2.3.2. In-situ stress variation

We now examine the observation that DITFs frequently disappeared in the clay- and organic-rich section of the lower Barnett shale. Because DITFs form when the tensile circumferential stress, $\sigma_{\theta\theta}$, exceeds the tensile strength of the formation, the occurrence of DITFs was controlled either by the variation in the formation tensile stress or the variation in $\sigma_{\theta\theta}$ around the wellbore. The former is unlikely because tensile strength of rocks vary little from each other, ranging between 0 and 10 MPa. Also, even if the small variation in tensile strength was enough to control DITF occurrence, the fact that clay- and organic-rich shales are generally weaker than carbonate-rich shales (Sone and Zoback, 2013b) suggests that DITFs should form more frequently in high-gamma ray formations, contrary to our observation in Figs. 4 and 5. Therefore it is the variation in $\sigma_{\theta\theta}$ along the well that causes the frequent appearance and disappearance of the DITFs.

The observed absence of DITFs implies that locally, the in-situ stress state can be to the right of the $T_0 = 0$ contour in Fig. 6. One way such crossover can occur is by the increase in mud pressure, $P_{\text{m}}$, or the formation pore pressure, $P_{\text{p}}$. However, we do not believe these are the causes because the mud weight is not changed so frequently. Also if there is any local variation in the pore-pressure gradient, we would expect higher pore pressure in the clay- and organic-rich layers where permeability is low and hydrocarbons are generated. However, this would create more DITF occurrences in the high gamma ray formations, again contrary to our observation in Figs. 4 and 5.

Therefore the most likely cause of the rapid appearance and disappearance of DITFs is the change in the horizontal stress magnitudes, either by a decrease in magnitude of $S_{\text{hmax}}$ and/or an increase in magnitude of $S_{\text{hmin}}$ (Fig. 6). Both of these effects result in a decrease in horizontal stress anisotropy, $S_{\text{hmax}} - S_{\text{hmin}}$ and a stress state closer to isotropic. This leads us to conclude that the stress state within the clay- and organic-rich layers is more isotropic and it is this stress variation that is causing the frequent appearance and disappearance of the DITFs.

3. Variation in viscous properties of shale gas reservoir rocks

Motivated by the observation that the stress state is closer to isotropic in clay- and organic-rich layers within the lower Barnett shale, we focus on the viscous (ductile) deformational properties of shale gas reservoir rocks to explain this phenomenon. Viscous rocks exhibit time-dependent creep deformation in response to applied stress, which also inherently relates to their ability to relax stress over time as tectonic loading is occurring. It is also found from creep experiments of shaly sands with varying clay content that time-dependent creep is enhanced by the increase in clay content (Chang et al., 1997). Given the correlation between lithology and horizontal stress difference in the previous section, it is important to investigate the variation in viscous deformational properties of shale gas reservoir rocks. Accordingly, Sone and Zoback (2013a, 2013b) performed laboratory experiments using shale gas reservoir rocks simultaneously characterizing their
elastic, ductile, and brittle properties. Sone and Zoback (2014) further interpreted the ductile creep behavior in the framework of linear viscoelastic theory to evaluate its long-term effect on in-situ stress. Here we briefly review key conclusions from these studies which form the basis for the stress analysis in the following sections.

Fig. 7 shows examples of creep data from three different shales. Under constant confining pressure, a step of axial differential stress was applied to cylindrical samples whose cylinder axis was either perpendicular or parallel to the bedding plane. The axial differential stress was then held constant over periods of up to 2 weeks to observe the time-dependent deformational behavior. Note that 2 subgroups of samples were tested for Barnett, Haynesville, and Eagle Ford samples where subgroup 1 (e.g. Barnett-1) has higher clay and organic content than subgroup 2 samples. It was found that the magnitude of creep strain in each sample was proportional to the applied stress (Sone and Zoback, 2013b). Therefore, the creep strain data normalized by the applied stress magnitude yields the creep compliance data, which can be used to compare the tendency to creep between different samples as in Fig. 7. We see from the results that creep deformation occurs regardless of sample orientation although there do exist a considerable anisotropy in creep. Also, creep deformation is greater for samples with higher clay and organic content. Evidence for pore volume reduction during creep suggests that creep
deformation is taking place within the clay and organic material which are the relatively porous components in the rocks.

If we take the creep compliance data at 3 h, \( S_{\text{creep}} \) (3-h creep compliance), as the quantity representing the tendency to creep for each sample, we find that \( S_{\text{creep}} \) correlates well with the elastic Young’s modulus, \( E \), of the rock (Fig. 8). This correlation between two fundamentally different deformational properties was explained by appealing to the internal stress partitioning that occurs amongst the mineral constituents, and by forward calculating the effective creep compliance of the samples based on the stress partitioning. Viewing shales as a binary mixture of “soft” and “stiff” components, each composed of clay/kerogen and quartz/feldspar/pyrite/carbonates, respectively, the following analytical relation between Young’s modulus and 3-h creep compliance were derived by Sone and Zoback (2013b).

\[
S_{\text{creep}}(E) = A_1 \frac{E}{E} + A_2
\]

![Fig. 6. In-situ stress state at 8500 ft depth constrained by the frictional strength and required strength for given wellbore failure. The gray area describes the allowable stress state when DITFs are present and breakouts are not present.](image)

\[ A_1 = \frac{E_{\text{soft}} E_{\text{soft}}}{E_{\text{soft}} - E_{\text{soft}}} (S_{\text{soft}} - S_{\text{soft}}) \]
\[ A_2 = \frac{S_{\text{soft}} - S_{\text{soft}}}{E_{\text{soft}} - E_{\text{soft}}} \]

where \( E_{\text{soft}} \) and \( E_{\text{soft}} \) are the effective Young’s moduli of the soft and stiff components, respectively, \( S_{\text{soft}} \) and \( S_{\text{soft}} \) are the effective 3-h creep compliance of the soft and stiff components, respectively. The equation explains the laboratory data well (Fig. 8, solid curve) and forms the basis for estimating ductile properties from elastic properties.

Sone and Zoback (2014) further analyzed the creep experiment data to capture the time-dependent constitutive relation between stress and strain. It was found that the total compliance response \( J(t) = \varepsilon(t)/\Delta \sigma \), both elastic and creep, of all samples can be adequately described by a power law function of time.

\[ J(t) = B t^n \]  

In Eq. (4), \( B \) describes the compliance response at \( t = 1 \) s, which essentially is the elastic response of the rock. Therefore, the

![Fig. 7. Examples of creep compliance data (creep stain divided by magnitude of stress step). (1) Barnett shale samples with different clay and organic content. (2) Haynesville shale samples with different clay and organic content. (3) Eagle Ford shale samples with similar mineralogy but different orientation.](image)

![Fig. 8. 3-h Creep compliance data plotted against Young’s modulus. Solid black curve shows analytical prediction of the overall correlation by Sone and Zoback (2013b) with \( E_{\text{soft}} = 5.4 \) GPa, \( E_{\text{soft}} = 86.9 \) GPa, \( S_{\text{soft}} = 1 \) e -4 MPa -1, \( S_{\text{soft}} = 0 \) MPa -1. Dashed black curve is the analytical prediction adjusted to fit the Barnett shale data by lowering the soft component 3-h creep compliance to \( S_{\text{soft}} = 0.4 e -4 \) MPa -1.](image)
A log \( Zoback, 2014 \)). Therefore, only the average strain rate is important for the stress analysis due to the relatively shorter time scale required for the time-dependent stress relaxation compared to the entire geological history.

4.2. Vertical profile of the horizontal stress difference

4.2.1. Applying viscous relaxation model to sonic log data

Here we apply the viscoelastic stress analysis to the entire section of the borehole studied in Section 2 with the aid of sonic log data to predict a continuous vertical profile of horizontal stress. In Section 5, we will compare this stress profile with one using a widely used extended Eaton model that incorporates formation anisotropy and regional strain. Our starting point is that the difference between the 2 horizontal principal stresses, \( S_{(Hmax - Hmin)} \), arises from the interaction between horizontal strain difference, \( \varepsilon_{diff} = \varepsilon_H - \varepsilon_V \), that accumulates over geological time and viscous relaxation. Another constraint on principal stress differences in the crust is its frictional strength (e.g., Townsend and Zoback, 2000), which will be mentioned briefly below.

To obtain a continuous profile of the power-law constitutive parameters \((B\) and \(n\)) for use in Eq. (5), we first utilize the fact the 1/\(B\) is roughly equal to the Young’s modulus of the rock. Because we are interested in horizontal (bedding-parallel) deformation, we need to estimate the horizontal Young’s modulus, \(E_h\), from vertical velocities measured by the sonic logs. This is done by utilizing the empirical relations found in Sone and Zoback (2013a) between Thomsen anisotropy constants (Thomsen, 1986) and vertical velocities. See Appendix A for details on how the anisotropic elastic constants are determined. The resulting profile of \(B\) is shown in Fig. 10 column (b). Next, we utilize the relation between 3-h creep compliance and Young’s modulus (which is independent of orientation) in Fig. 8 to obtain a relation between \(B\) and \(n\).

Because 1/\(B = E\), Eq. (3) yields

\[
S_{creep} = \frac{A_1}{E} + A_2 = BA_1 + A_2
\]

From Eq. (4), \(S_{creep}\) can also be described using the power-law constitutive parameters as

\[
S_{creep} = J(3600 \times 3) - J(1) = B \times \left(10, 800^{n - 1}\right)
\]

Eqs. (6) and (7) yield the relation between \(B\) and \(n\) is

\[n = \frac{\log \left(\frac{A_1 + (A_2/B) + 1}{\log(10, 800)}\right)}{\log(10, 800)}\]  

This equation should forward predict the general relation between \(B\) and \(n\) for the entire dataset shown in Fig. 9 assuming all samples are adequately described by a binary mixture of soft and stiff components, required by Eq. (3).

As shown in Fig. (9), Eq. (8) captures the increasing trend of \(n\) with increasing \(B\), suggesting the first-order validity of the shale model by Sone and Zoback (2013b) and also the description of the time-dependent deformation by a power-law constitutive relation. However, it is also evident that \(n\) for the Barnett shale samples are generally lower than the predicted trend. The same discrepancy was in fact already evident in Fig. 8 where the Barnett samples had smaller \(S_{creep}\) values compared to the overall trend. This implies that Barnett samples are less ductile compared to samples from other reservoir, probably due to differences in the properties of the constituent minerals as well as its different diagenesis and maturation histories.

Because our primary interest for the stress analysis is to best capture the properties of the Barnett shale samples, we generated a specific trend between \(S_{creep}\) and Young’s modulus for the Barnett shale data by lowering the 3-h creep compliance of the soft component to 0.4e – 4 [ MPa -1]. By doing so, the adjusted trend now passes through the Barnett shale \(S_{creep}\) data (Fig. 8, Fig. 9. Power-law constitutive parameters, \(B\) and \(n\), determined for each sample. Solid and dashed black lines are trends that are projected from those in Fig. 8.
dashed curve). The relation between $B$ and $n$ was then generated using this adjusted trend that matches the Barnett shale 3-h creep compliance data. The resulting relation between the power-law constitutive parameters is shown by the dashed curve in Fig. 9, which agrees well with the $B$ and $n$ values measured for the Barnett shale samples. We use this new relation between $B$ and $n$ as the Barnett-specific trend to obtain a continuous profile of $n$ from the profile of $B$ (Fig. 10 column (b)).

Finally, following Sone and Zoback (2014), we assign the average tectonic strain rate to be $\varepsilon_{\text{diff}} = 10^{-13} \text{s}^{-1}$ and the effective duration of the tectonic deformation to be 150 Myr. This results in total differential strain of about $\varepsilon_{\text{diff}} = 4.7e^{-4}$. The resulting profile of horizontal stress differences is shown in Fig. 10 column (c).

### 4.2.2. Extrapolation to limestone formations and quality of sonic log data

In Fig. 10, we have calculated the stress difference profile through the entire log including the Marble Falls, Forestburg, and Viola limestone formations because we need to know how the stress state compares between different formations. However, the power-law behavior and the associated constitutive parameters were constrained using laboratory data from the Barnett shale samples.

To address this, Fig. 11a shows the composition of each lithological units obtained by the elemental capture spectroscopy log in a ternary diagram together with the composition of the Barnett shale samples used in the laboratory experiments constrained by powder X-ray diffraction and pyrolysis. We can see that both upper and lower Barnett shale exhibit a wide range of lithology (cyan and blue dots) ranging from nearly pure carbonate rocks to low-carbonate shales with up to 20–40% clay content. Thus, the samples studied cover much of the range of composition spanned by the Marble Falls, Forestburg and Viola limestone formations. In addition, the compositions of our laboratory samples cover most of the compositions spanned by the Barnett shale log data. Fig. 11b compares the $V_p$ and $V_s$ sonic log data from each lithological units and the ultrasonic velocity data of laboratory Barnett shale samples. Again,
the ranges of sonic velocity exhibited by the 3 limestone formations are included within the range spanned by the Barnett shale, and the laboratory samples roughly cover the whole spectrum of log velocities. Therefore the composition and elastic properties of the limestone formations appear to be within those encountered in the Barnett shale laboratory data, and therefore leads us to believe that the extrapolation of the stress analysis to adjacent limestone formation is not unreasonable.

Because the stress profiles calculated in this study heavily rely on the sonic log data, it is also worthwhile to assess the quality of the sonic measurements in the $V_p$ vs. $V_s$ plot. We find that the velocity data generally lies around and between known empirical trends for brine saturated shales and carbonate rocks (Castagna et al., 1993), consistent with compositions encountered in the studied well (Fig. 11b). The low $V_p$ data that lies on the left side of the shale trend is also consistent with $V_p/V_s$ data of organic-rich shales from other studies (Lucier et al., 2011; Vernik and Milovac, 2011). However, there exists some data with unrealistic high $V_p/V_s$ ratios. The depths at which these data appear are shown by magenta circles in Figs. 10 and 13. Horizontal Poisson’s ratio calculated for these intervals have negative values (and vertical Poisson’s ratio takes values above 0.5). Thus, it is clear that these are spurious data.

4.3. Profile of the horizontal stress magnitude

Using the horizontal stress difference profile obtained above, we calculate profiles of the horizontal stress magnitudes by imposing a constraint that the faulting style (that is the relative stress magnitudes) should remain constant through the sedimentary section, an approach sometimes used to interpolate sparse data of stress measurement along a well (e.g., Zoback, 2007). The justification for this assumption is that if geologic deformation in an area is characterized by pure normal faulting ($S_v > S_{t(max)} > S_{h(min)}$), or a combination of normal and strike-slip faulting ($S_v \approx S_{t(max)} > S_{h(min)}$), or pure strike-slip faulting ($S_{t(max)} > S_v > S_{h(min)}$), etc., it requires constant relative stress magnitudes with depth. Quantitatively, we accomplish this by considering how close the intermediate principal stress is to the maximum and minimum principal stresses. Following Angelier (1979), we describe such condition using the parameter $\phi$

\[
\phi = \frac{S_v - S_3}{S_1 - S_3}
\]

where $S_1$, $S_2$, and $S_3$ are the three principal stress magnitudes in order of decreasing magnitude. $\phi$ varies from 0 to 1 as $S_2$ increases from $S_2 = S_3$ to $S_2 = S_1$, and contours of constant $\phi$ are shown on the stress polygon in Fig. 12. As originally pointed out by Angelier (1979), $\phi$ determines the slip direction on a particular fault and fault orientations favorable for slip under a given stress state. Therefore, $\phi$ governs the kinematics of faulting and becomes a measure for faulting style within a particular faulting environment. If different lithological units are intact within a sedimentary section, we may expect that the kinematics is uniform. In other words, $\phi$ is uniform with depth in order to satisfy regional crustal strain constraints.

Although we evaluate the stress relaxation between the 2 horizontal principal stresses in our model, laboratory creep data on vertical samples show that viscoelastic stress relaxation also occurs between vertical and horizontal stresses. The net result of simultaneous differential stress relaxation between all principal stresses is a stress state moving towards isotropic state ($S_v = S_{t(max)} = S_{h(min)}$), which coincides with a constant $\phi$ stress path in the stress polygon (Fig. 12). Thus, we note that the temporal effect of viscoelastic stress relaxation will not affect the spatial constraint on $\phi$ discussed above because stress relaxation occurs along constant $\phi$.

By imposing a constraint that $\phi$ stays constant along the well within the strike-slip faulting regime, we can calculate the absolute magnitude of the horizontal stresses based on the profiles of vertical stress ($S_v$) and horizontal stress difference ($S_{h(max)} - S_{h(min)}$).

\[
S_{h(min)} = S_v - \phi(S_{t(max)} - S_{h(min)})
\]

\[
S_{t(max)} = S_v + (1 - \phi)(S_{t(max)} - S_{h(min)})
\]

From the stress magnitudes constrained at 8500 ft depth in Fig. 6, in this particular case we are considering, $\phi$ lies somewhere between about 0.5 and 1.0 in a strike-slip/normal faulting environment (Fig. 12). Based on these observations, 2 profiles of the 2 horizontal principal magnitudes were calculated for a constant stress ratio of $\phi=0.6$ and $\phi=0.9$, shown in Fig. 10 columns (d) and (e), respectively.

The 2 stress profiles generally show similar pattern of variation as they are based on the same vertical stress and horizontal stress.
difference profiles. The difference lies in how the horizontal stress fluctuation is distributed between $S_{\text{hmax}}$ and $S_{\text{hmin}}$. Because $\phi = 0.9$ implies similar magnitudes between $S_h$ and $S_{\text{hmax}}$ in a strike-slip faulting regime, most of the fluctuation in horizontal stress differences is accommodated by the fluctuation in $S_{\text{hmin}}$, whereas in the case of $\phi = 0.6$, both $S_{\text{hmax}}$ and $S_{\text{hmin}}$ fluctuate by about the same amount. A notable common feature in both cases is that $S_{\text{hmin}}$ decreases abruptly as the well enters the Viola limestone below the Barnett shale. This is important information for fracture containment as will be discussed in Section 6.3.

5. Stress profiles based on an extended Eaton model

To compare our results from the viscous relaxation model with linear elastic models, we also generated a stress profile based on the extended Eaton model shown in Eq. (2) proposed by Thiercelin and Plumb (1994). Note that $\varepsilon_h$ in Eq. (2) is the in-plane horizontal Poisson's ratio, $\nu_{xy}$, when the $x_2$-axis is the symmetry axis perpendicular to the bedding plane. Therefore similar to Section 4.2, the anisotropic elastic constants are determined from the vertical sonic measurements (Fig. 13 columns (b) and (c)) using empirical relations discussed in Appendix A. We assign a constant value of 1 to the Biot’s coefficient, $\alpha$, for simplicity. The same vertical stress and pore pressure profiles as the previous section are used for the calculation.

Fig. 13d shows the stress profile obtained using the extended Eaton model. To generate a stress profile comparable to the profile from the viscous relaxation model, we assigned horizontal strain values $\varepsilon_h = 4.7e - 4$ and $\varepsilon_h = 0$ to match the total horizontal differential strain used in the viscous relaxation model. In panel (d), the gray data shows the horizontal stress from gravitational loading corresponding to the first term in Eq. (2). The red and green data shows $S_{\text{hmin}}$ and $S_{\text{hmax}}$ respectively, which results from adding the tectonic stresses due to tectonic deformation, the second terms on the right side of Eq. (2), to the gravity-induced stress in gray. The resulting stress profile agrees with the constraint on $S_{\text{hmin}}$ from a nearby ISIP data and the range of $S_{\text{hmax}}$ inferred from DITF presence. Therefore, the choice of horizontal strain values, $\varepsilon_h = 4.7e - 4$ and $\varepsilon_h = 0$, also appears to be reasonable estimates for the extended Eaton model as well.

6. Discussion

6.1. Comparison with nearby ISIP and frictional limits

We first discuss the consistency of the absolute magnitudes of the stress profiles by comparing the profiles with nearby ISIP data and limits on $S_{\text{hmax}}$ imposed by the frictional strength of the rocks. For the viscous relaxation model, the stress profiles generated were based on constant relative stress ratios ($\phi = 0.6$ and $\phi = 0.9$) implied from Fig. 12, but the stress magnitudes were not calibrated against any direct measurements of stress although a long-term geologic strain rate was assumed. Fig. 10 panels (d) and (e) shows that $S_{\text{hmax}}$ at 8500 ftdepth from both stress profiles lie within the constrained range from DITF presence and breakout absence. On the other hand, the $S_{\text{hmin}}$ profile for $\phi = 0.9$ clearly agrees better with the $S_{\text{hmin}}$ constrained from nearby ISIPs compared to the profile for $\phi = 0.6$. Thus the profile with $\phi = 0.9$ appears to be a better calibrated stress profile. However, $S_{\text{hmin}}$ approaches the pore pressure gradient at some depths when $\phi = 0.9$, especially within the Bench group and Forestburg limestone, which may not be realistic. In addition, for frictional coefficients between $\mu = 0.6$ and 0.8 (Table 1), assuming strike-slip faulting regime, ratios between the effective horizontal stresses are limited to

$$\frac{\sigma_H - \frac{S_{\text{hmax}} - P_p}{S_{\text{hmin}} - P_p}}{\sigma_h} \leq \left( \sqrt{\mu^2 + 1 + \mu} \right)^2 \approx 3.1 - 4.3 \quad (11)$$

We find that the stress profile with $\phi = 0.9$ frequently show effective horizontal stress ratio ($\sigma_H/\sigma_h$) that exceeds the upper limit defined in Eq. (11), while $\sigma_H/\sigma_h$ is within the upper limit throughout the data for $\phi = 0.6$. Thus, although the stress profile for $\phi = 0.9$ matches the nearby ISIP data, $\sigma_h$ needs to be lowered in order to satisfy the limits on effective stress magnitudes imposed by the frictional strength and $P_p$. After several iterations, we find that the profile generated with $\phi = 0.8$ produces a $S_{\text{hmin}}$ profile reasonably close to the ISIP data without violating the frictional limit on the effective stress ratio. This stress profile generated with $\phi = 0.8$ is used later for comparison with stress profile predicted from the extended Eaton model.

For the extended Eaton model, the stress profile generated using horizontal strain values $\varepsilon_h = 4.7e - 4$ and $\varepsilon_h = 0$ seem to agree well with both the ISIP data and the inferred range of $S_{\text{hmax}}$ in the Barnett (Fig. 13). Although these horizontal strain values were chosen to conveniently match the differential strain ($\varepsilon_{\text{diff}}$) in the viscous relaxation model, we note that these horizontal strain values may not be the unique combination of values that could satisfy the ISIP data and the constrained range of $S_{\text{hmax}}$ values. Therefore the choice is somewhat arbitrary. However, changes in the horizontal strain values do not affect the bedding-scale trend of the horizontal stress magnitudes which are more influenced by the elastic properties in the extended Eaton model.

6.2. Intra-reservoir stress variation and DITF occurrence

In this section, we revisit the observation of DITF occurrence to check if the stress profiles agree quantitatively with the frequent appearance and disappearance of DITFs. Using the stress profile obtained from the viscous relaxation model ($\phi = 0.8$) and extended Eaton models, we calculate the continuous profile of minimum circumferential stress, $\sigma_{\text{circ}}$, around the wellbore using the following equation (Brudy and Zoback, 1999) to assess the correspondence...
The predicted minimum circumferential stress for the viscous relaxation model such that DITFs disappear where $\sigma_{\theta\theta}^{\text{min}}$ is positive (compressive). On the other hand, the $\sigma_{\theta\theta}^{\text{min}}$ profile from the extended Eaton model does not show as much bed-to-bed variation in stress and the $\sigma_{\theta\theta}^{\text{min}}$ magnitude is always positive which should suggest no DITF occurrence. Thus, although the extended Eaton model is also calibrated against ISIP data and frictional limits, the model does not agree quantitatively with DITF occurrence away from the depth where the profile was calibrated.

Fig. 15 shows histograms prepared similarly to Fig. 5 with the horizontal axis replaced by the predicted minimum circumferential stress. For the viscous relaxation model, we see that the $\sigma_{\theta\theta}^{\text{min}}$ values are consistently positive, but the 2 histograms for DITF presence and absence have
the same shape and peak position. Thus, the extended Eaton model does not capture the depth dependent stress variation as in the viscous relaxation model.

6.3. Implications for vertical fracture propagation

Fig. 16 compares stress profiles obtained from the viscous relaxation model ($\phi=0.8$) and the extended Eaton model. Comparison between the $S_{\text{min}}$ profiles indicates that the 2 models predict markedly different fracture gradient profiles, and thus would predict very different fracture height growth. Both $S_{\text{min}}$ profiles show relatively small contrast in fracture gradient at the top boundary of the Barnett shale with the Marble Falls limestone. However, there is a pronounced difference in how the fracture gradient changes as the well enters into the Viola limestone. The extended Eaton model shows an increase in $S_{\text{min}}$, which would imply that the Viola limestone would act as a barrier for fracture propagation, whereas the viscous relaxation model shows the opposite trend implying that hydraulic fractures can propagate into the Viola limestone.

There are no direct stress measurements to validate these predictions, however it is known that hydraulic fracture propagation into the underlying limestone layer, either Viola or Ellenburger limestone depending on the location within the Fort Worth Basin, is a common problem encountered during stimulation in the Barnett shale. Fisher and Warpinski (2011) compiled depths of the shallowest and deepest microseismic events recorded during each hydraulic

Fig. 14. The same log section as Fig. 4 displayed together with the minimum circumferential stress ($\sigma_{\theta \theta}^{\text{min}}$) profiles predicted from the viscous relaxation and extended Eaton models.

Fig. 15. Histograms of minimum circumferential stress ($\sigma_{\theta \theta}^{\text{min}}$) for depths where DITF is present (black) and absent (gray): (a) prediction from viscous relaxation model and (b) prediction from an extended Eaton model.
fracturing operation in the Barnett shale to investigate how the hydraulic fractures propagate upward and downward from the injected depths. When the Barnett shale lies between 7700 and 8750 ft depth, their compilation indicates that the upward propagation of hydraulic fractures into the Marble Falls is rare but the downward propagation is quite common, oftentimes exceeding 1000 ft downward propagation. Therefore, Fig. 16 shows that the prediction of the viscous relaxation model is consistent with these field observations whereas the extended Eaton model is not.

6.4. Differences and limitations of the 2 models

Both models for stress prediction attempt to quantify the magnitude of horizontal stress caused by tectonic loading in a 1-dimensional layered model. However, the obvious advantage of the viscous relaxation model is that it takes into account realistic time-dependent rock behavior which was constrained through laboratory experiments using core samples. It is with this effect of viscous differential stress relaxation that we are able to predict a more isotropic stress state within clay-rich layers, thus explaining the DITF absence in the clay-rich layers as evident in Figs. 14 and 15. Thus it is more advantageous to employ the viscous relaxation model to capture the bed-to-bed stress variation in the case studied here.

In addition to the differences in rock constitutive behavior considered by the 2 models, the 2 $S_{hmin}$ profiles are also fundamentally different in how the horizontal stresses are related to the vertical stress. For the extended Eaton model, the isotropic horizontal stress arising from gravitational loading is first calculated from the vertical stress profile (gray data in Fig. 13d), and

![Diagram](image-url)
then the horizontal tectonic stresses are added. As a result, the fluctuation in $S_{\text{hmax}}$ for the extended Eaton model reflects the variation in horizontal tectonic stresses and the variation in the gravitational term $S_g$. While this is a somewhat direct approach which attempts to sum all sources of horizontal stresses, it is based on a model that does not capture realistic time-dependent rock behavior or geologic loading. Stresses in the earth do not arise due to instantaneous gravitational and tectonic stresses.

In the viscous relaxation model, the horizontal stress differences are superimposed on the vertical stress profile using constraints about where $S_e$ should lie relative to $S_{\text{hmax}}$ and $S_{\text{hmin}}$. The constraint is provided from the view that the kinematic faulting style is spatially homogeneous ($\phi=$ constant) within a continuous sedimentary section. Thus it does not aim to resolve all processes that lead to the current state, but rather aims to capture the outcome self-organized by the rock rheology and kinematic conditions which sometimes are better constrained than tracking every detail of the tectonic history. The current study shows that the viscous relaxation model appears to be more accurate for estimating subsurface stress states and its spatial variation.

However, despite the apparent advantage of the viscous relaxation model, further analysis of field and laboratory data in various settings is required for validation. As Fig. 15a shows, there is significant overlap between the two histograms for DITFs presence and absence, indicating the limited accuracy of the stress profile by the viscous relaxation model. It is still difficult to precisely quantify the error associated with the viscous relaxation model as there are many aspects of the model with significant uncertainty. For example, as seen in Eq.(5), the stress magnitudes predicted by the viscous relaxation model are sensitive to the choice of tectonic strain rate. While the value used here is reasonable from a tectonic point of view, one might consider this a free parameter that could be determined by fitting predicted values of $S_{\text{hmin}}$ to measured values. Also, constitutive relations used in the viscoelastic calculations were derived based on laboratory experiments conducted using room-dry samples at room temperature. Further experimental studies are needed in order to understand how the presence of pore fluids and elevated temperature affects the creep and stress relaxation behavior of these rocks. The physical mechanism responsible for the time-dependent deformation also needs to be understood and discussed carefully because such understanding is the only way to assure validity of extrapolating laboratory results to geological time scales.

7. Conclusion

From examination of geophysical and image log data from a vertical well in Barnett shale, we observed that the horizontal stress difference ($S_{\text{hmax}}-S_{\text{hmin}}$) along the well varies rapidly within the sedimentary unit in response to the variation in lithology. In light of laboratory creep experiments conducted on Barnett shale and other shale gas reservoir rocks which show a dependence of viscous properties on rock composition, we suggest that the variation in stress state is caused by the variation in viscous properties of the formation. To test this hypothesis, we first calculated the horizontal stress difference along the well that would arise from a profile of viscous rock properties and an estimate on the average horizontal tectonic deformation history in the Barnett shale based on viscoelastic theory. Then under the assumption that the faulting style, $\phi=(S_2-S_3)/(S_1-S_3)$, should be uniform within the sedimentary section, the absolute magnitudes of the 3 principal stresses were calculated along the well.

The stress profile obtained based on the above viscoelastic approach appears to be valid based on 2 lines of evidences. First, the profile of minimum circumferential stress around the well calculated based on the viscoelastic approach reasonably explains the frequent appearance and disappearance of DITFs. The tendency to form DITFs clearly increased as the predicted minimum circumferential stress became negative (tensoidal). Secondly, the obtained $S_{\text{hmin}}$ (fracture gradient) profile agrees with observations from microseismic studies that hydraulic fractures propagate downward into the limestone formations underlying the Barnett shale in this region. The viscous relaxation model predicts a decrease in fracture gradient as the well enters the underlying limestone formation which is a feature not captured by extended Eaton models. Our model provides a method for using laboratory observed time-dependent viscous properties for in-situ stress predictions. More study is needed to fully understand the limitation and validity of the model.

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Appendix A. Estimating anisotropic elastic constants from vertical well log

Both the viscous relaxation and extended Eaton models require anisotropic elastic constants to calculate stress profiles along the well. As Eqs. (2), (6), and (12) show, these are the vertical and horizontal Young’s modulus ($E_v, E_h$) and the vertical and horizontal Poisson’s ratios ($\nu_v, \nu_h$). However, $V_p$ and $V_s$ log data only provides 2 elastic constants that are not sufficient to determine all 4 elastic constants required for the analysis. Therefore some empirical relations and approximations were used to estimate the missing parameters.

We treat the shale as a vertical transversely isotropic (VTI) medium with the $x_3$-axis being the axis of symmetry, where the stiffness tensor in the Voigt notation is given by

$$\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 \\
\end{pmatrix} = \begin{pmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66} \\
\end{pmatrix} \begin{pmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_3 \\
\epsilon_4 \\
\epsilon_5 \\
\epsilon_6 \\
\end{pmatrix} , \quad c_{66} = (c_{11}-c_{12})/2 \quad (A1)
$$

The 4 elastic constants ($E_v, E_h, \nu_v, \nu_h$) required for the analyses can be expressed using the elements of the anisotropic stiffness tensor $c_{ij}$ as

$$E_v = c_{33} - \frac{c_{13}^2}{c_{11}-c_{66}} \quad (A2)$$

$$E_h = c_{11} + \frac{c_{13}^2(c_{11}-4c_{66})-c_{33}(c_{11}-2c_{66})^2}{c_{33}c_{11}-c_{13}} \quad (A3)$$

$$\nu_v(\nu_{31}) = \frac{c_{13}}{2(c_{11}-c_{66})} \quad (A4)$$

$$\nu_h(\nu_{12}) = \frac{c_{33}(c_{11}-2c_{66})-c_{13}^2}{c_{33}c_{11}-c_{13}} \quad (A5)$$
Therefore the stiffness constants required for the problem is $c_{11}$, $c_{22}$, $c_{66}$, and $c_{12}$. Each of this stiffness constant is determined following the procedures below.

The constant $c_{33}$ is the vertical P-wave modulus, so it is determined directly from the measured vertical P-wave velocity ($V_p$) and the density ($\rho$):

$$c_{33} = \rho V_p^2$$  \hspace{1cm} (A6)

The constant $c_{11}$ is then determined using an empirical exponential relation between the Thomsen anisotropy parameter $\varepsilon$ and vertical P-wave velocity fitted to the laboratory data from Sone and Zoback (2013a)

$$\varepsilon = 147 \exp(-1.62 V_p^5)$$  \hspace{1cm} (A7)

The constant $c_{66}$ can be determined using the same logic for the S-wave modulus. Again using an empirical exponential relation between anisotropy parameter $\gamma$ and vertical S-wave velocity again fitted to the laboratory data from Sone and Zoback (2013a)

$$c_{66} = \rho (V_s^5 - 2.48 V_s)$$  \hspace{1cm} (A8)

The constant $c_{13}$ is determined using Eq. (A2) assuming that we can approximate $c_{13}$ from the vertical P-wave and S-wave modulus. That is, using the expression for Young's modulus in an isotropic rock

$$c_{13} = \sqrt{(c_{33} - E_{\text{approx}})(c_{11} - c_{66})}$$  \hspace{1cm} (A9)

Using these equations, we obtained a continuous profile of the vertical and horizontal Young's moduli and Poisson's ratios from sonic log $V_p$ and $V_s$ data.

We finally note that we did not attempt to make any static to dynamic corrections. This is because Sone and Zoback (2013a) showed that there is practically no difference between them for the studied rocks when the unloading/reloading static modulus is compared with dynamic measurements. However, the average Young's modulus, $E$, in Eq. (3) uses the 1st-loading Young's which is on average about 15% lower than the unloading/reloading Young's modulus (Sone and Zoback, 2013a). Thus $A_1$ in Eq. (8) is divided by 0.85 to generate the trend in Fig. 9 in order to account for the lower value of the 1st-loading modulus.

References


Moos, D., Zoback, M.D., 1990. Utilization of observations related to wellbore failure to constrain the orientation and magnitude of crustal stresses: Application to continental, Deep Sea Drilling Project, and Ocean Drilling Program boreholes. J. Geophys. Res. 95 (B6), 9,305–9,325.


