Estimating sedimentary and crustal structure using wavefield continuation: theory, techniques and applications

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SUMMARY
Receiver function techniques are widely used in imaging crustal and mantle structure beneath a seismic station. The weak P-to-S conversions at deep seismic structures are usually masked by strong shallow reverberations when unconsolidated sediments are present below the station, making it nearly impossible to utilize receiver function techniques. We develop a method to estimate sediment and crustal structures beneath a seismic station based on wavefield downward continuation and decomposition method. The method parametrizes velocity structure beneath the station with a stack of constant velocity layers overlying a homogeneous half-space, and approximates the teleseismic P wave and its coda by the structural response to an incoming plane P wave. Our method is based on the principle that the upgoing S wavefield is absent in the half-space, and searches for the optimum velocity and thickness of the layers that give the minimum S-wave energy flux from the half-space to the layers. An iterative grid-search algorithm from the top to the bottom layers is employed to implement the search. In this study, we only use models comprising either only one crust layer or two layers (sediment + crust) with a half-space mantle, although models with more layers are also implementable. The method is especially useful in resolving seismic structure beneath a station sitting on unconsolidated sediments. It not only can be used to determine the sediment thickness and velocity structure, but also provides an effective way to generate subsurface receiver functions, which are formed by deconvolving the upgoing P wavefield from the upgoing S waves at the top of hardrock crust, and thus are free from shallow reverberations. The technique is applied to various synthetic data generated with different types of velocity model and noise levels, and appears to have good capability in recovering the input models. We further applied this method to teleseismic data recorded at a station inside the Songliao Basin in northeast China. The estimated sediment thickness and velocity agrees well with the results of previous active-source studies. The subsurface receiver functions also show a superior power in exposing the Moho Ps conversions, resulting in a well-defined peak in the H-κ domain, which are absent in the regular receiver function data.

Key words: Body waves; Site effects; Computational seismology; Wave propagation.

1 INTRODUCTION
The shallow structure beneath a seismic station can have significant effects on its recorded waveform data. The structure acts like a filter that modifies all seismic signals going through it. For example, an unconsolidated sedimentary layer can generate strong reverberation waves due to the large velocity contrast between the unconsolidated sediments and the underlying hard rocks. At teleseismic distances, the reverberations recorded in the horizontal components can be as large as the direct P wave (Fig. 1a). These strong reverberations are characterized by narrowband frequency contents (Fig. 1c) and mask other relatively weak later arrivals, such as P-to-S converted phases at boundaries beneath the sediments, making it difficult to decipher the deep structure with techniques such as receiver function analysis that relies on these weak arrivals.

The receiver function technique is perhaps the most widely used method in passive seismology to image the crust and estimate Moho depth beneath a broad-band seismic station. In a layered medium, a teleseismic record can be considered as the summation of the direct
arrival and a series of conversions and reflections at boundaries below the station. Receiver functions are an attempt to approximate a Green’s function associated with structure beneath the receiver by deconvolving one component of a teleseismic signal from another to remove source signals from seismograms (Ammon 1991). For instance, deconvolving the vertical (or longitudinal) component from the radial (or in-plane transverse) component helps to isolate the P-to-S converted waves, generating a regular receiver function, known as the Ps receiver function. The narrowband reverberations from the base of unconsolidated sediments modify the spectrum of the radial component (Fig. 1c). The resulting receiver functions are usually dominated by the sedimentary multiples (Fig. 1a). This has been noted by many receiver function studies (e.g. Owens & Crosson 1988; Sheehan et al. 1995). For comparison, we show the vertical and radial components of the same event recorded at a station on bedrock in Fig. 1(b). We also plot the receiver function formed from the two components, which shows a clear Moho Ps conversion due to lack of sedimentary reverberations.

The strong sediment reverberations pose significant challenges to receiver function based techniques, such as the H-κ analysis (e.g. Zhu & Kanamori 2000; Niu et al. 2007) and the common conversion point (CCP) stacking method (e.g. Dueker & Sheehan 1997; Gilbert et al. 2003; Niu et al. 2005). The H-κ analysis is a grid-search based technique, which seeks the optimum crustal thickness (H) and Vp/Vs ratio (κ) that best explains the arrival time of the P-to-S conversion and two multiple reflection phases at the Moho. It is usually difficult to obtain a well-defined peak in the (H, κ) domain when strong shallow reverberations are present. The long-lasting reverberation sequence can also interfere with P-to-S conversions from deep boundaries within the mantle, such as the lithosphere–asthenosphere boundary (LAB) and the 410-km seismic discontinuity, creating artificial events at upper-mantle depths in the CCP stacked images. Removing the sediment multiples is thus critical for imaging crustal and upper-mantle structure beneath a receiver.

The strong near-surface multiples can be roughly modelled by a simple 1-D model, which consists of constant velocity layers with a very low velocity layer at the top, a crust layer in the middle and a half-space mantle in the bottom (Fig. 1d). The synthetics of this model (Fig. 1e) computed using the Thomson–Haskell method (Haskell 1962) exhibit the major features of the multiples shown in Fig. 1(a). Langston (2011) proposed a waveform fitting method to
estimate structure of the unconsolidated sediments beneath the MPH station located inside the coastal plain of the central United States. To do so, he first generated vertical and radial transfer functions by deconvolving the $P$-wave records of bedrock stations outside the coastal plain. He then modelled these transfer functions using a simple model parametrization of sediment structure with a fixed thickness estimated from other studies. Once the sediment structure is known, he showed that the surface records can be backpropagated to a depth beneath the sediments, and the total teleseismic response of the $P$ wave can be further decomposed into upgoing and downgoing $P$ and $S$ waves. The resulting upgoing Swavefield is free of sediment reverberations, and is comprised mainly of deep $P$-to-$S$ conversions. The method, however, requires a reference bedrock station nearby the sediment station to make the transfer functions.

In this paper, we propose an alternative technique to invert sediment and crustal structure using the same theory of wavefield continuation and decomposition. We first propagate the surface records downwards to the uppermost mantle, where the wavefield is decomposed into upgoing and downgoing $P$ and $S$ waves. The wavefield inside the uppermost mantle consists of the incident $P$ wave and the reflected downgoing $P$ and $S$ waves. We thus develop an inversion method that iteratively searches for sediment and crustal velocity structure that minimize the energy of the upgoing $S$ wave in the uppermost mantle. Our method does not rely on a reference station, which could be difficult to find in certain circumstance, and it makes no assumption on the thickness of the unconsolidated sediments. In the following sections, we will first review the theory of wavefield continuation and decomposition, and then describe our proposed method for estimating sediment and crustal structure. We also show test results with synthetic data in order to demonstrate the effectiveness as well as the limitations of the proposed method. Finally, we apply this method to one broad-band station in the Songliao Basin in northeast China. With the estimated sediment structure, we downward continue the surface wavefield to the base of the sediments and recompute receiver functions by deconvolving the upgoing $P$ wave from the upgoing $S$ wavefield. The subsurface receiver functions are free of sediment reverberations and show clear $P$ to $S$ Moho conversions, and thus can be used in the regular receiver function imaging studies.

2 METHOD

2.1 Wavefield downward continuation and decomposition

The $P$ waveforms at teleseismic distances of $30^\circ$–$90^\circ$ are known to be relatively simple due to lack of triplications and diffractions associated with the upper-mantle discontinuities and the core–mantle boundary, respectively. Also the coda wavefield of the direct $P$ wave is composed mainly of $P$-to-$S$ converted waves and multiples at nearly horizontal boundaries beneath the receiver, as shown by many receiver function studies and the above synthetic example. Thus, the direct $P$ wave and its coda at teleseismic distances can be approximated by the response of a 1-D velocity structure to an incoming plane $P$ wave with a relatively small incident angle. However, when structure beneath the station is complicated, the above assumption for a plane-layered model might not be valid. In general, wavefield motions in a medium consisting of isotropic homogeneous layers can be decomposed into vertical, radial and transverse components. The first two and the third components can be modelled by decoupled $P$-SV and SH systems, respectively. Thomson (1950) and Haskell (1953) represented a vertically heterogeneous medium by a stack of homogeneous layers overlying a homogeneous half-space, and developed the well-known propagator matrix method. Below, we briefly review the method and show how we use it to search for sediment and crustal structure, as well as to remove sediment reverberations from receiver function data.

Let $v_s$ and $v_p$ represent the velocity in the radial (positive in the wave propagation direction) and vertical directions (positive downward), respectively, $\tau_{rr}$ and $\tau_{zz}$ are the shear and normal stresses on a horizontal surface. These velocity and stress form a 4-D vector, known as the motion-stress vector:

$$\mathbf{f} = (v_p, v_s, \tau_{rr}, \tau_{zz})^T.$$  \hspace{1cm} (1)

Here, $T$ denotes a transpose. In the frequency ($\omega$) and ray parameter ($\rho$) domain, the motion-stress vector satisfies the following vector differential equation (Gilbert & Backus 1966):

$$\frac{df}{d\xi} = -i\omega A f,$$  \hspace{1cm} (2)

where $A$ is a $4 \times 4$ matrix,

$$A = \begin{pmatrix} 0 & p & 1/\mu & 0 \\ p\lambda/(\lambda + 2\mu) & 0 & 0 & 1/(\lambda + 2\mu) \\ \rho - p^2\gamma & 0 & 0 & p\lambda/(\lambda + 2\mu) \\ 0 & \rho & p & 0 \end{pmatrix}.$$  \hspace{1cm} (3)

Here, $\lambda$ is the Lamé constant, $\mu$ is the shear modulus and $\gamma = 4\mu(\lambda + \mu)/\lambda + 2\mu$.

The matrix $A$ can be represented by the eigenvector decomposition

$$A = M \begin{pmatrix} -q_p & 0 & 0 & 0 \\ 0 & q_p & 0 & 0 \\ 0 & 0 & -q_p & 0 \\ 0 & 0 & 0 & q_p \end{pmatrix} M^{-1},$$  \hspace{1cm} (4)

where $M$ is a matrix consisting of the four eigenvectors corresponding to the motion and stress of the downgoing $P$, upgoing $P$, downgoing $S$ and upgoing $S$ waves

$$M = \begin{pmatrix} \alpha p & \alpha p & \beta q_p & \beta q_p \\ \alpha q_p & -\alpha q_p & -\beta p & \beta p \\ -2\alpha \mu q_p & 2\alpha \mu q_p & -\beta \mu \eta & \beta \mu \eta \\ -\alpha \mu \eta & -\alpha \mu \eta & 2\beta \mu q_p & 2\beta \mu q_p \end{pmatrix}.$$  \hspace{1cm} (5)

Here, $q_p = \sqrt{\alpha^2 - \beta^2}$ is the vertical slowness of $P$ wave, $q_s = \sqrt{\beta^2 - \rho^2}$ is the vertical slowness of $S$ wave, and $\eta = \beta^2 - 2\rho^2$.

The inverse matrix of $M$ can be written as

$$M^{-1} = \frac{1}{\rho} \begin{pmatrix} \mu p/\alpha & \eta \mu/2\alpha q_p & -p/\alpha q_p & -1/2\alpha \\ \mu p/\alpha & -\eta \mu/2\alpha q_p & p/2\alpha q_p & -1/2\alpha \\ \eta \mu/2\beta q_p & -\mu p/\beta & -1/2\beta & p/2\beta q_p \\ \eta \mu/2\beta q_p & \mu p/\beta & 1/2\beta & p/2\beta q_p \end{pmatrix},$$  \hspace{1cm} (6)

where $\rho$ is the density.

Based on Haskell (1953), in an homogeneous layer, the solution of the vector differential eq. (2) at any depth, $z$, can be computed
from the motion-stress vector at a reference depth, z₀, with a matrix multiplication
\[ f(z) = P(z, z₀)f(z₀). \]  
(7)

Here, \( P(z, z₀) \) is a 4 × 4 matrix, known as the propagator matrix, and can be represented by the eigenvector matrix \( M \) and its inverse \( M^{-1} \)
\[ P = M \begin{pmatrix} e^{i\omega z (z - z₀)} & 0 & 0 & 0 \\ 0 & e^{-i\omega z (z - z₀)} & 0 & 0 \\ 0 & 0 & e^{i\omega z (z - z₀)} & 0 \\ 0 & 0 & 0 & e^{-i\omega z (z - z₀)} \end{pmatrix} M^{-1}. \]
(8)

Since there is no stress on Earth’s surface, the motion-stress vector at Earth’s surface can be written as
\[ f₀ = (v₀, v₀, 0, 0)^T. \]
(9)

The motion-stress vector is also continuous at the boundaries between different layers, thus the motion and stress at any depth, \( z \), can be computed from surface records of the radial \((v₀)\) and vertical \((v₀)\) components, \( f(z) = P(z, z₀)(v₀, v₀, 0, 0)^T \), given that the structure above this depth is known. This procedure is known as wavefield downward continuation.

The elastic wavefield at each layer can be divided into four types of waves, upgoing \( P \), downgoing \( P \), upgoing \( S \) and downgoing \( S \) wave, respectively. As mentioned above, the motion-stress vector can actually be considered as the summation of the motion-stress eigenvectors corresponding to the four types of waves. Let the numbers \((P₁, P₁, S₁, S₁)\) denote how much of each of the four possible wave types is present in each layer, and then the motion-stress vector can be written as
\[ f = \begin{pmatrix} v_r \\ v_z \\ τ_{rz} \\ τ_{zz} \end{pmatrix} = M \begin{pmatrix} P₁ \\ P₁ \\ S₁ \\ S₁ \end{pmatrix} = Mw. \]
(10)

Here, \( w = (P₁, P₁, S₁, S₁)^T \) is referred to as the wavefield vector. The transformation (10) is known as wavefield decomposition (Kennet et al. 1978).

### 2.2 Minimization of upgoing S-wave energy flux and H–β search

As mentioned earlier, the teleseismic \( P \)-wavefield can be approximated by a pure plane \( P \) wave impinging at the bottom of a vertically heterogeneous structure beneath the receiver. The 1-D velocity structure can be parametrized by a stack of homogeneous layers overlaying a homogeneous half-space, and the wavefield can be described by the propagator matrix method. Based on eqs (7), (9) and (10), the wavefield at the half-space can be computed from the surface records \( v₀, v₀ \) and \( v₀, v₀ \)
\[ \begin{pmatrix} P₁ \\ P₁ \\ S₁ \\ S₁ \end{pmatrix} = M^{-1} P(z, 0) \begin{pmatrix} v₀ \\ v₀ \\ 0 \\ 0 \end{pmatrix}. \]
(11)

Here, \( P(z, 0) \) is the propagator matrix from surface to the half-space and is the product of the propagator matrix of each layer. We should emphasize here that within the half-space there is no upgoing \( S \) wave, so \( S₁ = 0 \). To illustrate how the upgoing \( P \) and \( S \) waves vary within different layers and the half-space, we show an example in Fig. 2. The model we used consists of two constant velocity layers (sediment and crust), and a half-space (mantle). The upgoing \( P \) and \( S \) waves inside the sediment, crust and mantle. Note the progressive complication of the two wavefields with upward propagation. The inverted triangle, square and triangle in (a) show the positions of the presumable wavefield records shown in (b).

Figure 2. (a) Schematic ray paths of the upgoing and downgoing \( P \) (solid lines) and \( S \) (dashed lines) waves inside the sediment, crust and mantle in response to an incoming \( P \) wave at teleseismic distance. Note there is no upgoing \( S \) wave inside the half-space mantle. (b) A comparison of the upgoing \( P \) (solid line) and \( S \) (dashed line) wavefields inside the sediment, crust and mantle. Note the progressive complication of the two wavefields with upward propagation. The inverted triangle, square and triangle in (a) show the positions of the presumable wavefield records shown in (b).

The energy flux associated with elastic wave propagation across a horizontal plane is defined as \( J_z = -(τ_{rz}v_r + τ_{zz}v_z) \) (downward positive). The energy flux of the up and downgoing \( P \) and \( S \) waves can be calculated using their motion-stress vector, which can be written as
\[ J^P_{z↓↑} = ±ρα²q_w |P₁|², \quad J^S_{z↓↑} = ±ρβ²q_p |S₁|². \]
(12)
The total energy flowing through a certain horizontal plane within a time window \([t_0, t_1]\) is the time integral of the energy flux

\[ E = \int_{t_0}^{t_1} J_c(t) \, dt. \]  

(13)

For the inverse problem of using surface records to estimate sediment and crustal structure, the absence of upgoing \(S\) wave in the mantle can actually be used as a constraint. We develop a grid-search based inverse method to estimate the optimum structure of the sediment and crust that minimizes the upgoing \(S\)-wave energy. We first assume that the \(P\)-wave velocity of the sediment and crust is known from independent observations, and then search the thickness \((H)\) and \(S\)-wave velocity \((\beta)\) of the sediment and crust separately in an iterative way. We refer this technique as to \(H-\beta\) search to mimic the \(H-\kappa\) stacking method.

3 SYNTHETIC TESTS

To illustrate the effectiveness and robustness of the method, we conducted tests with synthetic data generated from three velocity models. Details of the three models are listed in Table 1. The first model is a simple one-layer model, with a homogeneous crust on the top of a half-space mantle, which we refer to as the CM model. The second model (SCM) is a two-layer model with a constant velocity sediment and crust on the top of the half-space mantle. The third model has multiple layers of sediments with increasing velocity on top of the homogenous crust, which is referred as to the LSCM model. For each model, we generate vertical and radial seismograms at six different ray parameters, ranging from 0.050 to 0.075 km s\(^{-1}\) at a step of 0.005 km s\(^{-1}\), using the above Thomson–Haskell propagator matrix method. For each ray parameter, we generate two sets of synthetics with 1 and 15 per cent noise. A Gaussian function \((\exp(-4\pi t^2))\) is taken as the source time function. The seismograms are aligned at the peak of the direct \(P\) arrival, which is shifted to time zero.

The \(H-\beta\) search conducted in this section, we assume that the \(P\)-wave velocity and density are known, which are \(\alpha_s = 2.1\) km s\(^{-1}\) and \(\rho_s = 1.97\) g cm\(^{-3}\) for the sediment, and \(\alpha_c = 6.4\) km s\(^{-1}\) and \(\rho_c = 2.7\) g cm\(^{-3}\) for the crust. We also assume that mantle velocity and density structure is given \((\alpha_m = 8.0\) km s\(^{-1}\), \(\beta_m = 4.5\) km s\(^{-1}\) and \(\rho_m = 3.3\) g cm\(^{-3}\)). We only search layer thickness and \(S\)-wave velocity of the sediment and crust to minimize upgoing \(S\)-wave energy from the half-space mantle to the crust. As shown in Fig. 2, the incident \(P\) wave is propagated back to \(-5\) s, thus we take \([-10, 15\) s] as the time window to compute the energy flux of the upgoing \(S\) wave. As discussed later, we find that the assumed \(P\)-wave velocity and density values, as well as the selection in time-window length, have little effects on the search results.

3.1 The crust–mantle model

We search the optimum crustal thickness \((H)\) in the range of 30–40 km and shear velocity \((\beta)\) between 3.0 and 4.5 km s\(^{-1}\), with steps of 0.1 km and 0.01 km s\(^{-1}\) for \(H\) and \(\beta\), respectively. The results from the two sets of synthetics with 1 and 15 per cent noise are shown in Figs 3(a) and (b), respectively. The estimated \((H, \beta)\) from the data with high signal-to-noise ratio (SNR) is \((35.0\) km, 3.65 km s\(^{-1}\)), which matches perfectly with the input model. The estimated thickness and velocity from the data with 15 per cent noise are 34.9 km and 3.63 km s\(^{-1}\), respectively, which also show a very good agreement of the input values. Since the model has only one layer that needs to conduct the \(H-\beta\) search, no iteration is necessary here.

3.2 The sediment–crust–mantle model

Since both the sediment and crust thickness and velocity need to be estimated, we divide the \(H-\beta\) search into two steps. We first search the thickness and shear velocity of the sediment by assuming a crust model, which is set to \(H = 30\) km and \(\beta = 3.5\) km s\(^{-1}\), both values are different from those of the input model \((35.0\) km, 3.65 km s\(^{-1}\)). The search range is 0.5–1.5 km and 0.3–1.3 km s\(^{-1}\) for \(H\) and \(\beta\), respectively, and the search steps for thickness and velocity are 0.01 km and 0.01 km s\(^{-1}\). Once the sediment structure is estimated, we move the grid search to the crust in order to seek the optimum crustal thickness and average \(S\)-wave velocity. The search ranges and steps for the crust are similar to the crust–mantle case in the last section. The results of the first round of search with the high SNR data are shown in Figs 4(a) and (b), respectively. The estimated sediment thickness is 0.92 km, which is about 2.2 per cent different from the input value, 0.9 km. The measured shear velocity of the sediment is 0.78 km s\(^{-1}\), similar to the input value. The \((H, \beta)\) estimate of the crust is \((35.1\) km, 3.67 km s\(^{-1}\)), which are only (0.3 per cent, 0.5 per cent) different from the input value \((35.0\) km, 3.65 km s\(^{-1}\)).

With the improved crustal model, we start a new search for a better sediment model, and then use the new sediment model to further update the estimates of crustal thickness and velocity. This procedure is repeated until the measurements of \(H\) and \(\beta\) become stable, that is, remain unchanged. Usually, we only need two iterations to reach the stable measurements. Figs 4(c) and (d) show the results of the second iteration with the 1 per cent noise data. The estimated thickness and \(S\)-wave speed match the input values of the sediment and the crust.

For comparison, in Figs 4(e)–(h) we show the results of the first and second iterations using data containing high-level of noise, which have an average SNR of \(~8\), comparable to the data we used in the next section. The estimated \((H, \beta)\) is \((0.91\) km, 0.79 km s\(^{-1}\)) and \((35.2\) km, 3.67 km s\(^{-1}\)) for the sediment and crust, respectively, which are no more than 1 per cent different from those of the input model.

3.3 The linear sediment–crust–mantle model

We further compute synthetics using a non-uniform sediment model, in which the \(P\) and \(S\)-wave velocity increases linearly with depth, from \(\alpha_s = 1.56\) km s\(^{-1}\) and \(\beta_s = 0.58\) km s\(^{-1}\) at Earth’s surface to \(\alpha_s = 2.76\) km s\(^{-1}\) and \(\beta_s = 1.03\) km s\(^{-1}\) at the bottom of the sediment layer located at 0.9 km deep. The \(V_p/V_s\) ratio is kept nearly as a constant 2.69 and the density is set to 1.97 g cm\(^{-3}\) within the sediment layer. The model parameters are listed in Table 1. We first generate synthetics with this model and use the \(H-\beta\) method to search for an equivalent homogeneous sediment layer that minimizes the upgoing \(S\)-wave energy flux from the half-space mantle to the crust. We employ the same procedure, the same parameters

Table 1. Models used in synthetic tests.

<table>
<thead>
<tr>
<th>Model</th>
<th>(H) (km)</th>
<th>(\alpha) (km s(^{-1}))</th>
<th>(\beta) (km s(^{-1}))</th>
<th>(\rho) (g cm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CM Sediment</td>
<td>35.0/∞</td>
<td>6.40/8.00</td>
<td>3.65/4.50</td>
<td>2.70/3.30</td>
</tr>
<tr>
<td>2. SCM Sediment</td>
<td>0.9</td>
<td>2.10</td>
<td>0.78</td>
<td>1.97</td>
</tr>
<tr>
<td>3. LSCM Sediment</td>
<td>0.9</td>
<td>1.56+1.345z^2</td>
<td>0.58+0.5z^2</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>35.0/∞</td>
<td>6.40/8.00</td>
<td>3.65/4.50</td>
<td>2.70/3.30</td>
</tr>
</tbody>
</table>

\(z\) is the depth in kilometres.
except for an average $P$-wave velocity of $\alpha_s = 2.36$ km s$^{-1}$ to iteratively search for the thickness and $S$-wave velocity of the sediments and crust.

We select the synthetic data generated with 15 per cent random noise, and the $H$--$\beta$ search converges at the second iteration. The results are shown in Fig. 5. The estimated sediment thickness is 0.90 km, which is similar to the input value (Fig. 5a). The measured $S$-wave velocity of the sediment is 0.79 km s$^{-1}$ (Fig. 5a), which is close to the average velocity, 0.81 km s$^{-1}$ (Fig. 5b). In Fig. 5(c), we show the $(H, \beta)$ estimate of the crust, which is (35.2 km, 3.68 km s$^{-1}$), and is less than (0.6 per cent, 0.9 per cent) different from the input value (35.0 km, 3.65 km s$^{-1}$).

### 4 DATA EXAMPLES

We apply the inversion method to a station (NE68) located inside the Songliao Basin in northeast China. The station was deployed as part of an international collaboration among United States, Japan and China, and it was operational from 2009 September to 2011 August. The station is located inside the Changling graben-depression, also known as the Changling sag, in the central western part of the basin, where sediment thickness can reach as much as 9 km. The unconsolidated Quaternary and Neogene sediments are composed primarily of soil, sand and gravel with a maximum thickness of $\sim$510 m (Feng et al. 2010). From the 2-yr continuous records of the station, we collect waveform data of 305 teleseismic events with a
magnitude $\geq 5.5$. To reduce the uncertainties in the search, we only select seismograms with an SNR $\geq 5$. Here, the SNR is defined as the square root of the power ratio between the $P$-wave signal and the background noise on the vertical component. Only 112 teleseismic events are finally chosen for the inversion. As shown in Fig. 1(a), the radial component of all the events shows strong sediment reverberations that mask weak $P$ waves conversions from deep boundaries. We use the maximum $P$-wave amplitude of the vertical component to normalize the seismograms of each event before the search.

In order to compute the upgoing $S$ wave within the mantle, we assume that the $P$- and $S$-wave velocities of the half-space mantle are 8.0 and 4.5 km s$^{-1}$, respectively, with a density of 3.3 g cm$^{-3}$. We set the $P$-wave velocity and density to be (2.1 km s$^{-1}$, 1.97 g cm$^{-3}$) for the sediment and (6.4 km s$^{-1}$, 2.7 g cm$^{-3}$) for the crust. The initial $S$-wave velocity and thickness of the crust are set to be 3.5 km s$^{-1}$ and 30 km, respectively, when we start to search the $(H, \beta)$ of the sediment. We use the similar search steps employed in the synthetic tests, which are 0.01 km and 0.01 km s$^{-1}$ in the case of the sediment, and 0.1 km and 0.01 km s$^{-1}$ in the case of the crust. Only two iterations are needed before we reach stable measurements of $(H, \beta)$.

The results of the first and second iterations are shown in the top and bottom rows of Fig. 6. The final estimate of the sediment thickness is 0.31 km with an $S$-wave velocity of 0.51 km s$^{-1}$. The measured crustal thickness and average $S$-wave velocity are 34.8 km and 3.70 km s$^{-1}$, respectively. With the assumed $P$-wave velocities, the average $Vp/Vs$ ratio, $\psi$, is 4.118 for the sediment and 1.730 for the crust. The estimated thickness of the unconsolidated sediment agrees well with results from active source data (Feng et al. 2010).

5 SUBSURFACE RECEIVER FUNCTIONS

As described in Section 2, the wavefield-downward-continuation technique allows us to compute the wavefield vector at any depth beneath the receiver. One application of the wavefield downward continuation is to strip off the unconsolidated sediment layer and form pseudo-receiver functions at the top of the crust, which are referred as to subsurface receiver functions hereafter. In principle, this can be done by deconvolving the upgoing $P$ wave from upgoing $S$ waves at the top of the crust. In Fig. 7, we show a comparison of the ray path geometry of the teleseismic wavefield in response to an incoming plane $P$ wave inside the SCM and CM models that are used in the synthetic tests in Section 3 (Table 1).

The top four traces in the bottom of Fig. 7 are the presumed records of the upgoing $P$, downgoing $P$, upgoing $S$ and downgoing $S$ wavefield at the top of the crust. The first arrivals on the downgoing $P$ and $S$ components are the $PP$ and $PS$ reflections of the upgoing direct $P$ wave at the top of the crust. These first arrivals on the SCM synthetics are much more complicated than those on the CM data. More specifically, the $PP$ and $PS$ reflections of the SCM data appear to be followed by the first-order multiples of the sediment, which are absent in the CM synthetics. The two later arrivals on the upgoing $S$ wave records are the two crustal multiples, $2p1s$ and $2p2s$ (Niu & James 2002). They also appear to be succeeded by the first-order sediment multiples on the SCM synthetics. The wiggles after these main arrivals are the high-order sediments multiples. The $Pn$ conversion at the Moho, however, is isolated from shallow reverberations, as its waveform on the records of the two models appears to be very similar.

The SCM subsurface and the CM surface receiver functions are shown at the bottom of Figs 7(c) and (d), respectively. Both receiver
functions possess a clear and simple $P_s$ conversion, suggesting that it is appropriate to constrain crustal structure with subsurface receiver functions using various seismic imaging techniques, such as CCP stacking and Kirchhoff migration (e.g. Wilson & Aster 2005). The major difference between the SCM and CM synthetic receiver functions lies in the two crustal reverberations. The SCM synthetic receiver function shows clear sediment reverberations, which are absent in the CM synthetic data. The shallow reverberations are, however, always preceded by the crustal multiples, thus they are not expected to affect the regular $H-\kappa$ analysis (Zhu & Kanamori 2000). Like the crustal multiples, their presence does affect the mapping of deep structures in the upper mantle. However, in principle, they can be eliminated if the subsurface receiver functions are formed at the top of the mantle, which is beyond the scope of this paper but will be pursued in future studies.

In order to demonstrate the improvement of subsurface receiver functions, we first compute six synthetic records at ray parameters varying from 0.050 to 0.075 km s$^{-1}$ at a step of 0.005 km s$^{-1}$ using the SCM and LSCM models. The surface receiver functions formed with these synthetics are shown in Figs 8(a) and (e), respectively. We then propagate the surface synthetic records to the top of the crust using the corresponding sediment structure estimated in the Section 3, and compute the subsurface receiver functions by deconvolving the upgoing $P$ wave from the upgoing $S$ wave (Figs 8c and g). In particular, we use the equivalent uniform-sediment layer model in backpropagating the LSCM synthetics to the top of the crust. In general, the $(H, \kappa)$ values estimated from the regular SCM (37.5 km, 1.84) and LSCM (38.0 km, 1.80) receiver functions appear to be greater than the true values (35.0 km, 1.75), while the subsurface receiver functions resulting from the SCM and LSCM synthetics provide $(H, \kappa)$ estimates closer to the true value (35.0 km, 1.75), suggesting that the subsurface receiver functions are better than the regular ones in estimating Moho and crustal structure when unconsolidated sediments are present beneath the receiver.

The improvement of subsurface receiver functions is further demonstrated from the field data of the NE68 station. In Fig. 9, we show a comparison of the regular and subsurface receiver functions, as well as their $H-\kappa$ stacking results. Large monochromatic oscillations dominate the first $\sim 10$ s of the regular receiver functions (Fig. 9a), making it impossible to separate the weak Moho $P_s$ conversion from the oscillations. Multiple peaks with unrealistic crustal thickness and $V_p/V_s$ values are present in the $(H, \kappa)$ domain (Fig. 9b). These oscillations are effectively eliminated on the subsurface receiver functions, such that the Moho $P_s$ conversion is well exposed (Fig. 9c). The corresponding $H-\kappa$ stacking also yields a reasonably well-defined peak in the $(H, \kappa)$ domain. The estimated crustal thickness and $V_p/V_s$ ratio are 35.0 km and 1.730 (Fig. 9d),
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Figure 6. The $H-\beta$ search results of the station, NE68, located at the southwestern part of the Songliao Basin. The first and second iteration results are shown in the top and bottom rows of the figure, with left and right sides indicating measurements of the sediments and crust, respectively.

respectively, which are close to the values of (34.8 km, 1.739) from the $H-\beta$ search shown in Fig. 6.

6 DISCUSSION

The proposed technique here approximates the mantle with a homogeneous half-space and utilizes the principle that the upgoing $S$ wavefield is absent in the half-space. There are known velocity discontinuities, such as the LAB, the 410- and 660-km discontinuities in the mantle, which can generate upgoing $S$ waves from the incoming $P$ waves. The $Ps$ conversions at the 410- and 660-km discontinuities arrive at approximately 43 and 67 s after the direct $P$ wave, which are located outside the time window $[-10\ s, 20\ s]$ used in the $H-\beta$ search, and thus are expected to have no influence on the search results. The $Ps$ conversion at the LAB, on the other hand, roughly falls into our targeted time window, thus our assumption that there is no upgoing $S$ wavefield in the uppermost mantle is no longer valid when such $Ps$ conversions are present in the data. We thus investigate whether and how the $Ps$ conversion at the LAB affects the $H-\beta$ search results.

We take the CM model and create a series of models by dividing the mantle into a high-velocity mantle lithosphere with varying thickness and an underlying half-space asthenosphere with a velocity 10 per cent lower than that of the lithosphere. For each model, we compute synthetics and apply the $H-\beta$ technique to estimate the Moho depth and $S$-wave velocity. The estimated Moho depth and $S$-wave velocity are, respectively, shown in Figs 10(a) and (b) as a function of the LAB depth. In most cases, the $H-\beta$ search finds the correct model, but we also observe some deviations between the $H-\beta$ estimates (yellow solid line) and the input model values (white dashed line) in certain LAB depth ranges. The deviations are, however, less than 0.3 per cent, and are likely caused by an interference between the $Ps$ conversion and the crustal multiples, $2p1s$ and $1p2s$.

As shown in Figs 10(c)–(h), when the LAB is located out of these depth ranges, the $Ps$ conversion at the LAB arrives either before $2p1s$ or after $1p2s$. As such it does not affect the wavefield downward continuation and the decomposition of these two multiples. The reason is that the effectiveness of the $H-\beta$ analysis relies on the cancellation of the upgoing $S$ waves associated with three phase pairs: (1) the direct $P$ and Moho $Ps$ conversion; (2) the surface reflection $PP$ and $2p1s$ and (3) the surface reflection $PS$ and $1p2s$ (Fig. 10). At the right Moho depth (Fig. 10c) and $S$-wave velocity (Fig. 10d), the two associated upgoing $S$ waves in each pair possess the same amplitude but opposite polarity, leading to a complete cancellation of each pair (the red lines in Figs 10c and d). The only component left in the upgoing $S$ wavefield is the $Ps$ conversion at the LAB. On the other hand, if the crustal model employed in the
wavefield downward continuation and decomposition is incorrect, then the two wave components in each pair cannot completely cancel with each other. The three incompletely cancelled upgoing S-wave pairs have non-zero energy, $E_{\text{pairs}}$, which appears to be larger than that of the LAB conversion, $E_{\text{lab}}$. The total energy of the upgoing S wavefield, $E_{\text{total}} = E_{\text{pairs}} + E_{\text{lab}}$, thus has a minimum at $E_{\text{pairs}} = 0$ which occurs if the true crustal model is used in the downward propagation of the wavefield. The above scenario occurs only when the LAB Ps conversion is isolated from the two crustal multiples. If the LAB Ps conversion interferes with the two crustal multiples, then the minimum can occur at a Moho depth and S-wave velocity (the blue lines in Figs 10e–h) slightly different from the input model values (the red lines in Figs 10e–h). This means that the total energy $E_{\text{total}}$ can be smaller than $E_{\text{lab}}$ due to the interference. However, since the interference only happens near the true crustal model, and $E_{\text{pairs}}$ is generally larger than $E_{\text{lab}}$, therefore we expect that the minimum of $E_{\text{total}}$ still occurs near the true crustal model.

In the $H$–$\beta$ analysis, we also assume that the P-wave velocity of the sediment and crust can be obtained from independent data, such as from active source data. In some cases, such a priori knowledge of P-wave velocity may not be available, thus it is important to know how the assumed P-wave velocity affects the $H$–$\beta$ search results. In the case of the CM model, we vary the P-wave velocity from −5 to 5 per cent (i.e. 6.08–6.72 km s$^{-1}$) with a step of 1 per cent. For each assumed P-wave velocity, we run the $H$–$\beta$ search to estimate the crustal thickness and shear velocity. We further compute the deviations of the estimates of the two parameters from those measured with the correct P-wave velocity. The estimated $H$ and $\beta$ show a positive correlation with the reference P-wave velocity used in the $H$–$\beta$ search (Figs 3c and d). There is some scattering in the results computed from the data with 15 per cent noise in Fig. 3(d), which is likely caused by noise as it is less prominent in the low noise data shown in Fig. 3(c). If we assume that the uncertainty in the reference P-wave velocity is less than 5 per cent, then the corresponding errors in the estimated crustal thickness and S-wave velocity are also expected to be $\leq$5 per cent. Because of the positive correlation, the estimated $Vp/Vs$ ratio appears to be not affected by the reference P-wave velocity model. The $H$–$\kappa$ method also assumes a P-wave velocity in searching Moho depth and crustal $Vp/Vs$ ratio, and it is well known that the dependence of $(H, \kappa)$ measurements on the reference P-wave velocity in the crust shows a similar pattern (Zhu & Kanamori 2000; Chen et al. 2010).

As expected, the assumed sediment P-wave velocity has the same effects on the sediment $(H, \beta)$ estimates in the SCM model (Fig. 11a). On the other hand, the assumed P-wave velocity in the sediment has negligible effects on the $(H, \beta)$ estimates of the crust (Fig. 11a) of the SCM model. The assumed P-wave velocity inside one layer seems to only affect the estimates of its own layer, but not the other layers. In the SCM model, when the reference P-wave velocity inside the crust is changed, the measured $H$ and $\beta$ inside sediment remain nearly similar ($\leq$1 per cent) except for some noise-induced scattering ($\leq$3 per cent) shown in the high-noise data (Fig. 11b), while effects on the $(H, \beta)$ estimate of the crust appear to be similar to those shown in Figs 3(c) and (d).
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Figure 8. The regular receiver functions (a) and the corresponding $H$–$\kappa$ stacking results (b) of the SCM model are shown against with those (c, d) of the subsurface receiver functions of the same model. The synthetics are computed with the Thomson–Haskell propagator matrix method and include 1 per cent random noise. Note the shallow reverberations shown in the regular receiver functions in (a), which are effectively removed such that the Moho $P_s$ is clearly exposed in the subsurface receiver function data. The shallow reverberations seem to have strong effects on the $H$–$\kappa$ stacking, resulting in an incorrect estimate of the two parameters. The $H$–$\kappa$ stacking of the subsurface receiver function data, on the other hand, shows a well-fined peak in the $(H, \kappa)$ domain and yields measurements close to the input values. Panels (e)–(h) are similar to (a)–(d), except for the synthetics are generated with the LSCM model. It should be emphasized here that the subsurface receive functions are generated by wavefield downward continuation using a sediment model with constant velocity and density structure.
Figure 9. The regular and subsurface receiver functions generated from data recorded at the station NE68 are shown in (a) and (c) for comparison. Note that shallow reverberations dominate the first ∼10 s of the regular receiver function data, which mask the Moho Ps converted phase. The subsurface receiver functions, on the other hand, show a clear Moho Ps conversion at around 4.2 s. Panels (b) and (d) are the corresponding H–κ stacking results of the regular and subsurface receiver function data, respectively.

Our estimate of (H, β) does not seem to be significantly influenced by the assumed mantle P- and S-wave velocities. The results calculated from the CM and SCM models are shown in Figs 3(e)–(h), and Figs 11(c)–(d), respectively. Even with the high-noise data, uncertainties in (H, β) estimates introduced by the reference P- and S-wave velocities are less than ±5 per cent for sediment and ±2 per cent for crust, which correspond to variation of (0.045 km, 0.039 km s⁻¹) and (0.7 km, 0.073 km s⁻¹) for sediment and crust, respectively, if we assume a (H, β) of (0.9 km, 0.78 km s⁻¹) for the sediments and (35 km, 3.65 km s⁻¹) for the crust. For the SCM model, we also investigate the uncertainties in the measured (H, β) caused by the initial crustal S-wave velocity (β_c0) and crustal thickness (H_c0), which appear to be almost zero (≤1 per cent, Figs 11e and 10f).

We have also tested the robustness of the (H, β) estimates for various LSCM models with different slopes. We find that we can recover the input sediment thickness and the crust structure (H, β) when the slope of the S-wave velocity inside the sediments is less than 1.5 km s⁻¹ per kilometre. For more rapid increase of S-wave velocity, the sediment reverberations become more complicated, such that it is difficult to represent the LSCM models with a constant velocity layer.

In order to investigate the influence of selection of time window in computing upgoing S-wave energy on (H, β) measurements, we have employed three time windows [−10 s, 10 s], [−10 s, 15 s] and [−10 s, 20 s] in conducting the H–β analysis. We obtain nearly the same (H, β) measurements with the synthetic data. As our synthetic data are generated with a half-space mantle, so it is expected that any of the above three time windows contain no Ps conversions from deep boundaries in the upper mantle. However, as shown in Fig. 10, even with the presence of Ps conversion from mantle discontinuities, the measurements seem to be stable regardless which of the three windows is chosen. In general, when noise is present in the data, it is better to choose a time window that contains the two crustal multiples. At the station NE68, we also find that the (H, β) values measured with the three different time windows are nearly identical, suggesting that the H–β technique proposed here is valid regardless whether weak Ps conversions at shallow mantle structures are present in the analysing time window.

7 CONCLUSIONS

We have developed a new method to determine crustal structure beneath a station located on unconsolidated sediments using teleseismic data. The method parametrizes velocity structure beneath the station with a stack of constant velocity layers overlying a homogeneous half-space and utilizes the theory of wavefield downward
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Figure 10. Synthetic tests showing how the presence of the LAB $P_s$ affects the $H-\beta$ analysis. The upgoing $S$-wave energy normalized by its minimum value is shown as a function of crustal thickness and LAB depth in (a), and crustal shear wave velocity and LAB depth in (b). The vertical yellow solid lines and white dashed lines show the measured and input crustal thickness and $S$-wave velocity. The three horizontal lines indicate the three LAB models that are used to illustrate the backpropagated wavefield and the interference between the LAB $P_s$ and the crustal multiples, which are shown in (c)–(h). The upgoing $S$ wavefield at the uppermost mantle and its integrated energy are shown as a function of Moho depth (c, e, g) and crustal shear velocity (d, f, h) used in the downward continuation. The true $S$-wave velocity is used in computing the wavefield shown in (c), (e) and (g), and the true Moho depth is employed in (d), (f) and (h). In each figure, the horizontal red and blue lines represent the input and measured values of crustal thickness or shear velocity, respectively. The LAB in (c–d), (e–f) and (g–h) has a depth of 100, 118 and 158 km, respectively. Note the interference of LAB $P_s$ with $2p1s$ in (e–f) and $1p2s$ in (g–h), which slightly affects the measurements of the Moho thickness and crustal shear velocity using the $H-\beta$ analysis.
continuation and decomposition. We implement the method with a grid-search algorithm to seek the optimum thickness and $S$-wave velocity that minimizes the energy of the upgoing $S$-wave from mantle to crust. The search is performed progressively from the top to the bottom layers, and runs iteratively to improve the fits of wavefield in uppermost mantle. We conduct extensive synthetic tests and confirm that the method can robustly retrieve the input thickness and $S$-wave velocities of the sediment and crust for various models. We also apply the method to one broad-band seismic station inside the southwestern part of the Songliao Basin in northeast China, and obtain a sediment thickness that agrees well with active source data. We generate subsurface receiver functions by deconvolving the upgoing $P$ wavefield from the $S$ wavefield at the top of the hardrock crust, which show a significant improvement in resolving the crustal structure than the surface receiver function data. With the new approach proposed here, we can recover crustal structure and $H-\kappa$ estimates that would not be resolvable with standard receiver functions recorded at stations sitting above thick sedimentary layers. Also the subsurface receiver functions, which are free from shallow reverberations, are expected to provide better images of the deeper mantle discontinuities, with data recorded by stations at sedimentary basins and even by ocean bottom seismometers (OBS).

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