Constraining geologic surface interpolations with additional physical parameters
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CMG Seminar

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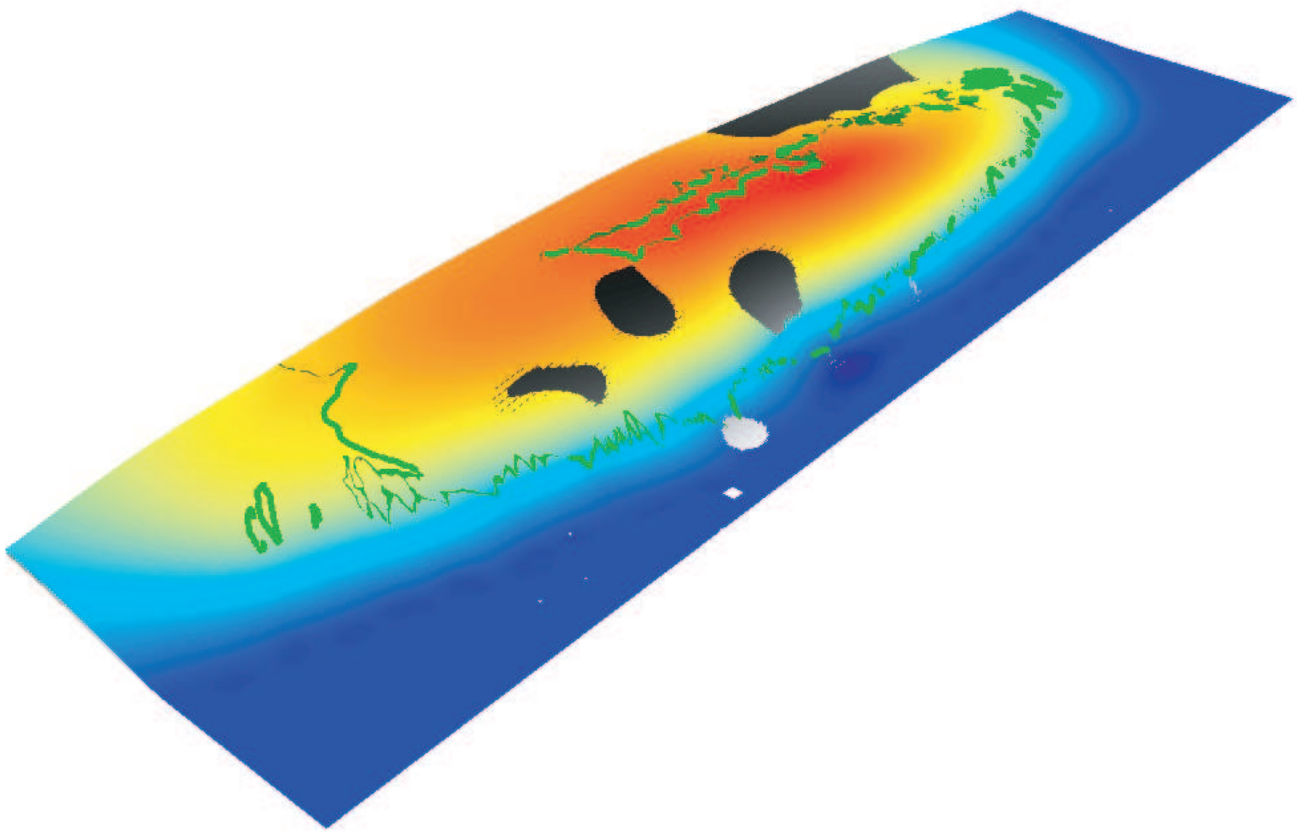
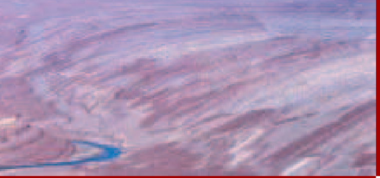
Background I

- Investigate fold curvature and fracturing relationship
- LiDAR data collected at Raplee Ridge, Utah
- Sparse coverage due to erosion
- Need to interpolate distinct layers

Background II

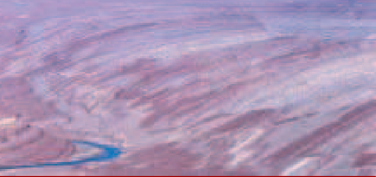
- Generic minimum curvature spline interpolation
- Unphysical results





Overview

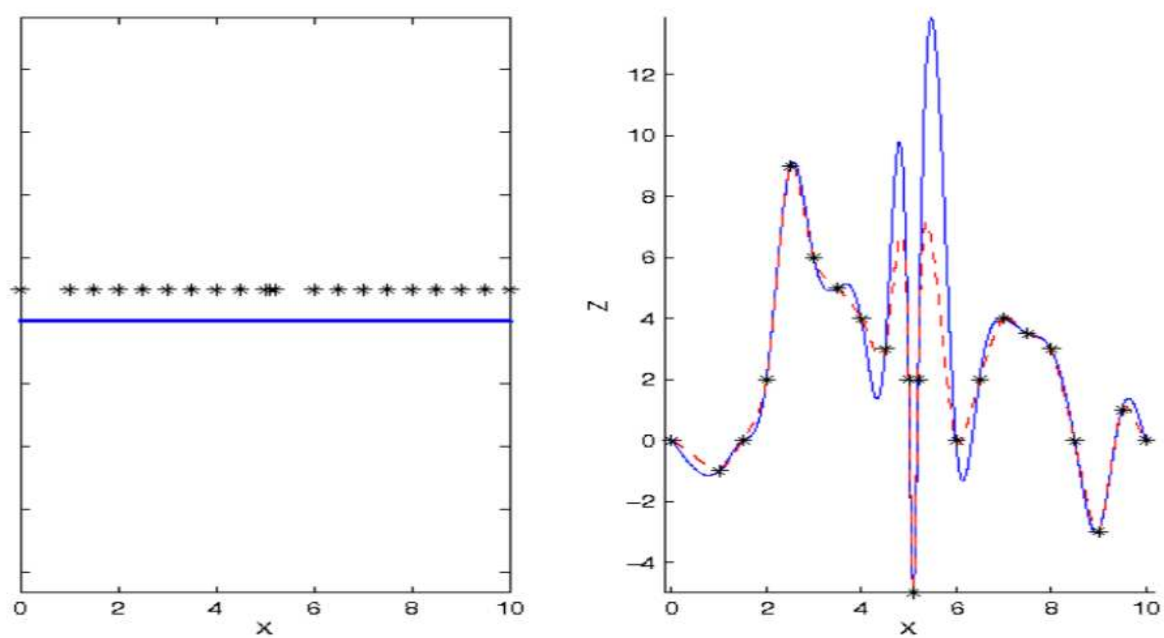
- Problem definition
  - Green function methods
  - Available additional data
  - Numerical implementation
- Results
- Current and future work
  - Regularization
  - Solvers
- Summary



Minimum curvature splines

- Mathematical analog of wooden splines
- Reduces elastic strain energy in surface
- Solves the biharmonic equation (linear PDE):

$$\begin{aligned} \nabla^4 z\left(x,y\right) &= 0 \\ \nabla^4 z\left(x,y\right) - T\nabla^2 z\left(x,y\right) &= 0 \end{aligned}$$



Green function methods I

- Particular solution for a point s(f)ource in 1-D

$$
\begin{aligned}
\frac{d^4 z(x)}{dx^4} &= \alpha \delta(x) \\
z(x) &= \alpha |x|^3
\end{aligned}
$$

- General solution: linear combination of point sources

$$
z(x_i) = \sum_{j=1}^{n} \alpha_j |x_i - x_j|^3 \quad \text{where } i = 1, \ldots, n
$$

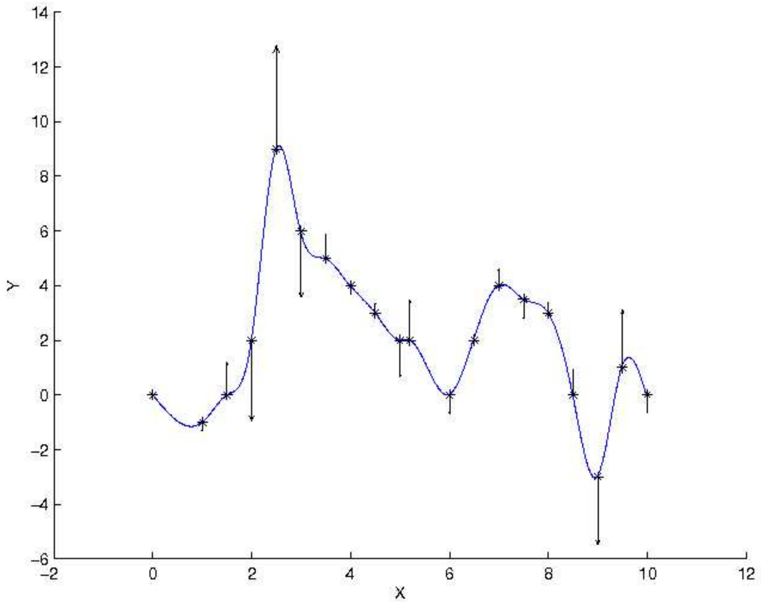
and α_j are the point force strengths at each point

Green function methods II

- Solve for point force strengths

$$\alpha_j = G_{ij}^{-1} z(x_i),$$

where $G_{ij} = |x_i - x_j|^3$



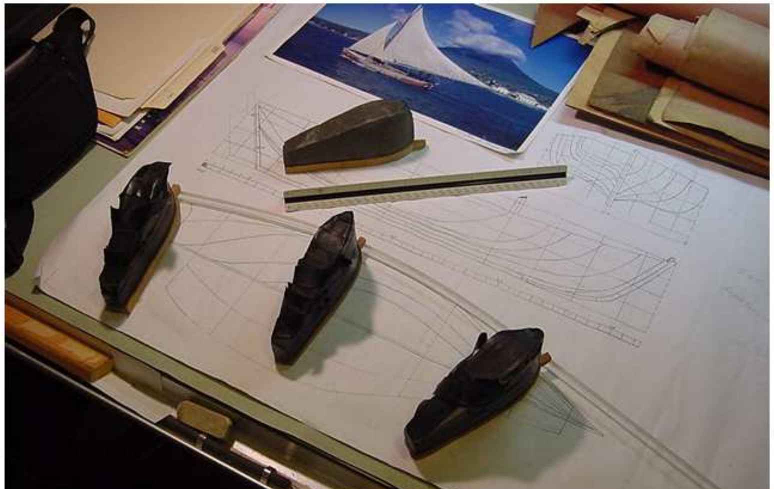
- Solve for displacement / elevation, $Z(X_I)$, on grid

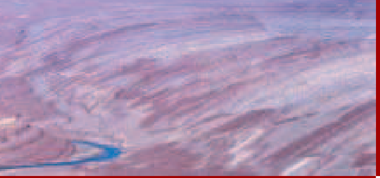
$$Z(X_I) = G_{Ij}\alpha_j$$

where $I = 1, \ldots, N$

- Putting it all together

$$Z(X_I) = G_{Ij} G_{ij}^{-1} z(x_i)$$





Problem formulation

- Numerous lithologic layers at Raplee Ridge
- Want to couple interpolations of two or more layers
- Consider two layers only:

Let w_i and W_I be the horizontal positions of data and grid points

$$\left\{ \begin{array}{c} z_I^{top} \\ z_K^{bot} \end{array} \right\} = \left[\begin{array}{cc} G^{top}\left(W_I, w_j\right) {G^{top}}^{-1}\left(w_i, w_j\right) & 0 \\ 0 & G^{bot}\left(W_K, w_l\right) {G^{bot}}^{-1}\left(w_k, w_l\right) \end{array} \right] \left\{ \begin{array}{c} z_j^{top} \\ z_k^{bot} \end{array} \right\}$$

$$x = Ad$$

- Data is contained in d, A relates data to model, x
- General formulation:

$$A^{\dagger}x = d$$

Thickness constraints

- Lower bounds of apparent thickness (vertical separation)

$$Z_I^{top} - Z_K^{bot} = \lambda$$

- Constraining the vertical separation:

$$[I \quad -I]\,x \geq \lambda, \qquad \text{where } \lambda \equiv \text{apparent thickness}$$

Quadratic programming

- Minimize $A^\dagger x = d$ in a least square sense:

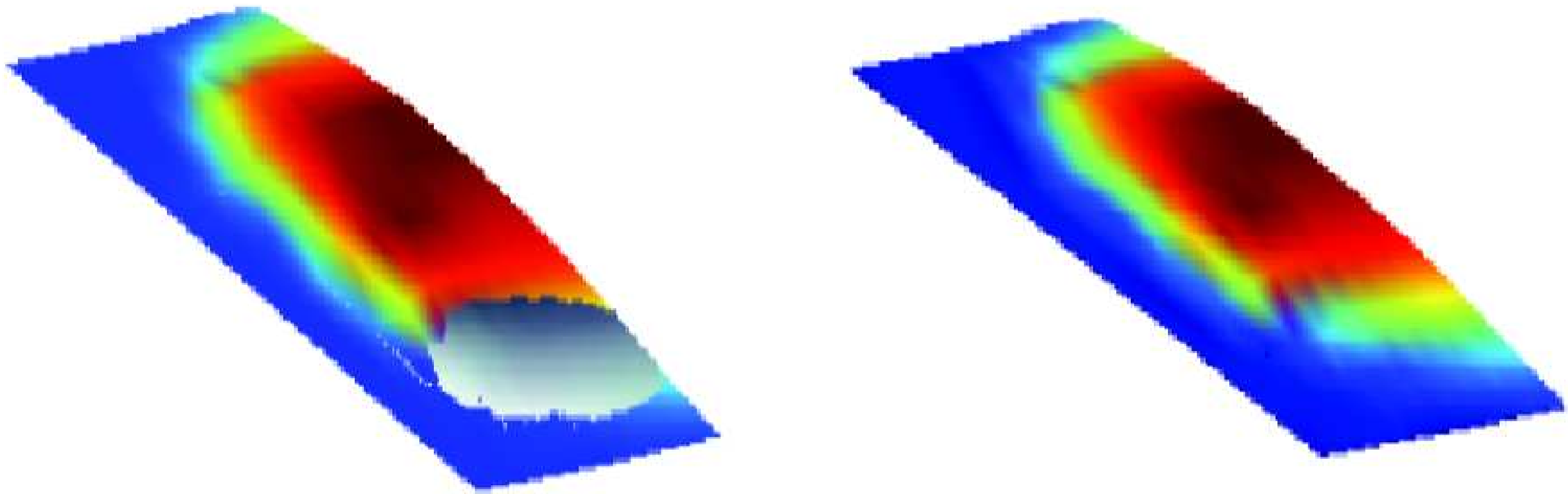
$$\min_x \quad \frac{1}{2} x^T \left(A^\dagger\right)^T A^\dagger x + x^T \left(A^\dagger\right)^T d$$

- subject to the constraint $\lambda \geq 0$

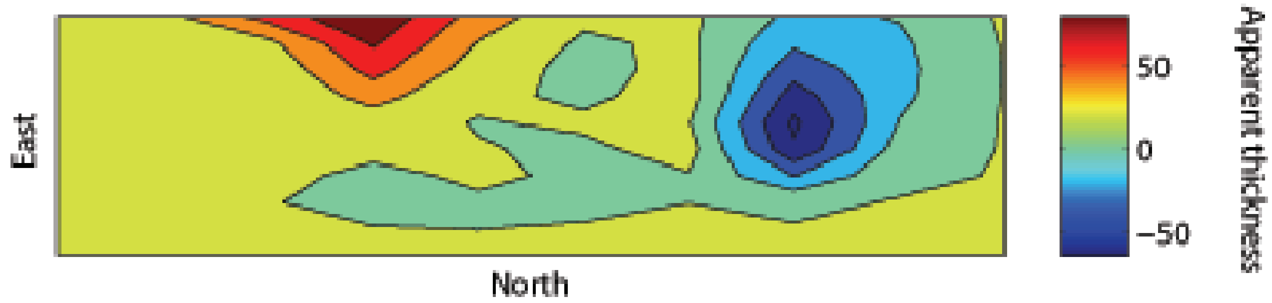
$$[I \quad -I]\, x \quad \geq \quad \lambda$$

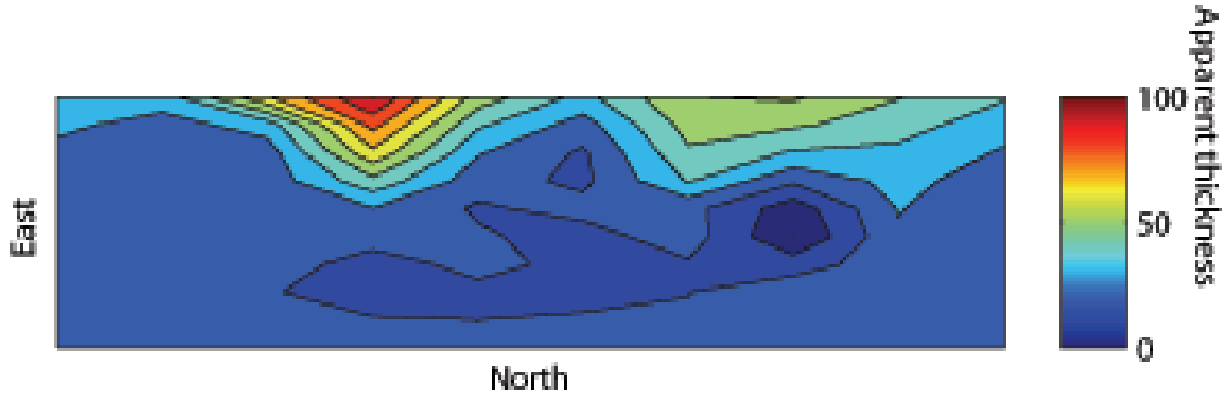
- Quadratic programming with inequality constraints:
  - Quadratic objective function
  - Linear constraint

Interpolations



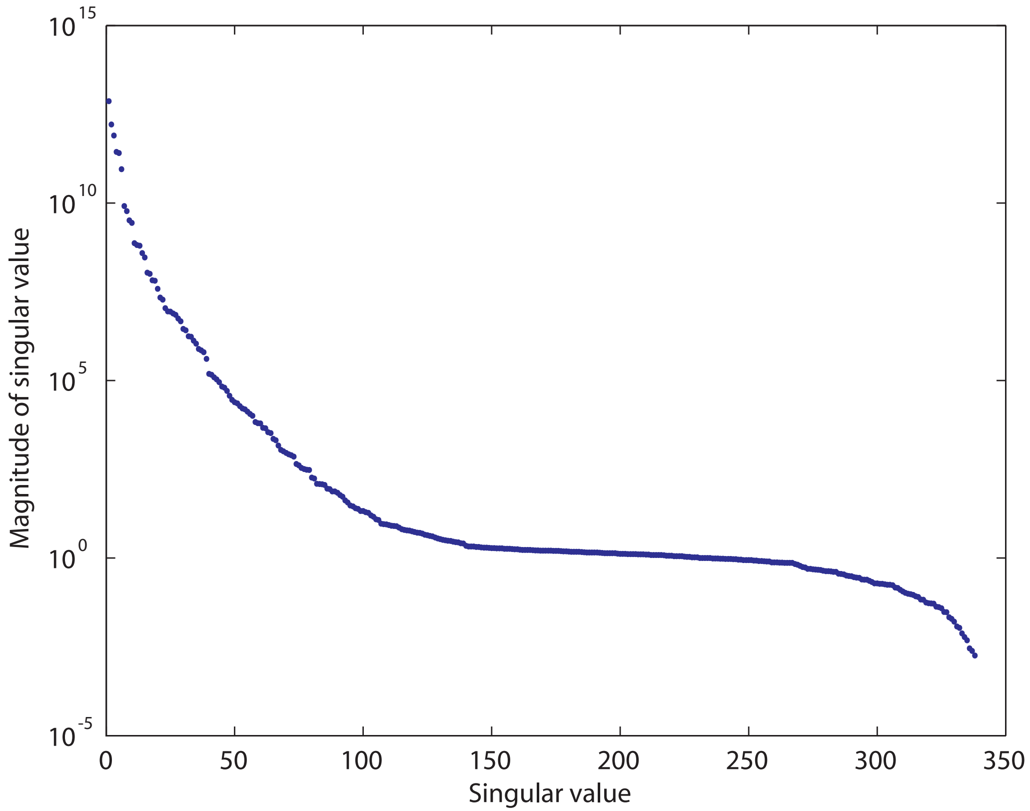
Apparent thicknesses





Numerical problems

- Closely spaced data points
- Small and large singular values
- Truncation of small values
- Trade-off between truncation and resolution



Regularization scheme

- Point forces can be regularized
- General regularizations
  - Sum of all point forces
  - Sum of all moments about origin

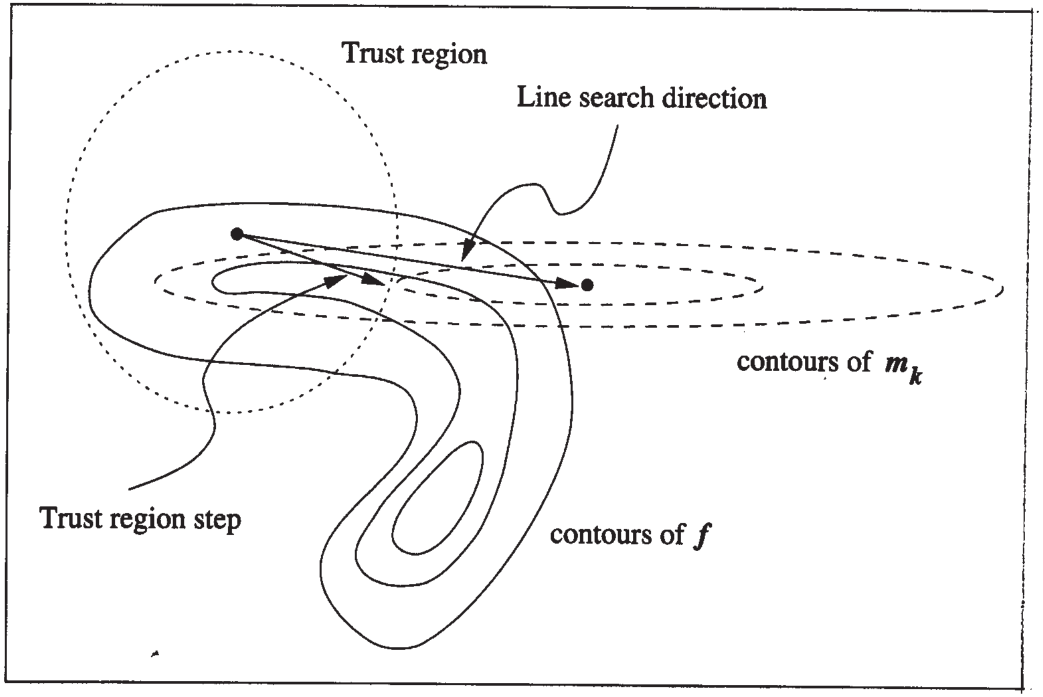
$$\begin{aligned} \sum \alpha_j &= 0 \\ \sum \alpha_j w_j &= 0 \end{aligned}$$

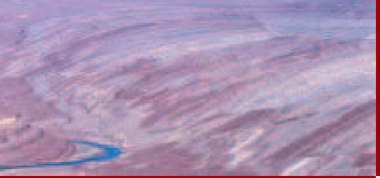
- Defines a subspace of admissible functions that regularizes the problem definition
- Plenty of additional functions available (polynomials of order 3 or less)
- Regularized Green functions:

$$F(w_i, w_j) = G(w_i, w_j) + \sum_{|\gamma| \leq 3} p_\gamma w^\gamma$$

Solution methods

- Line search methods:
  - Search direction
  - Step size
- Trust region methods
  - Direction and step size chosen simultaneously





Summary

- Overcome non-physical results
- Formulation can be extended to numerous layers
- Plenty of applications
- Large scale solver and regularization may do the trick