Ductile Folding of Sedimentary Rocks

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Abstract
This paper deals with numerical simulations of ductile folding of sedimentary rock layers with kilometer-scale dimensions. We use nonlinear continuum mechanics and the finite element method to understand the mechanics of large deformation leading to strain localization. Rock layers are subjected to combined bending and either extension or compression. Depending on the nature of the accompanying lateral deformation (i.e., whether it is extension or compression), localized deformation can manifest itself at different scales in the deforming mass. These studies are useful for integrating the mechanical and geological concepts and principles in order to formulate models constrained by available geological data.

Introduction

Two main types of deformation during folding of rocks have been observed: ductile and brittle. In general, ductile deformation takes place at greater depths, where the confining pressure is higher and the manifestation of heat is important. On the other hand, brittle deformation occurs at shallower depths, where the confining pressure is lower and the temperature is cooler. This paper focuses on the folding mechanics of granular rocks under high confining pressures, particularly the formation of deformation bands during the folding process. The study is carried out using classical bifurcation theory together with a three-invariant plasticity model in the finite deformation regime.

Ductile deformation is a common folding mechanism. Experiments have shown that various types of rocks deform to large plastic strains without breaking when deformed very slowly under high pressures. Moreover, it has been observed that during folding significant deformation may localize in narrow regions called deformation bands. In order to study these aspects, we develop a basic model for folding of multi-layers of ductile rock in the framework of nonlinear continuum mechanics.

Rocks are cohesive-frictional materials characterized by inelastic deformations, shear-induced dilatancy, and non-associative plastic flow. Yielding of rocks is pressure-dependent and non-symmetric in the deviatoric stress plane. Thus, a non-associative three-invariant plasticity model is appropriate to describe its mechanical behavior. These features are captured by the Matsuoka-Nakai (MN) yield criterion, a smooth version of the Mohr-Coulomb yield criterion. In this work, we utilize a shifted version of the MN yield criterion in order to capture the cohesive response of rocks.

A return mapping algorithm scheme in principal stress space is used to integrate the stresses over discrete loading increments, as well as to predict the onset of localization during the deformation process (Borja et al. 2003). The algorithm is based on a spectral representation of stresses and strains for finite deformation. The analysis of the boundary value problem developed for the folding model is performed with a finite element code. The finite deformation calculation is carried out using a Lagrangian description.

Description of Mechanical Model

The mechanical model involves a composite made up of individual continuous layers of approximately constant thickness but with distinct constitutive properties. A representative substructure contains an elastoplastic layer surrounded by two elastic layers and the trio would be capable of bending and stretching/compressing into prescribed geometrical configurations. As the layers fold their geometrical configurations change and the directions of the principal axes rotate. Tracking and capturing the evolving configurations of the layers and their structural features constitute a challenging aspect of the modeling and simulation. The most suitable approach for capturing finite deformation effects is to adopt a Lagrangian description. In addition to updating the position vector of every material point in the deforming strata, the constitutive equation also must be formulated so that it satisfies the axiom of objectivity.

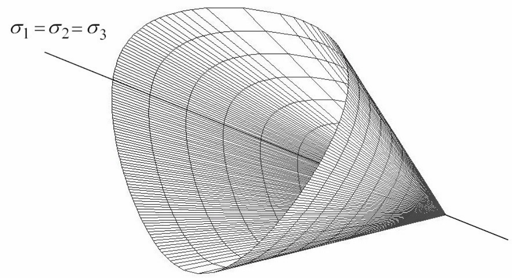


Figure 1. Matsuoka-Nakai yield surface.

Plasticity pertains to inelastic deformation, including dilatancy, deterioration of the tangent stiffness, and deformation-induced anisotropy. Geological layers exhibiting excessive deformation with very few shear fractures likely deformed in a more ductile manner, and these layers are the subject of investigation in this paper. In contrast, geological layers exhibiting several extension fractures likely behaved in elastic brittle manner and are not covered in this study. In this paper we model the deformation response of the elastoplastic layer using the Matsuoka-Nakai (1974) plasticity model. The MN yield surface is shown in Figure 1 and has the shape of a right smoothed triangular cone centered about the hydrostatic axis. The flat portion of the cone represents the tension side (lower yield stress), whereas the protruded portion represents the compression side (higher yield stress). The MN plasticity model represents the yield properties of rocks more realistically and is incorporated into the finite element program via multiplicative plasticity. The numerical integration of this constitutive model is done implicitly by a return mapping along the directions of the principal elastic stretches according to the algorithm presented by Borja et al. (2003).

We investigate failure at each numerical integration point in the finite element model using the localization condition of Rudnicki and Rice (1975). Here we equate "failure" with the emergence of a shear band. The localization condition entails tracking the evolution of the determinant function at each Gauss integration point. Initially, the determinant function is positive, but at some point in the deformation history the value of the function decreases to zero and even becomes negative. The onset of localization corresponds to the instant in the loading history at which this function becomes equal to zero, and the unit normal to the potential shear band that yields this zero determinant value defines the orientation of the band. Because the determinant function depends on an elaborate constitutive model, the search for the zero determinant is carried out numerically.

Numerical Simulations

We present two numerical examples simulating folding of an elasto-plastic rock layer sandwiched between two softer elastic layers (Figure 2). In the first simulation a rock layer was subjected to bending and extension. In the second simulation the same rock layer was subjected to bending and compression. Due to vertical symmetry it suffices to consider only half of the domain. The finite element mesh is 600 m wide and 100 m thick, and consists of 3,840 constant strain triangular elements. The inner rock layer is 80 m thick and the outer layers are 10 m thick each. We assumed a no-slip condition at the boundary between rock layers. The top boundary was subjected to a pressure load of 40 MPa corresponding to the overburden at the time of folding, whereas the vertical sides and the bottom boundary were constrained with roller supports. The loading history of the simulations consisted of two stages. In the first stage we applied the vertical load in 100 increments, and in the second we simulated folding by prescribing displacements at the boundaries in 200 increments.

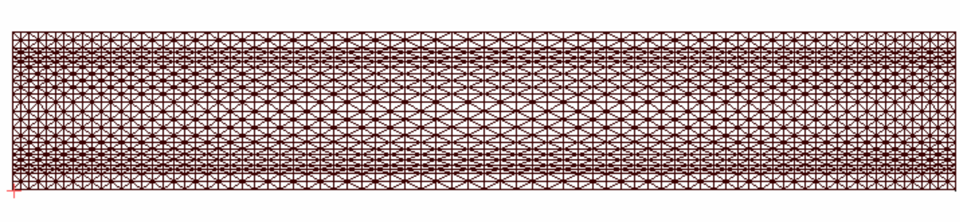
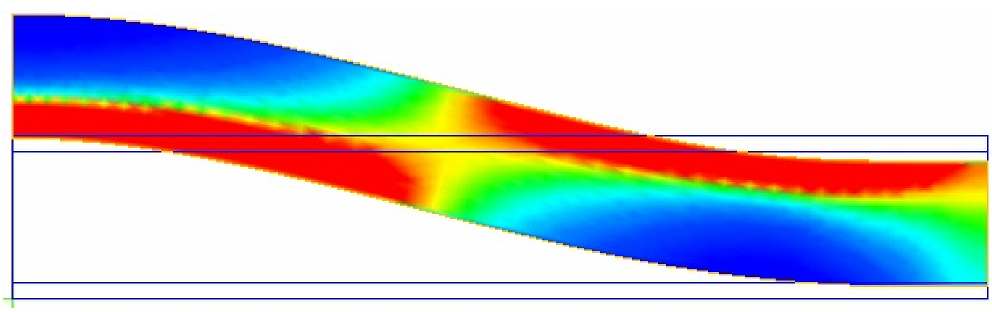


Figure 2. Finite element mesh for plane strain simulation of ductile rock folding.

We assumed the following material parameters for the inner layer: Young's modulus of elasticity E = 1,000 MPa; Poisson's ratio $\nu = 0.25$; initial and maximum size of the MN yield surface corresponded to friction angles of $\phi = 16^{O}$ and $\phi = 30^{O}$ respectively (considering that the MN yield surface passes through all six corners of the corresponding Mohr-Coulomb yield surface); and non-associative parameter $\alpha = \psi/\phi = 2/3$, where ψ is the dilatancy angle. The elastic outer layers had Young's modulus E = 100 MPa and Poisson's ratio $\nu = 0.25$.



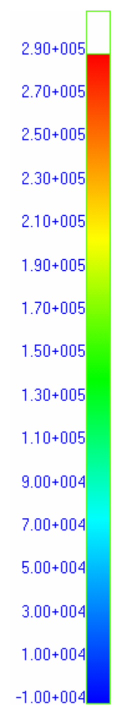


Figure 3. Contour of determinant of acoustic tensor at onset of localization for bending/extension simulation (only inner layer is shown).

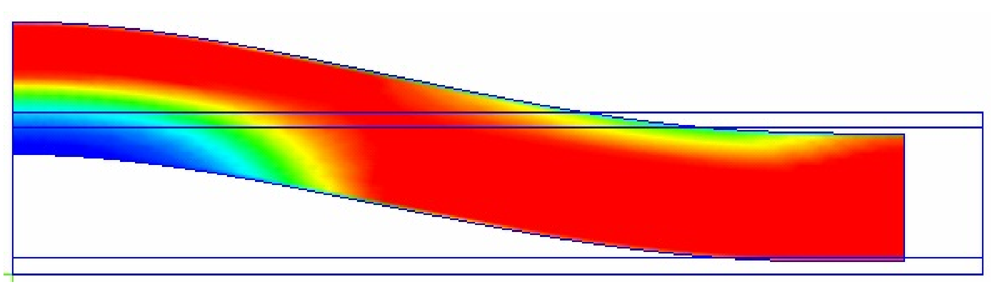


Figure 4. Contour of determinant of acoustic tensor at onset of localization for bending/compression simulation (only inner layer is shown).

Figures 3 and 4 show contours of the localization function, defined as the determinant of the acoustic tensor and used herein to detect shear band instability (Rudnicki and Rice 1975). These contours are superimposed on the deformed configurations at the onset of localization (i.e., at end of simulations). We observe that for the bending/extension simulation (Fig. 3), the onset of localization was predicted at the upper left corner of the inner layer, whereas for the bending/compression case (Fig. 4) localization was detected at the lower left corner and at an earlier time step.

In Figure 3 the vertical displacement at the upper left-hand corner is 90.2 m. The Gauss point located nearest to this corner, which localized first, was subjected to a vertical compression induced by the gravity load and a horizontal extension resulting from the finite stretching of the rock strata as they folded. Hence this Gauss point was under a Rankine active state of stress. At this Gauss point the potential shear band was calculated to be oriented at 57^{O} with respect to the horizontal axes of the current configuration.

On the other hand, the elastoplastic layer shown in Figure 4 behaved differently. At the end of the simulation the vertical displacement on the left side was approximately 67.8 m, while the horizontal displacement at the right end was 51.8 m to the left. The stress state at the onset of localization was similar to a Rankine passive state of stress: the horizontal compressive stress (due to bending/compression effects) was larger than the vertical compressive stress, and the orientation of the shear band with respect to the horizontal axes was calculated to be flatter at approximately 34^{O}. These shear band orientations agree well with the predictions using the formula proposed by Arthur et al. (1977).

Conclusions

Finite element modeling using nonlinear continuum mechanics combined with theory of plasticity can be a powerful tool to analyze complex geological processes such as ductile folding of sedimentary rocks and the accompanying shear band-type instability. Work is underway to apply the technique not only to deformation problems but also to analyze global instability that could result in multiple global solutions similar to those encountered in the buckling of plates and shells.

Acknowledgements

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