A transition from frictional weakening to thermal weakening during earthquake nucleation

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Earthquake nucleation and unstable fault slip requires a loss of frictional strength \( \tau = \mu(\sigma - p) \) with slip or slip rate. Here \( \tau \) is the shear resistance to slip, \( \mu \) is the friction coefficient, \( \sigma \) is the fault normal stress and \( p \) is the pore fluid pressure within the fault zone. A substantial amount of complex physics underlies this seemingly simple relation. Extensive laboratory and theoretical work has been conducted to understand changes in \( \mu \) with slip rate, slip, and slip history, giving rise to rate and state friction laws. At high slip rates flash heating of microscopic asperity contacts may result in substantial decreases in friction coefficient. The normal stress \( \sigma \) may vary with slip if the fault is not planar, or with gradients in slip if the elastic properties vary across the fault. Finally, pore fluid pressure may vary with slip due to a number of inelastic processes associated with shear in the fault zone, including dilatancy, pore compaction, and shear heating induced thermal pressurization. Shear heating increases \( p \) and, if dilatancy and pore pressure diffusion are limited, will cause fault strength \( \tau \) to decrease.

We are investigating the coupling between friction, thermal pressurization, pore fluid and thermal transport, and dilatancy. Some of the questions we are addressing are: 1) Can shear-heating induced pore-pressurization nucleate unstable slip on a fault that exhibits either no frictional weakening, or steady-state velocity strengthening? 2) If the fault is steady-state velocity weakening, does the frictional instability or the shear heating instability dominate? 3) If frictional weakening nucleates the instability at what point (displacement, slip speed) does thermal weakening nevertheless become the dominant weakening mechanism?

The tendency for pore fluid to escape the fault core depends on the permeability of the fault and the surrounding rocks. Field and drill core observations indicate that mature faults have a thin (~ 1 mm) shear zone on which slip is concentrated, embedded within a narrow (~ 0.1 m) fault core with permeability of order \( 10^{-21} \) to \( 10^{-19} \) m\(^2\), surrounded by rock of variable but higher permeability (see Figure 1). The governing equations for the system are listed below.

**Rate and state constitutive equations:**

\[
\tau = (\sigma - p)\left[\mu_0 + a \log \frac{\nu}{\nu^*} + b \log \frac{\theta v^*}{d_c}\right]
\]  

(1)
where \( v \) is slip speed, \( \theta \) is the state variable, \( a \) and \( b \) are material constants, \( v^* \) is an arbitrary normalizing constant, and \( \mu_0 \) is the nominal friction. State evolution is governed by

\[
\frac{d\theta}{dt} = 1 - \frac{\theta v}{d_c}\label{eq:2}
\]

where \( d_c \) is the characteristic displacement over which the state variable evolves.

**The temperature** \( T \) **is governed by**:

\[
\frac{\partial T}{\partial t} - c_{th} \frac{\partial^2 T}{\partial y^2} = 0 \quad y > 0, y < -h\label{eq:3}
\]

where \( c_{th} = K/\rho c_v \) is the thermal diffusivity, \( K \) is thermal conductivity, \( \rho \) is the density, \( c_v \) is specific heat capacity. Inside the shear zone,

\[
-2K \frac{\partial T}{\partial y}|_{y=0} + \rho_{total} c_v h \frac{\partial T}{\partial t} = \tau v \quad -h < y < 0\label{eq:4}
\]

the rate of frictional heat generation \( \tau v \) is balanced by conduction into the surroundings (first term), and accumulation of thermal energy in the fault (second term). In the limit of vanishing fault thickness the temperature change on fault due to shear heating is given by

\[
T(y = 0, t) = \frac{1}{\sqrt{2\pi} \rho c} \frac{1}{\sqrt{2c_{th}(t-t')}} \int_0^t \frac{\tau(t')v(t')dt'}{\sqrt{2c_{th}(t-t')}}\label{eq:5}
\]
The pore fluid pressure $p$ is governed by: For an adiabatic fault core the pore fluid pressure follows

$$\frac{\partial p}{\partial t} = \frac{\Lambda \tau v}{\rho_{\text{total}}c_v h} - \frac{\dot{\phi}_{\text{plastic}}}{\beta} - \frac{1}{t_f} (p - p^\infty).$$  \hspace{1cm} (6)$$

where $\Lambda$ is the ratio of thermal expansivity to compressibility, $\dot{\phi}_{\text{plastic}}$ is dilatancy, $\beta$ is compressibility, and $t_f$ is the characteristic diffusion time for pore fluid flow. The terms on the right hand side of (6) represent respectively thermal pressurization, dilatancy, and pore fluid diffusion. If the thermal and hydraulic properties are homogeneous, in the limit of zero fault zone thickness that temperature and pressure are uniquely related through

$$p(y = 0, t) = \frac{\Lambda T(y = 0, t)}{1 + \sqrt{c_h/c_{th}}}.$$  \hspace{1cm} (7)$$

where $c_{th}$ and $c_h$ are the thermal and hydraulic diffusivity, respectively.

1 Can Nucleation Occur by Shear Heating Alone?

We find that faults that are steady state velocity strengthening ($d\mu_{ss}/d\log v > 0$) are linearly stable at all wavelengths to adiabatic perturbations if the wall rock permeability exceeds a critical value given by

$$c_{\text{crit}}^* = \frac{1}{t_f} = \frac{\mu_0 v_0}{(a - b)L_p}.$$  \hspace{1cm} (8)$$

Taking $\mu_0 \approx 0.6$, $L_p \sim 0.01$m, $a - b \approx 4 \times 10^{-3}$, and $v_0 \sim 30$ mm/yr $\approx 10^{-9}$ m/s, and recalling that $c^* = 2c_w/hh_w$, with $h = 3$mm and $h_w = 100$ mm, leads to a critical diffusivity of $2 \times 10^{-9}$ m$^2$/s. This is several order of magnitudes less than estimated for the fault core, demonstrating that faults are sufficiently well drained to surpress shear heating from nucleating unstable slip.

2 When do shear heating effects become significant?

While shear heating effects are not significant near neutral stability, they will become increasingly important as the slip speed increases. The effects of shear heating become significant when the rate of thermal pressurization exceeds the rate of pore-pressure diffusion. For adiabatic deformation of a thin shear zone the rate of pressurization due to shear heating is $(\sigma - p)v/L_p$, so that a characteristic time for the effective stress to drop to zero is $L_p/v$. The ratio of thermal pressurization time to diffusion time is thus $L_p c^*/v$, and we expect that thermal effects will become significant when this ratio is order unity.

We expect that thermal pressurization effects will become significant when the rate of pore pressure build up exceeds the hydraulic diffusion rate. This occurs for slip rates in excess of $v > 2L_p c_h/hh_w$ where $L_p$ is the critical weakening length appropriate for undrained and adiabatic conditions. Taking reasonable estimates of the parameters leads
to critical slip speeds of order 1 mm/s. When slip rates exceed this value the fault becomes undrained and the shear strength begins to drop exponentially. The critical displacement at which shear heating effects become important is given by $u_{crit} = d_c \ln(L_p/t_f v^\infty)$. For $L_p \sim 10$ mm and $t_f$ of order $10^1$ to $10^2$ seconds, the critical displacement is of order 0.1 mm.

Figure 2 shows computed strength as a function of displacement. Initially the fault strength follows the elastic unloading line. This reflects the fact that at low velocities where inertial effects are insignificant, quasistatic equilibrium requires that the shear stress and fault strength are equal. At some point the slip speed becomes sufficiently fast the further slip is undrained and adiabatic. The theoretical value of $u_{crit}$ does a good job of predicting the onset of thermal weakening.

![Figure 2: Fault strength (MPa) as a function of displacement (m), for two cases, one including the effect of shear heating, the other isothermal. The vertical line marks the critical displacement at which shear heating effects are expected to become significant.](image)

The previous result is restricted to the case of adiabatic deformation. How much does heat conduction out of the shear zone delay the onset of shear heating instability? To address this question we consider the fault to be infinitesimally thin, and the thermal and hydraulic diffusivities to be spatially uniform. We ask at what stage in the acceleration of fault slip does the rate of thermal weakening exceed the rate of frictional weakening. This leads to a critical velocity beyond which the fault is weakening faster due to thermal pressurization than by frictional processes,

$$v_{crit} = \frac{1}{\pi d_c} \left[ \frac{4(b-a) \rho c_v (\sqrt{c_{th}} + \sqrt{c_{hyd}})}{\mu^2 \Lambda} \right]^2.$$

Figure 2: Fault strength (MPa) as a function of displacement (m), for two cases, one including the effect of shear heating, the other isothermal. The vertical line marks the critical displacement at which shear heating effects are expected to become significant.

Notice that increasing either the hydraulic or thermal diffusivity diminishes the rate of thermal weakening and increases $v_{crit}$. Increasing $(b-a)/d_c$ makes the fault more frictionally unstable, thereby increasing $v_{crit}$. On the other hand increasing the thermal pressurization factor $\Lambda$ decreases $v_{crit}$.
The greatest uncertainty in the parameters is in the frictional weakening distance $d_c$ and the hydraulic diffusivity, $c_{hyd}$. Taking reasonable estimates; $d_c \sim 50 \mu m$ and $c_{hyd} \sim 3 \times 10^{-6} m^2/s$, along with $b - a = 4 \times 10^{-3}$ yields an estimate of $v_{crit}$ of 0.7 mm/s. Increasing the hydraulic diffusivity by a factor of 3 to $10^{-5} m^2/s$ increases the critical slip speed to 2 mm/s. Further decreasing $d_c$ to 10$\mu$m would increase $v_{crit}$ to 10 mm/s, which is essentially an upper bound on this parameter. A plausible range of values is from the order of $10^{-2}$ mm/s to $10^1$ mm/s.

3 Conclusion

Our results suggest that the seismic weakening process may well be divided into two distinct regimes: an early nucleation regime dominated by rate and state frictional weakening, followed by a transition to thermal pressurization late in the nucleation process. The transition to shear heating induced thermal pressurization occurs at slip speeds of order 1 mm/s and slips of order 0.1 mm. Thus, thermal effects are likely to dominate late in the nucleation process, well before faults are radiating seismic waves, as well as during fast seismic slip. By the time shear heating effects dominate inertial slip is imminent ($\sim 10^{-1}s$), so that time to failure calculations based on rate-state friction alone are not biased by thermal pressurization. Thermal effects including shear induced thermal pressurization and flash heating may dominate during the fast (order 1 m/s) slip in the earthquake.