Exercise: infinitesimal strain during the great San Francisco earthquake of 1906

Reading: Fundamentals of Structural Geology, Ch 5

When adjacent particles in a rock mass displace in different directions or by different amounts the rock has strained. For example, Figure 1 shows the displacements of benchmarks near the trace of the San Andreas Fault during the great 1906 San Francisco earthquake. If the gradients in displacement on each side of the fault were zero (a homogeneous displacement field described as ‘rigid’ motion), all of the strain components would be zero. The spatial variations in magnitude and direction of the displacement vectors in Figure 1 clearly indicate that the motion was not that of two rigid blocks sliding past one another (i.e. not rigid plate tectonics). The crust of the earth near the San Andreas Fault strained during this earthquake event.

Figure 1. Map of the horizontal displacements of twelve monuments near Point Arena, CA, during the 1906 San Francisco earthquake (Lawson, 1908; Pollard and Segall, 1987).
The horizontal displacements illustrated in Figure 1 were calculated from the changes in horizontal angles between the 12 stations that were surveyed in 1891 and again in 1907. The 30 angle changes have been analyzed by Matthews and Segall (1993) to produce the data provided in the text file arena_data.txt. The columns of the data table contain:

latitude, longitude, north component of displacement, east component, semi-major axis of 95% confidence ellipse, semi-minor axes, azimuth of major axis, station name

1) Use the information in arena_data.txt to plot the displacement vectors for the twelve stations on a map of the Pt. Arena region using geographic coordinates (longitude, latitude) converted to decimal numbers. Include a straight-line representation of the fault trace, taken from measurements on Figure 1 of two points on the fault. Also include a scale bar for the displacement vectors.

2) Convert the geographic coordinates for the twelve stations and the two points on the fault to UTM coordinates. Use the Matlab m-script function deg2utm found at Matlab Central File Exchange. Plot the map again with stations, fault and scale using the UTM grid. Compare the two plots and explain why the map using geographic coordinates is distorted (both should be plotted using the axis equal command).

3) Starting with the standard ‘two point’ form for a straight line, deduce an equation for the fault trace using UTM coordinates. Convert this to the ‘slope y-intercept’ form and give numerical values for the slope and intercept. Resolve the displacements at each station into a direction parallel to the San Andreas Fault. Plot these resolved displacement vectors on the map from part 2) and give their components relative to the UTM coordinates.

4) Calculate the perpendicular distance from the trace of the fault to each station. Plot a graph of the resolved displacements (ordinate) versus the perpendicular distance (abscissa). This distribution will be used in the subsequent parts of the exercise to estimate the depth of faulting, stress drop, and strain at Earth’s surface.

To explore the relationship between fault geometry, slip distribution, and displacement and strain at Earth’s surface consider a mechanical model for a strike-slip fault proposed by Knopoff (1958). Figure 2a shows a map view of Knopoff’s conceptual model with the origin of coordinate axes at Earth’s surface on the fault trace. Also shown is an elevation view (Figure 2b) with line of sight parallel to the fault. The depth \( D \) measures the part of the fault that slipped during the earthquake. \( D \) is the only length scale in this two dimensional model because the fault is taken as very long in trace length (hundreds of kilometers) compared to the rupture depth (about ten kilometers).

The remote shear stress, \( s(r) \), acts to promote slip on the fault and this is resisted by the shear stress, \( s(f) \), in the fault zone. Presumably the fault shear stress is related to the frictional strength of the rock material in the fault zone, a quantity we return to in Chapter 9. Slip on the fault is also resisted by the stiffness of the elastic material surrounding the
fault. This stiffness is measured by the so-called shear modulus of elasticity, $G$, a quantity we return to in Chapter 8.

Knopoff used elasticity theory, a subject we take up in Chapter 8, to derive an expression for the displacement field at Earth’s surface during a slip event on the model fault:

$$u_x = \pm \left[ \frac{s(r) - s(f)}{G} \right] \left[ \left( y^2 + D^2 \right)^{1/2} - |y| \right]$$  \hspace{1cm} (1)

The plus sign is used for $y > 0$ and the minus sign for $y < 0$. The displacement is directly proportional to the difference between the remote stress, $s(r)$, and the fault stress, $s(f)$, referred to as the stress drop associated with faulting. The displacement is inversely proportional to the elastic shear modulus, $G$, a measure of rock stiffness.

5) Solve (1) for the relative displacement, $\Delta u_x$, (slip) across the fault and use this to write a non-dimensional equation for the displacement distribution and plot this distribution.

Given the observed slip on the San Andreas fault in the Point Arena area of $\Delta u_x = 5$ m (Lawson, 1908), use the displacement parallel to the fault at each station and Knopoff’s model to estimate the depth of faulting. Calculate the mean value of the twelve depth estimates, and use this value to plot the model displacement vectors on a map of the Pt. Arena region, along with the displacement vectors calculated from the geodetic data. Discuss the correlation or lack thereof.

Figure 2. Conceptual model for a strike-slip fault (Knopoff, 1958). a) Map view. b) Vertical cross-sectional view.
6) Based upon your estimates of $D$ and a shear modulus, $G = 3 \times 10^4$ MPa, taken from laboratory rock mechanics testing, estimate a range of stress drops for this earthquake. Give the calculated stress drops and their mean value. We note that others have estimated the stress drop to be between 4.7 MPa and 19.0 MPa (Chinnery, 1967).

7) The angle $(\pi/2) - \beta$ is a measure of the shearing distortion referred to as the angle of shear. Use trigonometry to relate the angle of shear to the differential displacement, $\Delta u_x$, and the differential height, $\Delta y$, of the element. Then relate the angle of shear to the partial derivative $\partial u_x / \partial y$. Compare your result to the definition of the infinitesimal strain tensor components given in (5.118). Explain why some components are zero for the Knopoff fault model, and identify which component corresponds to the angle of shear in the horizontal plane.

8) Use Knopoff’s equation (1) for the displacement field near the model strike-slip fault and the kinematic equations to derive an equation for the distribution of shear strain away from the fault. Take care that the proper signs are preserved in the derivative. Analyze this distribution of infinitesimal shear strain to determine the shear strain on either side of the fault, $y = 0^\circ$, and at great distances from the fault, $y = \pm \infty$. Explain why your results make sense given the distribution of displacements during the seismic cycle. Draw a sketch of the seismic cycle and use this to address the question.

9) Prepare a dimensionless graph of the distribution of angle of shear over the range of distances $-5 \leq y/D \leq +5$. Evaluate the range of angular changes you might expect for the next great earthquake along the San Andreas fault at distances of 0, 5, 10, 15, and 20 km. Suggest a design for a surveying network that would enable you to measure the distortion of the ground surface near the fault.

10) In two dimensions two orthogonal directions are associated with extreme values of the normal strains. These are referred to as the principal strains and the orientations of the line elements associated with these strains are given by:

$$\gamma_1 = \frac{1}{2} \tan^{-1} \left[ \frac{2\varepsilon_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}} \right], \quad \gamma_2 = \gamma_1 + \frac{\pi}{2}$$ (2)

Calculate the orientations of the principal strains near the model fault. The magnitudes of the principal strains are given by:

$$\varepsilon_1 = \frac{1}{2} \left( \varepsilon_{xx} + \varepsilon_{yy} \right) + \left[ \frac{1}{4} \left( \varepsilon_{xx} - \varepsilon_{yy} \right)^2 + \varepsilon_{xy}^2 \right]^{1/2}$$ (3)

$$\varepsilon_2 = \frac{1}{2} \left( \varepsilon_{xx} + \varepsilon_{yy} \right) - \left[ \frac{1}{4} \left( \varepsilon_{xx} - \varepsilon_{yy} \right)^2 + \varepsilon_{xy}^2 \right]^{1/2}$$ (4)

Calculate the magnitudes of the principal strains near the model fault. Explain how you would take advantage of this information to design a monitoring program to capture the distribution of interseismic strain.
References Cited

