PAPER 0

ANALYZING DIFFRACTIONS AND REFLECTIONS BY WAVE EQUATION MODELING

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ABSTRACT

In this paper, we show an example of synthesizing reflection from diffractions. It is accomplished by solving the velocity-density acoustic wave equation by Fourier transform method or, as it is often called, the pseudospectral method. It is shown that as discrete diffractors are lined up, their diffractions collapse to a reflection.

INTRODUCTION

In the simulation carried out in this paper, we concern with the acoustic wave equation,

\[ \frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\rho} \frac{\partial P}{\partial y} \right) = \frac{1}{c^2 \rho} \frac{\partial^2 P}{\partial t^2} + S \]

where \( P(x,y,t) \) represents the pressure, \( \rho(x,y) \) the density, \( c(x,y) \) the wave velocity, and \( S(x,y,t) \) the source term which equals the divergence of the body force divided by the density.

In numerically solving the acoustic wave equation (1), we apply a highly accurate method -- the pseudospectral method (Kosloff and Baysal, 1982). The spatial derivatives are calculated by Fourier transforms. The time derivative is calculated by a second order finite-difference. By this method, the spatial derivatives are calculated exactly, the only numerical error comes from the temporal finite-differencing. We choose the time sampling interval to be sufficiently small that the Courant number is less than 0.1, then modeling by this method is considered to be exact.
MODELING EXAMPLES

Figure 1 shows the modeling geometry. Source S is at the left well. Receivers are at the right well. The medium is homogeneous, with a constant velocity of 18,000 ft/s and density 3 g/cm³. The source wavelet is the first derivative of the Gaussian function with a dominant frequency of 500 Hz (Alford et al., 1974). The two wells are separated by ten wavelengths. Diffractors are put at the middle between the two wells by changing the velocity of the nodes to be 9,000 ft/s and density 2 g/cm³. The lengths of the aligned diffractors to modeled are quarter, half, one, one and half, two, four, eight, and ten wavelengths.

Figure 1  Modeling geometry. Diffractors are created by changing the velocity and density at the designated nodes.

Figures 2 and 3 show the modeling results of the different diffractor dimensions. Figure 2 shows that when the dimension of the diffractor is less than or equal to one wavelength, its modeling responses are in the forms of diffractions. Figure 3 shows that as the dimension of the diffractors is increasing to be larger than one, its modeling responses tend to be the reflection, i.e., the diffraction responses of the individual diffractors collapse to a reflection.

CONCLUSIONS

The relationship between reflection and diffraction has been analyzed by an accurate numerical wave equation modeling. When the dimension of the diffractor is less than or equal to one wavelength, its modeling responses are in the form of diffractions. As the dimension of the diffractors is increasing to be larger than one, its modeling responses tend to be the reflection, i.e., the diffraction responses of the individual diffractors collapse to a reflection.
Figure 2. Modeling results when the lengths of the diffractors are quarter, half, one, one and half wavelengths. In (a), 1 is the direct arrival, 2 the diffraction. Similar event identifications apply to (b), (c), and (d).
Figure 3  Modeling results when the lengths of the diffractors are two, four, eight, and ten wavelengths. The distance between the two wells is ten wavelengths. As the dimension of the diffractors is increasing to be larger than one, its modeling responses tend to be the reflection.
REFERENCES

