PAPER K

BAND-PASS DECONVOLUTION FOR SHORT WAVELETS

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ABSTRACT

In this paper we present a method for band-pass deconvolution. Band pass deconvolution is very important for reliable estimation of the effective source wavelet, since it minimizes the effect of the out-of-band noise on the phase inside the band. The deconvolution operator is defined as the solution of a constrained least squares problem and an exact solution is given. This is a time domain method and will be particularly useful for short wavelets. The problem is present in the cross-well tomography data acquisition with vibroseis-like source, when the sweep is very short so that crosscorrelation does not effectively produce a compact wavelet. The method will also be useful for deconvolution after correlation, since the effective wavelet is normally very short.

INTRODUCTION

The source for the cross-well tomography project is often a high frequency vibroseis-like band-limited source. Deconvolution must take into account the lack of energy at low and high frequencies. Band-pass deconvolution will be desirable in order not to whiten the data in the noisy part of the spectrum and to minimize the effect of such noise in the phases in the high S/N band-pass.

Band-pass deconvolution has been studied by Deeming(1987). Deeming's method modifies the estimated autocorrelation so that the prediction filter is band-limited. Essentially the modification puts high energy outside of the frequency band of interest, so that the one-step-ahead prediction filter does not contain energy at those frequencies.

Here we will treat the band limited prediction problem as a constrained least squares problem and we give the exact solution to the problem.

MATRIX FORM OF THE PREDICTION PROBLEM

The prediction problem consists of finding a convolutional filter that predicts the data from its past. The one-step-ahead prediction filter turns out to be the inverse of the source wavelet and it forms the basis for spiking deconvolution.

Convolution can be represented with the following matrix form:

$$\begin{bmatrix} x_1 & 0 & \cdots & 0 \\ x_2 & x_1 & \ddots & 0 \\ x_3 & x_2 & \ddots & 0 \\ x_4 & x_3 & \ddots & x_1 \\ x_5 & x_4 & \ddots & x_2 \\ \vdots & x_5 & \ddots & x_3 \\ x_N & \ddots & \ddots & x_4 \\ 0 & x_N & \ddots & x_5 \\ 0 & 0 & \ddots & x_5 \\ 0 & 0 & \cdots & x_N \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_M \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ \vdots \\ y_N \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Where x_k are the elements of one sequence and f_k the elements of the other, normally x_k is a time series and f_k is a filter. The output y_k is normally represented in terms of z-transforms as:

$$X(Z)\,F(Z) = Y(Z)$$

And the z-transform of the error $E(Z)$ is then:

$$E(Z) = X(Z)\,F(Z) - Y(Z)$$

In terms of matrices the prediction problem can be stated as follows:

$$\mathbf{X\,f - y = e}$$

For spiking deconvolution (one-step-ahead prediction) the elements of the desired output are:

$$y_k = x_{k+1}$$

For gap deconvolution with prediction distance g the desired output is then:

$$y_k = x_{k+g}$$

The elements of the filter **f** are obtained by minimizing the power of the error.

THE BAND-PASS CONSTRAINT

In matrix form the Discrete Fourier Transform is:

$$\mathbf{W\,f = F}$$

Where $\mathbf{W}$ is a matrix whose coefficients are $e^{2\pi i(k-1)(j-1)/N}$, $\mathbf{f}$ is the vector representation of the filter in the time domain and $\mathbf{F}$ is its Discrete Fourier Transform. Of course $\mathbf{W}$ and $\mathbf{F}$ are complex matrix and vector respectively, but can be reorganized to involve only real arithmetic. It is also possible to use the cosine or sine transform, in which case $\mathbf{W}$ is made up with either the real part or the imaginary part of the initial transformation. We will use the cosine transform, leaving for later the decision as to which is the best strategy to use.

We now define a constraint matrix $\mathbf{Q}$, formed with the rows of $\mathbf{W}$ that represent the frequencies outside the desired band. Then the fact that $\mathbf{f}$ is band-limited is represented with the following constraint equation:

$$\mathbf{Q\,f} = 0$$

In normal band-limited data, the rows of $\mathbf{Q}$ will be the first and last few rows of $\mathbf{W}$. This will effectively account for the undesired low and high frequencies.

BAND-PASS DECONVOLUTION

The band-limited deconvolution operator $\mathbf{f}$ is computed by solving the following Constrained Least Squares Problem:

$$\mathbf{X\,f} \approx \mathbf{y}$$

$$\mathbf{Q\,f} = 0$$

This is the same least squares problem defined by Deeming(1976), but with our constraints. The choice of $\mathbf{y}$ will either give spike deconvolution or gap deconvolution.

The solution is readily available (Claerbout, 1976). After some algebraic manipulation the solution is:

$$\mathbf{f} = \mathbf{f}_0 - \mathbf{R}^{-1}\mathbf{Q}^T(\mathbf{Q}\mathbf{R}^{-1}\mathbf{Q}^T)^{-1}\mathbf{Q}\ \mathbf{f}_0$$

where $\mathbf{f}_0$ is the solution to the unconstrained problem and $\mathbf{R} = \mathbf{X}^T\mathbf{X}$ is the autocorrelation matrix.

We note that in this method we are modifying the inverse of the autocorrelation matrix, whereas Deeming's method modifies the autocorrelation itself.

CONCLUSIONS

We have presented an exact deconvolution method that acts only in the pass-band of interest. The technique whitens the data only on the given pass-band, is implemented in the time domain and works with short filters.

We show that this method is equivalent to modifying the inverse of the autocorrelation function while Deeming's method modifies the autocorrelation itself.

In standard deconvolution the noise outside of the frequency band affects the phase in the frequency band. This is not the case with band-pass deconvolution, as the deconvolution operator will not introduce time shifts that are dependent on noise outside of the working frequency band. This is of course very important in seismic tomography in which timings are essential.

REFERENCES

Claerbout J. F. 1976, Fundamentals of Geophysical Data Processing, McGraw-Hill, New York.

Deeming T. J. 1987, Band-Limited Minimum Phase, Deconvolution and Inversion, M.H. Worthington Editor, Blackwell Scientific Publications.