PAPER I

DIFFRACTION TOMOGRAPHY RECONSTRUCTION USING CONSTRAINTS

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ABSTRACT

Adding independent information to the inversion of a given data set helps to reduce the nonuniqueness of the solution by reducing the null space of the measurements. We show in this paper, within the framework of reconstruction in Hilbert spaces, that prior information can be used in diffraction tomography reconstruction and the result should be an image whose support in the frequency domain has been expanded when compared with the support of the image obtained without using any prior information. This means that the null space of the reconstruction is reduced by adding high frequencies not present in the original data.

INTRODUCTION

Image reconstruction in Hilbert spaces is a general formulation that can be adapted to the solution of many inverse problems where the data are obtained by integrating the product of the unknown by a certain set of functions (sampling functions). The nature of the unknown and the set of functions define, of course, the kind of problem that we have to solve. In others papers in this volume (Michelena and Harris, 1990, paper F; Michelena, 1990, paper H) it is shown that when the unknown is the slowness model and the set of functions are the beam paths, the problem we are dealing with is tomographic traveltime inversion. It can be shown also that if the unknown is a signal and the sampling functions are shifted versions of another given 1D function, the problem we are facing is deconvolution. In Fourier reconstruction the sampling functions are complex exponentials and the unknown can be the Fourier transform of a particular function. In any of these cases, the formulation allows the use of prior information about the unknown in the form of images by simply using the given image as a weighting function in the definition of the inner product that describes the measurements. Examples of this procedure are given by Darling et al., (1983) and Michette et al., (1984) and Hall et al., (1982). More details can also be found in paper H in this volume.

Diffraction tomography reconstruction can be seen essentially as a problem of Fourier reconstruction where only the propagating waves are considered. The sampling functions in this case are complex exponentials with real wave numbers and the unknown is related with the velocity perturbation in an homogeneous background.

Therefore, this problem can also be considered as a reconstruction problem in Hilbert spaces. We show in this paper that when the problem is considered this way, prior information about the object profile can be easily incorporated, improving the frequency domain coverage of the reconstructed image. If successfull on this simple example, we expect the method can be applied to the more general problems of diffraction tomography for inhomogeneous background models on pre-stack migration.

PRIOR INFORMATION IN DIFFRACTION TOMOGRAPHY

The basic problem in diffraction tomography is the reconstruction of the object profile $O(x,z)$ from the scattered data. Fig. 1 shows the geometry for the cross well configuration. The velocity of propagation $C(x,z)$ in the object profile is related with the velocity of propagation in the homogeneous background through the equation

$$O(x,z) = 1 - \frac{C_0^2}{C^2(x,z)},$$

where C_0 is the velocity of the background medium, $C_0 = \omega/k$ (ω = angular frequency, k = wavenumber of the incident wave).

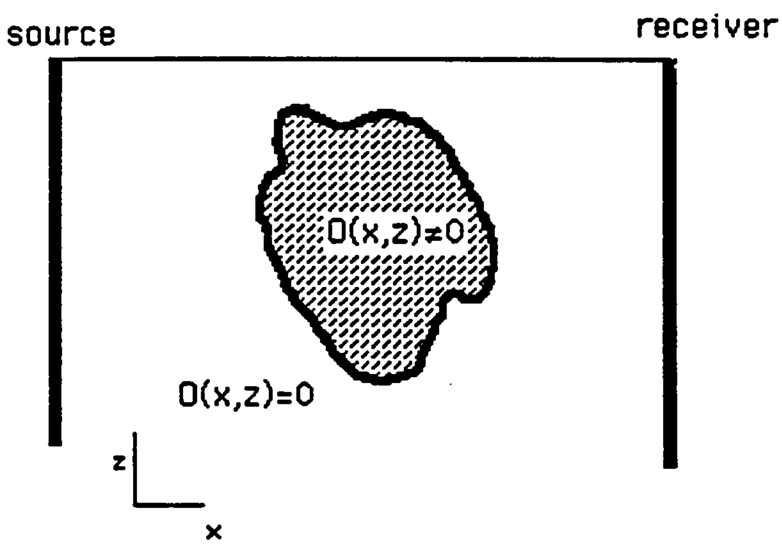

Figure 1: Tomographic configuration

Assuming that the scattered field is much smaller than the background field and applying the Born approximation (or Rytov in the same way) we obtain the following relationship between the spectrum of the scattered field and the 2D Fourier transform of the object profile $O(x,z)$ (Harris, 1987):

$$\hat{U}(x_g, k_g; x_s, k_s) = \pi k^2 (\gamma_s \gamma_g)^{-1} \hat{O}(\gamma_g - \gamma_s, k_g + k_s) U_0(\omega) \exp(-i\gamma_s x_s + i\gamma_g x_g) \quad (1)$$

where k_g and k_s are the wavenumbers along the geophone line and source line respectively and γ_g and γ_s the corresponding perpendicular wavenumbers ($\gamma_g^2 + k_g^2 = k^2$). The variable x_g identifies the position of the receiver line and x_s the position of the source line. $\hat{O}(\gamma_g - \gamma_s, k_g + k_s)$ is the 2D Fourier transform of $O(x, z)$

$$\hat{O}(\gamma_g - \gamma_s, k_g + k_s) = \int_{-\infty}^{\infty} dz dx O(x, z) \exp(-i(\gamma_g - \gamma_s)x - i(k_g + k_s)z). \quad (2)$$

$\hat{U}(x_g, k_g, x_s, k_s)$ is the spectrum of the scattered field along the source line and geophone line, or simply the "data". These equations were derived in 2D for line sources.

Substituting Eqn. 2 into Eqn. 1 we obtain,

$$\hat{U}(x_g, k_g; x_s, k_s) = \pi k^2 U_0(\omega)(\gamma_s \gamma_g)^{-1} \exp(-i\gamma_s x_s + i\gamma_g x_g) \int_{-\infty}^{\infty} dz dx O(x, z) \exp(-i(\gamma_g - \gamma_s)x - i(k_g + k_s)z). \quad (3)$$

Defining $\hat{\mathbf{U}}(k_x, k_z)$ as

$$\hat{\mathbf{U}}(k_x, k_z) = \hat{U}(x_g, \gamma_g; x_s, \gamma_s)(\gamma_s \gamma_g)/(\pi k^2 U_0(\omega)) \exp(ik_s x_s - ik_g x_g) \quad (4)$$

and $k_x = \gamma_g - \gamma_s$, $k_z = k_g + k_s$, we obtain the following compact relation between the object $O(x, z)$ and the data

$$\hat{\mathbf{U}}(k_x, k_z) = \int_{-\infty}^{\infty} dz dx O(x, z) \exp(-ik_x x - ik_z z). \quad (5)$$

Two important restrictions should be satisfied before applying the previous equations. The first one relates, as we said before, with the magnitude of the scattered fields relative to the direct field. The other restriction is that for preserving the Fourier transform relationship between the object profile and the data (Eqn. 5), it is necessary to consider only the case in which k_x is real, which means that we have to discard the effect of nonpropagating fields. This translates into limitations on the coverage of the spectrum of the object profile, as shown in Fig. 2 (Wu and Toksöz, 1987).

The conventional way of obtaining $O(x, z)$ as a function of $\hat{\mathbf{U}}$ is performing an inverse Fourier transform of the expression 5 considering only propagating waves (Devaney, 1984; Harris, 1987). A change of variables from the (k_x, k_z) domain to the (k_s, k_g) domain is needed. The inverse problem can be considered also as a problem of reconstruction in Hilbert spaces if we consider that the measurements are the projections of the object profile in the functions $\exp(-ik_x x - ik_z z)$.

$$\hat{\mathbf{U}} = < O(x, z), \exp(-ik_x x - ik_z z) > . \quad (6)$$

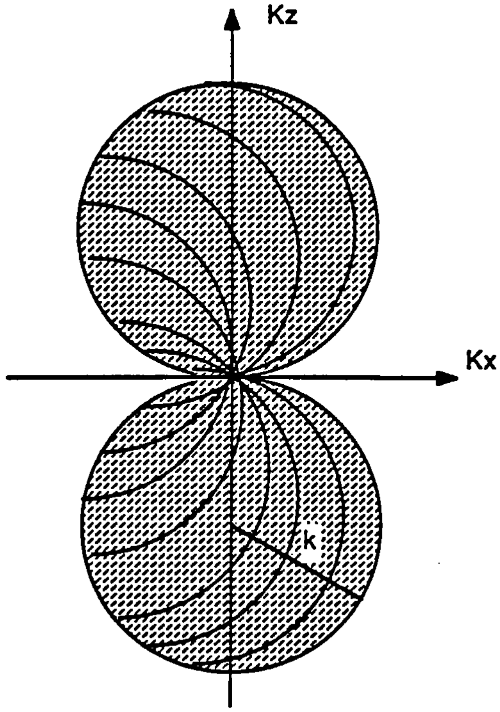

Figure 2: Region in the Fourier space in which the object profile is determined.

Let's denote $p(x, y)$ the prior information we have about the object $O(x, z)$. That information can be included in the reconstruction process as a weighting function in the inner product 6 that describes the generation of the measurements $\hat{\mathrm{U}}$. These measurements for each particular k_{x_i} and k_{z_j} can be described also as

$$< O(x,z), \beta_{ij}(x,z) >_p = \int_{-\infty}^{\infty} dxdz O(x,z)\beta_{ij}^*(x,z)/p(x,z), \qquad (7)$$

where

$$\beta_{ij}(x,z) = p(x,z)\exp(ik_{x_i}x + ik_{z_j}z).$$

The minimum norm estimate of the object profile $O(x, z)$ is

$$\tilde{O}(x,z) = p(x,z)\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}\exp(ik_{x_i}x + ik_{z_j}z). \qquad (8)$$

If we multiply both sides of Eqn. 8 by $\exp(ik_{x_m}x + ik_{z_n})$ and integrate, we obtain that the coefficients b_{ij} must satisfy the system of linear equations

$$\hat{\mathrm{U}}(k_{x_m}, k_{z_n}) = \sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}\hat{p}(k_{x_m} - k_{x_i}, k_{z_n} - k_{z_j}) \qquad (9)$$

where $\hat{p}(k_x, k_z)$ is the 2D Fourier transform of $p(x, z)$. The independent term $\hat{\mathrm{U}}(k_{x_m}, k_{z_n})$ is the scattered field compensated by a factor proportional to the deconvolution of the radiated source frequency spectrum $(4\pi k^2 U_0(\omega))^{-1}$ (Harris, 1987) and another factor which compensates for propagation effects $\exp(i\gamma_s x_s - i\gamma_g x_g)$. $\hat{p}(k_{xm} - k_{x_i}, k_{zn} - k_{z_j})$ is a block Toeplitz matrix.

If $p(x,z) = 1$, it results that the object is a simple Fourier inversion of the data. That is

$$\tilde{O}(x,z) = \frac{1}{(2\pi)^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{\mathrm{U}}(k_{x_i}, k_{z_j}) \exp(ik_{x_i}x + ik_{z_j}z). \tag{10}$$

where only the case k_{x_i} real is considered (propagating waves).

According to 8, the estimated of the object profile is the product of the function $p(x,z)$ (that reflects our prior knowledge about $O(x,z)$) by an unknown function $f(x,z)$, where $f(x,z)$ is

$$f(x,z) = \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij} \exp(ik_{x_i}x + ik_{z_j}z). \tag{11}$$

The Fourier transform of $f(x,z)$ has a support that is the same as the one of the reconstructed object (Fig. 2) when $p(x,z) = 1$. When $p(x,z) \neq 1$ the Fourier transform of the reconstructed object is simply

$$\hat{\tilde{O}}(k_x, k_z) = \hat{p}(k_x, k_z) * * \hat{f}(k_x, k_z), \tag{12}$$

where $**$ means 2D convolution. In this equation, the support of the function $\hat{f}(k_x, k_z)$ equals the support of the object reconstructed without constraints. The effect of the prior information can be easily understood when the function $p(x,z)$ is band limited in both directions x and z. In this case the reconstructed object has the spectrum

$$\hat{\tilde{O}}(k_x, k_z) = \Pi(k_x/W, k_z/W) * * \hat{f}(k_x, k_z), \tag{13}$$

where $\Pi(.,.)$ is the rectangle function and W its bandwidth. This particular case is illustrated in Fig. 3. In general, Eqn. 12 tells us that when we use prior information, the support area of the reconstructed object will contain always higher frequencies than the support of the object imaged without constraints.

The dependency of $\hat{\mathrm{U}}(k_x, k_z)$ on k_x and k_z in Eqn. 4 is not explicit. For making that dependency explicit we have to solve the equations (Harris, 1987)

$$k_x = \gamma_g - \gamma_s \qquad k_z = k_g + k_s$$

for k_g and k_s. The result is

$$k_s = \frac{k_z}{2} \pm \frac{k_x}{2K_T}(4k^2 - K_T^2)^{1/2}$$

and

$$k_g = \frac{k_z}{2} \mp \frac{k_x}{2K_T}(4k^2 - K_T^2)^{1/2}$$

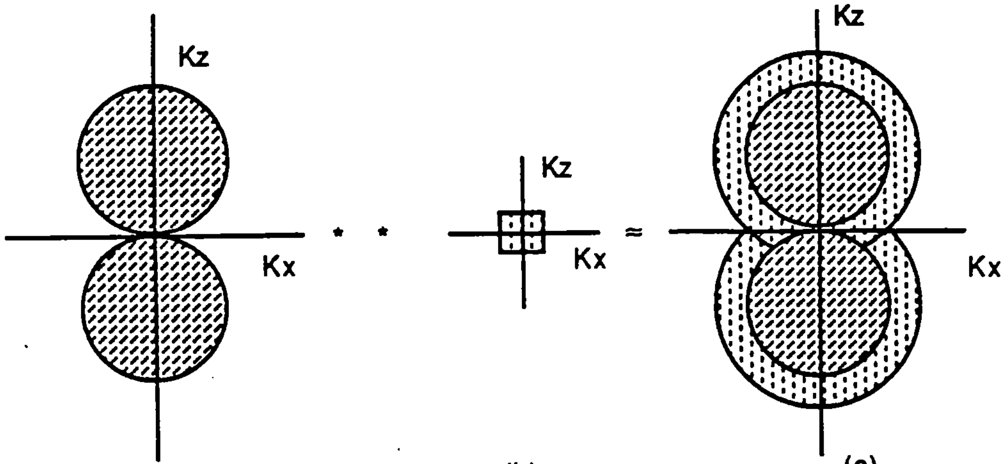

Figure 3: Effect of the prior information in the frequency domain coverage of the reconstructed object.

where $K_T^2 = k_x^2 + k_z^2$.

The effect of substituting these equations into 4 is to interpolate the data from the circles (Fig. 2) where it is originally acquired in the frequency domain to the regular grid (k_x, k_z). After making these substitutions, we obtain the independent term needed for solving the system of equations 9. This procedure contrasts with those proposed by Devaney (1984), Harris (1987) and Wu and Toksõz (1987), where the inversion is performed using directly the data from the circles of Fig. 2 and therefore, a tomographic filter (Jacobian) is needed before the backpropagation.

CONCLUSION

We have presented the way of introducing prior information in diffraction tomography when the problem is considered within the framework of reconstruction in Hilbert spaces. The effect of the prior information is to increase the frequency domain coverage of the reconstructed object. The data needed for the inversion is obtained from the original data by analytical interpolation in the frequency domain and then no tomographic filter is required.

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