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Utilization of mud weights in excess of the least principal stress in extreme drilling environments

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ABSTRACT: We address the theoretical possibility of drilling with mud weights in excess of the least principal stress for cases of particularly severe wellbore instability. Tensile fractures initiate at the wellbore wall at P_{frac}, they link up to form large axial fractures sub-parallel to the wellbore axis at P_{link}, and they propagate away from the wellbore at P_{grow}. In general, our modeling shows that P_{frac} and P_{link} can be maximized by drilling the wellbore in an optimally stable orientation, and P_{grow} can be maximized by using "non-invading" drilling muds, that is, those that prevent fluid pressure from reaching the fracture tip (i.e., if solids in the mud form bridges within the fracture).

1 INTRODUCTION

In this paper we investigate theoretically the circumstances under which it may be possible to drill with mud weights in excess of the least principal stress in extreme drilling environments. To accomplish this we must avoid lost circulation due to the initiation and propagation of hydraulic fractures. We consider the case of an arbitrary-oriented well with a perfect mud cake such that in the absence of hydraulic fracturing, no drilling fluids leave the wellbore. We consider a three fold strategy to increase, to the greatest degree possible, wellbore pressure during drilling. To accomplish this we utilize the facts that: (i) Wellbore pressure at fracture initiation varies with wellbore orientation, i.e., inclination and azimuth (Daneshy 1973, Hayashi et al. 1985, 1997, Peska & Zoback 1995). (ii) As deviated wells are generally not parallel to one of the principal stresses, multiple tensile fractures form at the wellbore wall in "en-echelon" pattern on opposite sides of the wellbore wall (Brudy & Zoback 1993). Wellbore pressure and wellbore orientation determine whether these multiple fractures link up or not (Weng 1993). (iii) When drilling with "high solids" water-based muds, pressures in the wellbore may not reach the fracture tip due to the narrow width of fracture and the bridging of solids within it (Black 1986, Fuh et al. 1992, Morita et al. 1996). Taking account of these facts, we show a theoretical model to estimate the critical pressures which dominate hydraulic fracture initiation and propagation. First, the pressure necessary to initiate fractures at the wellbore wall. Second, that required to link the inclined tensile fractures near the wellbore wall. Third, that required to extend the fracture unstably away from the wellbore. To the degree to which we can identify conditions which raise these pressures above the least principal stress, we can raise mud weights to deal with problems of extreme wellbore instability or in cases of extremely high pore pressure where the difference between the pore pressure and fracture gradient is quite small.

2 FRACTURE MODEL

2.1 *Tensile fracture initiation*

Let us consider stress state in a plane tangent to an arbitrarily oriented wellbore (Fig. 1). In the plane, the tangential stress s_t is given by

$$s_t = \frac{1}{2}\left(s_{zz} + s_{\theta\theta}\right) + \frac{1}{2}\left(s_{zz} - s_{\theta\theta}\right)\cos 2\omega + s_{\theta z}\sin 2\omega \quad (1)$$

where s_t deviates by the angle ω from wellbore axis, and $s_{\theta\theta}$, s_{zz} and $s_{\theta z}$ are stress components at wellbore wall in a wellbore cylindrical coordinate system (r, θ, z) (Hiramatsu & Oka 1962, Fairhurst 1968). Compressive stresses are positive in this paper.

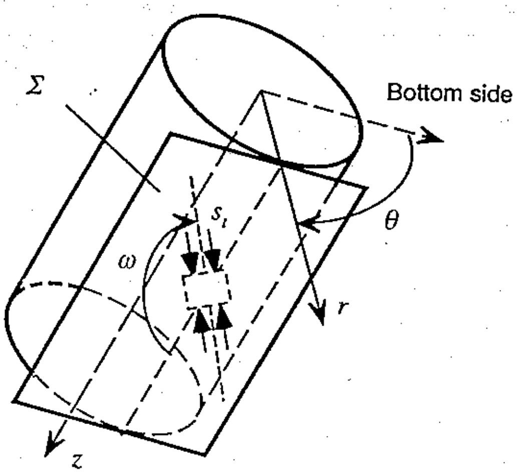

Figure 1. Coordinate system and the tangential stress s_t in a plane tangent to the wellbore (plane Σ).

Note that $s_{\theta\theta}$, s_{zz} and $s_{\theta z}$ change with wellbore orientation and the state of remote stresses, and $s_{\theta\theta}$ also changes with wellbore pressure. According to the Terzaghi effective stress law, the effective stress component of s_t is given by

$$\sigma_t = s_t - P_p \tag{2}$$

where the effective stress is denoted as σ_t, and P_p is the formation pore pressure. The minimum value $\sigma_{t\,min}$ is given by

$$\sigma_{t\,min} = \frac{1}{2}\left(s_{zz} + s_{\theta\theta}\right) - \frac{1}{2}\sqrt{\left(s_{zz} - s_{\theta\theta}\right)^2 + 4s_{\theta z}^2} - P_p \tag{3}$$

Note that $\sigma_{t\,min}$ is a function of θ. If $\sigma_{t\,min}$ goes into tension at certain angles of θ, tensile wall fractures will initiate at these points if $\sigma_{t\,min}$ overcomes the tensile strength of rock, T_0 (Daneshy 1973, Hayashi et al. 1985, 1997, Brudy & Zoback 1993, Peska & Zoback 1995). In other words, the tensile fractures will initiate, when

$$\sigma_{t\,min} = -T_0 \tag{4}$$

T_0 is frequently expected to be very close to zero, since some pre-existing flaws or irregularities are usually present on wellbore wall (Bradley 1979). In this paper, T_0 is assumed to be zero. The wellbore pressure at fracture initiation, i.e., wellbore pressure which satisfies Equation (4), is hereafter referred to the initiation pressure and denoted as P_{frac}.

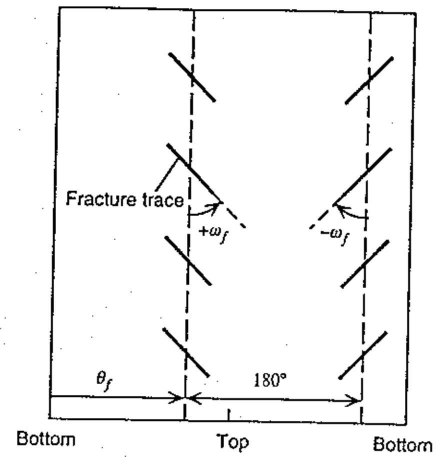

Figure 2. Unwrapped image of an arbitrarily-oriented wellbore with drilling induced tensile wall fractures (after Brudy & Zoback, 1993).

Every two points on the wellbore wall identified by θ and θ+180 ° have the same $s_{\theta\theta}$, s_{zz}, and the same $s_{\theta z}$, but with opposite signs (Hiramatsu & Oka 1962, Fairhurst 1968). Equation (3) shows that the sign of $s_{\theta z}$ does not play a role in value of $\sigma_{t\,min}$, since $s_{\theta z}$ appears in a quadratic form. This means that if $\sigma_{t\,min}$ satisfies Equation (4), the fracture initiates normal to $\sigma_{t\,min}$ at $\theta = \theta_f$ and at $\theta = \theta_f + 180°$. The fractures deviate from wellbore axis by the angle ω_f (see Fig. 2), where

$$\omega_f = \frac{1}{2}\tan^{-1}\left(\frac{2\,s_{\theta z}}{s_{zz} - s_{\theta\theta}}\right) \tag{5}$$

This equation shows that if the fracture angle at $\theta = \theta_f$ is $\omega = \omega_f$, its angle at $\theta = \theta_f + 180°$ is $\omega = -\omega_f$ because of the difference in the sign of $s_{\theta z}$. As a result, the fractures will appear as en echelon sets in wellbores whose axis is inclined to all of the principal stress axes (Brudy & Zoback 1993, 1998). Such fractures have been observed in numerous wellbores (e.g., Brudy & Zoback 1998, Kuriyagawa et al. 1989, Okabe et al. 1996).

2.2 *Fracture link-up*

As long as the wellbore axis deviates from the principal stress directions, the fractures will initiate at the angle ω_f towards the wellbore axis as shown in

Figure 2. As long as the wellbore pressure is lower than a critical value (discussed below), the fractures will propagate over a relative narrow range of angles along the wellbore wall and will not link up due to non-uniform distribution of $\sigma_{t\min}$ (Brudy & Zoback 1993) and interference between adjacent fractures (Weng 1993). A small fracture length along the wellbore wall suggests a small fracture opening at the wellbore wall especially as compared with the opening of large axial hydraulically-driven fractures once the small fractures have linked up. Conversely if the wellbore axis is nearly parallel to one principal stress, the fractures will initiate in parallel to the wellbore axis and form a pair of axial fractures on opposite sides of the wellbore. The opening of the fractures is again relatively quite large and link-up is not a factor. If the fracture is modeled as a so-called PKN fracture (Nordgren 1972), the fracture opening is proportional to the fracture length along the wellbore wall. Thus, if the length of an inclined fracture and an axial fracture is 0.1 m and 10 m, respectively, the opening of the axial fracture is approximately 100 times larger than that of the inclined fracture. If the fracture opening is small, the opening could be sealed by using appropriate drilling muds and significant lost circulation could be prevented.

Now let us consider the critical pressure, P_{link}, at which inclined tensile fractures at the wellbore wall would be likely to link up to form large axial fractures. In principle, the link-up phenomena will be dominated by the stress state in the plane Σ tangent to the wellbore. We disregard here existence of the fracture toughness of the rock because the compressive stresses acting in the plane Σ are so large that its effect on the fracture propagation is expected to be very large compared with the effect of the fracture toughness. The stresses acting parallel and normal to the fracture in the plane Σ are denoted as s_{para} and s_{norm}, respectively. Fluid pressure within the fracture is assumed to be same as the wellbore pressure P_w. In general, the fluid pressure in the fracture must be equal to or larger than s_{norm} in order for a fracture to grow. The following two cases (i) and (ii) can be considered at the fracture growing.

- Case (i) $P_w = s_{norm} > s_{para}$
- Case (ii) $P_w \geq s_{norm}$, $s_{para} \geq s_{norm}$

In this study, s_{para} and s_{norm} are taken to be approximated by

$$s_{para} = s_t \big|_{\omega=\omega_f} \tag{6}$$

$$s_{norm} = s_t \big|_{\omega=\omega_f+90°} \tag{7}$$

Note that s_{para} and s_{norm} will change with P_w in contrast to ω_f which is given by P_{frac}.

2.2.1 *Case (i)*

When P_w reaches s_{norm}, the fractures will start to grow. The fractures will grow by reorienting themselves to normal to s_{para} in this case, because fractures will tend to grow normal to the minimum compressive stress. As a result, the fractures will grow towards adjacent fractures and will link up finally to form the axial fractures. Therefore, the critical wellbore pressure at the link-up, P_{link}, can be estimated by solving the equation $P_w = s_{norm}$.

2.2.2 *Case (ii)*

Under a stress state that leads to $s_{para} \geq s_{norm}$, the fracture will always grow in a direction parallel to the initial fracture. However, as there is the interference between adjacent fractures, the fracture will reorient themselves to deviate from the direction of the initial fracture line under a certain combination of s_{para}, s_{norm} and P_w. Weng (1993) carried out a 2D analysis of the link-up problem of en-echelon fractures taking account the interference between them, and obtained a criterion which defines the link-up phenomena. Although his analysis was conducted originally for the case of fractures that link up in a region away from a wellbore, we adopt here this criterion to approximate fracture link-up near the wellbore wall. The criterion is expressed as

$$\omega_f \leq \omega_{crit} \tag{8}$$

where

$$\omega_{crit} = \sin^{-1}\left\{0.57\left(\frac{\Delta S}{\Delta P}\right)^{-0.72}\right\} \tag{9}$$

and

$$\Delta S = s_{para} - s_{norm} \tag{10}$$

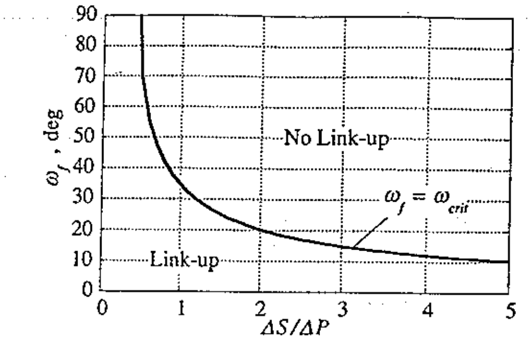

Figure 3. Critical fracture angle controlling the link-up of inclined fractures near the wellbore to form large axial fractures.

$$\Delta P = P_w - s_{norm} \tag{11}$$

The numeric constants in Equation (9) (i.e., 0.57 and 0.72) were obtained by numerical simulations of fracture link-up (Weng 1993). Figure 3 shows the critical angle ω_{crit} for fracture link-up calculated from Equation (9). For the inclined fractures with the angle ω_f above the critical angle curve, the fractures will not link up. For the inclined fractures with ω_f below the critical angle curve, the fractures will link up. However, ω_{crit} is a function of P_w. For initial fractures with a given ω_f, the critical wellbore pressure P_{link} at which the initial fractures just link up to form the axial fractures can be estimated by substituting ω_{crit} with ω_f in Equation (9) and solving the equation for P_w.

Thus, the procedure to estimate P_{link} is summarized as follows;

(a) For a given set of remote stresses and wellbore orientation, estimate ω_f using the formulae presented in the previous section.
(b) Estimate the wellbore pressure P_w which satisfies $P_w = s_{norm}$. Note that s_{norm} is a function of P_w. The estimated P_w is denoted here as P^*.
(c) If $s_{norm} > s_{para}$ at $P_w = P^*$, then $P_{link} = P^*$.
(d) If $s_{norm} \le s_{para}$ at $P_w = P^*$, then P_{link} is estimated by substituting ω_{crit} with ω_f in Equation (9) and solving the equation for P_w.

2.3 *Fracture propagation*

Once the inclined tensile fractures link up, the hydraulically driven fractures will tend to propagate away from the wellbore and reorient themselves to

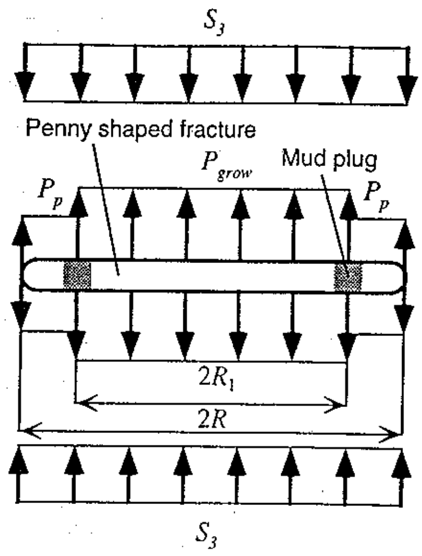

Figure 4. Definition of the invaded zone ($2R_1$) where the internal pressure is P_{grow} and the non-invaded zone (R-R_1) where the internal pressure is P_p during fracture growth.

be normal to the minimum remote stress S_3, since wellbore-induced stresses diminish quickly away from the wellbore (Kirsch 1898). The experiments carried out by Behrmann & Elbel (1990) indicated that this re-orientation can even take place within one wellbore diameter under certain stress conditions. Afterwards, the fractures will propagate keeping their surfaces normal to S_3 (Hubbert & Willis 1957).

Now let us consider the fluid pressure in the fracture which is necessary to drive fracture propagation away from a wellbore. To this end, the fracture is assumed to be large compared with wellbore size, and the fracture is modeled as a penny shaped fracture oriented normal to S_3. Pressure distribution in the fracture is assumed as shown in Figure 4, where we take into account the fact that drilling fluids containing solids may prevent pressure from reaching the fracture tip if the solids effectively plug the fracture due to its narrow width (Black 1986, Fuh et al. 1992, Morita et al. 1996). For simplicity, we assume that the pressure is uniform at P_{grow} except for a zone at the fracture tip where the drilling fluid does not reach and pressure remains equal to the pore pressure P_p. We refer to these zones as the invaded zone and the non-invaded zone, respectively. For this case, Abé et al. (1976) derived a theoretical relationship between P_{grow}, P_p

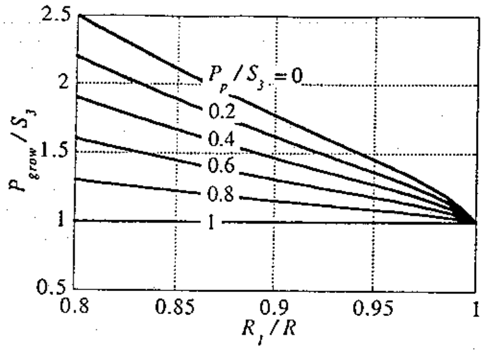

Figure 5. Variation of fracture propagation pressure with size of the non-invaded zone for different ratios of pore pressure to the least principal stress. Note that this is likely a lower-bound estimate of P_{grow}/S_3 as a uniform pressure distribution is assumed in the invaded zone (see text).

and S_3. The relationship is given by

$$\frac{P_{grow}-S_3}{S_3-P_p}=\frac{1}{1-\sqrt{1-\left(\frac{R_1}{R}\right)^2}}\times\left[\sqrt{1-\left(\frac{R_1}{R}\right)^2}+\sqrt{\frac{\pi}{4R}}\frac{K_{IC}}{S_3-P_p}\right] \quad (12)$$

where K_{IC} is the fracture toughness of the rock, and R and R_1 are radius of the fracture and that of the invaded zone, respectively. K_{IC} can be neglected for large size fractures such as the fracture which we consider here (Barenblatt 1962). Therefore we come up with a following equation:

$$\frac{P_{grow}}{S_3}=\frac{1-\frac{P_p}{S_3}\sqrt{1-\left(\frac{R_1}{R}\right)^2}}{1-\sqrt{1-\left(\frac{R_1}{R}\right)^2}} \quad (13)$$

The relationship between P_{grow}/S_3 and R_1/R obtained from Equation (13) is plotted in Figure 5. This figure shows that existence of the non-invaded zone contributes effectively to maximize P_{grow}. The fractures will not grow significantly as long as the wellbore pressure is lower than P_{grow}, even if tensile fractures are initiated at the wellbore. P_{grow} is hereafter referred to as the fracture propagation pressure. Note that Equation (13) gives a lower limit of P_{grow}. If a pressure gradient exists in the fracture due to fluid flow, P_{grow} will be even larger than that given by Equation (13) and shown in Figure 5.

Laboratory experiments by Morita et al. (1996) indicate that P_{grow} is higher with the water base muds because the non-invaded zone was larger than with oil base muds. Fuh et al. (1992) argue that special loss prevention material (LPM) can be utilized to enhance the non-invaded zone and report dramatically increased fracture propagation pressures - by as much as 8 ppg in one well and 3 to 6 ppg in others. In their paper Fuh et al. (1992) presented an equation which gives a theoretical relationship between P_{grow} and the fracture width at the inlet of the non-invaded zone (i.e., W_c), which is basically the same as Equation (12).

3 MODEL STUDY

The analysis presented above suggests that mud weight during drilling can be maximized without loss of circulation in two ways. First, because both the initiation pressure P_{frac} and the linking pressure P_{link} depend on wellbore orientation, P_{frac} and P_{link} can potentially be maximized by drilling in an optimal direction. Second, because drilling with a non-invading mud will inhibit fluid pressure from reaching the fracture tip (i.e., $R_1/R \leq 1$), this will increase P_{grow}/S_3, as illustrated in Figure 5. We simulate here how those critical pressures can be maximized for a model case where S_{Hmax}= 50 MPa, S_{hmin} = 45 MPa, S_v = 55 MPa and P_p = 42 MPa, (S_{Hmax}, S_{hmin} and S_v are the maximum and minimum horizontal stresses and the vertical stress, respectively). This stress state is a normal faulting stress regime and modest overpressure.

Figures 6 (a) – (d) show the variation of P_{frac} and P_{link} with wellbore orientation angles δ and ϕ, (δ is the azimuth of the horizontal projection of the wellbore measured clockwise from the S_{hmin} direction toward the S_{Hmax} direction and ϕ is the deviation of wellbore with respect to the vertical). The fracture angle ω_f and the propagation pressure P_{grow} are also plotted for comparison, although P_{grow} is independent on δ and ϕ. R_1/R is assumed to be 0.95 in these calculations and this value results in P_{grow} = 46.4 MPa in this case. As described above, smaller values of R_1/R would cause P_{grow} to increase. The shaded areas show the "mud window", the difference between the pore pressure P_p (= 42 MPa) and the least principal stress S_3 (= 45 MPa). While drilling, the mud pressure is usually kept in the mud window - above P_p and below S_3, to prevent lost circulation. Hence, the shaded areas represent the allowable range of wellbore pressure for keeping a wellbore stable in a conventional sense. Axial symmetry

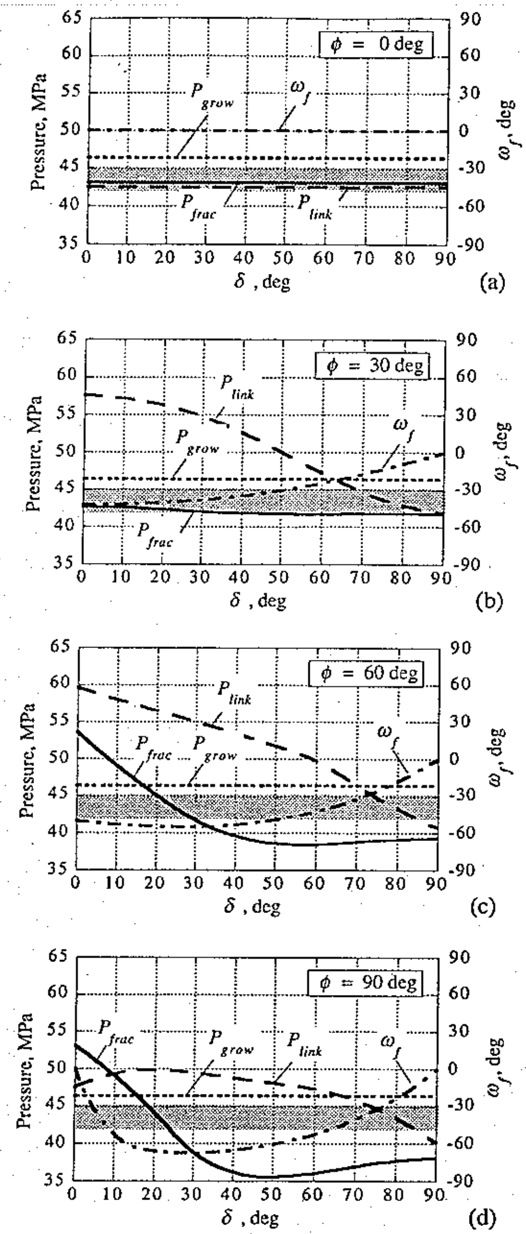

Figure 6. Variation of the critical pressures with wellbore orientation. δ is the azimuth of the wellbore and ϕ is the deviation of wellbore with respect to the vertical (see text).

with respect to δ enables us to present all the results using only the results in the range of δ = 0–90° for simplicity. Figure 6 (a) shows that if a vertical wellbore (ϕ=0°) is drilled, the critical pressures will be on the order $P_{link} < P_{frac} < P_{grow}$. The result suggests that if the wellbore pressure exceeds P_{frac}, the fractures will initiate at the wellbore wall and they will link up simultaneously. However, significant lost circulation will not occur as long as the pressure does not exceed P_{grow} (= 46.4 MPa). The situation will change dramatically as the wellbore inclination increases. While this affect is rather modest, in the case of deviated wells, ϕ = 30°, 60° (see Figs. 6(b), (c)), P_{link} is significantly larger than S_3 for many wellbore azimuths thus demonstrating the possibility of using directional drilling to enhance potential mud weights.

4 CONCLUSIONS

The analysis presented here illustrates the theoretical pressures at which inadvertent hydraulic fracture initiation and propagation might occur during drilling. Drilling with mud pressures greater than the least principal stress is obviously a dangerous thing to do as it presumes that there will be no pre-existing, permeable fractures or bedding planes that intersect the wellbore. Nonetheless, as drilling with mud weights greater than the least principal stress may be sometimes necessary, the analysis presented here can provide insight into how to accomplish this. In the hypothetical example considered, the best way to maximize mud pressures will be to utilize directional drilling to optimize P_{link}. In other cases, maximum mud pressures can be obtained by increasing the non-invaded zone at the tip of propagating fractures using fluid loss additives as noted by previous workers.

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