Generation of Multiple History Matched Models Using Optimization Technique

Manish Choudhary and Tapan Mukerji
Department of Energy Resources Engineering
Stanford University

Abstract

Uncertainty in the geological model presents a key challenge in development decisions. Production data from the field are acquired only at limited locations and are sparse. Time lapse seismic data is available field-wide but has limited resolution. In addition, increasingly production logging data is being recorded in wells, which provide information regarding vertical heterogeneity between wells. The available data set is still sparse for accurately modeling spatial distribution of reservoir properties. And hence, multiple geological realizations exists which match a given production history with varying forecast, all of which should be analyzed for decision making.

In this paper, we test two optimization algorithms for generating multiple history- matched geological models. The reservoir inversion problem has been formulated using optimization technique, with an objective of minimizing the variance between observations and output of numerical models using one, two and all three datasets as described above. Optimization is carried out in reduced model space. Model reduction is achieved by spatial principal component analysis (PCA), where optimization search space is projected to a subspace of much smaller dimension.

Local optimizers often tend to find solutions faster than global methods, though they can get trapped in local minima. Randomly generated multiple initial points can be optimized in parallel to locate multiple models matching history. Hook-Jeeves direct search (HJDS) algorithm, simultaneous perturbation stochastic approximation (SPSA) algorithm has been used for optimization and results are compared with rejection sampler. The minima points identified through optimization represent geological models which are consistent with the production history of the field. In our paper, a synthetic case study is where this methodology has been applied to facies based. The optimization process locates geological models which are consistent with production history but present a varying forecast.
Introduction

Development decisions in the petroleum industry are made with help of long term forecast from simulation models. Numerical simulation models describing field behavior require reliable estimates for various reservoir parameters which cannot be measured directly (Slater & Durrer, 1971). Limited data is available to accurately classify the spatial distribution of reservoir properties, resulting in multiple geological models matching the history dataset. Production data from the field is acquired only at limited locations and is sparse. Time lapse seismic data is available field-wide but has limited resolution. Increasingly production logging data (spinner survey (Kading, 1976)) is being recorded in wells which provide vertical resolution to the model and are increasingly being used for updating geological models (Yoelin & Howald, 1970; Vogelij, Leach, & Kapteyn, 1993; Panda & Nottingham, 2011). Decisions pertaining to selection of best possible development option should be based on performance analysis of all possible subsurface realizations which match the history dataset.

The process of optimization minimizes the error between observed data set and calculated data set by perturbing the input parameters in a systematic fashion. A petroleum reservoir can have close to million unknowns (porosity, permeability, facies etc.) and any optimization in the original space will be inefficient and would have extremely high computation costs. Many authors have proposed formulation of simultaneous matching of production and seismic data as an optimization problem (Huang, Meister, & Workman, 1998; Sarma, Durlofsky, Aziz, & Chen, 2006). (Echeverria & Mukerji, 2009) suggest an easy to implement scheme using principal components to reduce the optimization search space while maintaining geological consistency. The process of optimization would locate a model which has minimum variance across observed data points and estimated values, in the local space (local minima) or across the entire solution space (global minima). This process would only locate one solution model and hence does not provide any mean to capture production forecast uncertainty. This paper builds up on the earlier proposed optimization scheme of (Echeverria & Mukerji, 2009) in reduced dimension, to locate a range of geological models with widely varying production forecast, and hence provided a methodology to capture localized uncertainty around the optimal model.

Methodology

The inversion process is formulated as an optimization problem, with the aim to reducing the error between observations and calculated dataset, similar to the one proposed (Echeverria & Mukerji, 2009). Let $U^n$ denotes the n dimensional space in which the reservoir models are
defined, \( G \) is the set of admissible geological models which closely matches observed data and \( g \) denotes the indicator property (facies) or reservoir property (porosity and/or permeability) which is used to define the geological model. The system of models can be expressed as \( g \in G \cup U^n \). The optimization problem can be defined as in Equation 1

\[
g^* = \arg\min_{g \in G} \left\| O(m) - O_m \right\|^2
\]

where \( O_m \cup U^n \) comprises all observed data in the inversion, and \( O(m) \in U^n \) represents the numerically computed observables for the model \( g \). Varying weights/normalization for different components in the observables (production, seismic, flow survey etc.) can be included in the optimizer function (Euclidean norm in this case) to take into account variable uncertainty in the data acquisition & observations. The optimization problem is ill-conditioned due to much larger number of inversion parameters than measurements \( n \gg m \) and model reduction using principal component analysis have been proposed earlier. Optimization is carried out in subspace of lower dimension (principal components: \( n_R < m \)), statistical information for which is obtained from prior knowledge of the reservoir. The principal components also can be used to randomly generate geologically consistent reservoir model from randomly generated coefficients for each principal component. Local optimizers are strongly dependent on the initial guess but tend to be more efficient than the global optimizer. Multiple optimization run starting from different initial guesses can help to obtain multiple minima across the solution space, while using efficiency of local optimizers.

**Model Description**

**Geology**

A synthetic reservoir model was created to test the optimization algorithm for history matching and locate multiple realizations which match the simulated observed data within given tolerance. The reservoir is a box model made of 75 cells in X–direction, 50 cells in Y-direction and 25 cells in Z-direction. The reservoir was named “Zamba”. The reservoir structure dips towards south at an angle of 20 degrees with respect to horizontal. The reservoir consists of
two facies representing sand and shale. A channel training image was used to generate the reservoir structure using Single Normal Equation Simulation (SNESIM) algorithm of SGeMS\(^1\). Figure 1 shows a schematic of the reservoir structure and the training image used for generating the reservoir facies. Sand is shown as red while background shale is shown in blue. The reservoir has 8 wells placed in 3 rows. The Top row of well consists of 3 producers (Well 1, 2 and 3), Middle row contains 2 producers (Well 4 and 5) and Bottom row contains 3 injectors (Well 6, 7 and 8).

![Reservoir structure (Left) and Channel Training Image (Right). Red color denotes Sand facies and blue color denotes Shale facies](image)

**Petrophysics**

Both sand and shale have been considered porous and permeable in the reservoir model. The porosity in sand and shale facies were populated independently using Sequential Gaussian simulation (SGSIM) algorithm of SGeMS. Porosity variogram in sand was considered anisotropic with continuity along the channel direction, while shale variogram was less anisotropic with maximum, medium and minimum direction comparable with each other. Figure 2 shows the porosity distribution in the reservoir. Figure 3 shows the porosity histogram of the sand and shale facies. The effective porosity in sand varies from 16% to 33% while it varies from 3% to 16% in shales.

Permeability in the grid cells was computed using a porosity permeability relationship. A modified porosity permeability relationship from Norne field was used for the model. Equation 2 and Equation 3 represents the porosity permeability relationship used in the reservoir model for sand and shale respectively.

\[
\log k = 5.095 \phi + 2.0580 \tag{2}
\]

\[
\log k = 14.610 \phi + 0.1745 \tag{3}
\]

**Relative Permeability & Capillary Pressure**

The reservoir model is simulated for water flood scenario with both oil-water contact and gas-oil contact. A three phase relative permeability model was used to model flow across grid cells. Oil-water relative permeability for sand was generated using Corey functions with Corey exponent of 2.75 and 4 for oil and water respectively. The Corey exponent for oil and water in case of shale are 3.25 and 3.75 respectively. Connate water saturation in sand and shale is
assigned as 12% and 30% respectively. The relative permeability and capillary pressure model used in the model are shown in Figure 4.

![Relative Permeability and Capillary Pressure](image)

**Figure 4: Oil – Water relative permeability curve (Left) and Capillary pressure curve (Right)**

**Fluid Properties**

The reservoir simulation model included a three phase fluid system (Oil, Water and Gas). An isothermal black oil formulation was used to describe fluid behavior. Oil in the reservoir was assigned an API gravity of $30^\circ$ with a saturation pressure of 3600 psi. The gravity of the gas in the reservoir was assigned as 0.85. The salinity of water was assumed 50000 ppm. The reservoir temperature was set at $180^\circ$F. The model was initialized with an oil-water contact (OWC) within the structure at 6800 feet. No free gas existed in the model at the time of initialized. The key fluid properties are listed in Table 1

<table>
<thead>
<tr>
<th>Fluid Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil API</td>
<td>$30^\circ$ API</td>
</tr>
<tr>
<td>Gas Gravity</td>
<td>0.85</td>
</tr>
<tr>
<td>Separator Gas Gravity</td>
<td>0.90</td>
</tr>
<tr>
<td>Reservoir Temperature</td>
<td>$180^\circ$ F</td>
</tr>
<tr>
<td>Saturation Pressure</td>
<td>3600 psi</td>
</tr>
<tr>
<td>Water Salinity</td>
<td>50,000 ppm</td>
</tr>
</tbody>
</table>
Rock Physics Model

Rock physics model defines how seismic attributes change due change in pressure and saturation of the rock. An uncemented-sand (or “soft-sand”) is used for sand facies. The model assumes that the sand grains were deposited as a dense random pack of identical spherical grains and average number of contacts per grain between 5 and 9 (Mavko, Mukerji, & Dborkin, 2009). Seismic attributes are defined for this type of setting using Hertz-Mindlin theory (Mindlin, 1949). Sand is assumed to be made of Quartz, Feldspar and fragments, while shale is made of clay and quartz.

Seismic attributes for shale attributes are estimated using Gardner’s power law for ($V_p$) P wave velocity (Gardner, Gardner, & Gregory, 1974) and “Mudrock line” for ($V_s$) S wave velocity (Castagna, Batzle, & Eastwood, 1985). In-situ fluid properties are computed using Batzle-Wang relations (Batzle & Wang, 1992) which are used to compute seismic properties for rock saturated with fluid mixture using Gassmann fluid substitution (Gassmann, 1951). The key parameters of rock physics model are listed in Table 2. Seismic data will be used for future study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand Composition</td>
<td>80% Quartz, 15% Feldspar, 5% Rock fragments</td>
</tr>
<tr>
<td>Shale Composition</td>
<td>85% Clay, 15% Quartz</td>
</tr>
<tr>
<td>Sand Vp &amp; Vs</td>
<td>Uncemented Soft Sand Model</td>
</tr>
<tr>
<td>Shale Vp &amp; Vs</td>
<td>Gardner’s relation ($V_p$) &amp; Castagna relation ($V_s$)</td>
</tr>
</tbody>
</table>

Production Strategy

The “Zamba” reservoir model was simulated for water flood scenario. The top and middle rows of well were assigned as producers. The top row of the wells (Well – 1, 2 and 3) were completed with preformation in all 25 layers, while the middle row of wells (Well – 4 and 5) were completed in top 20 layers. The middle group of wells was constrained at a rate of 13000 bbl. per day, while top group wells were constrained with a maximum liquid rate of 8000 bbl. per day each. A lower rate constraint was assigned on top group of wells to minimize formation of secondary free gas due to pressure drop. All producers had a minimum bottom hole pressure (BHP) constraint 50 psi. Figure 5 shows the location of wells in the reservoir.
Water was injected through the bottom row of wells (Well – 6, 7 and 8) to push the oil to the producer and to maintain producers. The injectors were completed in all 25 layers and were set to inject at a rate of 15000 bbl. per day with a maximum injection pressure of 3500 psi. Injected water replaced the voidage created in the reservoir by the producers thereby maintaining pressure, at the same time pushing the water towards producers. The reservoir model was simulated for 4 years of water flood.

Oil, gas and water production rate were recorded from the well at every month. Production rates from every layer were recorded once every year simulating a production logging survey – spinner survey in the

**Model Decomposition and Reconstruction**

**Principal Component Analysis**

The multistart optimization methodology for history matching described in Section 2 is carried out in a reduced model space. A large set of reservoir models – 10,000 models were generated with the same training image using SNESIM algorithm in SGeMS software. The training image is indicative of the reservoir geology is a prior information available with geoscientists.
Principal component analysis was carried out the matrix of reservoir model to compute the major principal components. The principal components are set of values of linearly uncorrelated variables, such that first principal component has the largest possible variance and each succeeding component in turn has the next highest variance. PCA of the model set resulted in 10,000 principal components. Figure 6 and Figure 7 shows a plot of variance and cumulative variance of first 3000 principal components.

**Dimension Reduction**

Few of the models were reconstructed using limited set of principal components and were visually compared with true model for consistency. Figure 8 shows a reconstruction of layer 5 of the model number 1000 using 30, 70, 1000, 3000 and 5000 principal components. As it can be observed the reconstructed model matches the true model closely when it is reconstructed using 3000 principal components. As it can be inferred from Figure 7, the first 3000 principal components represent 90% of the total variance.
As it can be observed from the Figure 8, models reconstructed with more than 3000 major principal components closely match the true realization. This is in line with observation made by in other papers (Choudhary, Mukerji, & Echeverria, 2011; Echeverria & Mukerji, 2009) which suggests that models reconstructed with 25% of non-zero principal components closely match the true model. Additionally, it can be visualized that model reconstructed with 70 principal components reproduce the significant channels. The higher order principal components only add to finer details to these channels.

**Model Generation**

The computational capacity limits the number of dimensions in which optimization can be performed, hence a smaller subset needs to be selected, which at the same time should be able to reproduce the key features. For the purpose of optimization for this case study, models were reconstructed using first 3000 principal components, while optimization was only performed in the first 70 principal components. Coefficients for each principal component are selected sampled from a uniform distribution between minimum and maximum coefficient value obtained as part of principal component analysis. The minimum and maximum bound for coefficients of the 3000 major principal vectors is shown in Figure 9 along with coefficients for a randomly generated model.
Sand and shale facies is assigned in the model generated using with limited components using a threshold equal to prior sand proportion. Figure 10 represents two dimensional X-Y slices (Layer 1) for three random realizations generated using 3000 principal components.

History

Figure 11: Production history of the Zamba reservoir
The “Zamba” reservoir was simulated using Eclipse® for water injection for a period of 4 years with wells on rate and pressure constraint. Injected water maintains the pressure of the reservoir and pushes the oil towards the producers. The simulated production for 4 years is used as production history for purpose of optimization. Additionally, the layer wise flow rate data for the wells was recorded annually to simulate PLT survey. Figure 11 shows the field production history of the reservoir.

**Optimization**

**Workflow**

An automated workflow was developed in MATLAB® for the purpose of optimization. Facies for the reservoir is constructed using randomly generated coefficients and limited set of principal vectors. The facies model is used an input in SGeMS for populating porosity and permeability in the grid. Porosity for each facies is generated using SGSIM², while permeability is assigned using porosity-permeability relationship. The generated reservoir model is simulated for water flood in Eclipse® and observables compared against history. The mismatch between simulation output and history is used to perturb PCA coefficients, which in turn generates an updated reservoir model. Figure 13 shows a schematic of the workflow.

---

² SGSIM – Sequential Gaussian Simulation
**Observables & Response function**

The observables for reservoir model include both well level data and grid level data. Oil, water and gas phases are recorded for wells at the end of each month, while pressure and saturation data for entire grid is recorded on a yearly basis. The grid level data (pressure and saturation) is used to generate normal-incidence impedance grid which is also used for optimization. Additionally, layer-wise phase rates are also recorded for every well on a yearly basis to simulate PLT spinner surveys.

A weighted sum of the normalized variance of the model response with respect to observed data set is used as a response function for optimization. The weight for each data type can be altered based on data quality. Equation 4 represents the cost function used for optimization.

\[
Response \ Function = \sum_p \omega_p \left\{ \sum_{t=0}^{T} \frac{[R_p(t) - O_p(t)]^2}{O_p(t)^2} \right\}
\]

\(p = \text{Field WC, Well WC and PLT data}\)

Here \(R_p(t)\) denotes the model response for given parameter, \(O_p(t)\) denotes the observed history data, \(\omega_p\) represents the weight for given data type, \(WC\) represents water cut and \(PLT\) represents spinner survey data for wells.

<table>
<thead>
<tr>
<th>Flow Only</th>
<th>Flow &amp; PLT Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Water Cut −10</td>
<td>Field Water Cut − 10</td>
</tr>
<tr>
<td>Well Water Cut − 20</td>
<td>Well Water Cut − 20</td>
</tr>
<tr>
<td></td>
<td>Well PLT Data − 30</td>
</tr>
<tr>
<td><strong>Total - 30</strong></td>
<td><strong>Total - 60</strong></td>
</tr>
</tbody>
</table>

Optimization was performed for two different scenarios - Flow data only and Flow data & PLT data. As the “Zamba” reservoir is primarily under water flood, only water cut at field level and well level was included for purpose of optimization. In case of PLT dataset, the layer-wise phase rates (oil, water and gas) for all eight wells were included for computing the response function. The weights used for all the three scenarios are listed in Table 3.
Optimization Algorithm

Optimization was performed using a pattern search algorithm and a stochastic gradient algorithm. The optimization algorithms used are briefly described in the section below.

Hook Jeeves Algorithm

Hook Jeeves Direct Search (HJDS) is a gradient free optimization method, where approximate numerical slopes are calculated from function calls (Hooke & Jeeves, 1961; Gottfried & Weisman, 1973). The pattern search method consists of sequence of exploratory moves about a base point which, if successful, are followed by pattern moves. The purpose of exploratory moves is to acquire information about the function in the neighborhood of the base point. The base point is perturbed one dimension at a time and function value is evaluated resulting in general into a new base point. If no function reduction is achieved, the step length is reduced and procedure is repeated.

A pattern move attempts to speed up search using information from preceding base points. A translation in the best search direction is made and function is reevaluated. The alternating sequence of exploratory and pattern moves is continued until convergence or if the step length has reduced to specified small value.

SPSA Algorithm

Simultaneous perturbation stochastic approximation (SPSA) is a modified version of the Kiefer-Wolfowitz algorithm (Kiefer & Wolfowitz, 1952), where stochastic gradient for the function is estimated by perturbing one parameter at a time. Kiefer-Wolfowitz algorithm is unfeasible as evaluation of objective function for perturbation in every parameter is impractical in case of history matching.

A stochastic approximation (SA) for minimization uses some sort of random process to select the search direction for every iteration (Robbins & Monro, 1951). SPSA improves on the SA algorithm by simultaneously perturbing all model parameters randomly to generate a search direction for every iteration. The search direction can be random walk but is chosen downhill for every iteration. The algorithm behaves similar to steepest descent direction (Gao, Li, & Reynolds, 2007) and can be implemented with any simulator.
Results

A set of 10 randomly generated initial models was optimized using the optimization algorithms described above. The results for each of the scenario are discussed below.

Flow Only

The response function for "Flow only optimization" was computed using water cut measurement at field and well level. The process of optimization resulted in 7 models with appreciable match. Figure 14 shows the water cut response of the reservoir and individual wells. The blue lines denotes model which match history, grey lines denotes model which did not match history and red points denotes the history dataset.

Figure 13: Water Cut - Flow only Optimization
Red – History, Blue – Models matching History, Grey – Models not matching History

Figure 15 shows the XY slices of the top layer of models which converged during iterations along with the “True Model”. As it can be observed, the converged solutions are quite different from the true solution.
Flow and PLT Data

"Flow & PLT data" optimization used a higher weightage of the PLT data set. The PLT data provided the vertical resolution to the optimization problem. The process of optimization resulted in 8 models with appreciable match. A total of 7 out of 20 models matched the history. Figure 15 shows the water cut of the wells and the reservoir.
The PLT data match for Well 7 - an injector at the end of 4 years of simulation time is shown in Figure 16. Figure 17 shows the PLT data match for Well – 4 (producer) for all three phases. The last figure in the set of 4 figures represents the response of true model. The history-match optimization algorithm is able to capture significant peaks of the PLT dataset.
Figure 18 shows the XY slices of the reservoir for layer 15 and the E-type map generated from the models. Compared to E-type of PLT only case Figure 14, the E-type does not provide greater information about occurrence of sand facies in the reservoir.

Rejection Sampling Comparison

The results of optimization are compared with rejection sampler to qualitatively estimate the efficiency of optimization algorithms. The "rejection sampler" or acceptance-rejection method is called an "exact" sampler as it perfectly follows the Baye's rule. A rejection sampler requires exhaustive evaluation of very large set of prior models. All models which do not match the historical data set are rejected, while retaining subset of models which history (Caers, 2011).
For the purpose of rejection sampling, 10,000 prior models generated for PCA were simulated for water flood using Eclipse® and model response compared with the history. Rejection sampling was carried out for both cases - Flow only and Flow & PLT data. 117 models were selected during rejection sampling of flow only case scenario. 27 models were only selected for Flow & PLT optimization. The E-type of models obtained from rejection sampling is compared with E-type obtained from optimization in Figure 19 – Figure 22.

Algorithm efficiency

Convergence is slow with both the algorithm for a large model used in case study. While, Hook-Jeeves algorithm moves step by step towards the minima, SPSA algorithm uses stochastic gradient computed in randomly generated directions. The convergence for both the algorithms
is shown in Figure 23 for first 40 iterations. The lines in blue denote the flow only optimization scenario while lines in red represent the flow & PLT data optimization.

**Conclusion**

The technique of multi-start optimization in reduced dimension is a simple to use methodology for generating multiple geological models. The resulting models are consistent with production history of the field hence represent possible subsurface realizations. This method thus helps in capturing non-uniqueness and variability in subsurface modeling and is able to locate possible realizations which should be investigated prior to any development decision.

**References**


