Single Loop Inversion of facies from seismic data based on sequential simulations and probability perturbation method

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Abstract

The main objective of this work is to present a new methodology for seismic reservoir characterization that provides fine-scaled reservoir models of facies and reservoir properties, such as porosity, net-to-gross, and, possibly, fluid saturation.

The proposed iterative methodology is based on sequential simulations of discrete variables, namely sequential indicator simulation, and a stochastic optimization technique called probability perturbation method. At each step of the optimization we generate a facies model, distribute reservoir properties, calculate the corresponding elastic attributes through a rock physics model, compute synthetic seismograms and, finally, compare these synthetic results with the real seismic amplitudes. The stochastic optimization technique perturbs the probability distribution used to generate the initial model and obtains the most probable facies model through a relatively small number of iterations.

The method is applied to a real well profile, where three facies have been identified, and finally extended to a real 2D seismic section.
1. Introduction

One of the aims of reservoir modeling is to describe the spatial variability of reservoir properties, facies, and their petrophysical properties, such as porosity, permeability, net-to-gross, and fluid saturation. Reservoir models cannot be built independently from seismic data, because seismic amplitudes are the only available data to constrain reservoir models away from well control. However, seismic data do not provide direct information about reservoir properties but instead reflect the elastic contrasts in the subsurface. For these reasons, several techniques, both deterministic and stochastic, have been developed to estimate reservoir properties from seismic data (Bosch et al., 2010).

Reservoir properties can be estimated from inverted seismic velocities through deterministic or probabilistic techniques (Grana and Della Rossa, 2010). Usually, the probabilistic approach correctly propagates the uncertainty associated with input data and physical model approximations to the posterior probability of reservoir properties. However, the resolution of the estimated properties is the same as that of the seismic data. As a consequence, these methodologies require the integration with geostatistical methods to include seismic inversion results into reservoir models (e.g., Mukerji et al., 2001).

We propose here a new approach which aims at estimating fine-scaled reservoir models in a single loop inversion by combining geostatistical methods, such as sequential simulations (Doyen, 2007) and probability perturbation method (Caers and Hoffman, 2006), with classical geophysical models, such as seismic convolution and rock physics models (Mavko et al., 2009). Single loop inversion is mainly based on the iterative application of a forward model and the inversion step is performed using deterministic or stochastic optimization techniques. A similar approach has been presented in Gonzales et al., 2008, with the target being the direct inversion of facies with the integration of the rock physics model and multiple point geostatistics. In the said method, the optimization is performed directly on the realization and not on the associated probability distribution. In our approach, we propose to combine two geostatistical techniques: Sequential Indicator Simulation (SISim, Journel and Gomez-Hernandez, 1989) to generate several facies models, and a stochastic optimization technique, the Probability Perturbation Model (Caers and Hoffman, 2006), to perturb the probability used in SISim.

Single loop inversion is applied to a well log profile and to a 2D seismic section with the objective of reconstructing the actual facies classification. In this example we integrate into the method a further probability derived from seismic data by means of a traditional Bayesian approach.
2. Method and Theory: Single Loop Inversion

Single Loop Inversion is an emerging technique in seismic reservoir characterization that attempts to directly integrate the petroelastic model and facies classification into the seismic inversion workflow (Figure 1): high-resolution models of facies (or litho-fluid classes) are generated (for example, using Sequential Indicator Simulation, Journel and Gomez-Hernandez, 1989); then rock physics transforms are first applied to the simulated volumes to generate the corresponding volumes of the elastic properties, and then synthetic seismic traces are computed on these volumes by a convolutional model. Finally, thus-obtained synthetic seismic data are compared to real seismic data to evaluate the mismatch; the procedure is iterated by modifying the initial set of models until a good match between synthetic and real seismic amplitudes is obtained. This step is performed here, by using a stochastic optimization technique: the probability perturbation method (Caers and Hoffman, 2006). This approach provides a high-resolution reservoir model which honors seismic data.

The basic structure of the algorithm can be described as follows:
1) We select a random path of simulation; generate an initial realization of facies $F$, namely $i_0(u)$, using SISim and according to the selected variogram; and, then, distribute rock and elastic properties and compute synthetic seismic traces;
2) We perform a seismic-driven stochastic optimization. The objective function is the root mean square error between synthetic seismic ($d_{\text{synth}}$) and real seismic trace ($d_{\text{obs}}$) and it is evaluated as follows:

2a) Compute a new probability, $P(F \mid d_{\text{obs}})$, as a linear combination of the initial realization $i_0(u)$ and the prior probability $P(F)$ of the facies:

$$P(F \mid d_{\text{obs}}) = (1 - r^k) i_0(u) - r^k P(F)$$

2b) Combine the prior probability $P(F)$ and the indicator kriging obtained in SISim $P(F \mid G)$ (i.e., the probability of facies given hard data
for example, well data) to obtain the probability $P(F \mid G, d_{\text{obs}})$, by using the Tau-model (Caers and Hoffman, 2006):

$$P(F \mid G, d_{\text{obs}}) = \frac{a}{(a + bc)}$$  \hspace{1cm} (2)

where $a = (1 - P(F)) / P(F)$, $b = (1 - P(F \mid G)) / P(F \mid G)$ and $c = (1 - P(F \mid d_{\text{obs}})) / P(F \mid d_{\text{obs}})$;

2c) Generate a new model $i_{rs}(u)$ by sampling from $P(F \mid G, d_{\text{obs}})$;

2d) Distribute rock and elastic properties and compute synthetic seismic $d_{\text{synth}}(r^s)$;

2e) Calculate the objective function

$$O(r^s) = \| d_{\text{obs}} - d_{\text{synth}}(r^s) \|.$$  \hspace{1cm} (3)

The optimization step is performed within two nested loops: we change the random seed (i.e., the random path of simulation) until a good match is achieved between the synthetic seismic traces of the trial model and the real seismic traces (outer loop). At each step we perform a 1D optimization on the deformation parameter $rs$ of probability perturbation model to obtain the parameter that minimizes the error between the synthetic and real seismic data (inner loop). If the error of the new model is less than the error of the previous model, we accept the new model and we set $i_0(u) = i_{rs}(u)$, otherwise we change random seed, and we repeat the steps 2a) – 2e). We iterate this procedure until the error is less than a fixed tolerance value.

To speed up the convergence we can include a further probability in the Tau model (step 2b, Equation 2), for example, the probability of facies obtained by Bayesian seismic inversion, $P^*(F \mid d_{\text{obs}})$, where $P^*$ represents the pointwise probability of facies conditioned by seismic data at seismic scale, i.e. at low frequency (Grana and Della Rossa, 2010). The probability of facies at seismic scale can be obtained by using other methods and it can be conditioned by different data, for example seismic impedances $P^*(F \mid I_p, I_s)$, or AVO properties $R_0$ and $G_r$, $P^*(F \mid R_0, G_r)$ (see Mukerji et al., 2001). This step allows one to account for a secondary information, the probability of facies conditioned by seismic, which increases the convergence speed. This probability can be integrated into the workflow in different ways: additional probability in Tau-model, collocated cokriging, or Bayesian updating in SISim. In our example we used the Tau-model (Equation 2).

2.1. Seismic facies probability derivation

The probabilistic approach to seismic facies classification (to derive the probability of facies at seismic scale, $P^*(F \mid d_{\text{obs}})$) and petrophysical properties estimation consists of three main steps: seismic inversion to recover elastic attributes
from seismic amplitudes; estimation of petrophysical from elastic attributes; and facies classification to classify seismic facies from petrophysical properties. This method is based on different physical-mathematical models: rock physics models, seismic convolution, and linear approximations of Zoeppritz equation. These models cannot be deterministically inverted because the solution is not unique, which requires a probabilistic approach. The proposed methodology overcomes the common assumption of Gaussian distribution of petroelastic properties by using Gaussian Mixture Model (Grana and Della Rossa, 2010).

A full Bayesian approach has been adopted, based on the integration of the probabilities obtained from Bayesian seismic inversion and statistical rock physics model. The methodology can be summarized into 3 steps. First of all, a Bayesian seismic inversion is performed (Buland and Omre 2003) to obtain the probabilities of impedances from seismic amplitudes, relying on a linearized equation for reflectivity coefficients computation and a convolutional model (step a). Then a petrophysical properties probabilistic characterization is applied (Grana and Della Rossa 2010) to estimate the probability of porosity, clay content and water saturation (step b) by integrating the statistical rock physics model with the probabilities of impedances obtained from Bayesian inversion. Finally a probabilistic litho-fluid characterization is performed (Grana and Della Rossa 2010): the estimation of litho-fluid classes probabilities conditioned by seismic (step c) can be obtained combining petrophysical properties probabilities (step b), log-facies classification and seismic information from Bayesian elastic inversion (step a). The described approach allows to propagate uncertainty from seismic to facies domain and includes various sources of uncertainty: seismic low resolution, scale changes, model approximations, heterogeneity and natural variability.

In the following we will use $d_{obs}$ to indicate seismic data, $R$ for petrophysical properties (porosity, clay content and water saturation), $m^f$ for the elastic properties (impedances) at fine scale, $m^c$ for the elastic properties at coarse scale (seismic impedances), and $F$ for the litho-fluid facies. From elastic inversion of seismic data we obtain $P(m^c \mid d_{obs})$ at each point of the 3D volume, assuming that $m^c$ is distributed as a multivariate Log-Normal distribution. By means of statistical rock physics model we can estimate the probability of petrophysical properties conditioned by high resolution elastic properties $P(R \mid m^f)$ and the probability $P(m^f \mid m^c)$ including the probabilistic upscaling step. Finally we combine the three probabilities by means of the following equation (Grana and Della Rossa 2010):

$$P(R \mid d_{obs}) = \int_{R^f} P(R \mid m^f) P(m^f \mid m^c) P(m^c \mid d_{obs}) dm^f dm^c$$

(4)

The statistical rock physics model is based on the assumption of Gaussian Mixture distribution of petrophysical properties, which is a reasonable assumption whenever petrophysical attributes describe different litho-fluid classes features. If this is the case, the weights of the mixture can be interpreted as the probability of litho-fluid classes occurrence. From the posterior probability of petrophysical properties we can
infer the median or the maximum a posteriori of the probability of the petrophysical attributes. The methodology can be extended to the discrete domain, for litho-fluid classification based on seismic data and the results of Bayesian petrophysical inversion are used to predict litho-fluid classes. The posterior probability of litho-fluid classes has been computed as:

\[
P^* (\mathbf{F} \mid \mathbf{d}_{\text{obs}}) = \int_{\mathbf{R}} P(\mathbf{F} \mid \mathbf{R}) P(\mathbf{R} \mid \mathbf{d}_{\text{obs}}) d\mathbf{R} \tag{5}
\]

where the rock physics likelihood \( P(\mathbf{F} \mid \mathbf{R}) \) is calibrated using well data: petrophysical curves and log facies classification. The final results of the proposed methodology are the probability volumes of seismic litho-fluid facies.

In the proposed approach the probability \( P^*(\mathbf{F} \mid \mathbf{d}_{\text{obs}}) \) is integrated as an additional information in the Tau model:

\[
P(\mathbf{F} \mid \mathbf{G}, \mathbf{d}_{\text{obs}}) = a^2 / (a^2 + bcd) \tag{6}
\]

where \( d = (1 - P^*(\mathbf{F} \mid \mathbf{d}_{\text{obs}})) / P^*(\mathbf{F} \mid \mathbf{d}_{\text{obs}}) \).

3. Example

The methodology has been applied to an oil-saturated clastic reservoir, using partial-stack seismic data and well-logs data coming from two wells of the field. Four angle stacks are available, in addition to a complete set of well logs and interpreted curves at the calibration well location. First, the method has been tested at the well location to predict the facies distribution from seismic data. A litho-fluid classes classification has been performed using sedimentological information, core analysis, and clustering techniques. The actual classification consists of 3 litho-fluid classes: oil sand, water sand, and shale; and it has been estimated using petrophysical curves (porosity, clay content, and water saturation).

The results of the 1D application are shown in Figure 2, where we reconstruct the litho-fluid classification at the well location. The “low frequency” probability of facies, \( P^*(\mathbf{F} \mid \mathbf{d}_{\text{obs}}) \), conditioned by seismic data (Figure 2, left) has been computed following the approach proposed in Grana and Della Rossa (2010). The initial model has been generated using SISim with an exponential variogram model fitted on the experimental variogram of the actual classification. The number of iterations required to obtain a good match with the actual classification depends on the quality of the seismic data (signal-to-noise ratio), and on the extent of the hard data.
Figure 2 Single Loop Inversion results: (from left to right) litho-fluid class probabilities obtained from Bayesian seismic inversion and classification (Multi-step inversion), actual litho-fluid classification, initial model used in Single Loop Inversion, and optimized model.

Figure 3 Single Loop Inversion results: (from left to right) actual litho-fluid classification, four realizations (obtained after 25 iterations), cumulative frequency of 25 simulations, and litho-fluid classes frequency.

The integration of the secondary probability information reduces the number of iterations by a factor of 5. Here we show the result after 100 iterations (Figure 2,
right), but acceptable results can be obtained even with a fewer number of iterations. We then generate several models with this technique using a fixed number of iterations, 25 in the following example. By running the same methodology several times, starting with different initial models and following different random paths in the sequential simulation steps, we can obtain several models and approximate the posterior probability from the frequency of occurrence of each classified facies (Figure 3). The cumulative frequency distribution of facies (Figure 3) shows a good match with the actual classification at the well location. Finally, in Figure 4 we show some of the computed properties associated with the optimized facies models: synthetic seismograms, upscaled P-wave velocity, and upscaled porosity after 25 iterations.

We point out that the method cannot resolve the thin layers below the resolution of seismic data; however, the main advantage of this method, as compared to traditional classification of seismic facies, is that it provides detailed facies models with the same variogram as that of the actual classification and with the same seismic signatures.

![Figure 4](image)

**Figure 4** Seismic attributes and rock properties distribution from 25 simulations: (from left to right) seismic amplitudes (red curve is the actual seismic trace), P-wave velocity, and porosity.

We then extended the methodology to the 2D seismic section shown in Figure 5. Due to the low signal to noise ratio, only two angle stacks have been used (the near
stack corresponding to 20° and the far stack corresponding to 44°). As in the 1D application, the posterior probability of facies, $P^*(F \mid d_{obs})$, at seismic scale, has been derived using a full Bayesian approach (Grana and Della Rossa, 2010). The maximum a posteriori of this probability distribution has the same resolution of seismic data. However we integrated this probability as secondary information in Single Loop Inversion to improve the convergence of the algorithm. The variogram of the facies has been estimated using information from nearby fields of the same area.

The proposed methodology has been applied in this 2D case by extending the sequential simulation algorithm to the 2D section. The main result of this study is the optimized facies model. This result honors the prior information and the spatial continuity of the data. Furthermore, all the optimized realizations (an example is shown in Figure 6) honor the seismic data. The so obtained facies model has a higher resolution than the model obtained from the maximum a posteriori of the probability of facies directly inferred from seismic.

Figure 5 2D seismic sections passing through 2 wells, well A (calibration well) and well B: (top) angle stack 20°, and (bottom) angle stack 44°.
4. Conclusions

We presented a new methodology for facies and reservoir properties modeling which combines traditional geophysical models with geostatistical methods. The use of sequential simulations allows us to generate high resolution models, while probability perturbation method guarantees that the optimized model matches the real data with a fixed tolerance. The main advantage of this technique is that it provides high resolution models of facies and the associated properties. The method is fast, especially if secondary information is provided. This secondary information can be obtained from seismic data through other techniques. However if the secondary information is not taken into account, the convergence could be quite slow, especially in complex sequences of thin layers.

The application to the real well data shows that the methodology can be applied to complex reservoir with good results, even in thin-layer sequences.
References

- Doyen, P., 2007, Seismic reservoir characterization: EAGE.