Seismic velocity tomography with co-located soft data

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Abstract

In addition to uncertainties in different data acquisition methods, lack of sufficient data limits our ability to image the Earth’s subsurface. Consequently, the lack of certainty in the mapped image of the subsurface may lead to inaccurate geological interpretations for oil exploration or reservoir monitoring purposes.

A major part of these inaccuracies in seismic imaging is caused by uncertainties in wave-propagation velocity in the subsurface. Precise estimation of subsurface velocity is a requirement for high-quality imaging and reservoir modeling. Various techniques are developed to address the velocity-analysis problem for simple or complex geological structures in the subsurface. In areas with complex structures, and significant lateral velocity variations, estimating a geologically reasonable velocity remains an issue. In these areas, even advanced techniques such as seismic tomography are limited in yielding acceptable velocity estimation.

Reducing uncertainty by using co-located soft data can improve velocity estimates. The soft data can be given by either a non-seismic survey or training images provided by statistical methods. The challenge is the lack of analytical relationships between the different properties exploited by different geophysical techniques. Since all different surveys map the same geological structure in the subsurface, a measure of structural similarity between the two given data fields can create the link between different types of data. Cross-gradient functions as one of these structural similarity measures can be used to integrate different types of data. In this study, I aim to integrate the soft data in the seismic tomography problem to improve the subsurface velocity estimation.

1 Introduction

Seismic data contains a wide range of different uncertainties which directly affects the quality of the seismic images. Previous studies have tried to extract more information from raw seismic data to reduce the uncertainty in the seismic-imaging problem (Yilmaz 2001, Aki and Richards 2002). Since velocity analysis
plays a fundamental role in seismic imaging, uncertainties in velocities lead to significant inaccuracies in seismic images. Without an accurate velocity, seismic reflectors are misplaced, the image is unfocused, and seismic images can easily mislead earth scientists (Claerbout 1999; Clapp 2001). Defining a reliable velocity model for seismic imaging is a difficult task, especially when sharp lateral and vertical velocity variations are present. Moreover, velocity estimation becomes even more challenging when seismic data are noisy (Clapp 2001).

In areas with significant lateral velocity variations, reflection-tomography methods, where traveltimes are mapped to slowness, are often more effective than conventional velocity-estimation methods based on measurements of stacking velocities (Biondi 1990; Clapp 2001). However, reflection tomography may also fail to converge to a geologically reasonable velocity estimation when the wavefield propagation is complex.

Unfortunately, the reflection-tomography problem is ill-posed and underdetermined. Furthermore, it may not converge to a realistic velocity model without a priori information, e.g., regularization constraints and other types of geophysical properties in addition to seismic data (Clapp 2001). Better velocity estimation can be achieved by integrating co-located soft data, such as non-seismic geological data, in the reflection-seismic tomography problem.

The last missing part is an analytical relationship between different measured geological properties. Besides the conventional probabilistic relations, similarity-measurement tools can be used to enforce the structural information contained in soft data for seismic velocity estimates. Based on these tools, differences in two images are classified as structural differences and non-structural differences. Since gradient fields are a good choice for geometrical (structural) comparisons, the cross-gradient function is one useful similarity-measurement tool. This is true because the variations of geophysical properties can be described by a magnitude and a direction (Gallardo and Meju 2004, 2007).

Here, I use the cross-gradient function to integrate a given set of soft data—the resistivity field measured by magnetotelluric (MT) sounding in our case—into the reflection-seismic tomography problem. This integration requires consideration of differences in frequency in seismic and resistivity data. In the following sections I study the behavior of cross-gradient functions in different cases, followed by an overview of how an understanding of these differences can be used to improve velocity estimates given by seismic tomography.

2 The cross-gradient function: a structural similarity measure

As mentioned in previous section, integration of soft data into the seismic tomography problem can reduce model uncertainty and result in a better velocity estimation, especially in complex areas. Among the techniques for integrating different types of geological data, structural similarity-measurement tools may
be a good choice for our tomography problem. This is true because different geological acquisition methods are probing the same structures in the Earth’s subsurface. The cross-gradient function is one tool that measures the structural similarity between any two fields. Following Gallardo and Meju (2004), we can define the cross-gradient function for the tomography problem as

\[ g = \nabla r \times \nabla s, \]

(1)

where \( r \) and \( s \) can represent any two model parameters. In our case, they represent resistivity and slowness in our case, respectively. Zero values of the cross-gradient function correspond to points where spatial changes in both geophysical properties, i.e., \( \nabla r \) and \( \nabla s \), align. However, the function is also zero where the magnitude of spatial variations of either field is negligible, e.g., where either property is smooth. Note that the cross-gradient function is a non-linear function of \( r \) and \( s \) if both are unknowns. In a 2-D problem, \( g \) simplifies to a scalar function at each point, given by

\[ g = \frac{\partial s}{\partial x} \frac{\partial r}{\partial z} - \frac{\partial s}{\partial z} \frac{\partial r}{\partial x}, \]

(2)

where the model parameters are given in \( x - z \) plane. To compute the cross-gradient function, we can further simplify it by using first-order forward-differences approximation of the first derivative operators.

Figures 1(a) and 1(b) show the smooth Marmousi synthetic 2-D velocity model (Versteeg and Grau, 1991), a well-known 2-D synthetic dataset, and auto-gradient of it, respectively. Note that, the cross-gradient of a field with itself should be zero everywhere; however, since the figures are prepared with a first-order linear approximation of the cross-gradient function, it is not zero, especially in areas with sharp edges.

Although we expect different types of geological surveys to result in similar structural maps, in practice each survey method maps the subsurface through different filters and frequency contents. Typical frequencies in magnetotelluric data are much lower than those of seismic data (Kaufman and Keller, 1981). This difference in the frequency content of two fields may affect how the cross-gradient represents the structural similarity of two fields. To investigate the effect of different frequency content, Figure 1 is prepared where the cross-gradient of Marmousi velocity model and a smooth version of it are computed. In Figure 1(d) the smoothing factor is increased, which is equivalent to a lower cut-off frequency for lowpass filter. Note that because of the relatively sharp edges in the original velocity model, the cross-gradients in Figure 1 seem to include some structure as well as higher amplitudes with respect to Figure 1(b). However, this synthetic example is an extreme case of complexity and sharp edges. In simpler cases such as the Pillow model, which is shown in Figure 2(a) the cross-gradient with the smooth version of the velocity model leads to an acceptable similarity map (See Figure 2). The amplitude may be improved by using a higher-order linear approximation of the cross-gradient computation. These figures in general imply that the cross-gradient function can be used as a constraint for joint
data inversion problems or to integrate a priori information from other fields into the seismic tomography problem.

![Figure 1](image-url)

Figure 1: Frequency sensitivity of the cross-gradients function: (a) The Marmousi velocity model, (b) its auto cross-gradient. Cross-gradient of the Marmousi velocity model and its (c) smooth and (d) very smooth copies.

### 3 Reflection seismic-tomography

By definition, tomography is an inverse problem, where a field is reconstructed from its known linear path integrals, i.e., projections (Clayton 1984; Iyer and Hirahara 1993). Tomography can be represented by a matrix operator $\mathbf{T}$, which integrates slowness along the raypath. The tomography problem can then be stated as

$$\mathbf{t} = \mathbf{T} \mathbf{s},$$

where $\mathbf{t}$ and $\mathbf{s}$ are travel time and slowness vector, respectively (Clapp 2001). The tomography operator is a function of the model parameters, since the raypaths depend on the velocity field. Consequently, the tomography problem is
non-linear. A common technique to overcome this non-linearity is to iteratively linearize the operator around a \textit{a priori} estimation of the slowness field $s_0$ (Biondi, 1990; Etgen, 1990; Clapp, 2001). The linearization of the tomography problem by using a Taylor expansion is then given by

$$t \approx Ts_0 + \frac{\partial T}{\partial s} \bigg|_{s=s_0} \Delta s.$$  

Here, $\Delta s = s - s_0$ represents the update in the slowness field with respect to the \textit{a priori} slowness estimation, $s_0$. Equation (4) can be simplified as

$$\Delta t = t - Ts_0 \approx T_L \Delta s,$$  

where $T_L = \frac{\partial T}{\partial s} \bigg|_{s=s_0}$ is a linear approximation of $T$. A second, but not lesser, difficulty arises because the locations of reflection points are unknown and are function of the velocity field (van Trier, 1990; Stork, 1992). Clapp (2001) attempts to resolve some of the non-linearity issues with the introduction of a new tomography operator in the tau domain and use of steering.

Figure 2: Frequency sensitivity of the cross-gradients function: (a) The Pillow velocity model, (b) its auto cross-gradient. Cross-gradient of the Pillow velocity model and its (c) smooth and (d) very smooth copies.
filters. In addition to geologic models, other types of geophysical data can also be extremely important. In the following section, I show how the cross-gradient function can be used to add constraints to the seismic tomography problem.

4 Application of cross-gradient function in seismic tomography

Figure 3 shows the resistivity and corresponding velocity of a synthetic 2-D model with two anomalies (fast and slow) in a constant background. The velocity profile is computed using the Archie/time-average cross-property relation (Carcione et al., 2007) with arbitrary parameter values.

![Figure 3: Synthetic sinusoidal model with (a) two velocity anomalies and corresponding (b) resistivity model.](image)

Availability of an accurate estimate of the electrical resistivity profile suggests the use of the cross-gradient function as a constraint for the reflection-seismic tomography problem. In this case, we can write the cross-gradient function given in equation 2 as a linear operator $G$ on the slowness field, $s_0 + \Delta s$. We can then extend the linearized tomography problem by employing $G$ as an additional constraint. The objective function, $P(\Delta s)$, of this extended problem becomes

$$P(\Delta s) = ||\Delta t - T_L \Delta s||^2 + \epsilon_1^2 ||G(s_0 + \Delta s)||^2,$$

where $\epsilon_1$ is a problem-specific weight.

Figures 4(a) and 4(b) show the estimated velocity for the synthetic model with two anomalies with steering filters and cross-gradient constraint, respectively. Although this synthetic model may not be the best to start with, the enforced structure is clearly visible in Figure 4(b). Note that these figures are initial results from the tomography code. I expect to improve the results by both moving to a simpler case of synthetic model and integrating a second laplacian constraint to the tomography problem with cross-gradient constraint.
The important advantage of using the cross-gradient function instead of steering filters may not be very clear in this synthetic example. Steering filters are most effective for continuous anomalies with smooth boundaries. However, in the case of sharp boundaries, e.g., Gaussian anomalies or salt boundaries, the cross-gradient function is better able to handle the seismic tomography problem. Consequently, the cross-gradient function seems to be a suitable option in the tomography problem when steering filters are not effective. This is true because steering filters assume a priori knowledge of the model parameters, while the cross-gradient function uses the co-located soft data field to build this information.
5 Future work

This method may lead to improved subsurface interpretations in regions with more than one type of data acquisition. A marine field example of this case is provided by WesternGeco company, where a seismic survey is supported with a co-located magnetotelluric (MT) resistivity acquisition. Figure 5 shows a CMP gather of the seismic in the same survey area and a slice of the inverted resistivity field from the MT survey, respectively. I hope to improve the velocity estimations given by seismic data itself by including the co-located smooth resistivity map in the tomography problem. Note that the frequency content of seismic and resistivity are different, and the resistivity field provides only a low frequency estimation of the subsurface structure. However, we hope to enforce a reasonable geological structure on the output of seismic tomography problem by using this smooth image as the constraint.

This method is also extendable to seismic tomography constrained by training images, where we can also aim for different realizations of the velocity model by altering the co-located data or training image.

References


URL http://link.aip.org/link/?GPY/72/E193/1


Figure 5: Field data by WesternGeco company: (a) A seismic CMP gather from field data and (b) Inverted resistivity map from MT survey.