Construction of Hybrid Geostatistical Models Combining Surface Based Methods with Multiple-point Geostatistics: Use of Flow Direction and Drainage Area

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Abstract

The use of hybrid techniques aims at constructing new models which enable us to overcome the existing limitations in the reproduction of curvilinear structures. Improvements on a deepwater turbidite reservoir model combining several primary modeling methods are presented. In this approach, multiple points geostatistics (MPS) plays an important role in simulating and conditioning the realizations. Representation of flow direction and calculation of upslope areas on the rectangular grid elevation model are used to determine the drainage basin and MPS simulation area (SA). The SA is then used to relate the previously simulated surface with the current geological event being simulated. The procedure is based on representing flow direction as a single angle taken as the steepest downward slope on eight triangular facets centered at each grid point. For a given anchor point joining the channel to the lobe in the channel-lobe parameterization, we obtain the influence and dependence area in a sequential process. The contour of the newly generated SA along with the cumulative density functions (CDF) retrieved from a process-based model output are employed to introduce variability to the object-based generated training images (TIs). These images are used in the MPS simulation. This procedure is repeated as many times as lobes are to be simulated, only updating the current base topography. The improved model emphasizes using more realistic geological rules, especially on lobe orientation and erosion caused during the deposition of the geobodies. The lobe erosion process is simulated in such a way that flow direction and special topographic features related to the flow erosion power are accounted for. Objective rejection rules are taken into account in the implementation of the model. This gives rise to an automatic and user independent algorithm. This approach is potentially capable of representing the internal features of the geobodies. Finally, strategies on how to condition the model to facies and thicknesses are discussed.

1. Introduction

The proper spatial characterization of subsurface properties that control the movement of reservoir fluids is a key aspect of the decision making process involved in reservoir managing.
The property values and spatial distributions are considered as data by the reservoir simulator, hence the results obtained are fully dependent on the input data. Modeling important spatial features is a challenging area that has been explored in multiple ways. Classic geostatistics deals with the problem using two-point correlations through the variogram. These algorithms are computationally efficient, but the realizations lack realism. Multiple-point algorithms such as SNESIM (Strebelle, 2002) and FILTERSIM (Zhang et al., 2006) can reproduce simple curvilinear features with relative success, but currently are far from being able to reproduce complex features such as the ones found in turbidite outcrops. Process-based methods generate very realistic realizations by solving the flow and sediment transport equations, but only one realization can be obtained using an initial set of parameters and boundary conditions, and the simulations cannot be conditioned to hard data, since the realizations are constructed forward in time. Besides, obtaining each realization is extremely expensive in terms of the computational time. The approach presented here, based on the work of Michael et al., (2008), breaks down the problem of simulating complex geological features into a set of smaller problems that can be tackled using the most suitable approaches. The goals of hybrid models are to speed up the simulation process, generate multiple realizations for a fixed set of initial parameters, and be possible to condition to hard data. A workflow on how the hybrid approach should be addressed is proposed and the initial deepwater turbidite hybrid model by Michael et al. (2008) is modified to account for new aspects proposed in the methodology. One of the key new aspects is that the changes brought about by the previously simulated topography to the simulation of the new lobe are incorporated in the modified model, using surfaced-based modeling. Channels directions and catchment areas are computed using the method proposed by Tarboton (1997).

2. Hybrid Modeling Workflow

Figure 1 schematizes the methodology on how hybrid modeling approach should be faced for simulating specific depositional environments. First, we start off understanding the deposition environment and identifying the structures that have to be considered, these are the structures that play a role in the flow simulation. Once the geobodies that we want to model are identified, we continue with their parameterization. These parameters must fully describe the shape of the geobodies and be able to be obtained from any related source of data such as geologic studies or process-based simulation. The parameter values are drawn from the CDFs previously constructed using Monte-Carlo simulation.

![Figure 1: Hybrid modeling approach workflow](image-url)
The realization generation follows a sequential process that starts with the simulation of major features (i.e., lobes, channels, etc.) and continues with the intermediate features (i.e., intermediate grained units, levees, etc.). Then the two-dimensional realization (in most of the cases obtained using MPS algorithms) is post-processed to achieve the final thickness map which will be stacked on the current base surface. Consideration of the geologic processes must be accounted for in this part. Depositional layers will add thickness to the digital elevation model (DEM), whereas erosional processes will cut into the layers underneath in such a way that part of previous historical events will be removed. All this sequential process will be repeated as many times as the number of major geological features we want the realization to have.

3. Deepwater Lobe Reservoir Workflow

Turbidite systems characterization has gained importance as offshore exploration becomes more popular and feasible. Attempts to model complex turbidite systems using 2-point statistics have found difficulties in reproducing the complex features shown by outcrops and known to be present in turbidite systems. After 2-point statistics, object-based techniques were the immediate approach to the problem (Haldorsen and Lake, 1984; Haldorsen and Chang, 1986). Recently, surface-based (Deutsch et al., 2001; Pyrcz and Deutsch, 2003; Pyrcz et al., 2005) and ruled-based techniques using MPS (Michael et al., 2008) have been introduced, but there has not been any approach that combines these last two techniques. The approach herein presented, based on work of Michael et al., (2008), uses surface-based simulation to determine feasible areas on which the MPS simulation of geobodies is performed. The usage of MPS aims at obtaining an algorithm able to generate realization in a fraction of the time required by process-based approach (this is, able to run in few hours), reproduce curvilinear features, and possible to be conditioned to hard data.

Assuming the availability of CDFs for the parameters that describe the geometries of the geobodies we want to modeled, the combined surface-based and MPS approach to simulate turbidite lobes deposition (Figure 2) is as follows:

- Drawing of the anchor point location according to a Poisson process with a spatially variable intensity function (Lantuejoul, 2002). The so-called P-field (intensity function) changes after every major sedimentation event (lobe deposition).
- Computation of the influence area corresponding to the anchor point previously drawn from P-field using the infinite directions algorithm (Tarboton, 1997).
- Calculation of the dependence area relative to the points that are in the path that the flow from the anchor point would follow as it drains down respecting the infinite directions algorithm.
- Determination of angle range of plausible geobodies orientation considering the furthest n-percentile of points on the boundaries of the initial simulation area (dependence area) to the anchor point.
- Construction of object-based realization of the lobes considering the variability incorporated by using randomly drawn parameters from the CDFs available and the valid
angle range previously obtained. These object based realizations are used subsequently as training images in the MPS simulation of the shape of the geobodies.

- Final simulation area obtained by combining the initial simulation area with the TIs obtained in the previous step.
- MPS simulation of 2-dimensional geobody shapes. In the case when hard data are available, the simulation area and the MPS simulation are constrained to these data.
- 2D to 3D transform of MPS realization (one thickness and one erosion map for each accepted MPS realization).
- Stacking of the thickness and erosional maps on the surface being currently simulated.
- Presence or absence and thickness simulation of intermediate fine-grained unit.

Figure 2: Deepwater Lobe Reservoir Workflow (from Michael et al., 2008)

4. Construction of CDFs from Process Based Model

The statistics required to build the CDFs were obtained from a turbidite process-based model output by Tao Sun (left picture in Figure 3). After analyzing the model, it was possible to infer that most of the important features in the flow simulation can be characterized as lobes and channels. These channels join the sediment source to the lobe anchor point (red start in Figure3). Besides lobes and channels, the presence or absence of a thin fine-grained layer between lobe depositional events was modeled as well.
Figure 3: Process-based model (left). Lobe length and width (center). Channel width (right). (Modified from Michael et al., 2008).

Lobe formation did not occur immediately after the underlying lobe deposition ended. Considering that, a CDF of the elapsed time after each lobe deposition was built. If this time was longer than a given threshold, the intermediate grained unit was modeled assuming a constant depositional rate per year; otherwise, the following lobe depositional event was modeled without considering a layer in between. It is really important to account for these thin layers since the grain size of particles that are contained in it are much smaller than the average of the model, converting them into flow barriers.

5. Lobe and Channel Parameterization

The two dimensional shape of the lobe is fully parameterized by the width and length of the geobody (lemniscate). Even though this parameterization does not capture the geobody shapes in detail, it is a good approximation of the representation of the geologic structures that most control fluid flow. The following equations describe the boundaries of the lobes:

\[ r = a \cos(2\theta), x = r \cos(\theta), y = b \sin(\theta) \]

where \( a \) is the length, \( w \) is the lobe width, and \( b = 1.8(w/a) \). The angle \( \theta \) is given by the lobe orientation which is related to the base topography by the relative positions of the drainage basin with respect to the anchor point. The channels are described by their initial points (sediment source), end points (anchor points), and channel widths (drawn from a CDF).

6. Flow Direction Computation

Multiple algorithms have been developed for the calculation of flow directions. Among the most popular and widely used is ‘8 flow directions’ or D8 introduced by O’Callaghan and Mark (1984) The D8 approach has the disadvantages arising from the discretization of the flow direction in only 8 possible directions, separated by 45 degrees. Fairfield and Leymarie (1991) try to
overcome this problem by adding a random component and assigning a flow direction to one of the neighbors downslope. We picked out the so-called ‘Infinite Directions’ algorithm by Tarboton (1997) because of its capability to minimize dispersion (the flow coming from one grid is split up into at most two downslope neighbors), its simple an efficient grid based matrix storage, and its robustness in terms of handling irregularities in the DEM.

If we consider a single triangular facet (Figure 4) delimited by $e_1$, $e_2$, and $e_3$ ($e_i$ and $d_i$ are elevations and distances between pixels as labeled in Figure 4), the slope vectors $S_1$ and $S_2$ are given by the following expression:

$$
S_1 = (e_0 - e_i) / d_i
$$
$$
S_2 = (e_0 - e_j) / d_j
$$

$$
r = \tan^{-1} \left( \frac{S_2}{S_1} \right)
$$

$$
S = \sqrt{S_1^2 + S_2^2}
$$

where $r$ and $S$ are the magnitude and slope direction of the flow respectively. If $r$ is not in the range $[0, \tan^{-1}(d_2/d_1)]$, then $r$ has to be set as the direction along the appropriate edge and $S$ assigned as the slope along that edge.

$$
\text{if } r \notin \left(0, \tan^{-1}\left(\frac{d_2}{d_1}\right)\right) \text{ then }
$$

$$
\text{if } r < 0
$$
$$
r = 0
$$
$$
S = S_1
$$

$$
\text{if } r > \tan^{-1}\left(\frac{d_2}{d_1}\right)
$$

$$
r = \tan^{-1}\left(\frac{d_2}{d_1}\right)
$$

$$
S = \left(\frac{e_0 - e_2}{\sqrt{d_1^2 + d_2^2}}\right)
$$

The eight possible values for the eight facets depicted in Figure 4 are computed. The local angle associated with the largest downwards slope is selected and adjusted to reflect an angle counterclockwise from east.

$$
\rightarrow \text{the local angle with the largest downwards slope}
$$

$$
r_g = a_f r^* + a_c \frac{\pi}{2}
$$

The multiplier $a_f$ and constant $a_c$ depend on the facet selected. In the case of the facet shown in Figure 4, these two values are 1.
7. Drainage Basin

The drainage basin is computed using the upslope area calculation procedure proposed by Tarboton (1997). The calculation follows a recursive method that is an extension of the very efficient recursive algorithm for single direction (Mark, 1988). The infinite directions algorithm splits the flow from one grid-block into up to two downwards neighbors with the largest downslope. The upslope area from a particular cell will be given by its own area plus the upslope area of the upslope neighbors that have some fraction draining to the pixel for which the slope area is being calculated. This recursive algorithm has the particularity of being extremely fast in terms of computational time, which makes it suitable for being used in a iterative way in case that the conditioning of the model requires it.

The points that are in the path that a droplet would follow as it drains down respecting the infinite directions algorithm constitute the influence area (IA). We obtain the dependence area (DA) by calculating the upslope area associated to all the points in the IA (Figure 5). Below is the pseudo-code of the logic of the infinite direction algorithm:
Procedure DPAREA(i, j)
if Area(i, j) is known
then
    no action
else
    Area(i, j) = 1 (The area of a single pixel)
    for each neighbor (location in, ij)
        \( p = \text{proportion of neighbor (in, ij) that drains to the pixel (i, j) based on angle} \)
    if \( p > 0 \) then
        call DPAREA(i, j)
        Area(i, j) = Area(in, jn) + p × Area(in, jn)
    return

The DA will delimit the initial simulation (highlighted in green in Figure 5). The initial simulation area contours are used to incorporate variability in the object-based simulation of TIs. These TIs are stochastically built in such a way that they preserve the appearance of the geobody that wants to be represented, but having a high stochastic component.

![Figure 5: Influence and Dependence Area for a given anchor point.](image)

8. Anchor Point Probability Field

The location of the anchor point is drawn according to a Poisson process with a spatially variable intensity function (P-field) previously generated. This P-field changes after each major
depositional event. Since the lobe formation tends to occur close to the sediment source, for the first lobe depositional event, the P-field is constructed by considering a uniformly decreasing probability field starting from the sediment source. The probability intensity function becomes zero after the distance of the point is larger than half of the longest dimension of the simulation.

![Image of probability fields](image)

**Figure 6: Anchor Point P-Field dependence on previously simulated lobe.**

It was observed in the process-based simulation output that after a lobe deposition the subsequent lobe tends to deposit close to the previous, hence a P-field that decreases as the points location gets further from the previous MPS simulation area was computed. We obtained the joint intensity function of the decreasing trend from the sediment source P-field (used in the first lobe deposition) and the P-field constructed computing proximity distance on the previous lobe MPS simulation area using the *Tau Model* with parameters one for each P-field. After each lobe deposition, the P-field relating the location of the last lobe simulated is computed and then combined with the uniformly decreasing probability field to obtain the new probability field.

## 9. Lobe Orientation

The lobe orientation is related to the drainage basin. Each anchor point defines an influence and dependence area (Figure 7). The dependence area is considered as the drainage basin for that particular anchor point. The valid range of angle θ used for the object-based simulation is defined by joining the anchor point to the furthest n-percentile (usually n around 30%) of the points on the drainage basing boundary. This angle range gives rise to the orientation of the lobes of the TIs used in the MPS simulation.
Figure 7: Lobe: Possible angle range orientation based on drainage basin.

Figure 8 shows how the lobe directions in the TIs are related to the surface. This is required since a lobe deposition would follow the same orientation that a flow draining down from the anchor point would follow.

Figure 8: Object-based TI generation.

10. Simulation Area

The final simulation area is obtained by combining the initial simulation area (drainage basin) with the previously simulated TIs for the lobe currently being simulated. In the case of the channel zone (distance from the sediment source to the anchor point), since the drainage area
does not considers it, the union of all the channels part of the TIs are taken. With respect to the position where the lobes hypothetically are, the interception of the initial SA with the lobe part of the TIs built accounting for drainage basin separately is taken.

![Initial Simulation Area + Object-based Training Images = Final Simulation Area](image)

Figure 9: Final MPS Simulation Area.

11. **2D to 3D transform of MPS realization**

The 2D to 3D transform is carried out by applying a distance transform on the binary image of the geobody simulation (MPS realization). Different distance transforms can be used in this respect. Proximity transform is essentially a distance transform followed by an inverse normalization of the resulting distances such that all nodes of the transformed image range between [0,1] with 0 indicating the furthest node and 1 indicating a target object node (Arpat, 2005). In our case, we use the inverse of this distance transform, therefore the values in the image range in [0, 1] with lower number indicating the internal point that is closer to the ‘boundaries’ of the geobody.
The proximity transform brings some unwanted features to the [0, 1] valued image, such as shortening in the values in the channel zone. Therefore a compensation of low zones must be performed. Basically, it consists of increasing the proximity transformed values below a given threshold. Values lower than the threshold are augmented so that the geologic structures that they are meant to represent are preserved in comparison to the highest point in the geobody. Then, the [0,1] ranged image is smoothened out using a local varying mean algorithm, consequently the possible artifacts due to the low zones compensation are removed. Finally, the [0, 1] altitude map is multiplied by the maximum thickness value drawn from a CDF constructed with information obtained from the process-based model.

In the case of hard data conditioning the simulation, the altitude values can be used as a local varying mean guiding another 2-point geostatistics algorithm. In addition, different forms of the proximity distance transforms can be used. Another good approach, given the availability of an underlying topography, would be the use of geodesic distance transform for gray-scale images as suggested by Toivanen (1996).

12. Erosion

Since the erosion is considered as a consequence of the depositional event (lobes erode the surface on their way downhill), it is simulated in such a way that only the topography underneath the lobe is eroded. This way, there is no erosion where there is not a major depositional event.
above. The erosion process is simulated accounting for flow direction and special topographic features related to the flow erosion power. Topographic gradient and curvature are used to give erosion values at a given point in the topography under a lobe deposition. Locations with high gradient magnitude will be eroded more than the ones with low gradients, since the flow energy is assumed to be lower at locations with low gradients. With respect to the curvature profile, it is used in such a way that positive curvature indicates less erosional power than negative curvature, point at which the flow collide into surface barriers. The flow direction given by the direction of the flow in the influence area is considered in an 8-direction scheme (using infinite directions is impractical since the orientation of the channel along its whole length is described by an average direction). The same scheme is utilized to compute the gradient direction at any given location. At locations where the gradient direction coincides with the direction of the flow, under an 8-direction scheme, more erosional power is taken into account. The relative values will depend on the fluid properties such as viscosity, grain content, flow mass, etc. In this study arbitrary values are taken, although further corrections to erosion-fluid properties are known to be required to obtain a more precise depiction of what really occurs in a geologic depositional event.

Figure 11: Flow directions and curvature and gradient profiles used in simulating erosion.
By taking into account flow direction, gradient and curvature values, very realistic features present in erosional process, similar to the ones observed in landslides, are reproduced in the model. Figure 13 shows a close-up view of the features obtained by considering erosion just underneath a lobe deposition.

13. Rejection Rules

The rejection rules are almost as important as the geologic rules used in the hybrid approach. They are meant to ensure that the model realization is comprised of geobodies physically feasible and geologically realistic. These rejections rules must be strict but also must let the algorithm to run within a reasonable amount of time to make it really competitive with process-based simulation, which in general are known to run over extremely long time.
Some examples of rejection rules already implemented in the simulation code are shown in Figure 14 (we consider the sediment source to be in the center of the northernmost boundary). Most of these conditions are imposed on the MPS simulation. We require the simulated lobe to be connected to the sediment source (fig. on the left), the MPS simulation comprised of just one geobody (fig. in the center) and a feasible channel-lobe structure (fig. on the right). All the not-allowed lobe simulations are possible outputs of any MPS algorithm that respects the proportions of each facies required (in this case lobe/no lobe facies), hence it is necessary to check each simulation realization. Additionally, conditions are imposed on the direction and length of the channel that determines the dependence area. In general it is required that the channel flows away from the sediment source; but this is not always achieved directly since the topography changes without any constraint, rather only accounting for the depositional events.

![Figure 14: Examples of realizations that should be rejected.](image)

14. Simulation Results

Below one realization of a turbidity-lobe system is presented (Figure 15). This realization was obtained following the procedure fully described in this paper and is comprised of 11 lobes and 1 intermediate fine-grained unit. The lobe orientation follows the direction of flow and the drainage basin is respected for each lobe deposition, since it was imposed through the simulation area used in the MPS simulation.
The basal topography is updated after each depositional event, and then used in the erosion computation. The erosion process is linked to mathematical properties of the surface (gradient and curvature), as way of inferring the erosional power of the lobe given the direction followed by it downhill.

The elapsed time threshold related to the presence or absence of the intermediate layer was set at 1000 years, which is quite high considering the CDF used in this regards, which explains the low frequency of appearance of the intermediate grained unit. The threshold value is a user defined parameter, therefore higher frequency of low permeability layers can be achieved by lowering its value.

### 15. Model Conditioning

Despite being constructed utilizing the pseudo-discrete step given by each geobody deposition, the algorithm herein proposed has the capability of being conditioned to hard data. On the other hand, process-based algorithms can simulate geologic events very realistically, but since they are built further in time and just one realization can be obtained from a fixed set initial of parameters (typically there are no stochastic processes involved in the generation of the realization), they cannot be conditioned to hard data.

Some possibilities for conditioning the model to hard data are as follows:

- Data conditioning layer thicknesses.
- Data related to petrophysical properties distribution.
When the lobe simulation is required to be led in a particular direction, iterations on the influence and dependence area are necessary. The iterative process will stop when the simulation area contains the location where the hard data are located. With respect to the layer thicknesses, the thicknesses obtained using the selected distance transformed will be used as a local varying mean for the proper 2-point geostatistics algorithm (SGSIM, DSGS, etc.). In case of interface due to an erosional process, the final thickness of a lobe will be let to stand-by until the next lobe that happens to be deposited above erodes the top of the underneath lobe up to its thickness respects what is seeing in the data, such as Formation Multi-Imaging (FMI) log data.

16. Conclusions

A workflow for constructing hybrid geostatistical models was proposed. Hybrid models break down the problem of simulating complex geological features into a set of smaller problems that can be faced using the most suitable approaches. The goals of the hybrid models, in addition to simulating complex geologic features, are to speed up the simulation process, generate multiple and different realizations using a fixed set of initial parameters, and be possible to condition to hard data. The workflow starts by understanding the deposition environment and identifying the structures that have to be considered; these are the structures that play a role in the flow simulation. Then, the process continues with the parameterization of the important structures. These parameters must fully describe the geobody shapes and be able to be obtained from any related source of data such as geologic studies, laboratory experimental data or process-based simulation. The parameter values are drawn from the CDFs previously constructed using Monte-Carlo simulation. The realization generation follows a sequential process that starts with the simulation of major geobodies and continues with simulation of intermediate features. A two-dimensional realization (in most of the cases obtained using MPS algorithms) is post-processed to achieve the final thickness map which will be stacked on the base surface. Consideration of the geologic processes must be accounted for in this part. All this sequential process will be repeated as many times as the number of major geological events we want the realization to have.

The initial deepwater turbidite hybrid model by Michael et al. (2008) is modified to account for the new aspects proposed in the methodology. The new model considers the changes that the previously simulated topography will bring to the simulation of the new lobe by using surface-based modeling (Influence Area and Drainage Area). Additionally, the algorithm generates realistic realizations, which are comprised of lobe-channel structures, and intermediate fine-grained units. The proper simulation of the last is really important since its presence or absence might constitute a flow barrier.

Incorporating Surface-based and MPS algorithms makes the model potentially capable of representing the internal features of geobodies and becoming conditioned to facies and thicknesses.
References


