Estimating Value of Information in Spatial Decision Making for Reservoir Development

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Value of Information

• Gathering information is crucial in any decision making process

• Estimating the value of information (VOI) is therefore important for acquiring the right kind of information
Griffiths and King assessed the VOI in terms of time savings. Surveys in eight organizations asked professionals to indicate whether reading journal articles, books, or internal reports saved them any time.

<table>
<thead>
<tr>
<th>Reason for Time Savings</th>
<th>Journal Articles</th>
<th>Books</th>
<th>Internal Reports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall time savings</td>
<td>26</td>
<td>42</td>
<td>50</td>
</tr>
<tr>
<td>Avoided primary research</td>
<td>49</td>
<td>22</td>
<td>51</td>
</tr>
<tr>
<td>Stopped unproductive research</td>
<td>10</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>Modified research design</td>
<td>12</td>
<td>13</td>
<td>19</td>
</tr>
<tr>
<td>Modified analysis method</td>
<td>16</td>
<td>30</td>
<td>9</td>
</tr>
</tbody>
</table>

Table A-1. Information and Time Savings (%)
Value of Information

Information Entropy approach
versus
Decision theoretic approach

Both well established over many years
Measures of Information – Information Theory

Shannon Entropy:

\[ H(x) = -\int p(x) \log[p(x)] dx \]

Mutual Information:

\[ MI = \frac{H(x) - \int H(x \mid y) p(y) dy}{H(x)} \quad 0 \leq MI \leq 1 \]

reduction in uncertainty of \( x \) on observing the outcome \( y \) of an experiment
Experiment is valuable when

- *material* to the decision

Information from experiment may reduce uncertainty but is not valuable until it can change the decision.
Objective

Formulate a decision-theoretic model to estimate value of information in spatial decision making.
Spatial Decision Making

Decision problems involving:

• Choice of alternatives over space  
  (e.g. selecting sites for wells)

• The distinction of interest is spatially correlated  
  (e.g. facies, porosity)
Spatial Decision Making

Continuous spatial random field
  - porosity, saturation

Categorical spatial random field
  - facies
Value of Information: Decision Theoretic Approach

Literature:

Raiffa, H., (1968), Decision analysis, Addison-Wesley.

Matheson, J. E., (1990), Using influence diagrams to value information and control, [Oliver and Smith (Eds), Influence diagrams, belief nets and decision analysis, 25-48, Wiley.

Value of Information

A related problem: Spatial sampling design

Journel,

Zidek, Caselton, Le, & co-workers

Gelfand, Banerjee, et al.

……and others
Value of Information: Decision Theoretic Approach

Literature:

Houck & Pavlov (2006):
VOI for CSEM data at global reservoir level

Bickel et al. (2006):
VOI for seismic attributes at local level

Need to incorporate spatial dependence
Value of information: Decision Theoretic Approach

We build on the concepts presented by earlier workers and present models incorporating spatial dependence within a decision-theoretic framework.
Values associated with decisions

• Prior Value

• Value with Perfect Information

• Value of Experiment
Example

Simple example: no spatial dependence

Treasure hunt

Revenue = \( R = \$10000 \) if treasure!
\( P(\text{treasure}) = p = 0.2. \)
Cost = \( C = \$1000 \) for digging

Prior value =
\[
\text{PV} = \max(0, pR - C)
= 0.2 \cdot 10000 - 1000 = 1000.
\]
Simple Example

Concept of clairvoyance: Perfect information

Value with free clairvoyance
\[ VFC = p(R-C) = 0.2(10000 - 1000) = 1800. \]

Value of perfect information
\[ VOPI = VFC - PV = 1800 - 1000 = 800. \]
Simple Example

Concept of experiment: Imperfect information

\( D = \{0, 1\} = \{\text{Detector displays “no treasure”, Detector displays “treasure”}\} \).

Sensitivity = \( P(D = 1|Tr = 1) = P(D = 0|Tr = 0) = 0.9 \).

Posterior prob = \( P(Tr \mid D) \)
Simple Example

Concept of experiment: Imperfect information

\[
VFE = \sum_{d=0,1} \max[0, P(Tr = 1|D = d)R - C]P(D = d)
\]

Value with free experiment

\[
VFE = 1540.
\]
Simple Example

Prior value: $PV = pR - C = 1000$
Value with free clairvoyance: $VFC = p(R - C)$
Value with free experiment: $VFE$

Value of (imperfect) Information

$VOI = VFE - PV = 1540 - 1000 = 540$. 
Spatial System

• Variable $x_i$ on a spatial lattice $n_1 \times n_2$

• Probability of favorable outcome: $P_i(x_i = 1) = p_i$

• Cost if site $i$ is selected: $C_i$
  (note: not cost of experiment)

• Revenue: $R_i$
Values associated with spatial decisions

• Prior Value

• Value with Perfect Information

• Value with Free Experiment

• Value of Information
Values associated with decisions

- All calculations assume, for the sake of clarity, that the decision maker is risk-neutral.

- Model can incorporate risk aversion or risk seeking behavior.
Spatial Lattice Model

- Brine saturation $s = (s_1, \ldots, s_n)$ on a lateral grid.
- Porosity $= (\phi_1, \ldots, \phi_n)$ on a lateral grid.

$\phi_i \sim \{\phi_i, s_i\}$
Spatial Lattice Model

- Gaussian Markov Random field (GMRF) for logistic transforms of porosity and saturation
- Exponential Spatial Covariance $\Sigma$
Spatial Lattice Model

- Each node is a possible selection site.
Spatial Decision Making

• Prior: Lattice model for brine saturation and porosity in reservoir - with spatial correlation.

• Experiment:
  – Seismic data (Amplitude Versus Offset)
  – Electromagnetic data (CSEM)
Expected Prior Value (PV)

Expected profit = $p_i R_i - C_i$

Choose site only if expected profit > 0

$$V_i = \max\{0, [p_i R_i - C_i]\}$$

$$PV = \sum_{i} V_i$$
Value with Free Clairvoyance (VFC)

Decision maker has perfect information about latent variable

\[ VFC = \sum_{i} p_i \max \{0, (R_i - C_i)\} \]

\[ VFC = \sum_{i} p_i (R_i - C_i) \quad \text{For } R > C \]
Value of Perfect Information (VOPI)

\[ \text{VOPI} = \text{VFC} - \text{PV} \]

Value of Perfect Information is an upper bound on the Value of Information
Value with Free Experiment (VFE)

Experiment performed and data $y_i$ is observed

Conditional probability of favorable outcome: $P_i(1|y_i)$
Expected Posterior Value

Prior:

\[ V_i = \max \{ 0, [ p_i R_i - C_i ] \} \]

Posterior:

\[ V_i^{post} = \int \max \{ 0, [ p_i (1 | y_i) R_i - C_i ] \} P(y) dy \]

\[ VFE = \sum_i V_i^{post} \quad \text{Value with free experiment} \]
Expected Posterior Value: Continuous variable

Prior:

\[ V_i = \max\{0, [E(\phi_i)R_i - C_i]\} \]

Posterior:

\[ V_i^{post} = \int \max\{0, [E(\phi_i | y_i)R_i - C_i]\} P(y) dy \]

\[ VFE = \sum_{i} V_i^{post} \quad \text{Value with free experiment} \]
Value of Information

VOI = VFE - PV

Value of Information = Value with Free Experiment – Prior Value
Geology and Physics Relations

Prior:

\[ V_i = \max \{0, [E(\phi_i)R_i - C_i] \} \]

Posterior:

\[ V_i^{post} = \int \max \{0, [E(\phi_i | y_i)R_i - C_i] \} P(y) dy \]

Related by physics and wave propagation forward models.

Spatial random field
Likelihood of experiment

• Seismic data $y = (y_1, \ldots, y_n)$ at all grid cells.

• $y_i = (\text{Zero-offset reflectivity, AVO gradient})$, or similar.

Gassmann’s relations $(s_i, \phi_i)$  $(\mu_i, K_i, \rho_i)$

Aki–Richards approximations $(\mu_i, K_i, \rho_i)$  $(R_0, G)$
Likelihood of experiment

- EM data $y = (r_1, \ldots, r_n)$ along a sail line.
- $r_i = \text{inverted resistivity}$

Archie’s relations $(s_i, \phi_i) \rightarrow r_i$

- Averaging over multiple cells to simulate lower resolution
Likelihood

Local geology and deposition sets the expected rock physics model and geophysical response for different porosity and saturation.
Computations

Computations of expectations and conditional expectations by Monte Carlo simulations of
  - spatial random fields
  - forward modeled datasets over the simulated fields.
  - posterior conditional expectation by iterated linear approximation
Spatial Models and Rock Physics Relations

Posterior:

\[ V_{i}^{post} = \int \max \{0, [E(\phi_i | y_i)R_i - C_i] \} P(y)dy \]

Iterated linear approx

Monte Carlo Integration
AVO Example

Spatial random field: porosity, saturation

Data: AVO intercept & gradient, $R_0$ & G

$P(R_0, G)$ from forward model
AVO and CSEM Example

$P(R_0, G|m)$ from forward model

$P(r|m)$ from forward model

Seismic AVO data.

CSEM resistivity data.

$G$

$R_0$
AVO Example

\[ P(\text{porosity, saturation} \mid R_0 = 0.05, G = -0.05) \]
AVO Example: VOI

- VOI
- Prior Value
- Posterior

Better data quality
AVO Example: VOI

Better data quality
AVO Example: VOI

Better data quality

VOI

Prior Value

Posterior

Cost of experiment

Perturbation factor

Better data quality
AVO Example: VOI

Better data quality
Chance of Knowing

\[ CK = \frac{VOI}{VOPI} \]

\[ 0 \leq CK \leq 1 \]

Rates worth of an experiment and compares different experiments on the variable of interest

e.g. Seismic and CSEM
Value of Information

Rates worth of an experiment and compares different experiments
e.g. Seismic and CSEM

\[
\frac{(VFE - PV)}{PV}
\]
Measures of Information

MI: Information theoretic

VOI: Decision theoretic

MI:
- how much uncertainty can be reduced by an experiment
- not linked to decision

VOI:
- how much decision maker should pay for the experiment
- linked to decision
- more complete for valuing information
- more difficult to estimate
Measures of Information

Experiment is valuable when

- *relevant* to the distinction of interest
- *material* to the decision

Information from experiment may reduce uncertainty but is not valuable until it can change the decision.

Entropy based measures address relevancy alone. VOI addresses relevancy and materialness of experiment.
Summary and Conclusions

Demonstrated a method for computing the Value of Information (VOI) for a spatial decision making – correlated variables

Information theoretic vs. Decision theoretic

VOI in a spatial setting involves complex interplay between the reliability of the experiment, spatial uncertainty and the nature of decision and values.