Geostatistical Integration of Coarse and Fine Scale Data, 
*BGEOST*: Applications and Results

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1 Summary

BGEOST (block geostatistics) is a geostatistical package written within the SGeMS software (Remy et al. 2007) for integrating data of different scales. It consists of three algorithms, BKRIG (block kriging estimation), BSSIM (block sequential simulation) and BESIM (block error simulation), and one utility program BCOVAR (block covariance calculation). These three algorithms take different approaches to integrate fine point-support data and coarse block-support data. BKRIG performs kriging estimation generating one single estimate "best" in the least square sense. BSSIM generates multiple equally probable realizations using direct sequential simulation (Journel 1994). BESIM performs simulation with conditioning done through the error simulation approach (Journel and Huijbregts 1978; Gloaguen et al. 2005; Liu and Journel 2007). The utility program BCOVAR computes block or point covariance values and block-to-point covariance maps (Kyriakidis et al. 2005; Liu et al. 2006). Details about the theory and usage of BGEOST can be found in Liu and Journel (2006, 2007); Liu et al. (2006); Liu (2005).

This report focuses on the analysis and applications of BGEOST to integrating point and block data. Synthetic downscaling or tomography case studies are developed on large 2D and 3D grids using the three programs. The programs perform as expected for point and block data integration, with both types of data honored. Target statistics, such as a histogram and covariance model, are also reasonably well reproduced. A comparative performance study of the three programs is provided.

2 2D downscaling case study

In previous reports (Liu and Journel 2006; Liu 2005), the case studies were developed on a small 2D field of size $50 \times 50$. The program then used was BDSIM. The present program BSSIM is a new version of BDSIM with many parts much improved and new features added. The programs BKRIG and BESIM are new. In this section, a larger 2D field of size $150 \times 150$ is created, and downscaling case studies are performed using all three programs.
2.1 The 2D reference data

Figure 1(a) and Figure 1(b) give the reference data set of size $150 \times 150$ and its histogram. It was generated using unconditional sequential Gaussian simulation (SGSIM) with the standardized semivariogram model:

$$\gamma(h_x, h_y) = 0.1 + 0.9 \text{Sph}\left(\sqrt{\left(\frac{h_x}{3500}\right)^2 + \left(\frac{h_y}{2000}\right)^2}\right)$$

(1)

where $\gamma$ denotes the semivariogram; $h_x$ and $h_y$ are the separation distances in $x$ and $y$ directions; $\text{Sph}(\cdot)$ denotes a spherical semivariogram function. The relative nugget effect value is 0.1 and the correlation ranges are 3500 in the $x$ direction and 2000 in the $y$ direction. This semivariogram model is the default input model for all following runs.

The values along the two columns ($i = 19$ and $i = 20$) in the reference model are retained as hard well data, see Figure 1(c). Figure 1(d) gives the histogram of the well data. Since the well trajectories do not traverse the top and the middle high-value areas seen on the reference model, the mean of the well data (0.14) is lower than the mean (0.17) of the reference model.

Equally-sized square-shaped block data are obtained by averaging point values taken from the reference model. The nine block data cover the entire field, see Figure 1(e). The color represents the block average value. The block data histogram is given in Figure 1(f). Note the lower block spatial variance due to the averaging effect.

In all following 2D case studies, unless specified otherwise the same parameter values are used to allow comparison. All estimations and simulations are conditioned to the same well and block data shown in Figure 1. The same standardized variogram model (1) is used. The point data search ellipse radii are 1.5 times larger than the corresponding variogram ranges. The fast FFT-integration hybrid approach (Liu et al., 2006) is used to compute all block-related covariances. Simple kriging is retained with the maximum number of conditioning blocks set at 9.
2D DOWNSCALING CASE STUDY

2.2 **BKRIG** results

**BKRIG** is a program for kriging estimation conditioned to both point and block data. It
has three conditioning options: conditioning only to point data (equivalent to a traditional
point kriging), conditioning only to block data, and last conditioning to both point and block
data. Two types of kriging, **SK** and **OK**, are accepted. There are two options for computing
the block covariance values: traditional integration approach and FFT-integration hybrid
approach. Block data can be entered either from a block data file or from pre-loaded point-
set objects. More details about **BKRIG** can be found in Liu and Journel (2007).

Figure 2 gives the SGeMS parameter setting for this **BKRIG** estimation study.

Figure 3(a) gives the kriging estimation result conditioned to both well and block data.
It is seen that the main patterns in the reference model Figure 1(a), such as the top and
middle high value areas and the bottom low value area, are reproduced. But the shape of
the high value heterogeneities is not reproduced, this is due to the lack of resolution of the
block data.

As expected, the mean of kriging map 0.168 is very close to the reference mean 0.169,
and the variance of kriging map is much lower than the reference variance (smoothing
effect of the kriging). The long tail of the reference histogram is not reproduced, compare
Figure 3(b) and Figure 1(b).

Figure 3(c) shows the kriging variance. At the well data locations, the variances are
zeros. Variances increase away from the data locations. The variances at the center of
block data are lower than those at the edge of block data.

The number of conditioning block data retained at each node is shown in Figure 3(d). In
the central area more blocks are retained, while close to the 4 corners only two conditioning
blocks are retained. The anisotropy of the variogram model is somewhat reflected on this
neighboring block number map.

From the kriging map, the estimated average block values are retrieved. The cross-plot
of these estimated block values vs. the input block data is given in Figure 3(e). The block
data are seen to be well reproduced with an average relative absolute error of 2.1%.

The run time for this study is 18 seconds on a 2.33GHz laptop. The memory cost is 57
megabytes.

2.3 **BSSIM** results

**BSSIM** is an algorithm for simulating point values conditioned to block and point data. The algorithm utilizes block-point kriging and direct sequential simulation (Liu and Journel, 2007).

Figure 4 gives the parameter settings for this **BSSIM** case study. The fully-random simulation path is selected (Figure 4(a)). This fully-random path is equivalent to the block-first path when the block data cover the entire field with no block overlap. In order to better reproduce the block data values, all previously simulated nodes located within a block crossing the current simulation node are retrieved as conditioning points (Liu and Journel, 2006; Hansen and Mosegaard, 2007). This option, however, would not be practical if there are many large blocks. As a compromise, one could set a maximum number for those informed nodes (Liu and Journel, 2007). In this study, this maximum value is set at 40 (Figure 4(b)). The histogram reproduction algorithm proposed by (Soares, 2001) is used. The target histogram is the reference histogram. The power model is used for histogram upper and lower tail extrapolation, see Figure 4(d).

Two **BSSIM** realizations are shown in Figures 5(c) and 5(e). Both realizations display the high-value areas and the low-value areas at their correct locations, consistent with the reference model (Figure 5(a)). Even though the conditioning block data cover the entire field, their lack of resolution leaves significant uncertainty as seen from the differences between the two realizations. Compared to the target histogram (Figure 5(b)), the shape, mean, variance and quantiles of the realization histograms (Figures 5(d) and 5(f)) are well reproduced.

The **E-type** map averaged from 50 **BSSIM** realizations is shown in Figure 6(a). As expected, this E-type map appears close to the **BKRIG** kriging result (Figure 6(b)).

Block data reproduction is checked in Figure 7. Figures 7(a) and 7(b) are the cross-plots of simulated block values versus input block data for the two realizations. The average relative absolute errors are 3.3% and 3.9%, respectively. Those errors could be reduced by
increasing the search ellipse or increasing the maximum number of conditioning points within crossing blocks. The block reproduction of E-type is checked in Figure 7(c). The block reproduction of the E-type is better than that provided by any single realization, as expected.

The variograms along the $x$ and $y$ directions are computed from the two realizations, see the red dot lines in Figure 8. The simulated realizations match the input variogram model.

The run time for each realization is 55 seconds and the memory cost is 59 megabytes.

2.4 **BESIM** results

*BESIM* utilizes an alternative data conditioning algorithm to integrate point and block data (Liu and Journel, 2007). It is based on the error simulation approach (Journel and Huijbregts, 1978; Gloaguen et al., 2005). In all rigor, a multi-Gaussian assumption is required to ensure full independence between the kriging estimates and the corresponding errors being simulated.

The parameter settings for *BESIM* are the same as for *BSSIM* (Figure 9). The main difference is that *BESIM* does not consider any previously kriged value within blocks.

Two *BESIM* realizations and their histograms are shown in Figure 10. Again, the high value and bottom low value areas are reproduced at the correct locations. The histograms are, however, poorly reproduced as expected. The histogram lower tail and upper tail extrapolation cannot be controlled in *BESIM*. The *BESIM* E-type (Figure 11(a)) averaged from 50 realizations is almost the same as the *BKRIG* estimation map (Figure 11(b)). The input block data are well reproduced in both realizations and in the E-type map, see Figure 12. The variograms along both $x$ and $y$ directions computed from the two realizations (red dots in Figure 13) also reasonably reproduce the input variogram model (the black curves).

*BESIM* calls for two krigings, an original kriging providing the estimate $Z_{\text{K}}^*(u)$ based on original data, and a second kriging providing the simulated estimate $Z_{\text{K-S}}^*(u)$. Both krigings have the same data configuration, thus the block-point kriging system needs to
be solved only once at each node, with the resulting kriging weights being stored. These weights are retrieved whenever they are needed in all subsequent simulations. Hence, the first realization takes longer time. The CPU time for that first realization is 30 seconds and for all subsequent realizations it is only 13 seconds. The memory cost is 73 megabytes. Compared to BSSIM, BESIM is much faster but with a higher memory cost.

3 3D case study

3.1 3D downscaling case

The reference data

The reference 3D field is of size $51 \times 51 \times 30$. It has been generated using unconditional sequential Gaussian simulation with a standardized variogram model:

$$\gamma(h_x, h_y, h_z) = 0.1 + 0.9S\phi\left(\sqrt{\left(\frac{h_x}{1600}\right)^2 + \left(\frac{h_y}{1000}\right)^2 + \left(\frac{h_z}{500}\right)^2}\right)$$

(2)

Figures 14(a) and 14(b) give the reference model. The corresponding histogram is shown in Figure 14(e).

Eighteen equal-sized block data are retrieved from the reference model, see Figures 14(c) and 14(d). The histogram of these block data is given in Figure 14(f). The mean of block data identifies that of the reference model and the variance is much lower.

Four wells are extracted from the reference model at the locations $(i = 7, j = 7)$, $(i = 7, j = 47)$, $(i = 47, j = 7)$ and $(i = 47, j = 7)$, see Figure 14(g). Figure 14(h) gives their histogram. The mean of well data is a little less than that of the reference model.

The parameter settings for the following downscaling case studies are as follow. All point and block data are used for conditioning. The same variogram model (2) is used. The point data search ellipsoid radii are set equal to the variogram ranges. The FFT-integration hybrid approach is used to compute block-related covariances. Simple kriging is selected, with the maximum number of conditioning blocks set at 10.
**BKRIG results**

Figure 15 shows the parameter settings of BKRIG for kriging estimation. The kriging results are shown in Figures 16(a) and 16(b). Compared to the reference model (Figures 14(a) and 14(b)), the large heterogeneity areas are reproduced but with significant smoothing. The kriging variances are lower in areas close to wells or blocks (Figures 16(c) and 16(d)). The number of conditioning blocks is higher in the central area than along the edges of the field (Figures 16(e) and 16(f)). The mean of the kriging estimates is 0.193, a value close to the reference mean 0.191 (Figure 16(g)). The 18 block data values are well reproduced with an average relative absolute error of 1.9% (Figure 16(h)).

The CPU time of one BKRIG estimation run is 51 seconds and the memory cost is 265 megabytes.

**BSSIM results**

The parameter settings for the BSSIM simulation run is given in Figure 17. The maximum number of conditioning point data is 20. The maximum number of conditioning points within crossing blocks is 40. The minimum and maximum values for histogram tail extrapolation using the Soares approach is set to 0 and 0.49, respectively.

Two BSSIM realizations are given in Figure 18. It is seen that the central high value heterogeneity area is well captured in both realizations, with the uncertainty being displayed by the difference between the two realizations.

Comparing the histogram of realization 1 (Figure 19(a)) to the reference histogram (Figure 19(b)), it appears that the histogram shape, mean, variance and quantiles are well reproduced. The variograms along the $x$, $y$ and $z$ directions computed from realization 1 match reasonably the input variogram models (Figures 19(c), 19(d) and 19(e)). The block data values are reproduced with an average relative absolute error of 3.9% (Figure 19(f)).

In Figure 20, the E-type map averaged from 20 realizations is compared with the BKRIG kriging map. They are very close in terms of spatial pattern distribution and histogram.

The run time of one BSSIM realization is 253 seconds and the memory cost is 246 megabytes.
3D CASE STUDY

**BESIM results**

The parameter settings for BESIM case study is given in Figure 21. Two BESIM realizations are given in Figure 22. Again, the central high-value heterogeneity area is captured in both realizations, with some uncertainty.

The histogram of realization 1 (Figure 23(a)) does not reproduce the target reference histogram (Figure 23(b)). The variograms along the x, y and z directions computed from realization 1 match reasonably the input variogram model (Figures 23(c), 23(d) and 23(e)). The block data values are reproduced with an average relative absolute error of 2.2% (Figure 23(f)).

In Figure 24, the BESIM E-type averaged from 20 realizations is compared with the BKRIG estimation map. They are very close in terms of spatial pattern distribution and histogram.

The run time of the first BESIM realization is 103 seconds and that of any subsequent realization is only 38 seconds, much less than the run time of BSSIM realizations. The memory cost is 284 megabytes.

### 3.2 3D tomography case

**The reference data**

The 3D reference model for the tomography case study is the same as that of 3D downscaling case. The seismic slowness was simulated using unconditional sequential Gaussian simulation with the same variogram used for the 3D downscaling case, see expression 2. Figures 25(a) and 25(b) give the reference model. The corresponding histogram is shown in Figure 25(f).

Four wells are extracted from that reference model at locations \((i = 4, j = 25), (i = 25, j = 4), (i = 25, j = 46)\) and \((i = 46, j = 25)\). Figure 14(h) gives the well data histogram.

In this study, the rays are located on two planes determined by the well pair \((i = 4, j = 25), (i = 46, j = 25)\) and the well pair \((i = 25, j = 4), (i = 25, j = 46)\), see Figure 25(d). On each ray plane, there are 3 sources on one side and 4 receivers on the other side. The
ray paths are assumed straight lines. The ray configuration of each ray plane is shown in Figure 25(e). There is a total of 24 rays. The histogram of the ray data is given in Figure 25(g). The mean (0.39) of the ray data is close to that (0.38) of the reference model.

The parameter settings for this 3D tomography case study are as follow. All point and ray data are used for conditioning. The variogram model (2) is used with a point search ellipsoid of radii set equal to the variogram ranges. The FFT-integration hybrid approach is used to compute block-related covariances. Simple kriging is selected. The maximum number of conditioning blocks is set at 10.

**BKRIG results**

Figure 26 shows the parameter settings of BKRIG for kriging estimation.

The kriging estimation results are compared with the reference model in Figure 27. It is seen that the overall patterns of the reference model are reasonably reproduced although with smoothing. The kriging variances are lower in the area close to wells or rays (Figures 28(a) and 28(b)). The number of conditioning rays is higher in the central area than it is at the edge of the field (Figures 28(c) and 28(d)). In these last two figures, the neighboring block number spatial distribution is not symmetric because the ray configuration is not symmetric and the variogram model is anisotropic. The kriging mean estimate 0.392 is close to the reference mean 0.384 (Figure 28(e)). The 24 ray data are well reproduced by kriging with an average relative absolute error of 0.86% (Figure 28(f)).

The CPU time of one BKRIG estimation run is 39 seconds and the memory cost is 299 megabytes.

**BSSIM results**

The parameter settings for the BSSIM 3D case study is given in Figure 29. The maximum number of conditioning point data is 20. The maximum number of conditioning points within crossing rays is 40. The minimum and maximum values for histogram tail extrapolation using the Soares approach are set to 0.2 and 0.56, respectively.
Two BSSIM realizations are given in Figure 30. It is seen that the reference major heterogeneity areas are captured in both realizations, with some uncertainty.

Comparing the histogram of realization 1 (Figure 31(a)) with the reference histogram (Figure 31(b)), it appears that the histogram shape, mean, variance and quantiles are well reproduced. The variograms along the $x$, $y$ and $z$ directions computed from realization 1 match the input variogram models (Figures 31(c), 31(d) and 31(e)). The ray data are reproduced with an average relative absolute error of 2.0% (Figure 31(f)).

In Figure 32, the BSSIM E-type averaged from 20 realizations is compared with the BKRIG estimation map. They are very close in terms of spatial pattern distribution and histogram.

The run time of one BSSIM realization is 66 seconds and the memory cost is 301 megabytes.

**BESIM results**

The parameter settings for the BESIM 3D case study is given in Figure 33(b).

Two BESIM realizations are given in Figure 34. The reference high-value heterogeneity areas are captured in both realizations, with some uncertainty.

The histogram of realization 1 (Figure 35(a)) fails to reproduce the target histogram (Figure 35(b)). The variograms along the $x$, $y$ and $z$ directions computed from realization 1 match the input variogram models (Figures 35(c), 35(d) and 35(e)). The block data are well reproduced with an average relative absolute error of 0.88% (Figure 35(f)).

In Figure 36, the BESIM E-type averaged from 20 realizations is compared with the BKRIG estimation map. They are very close in terms of spatial pattern distribution and histogram.

The run time of first BESIM realization is 108 seconds and that of any subsequent realization is 38 seconds. The memory cost is 328 megabytes.
4  Comparison and analysis

4.1  BSSIM vs. BESIM

The pros and cons of BSSIM and BESIM are summarized in Table I using the results of the 2D downscaling case study.

- **BESIM** requires a Multi-Gaussian assumption to ensure that the kriging estimator is independent of the simulated errors. This is a severe limitation of BESIM unfortunately not always well understood.

- **BSSIM** can reasonably reproduce the input target histogram, while BESIM can not because of the previous limitation. Figure 37(a) shows the QQ plot of one BSSIM realization versus the reference model. The cross points are distributed along the 45° line indicating that the BSSIM realization histogram is close to the reference histogram. Figure 37(b) gives a similar QQ plot of one BESIM realization versus the reference model. The cross points now deviate significantly from the 45° line. The BESIM realizations can not be post-processed (e.g. through TRANS) without losing the block data reproduction.

- In both BSSIM and BESIM results, the block data are well reproduced. Figure 38 gives the histograms of block errors (simulated block values minus input block data) obtained from 50 realizations for each block used in the 2D case study. These block errors show no bias. The block data values of the 6 blocks on the left and right columns are slightly underestimated with negative error means. This is because the two wells crossing these two columns of blocks have means lower than the reference mean. The block data values of the 3 blocks in the middle column are overestimated with positive error means because there is no low-value well passing through them.

- The variogram model is well reproduced by both BSSIM and BESIM realizations.

- BESIM is much faster if many realizations are called for, see the plots of the CPU time versus number of realizations for both BSSIM and BESIM in Figure 39. The reason is as follows. In BESIM the block-point kriging system is solved only once
at each node with the conditioning data and weights stored when simulating the first realization; they are retrieved to be used in any subsequent realization. Thus, the only major CPU cost in any subsequent realization is the point-based unconditional simulation.

- The memory demand of BESIM, however, is larger because the local conditioning data and weights at each node need to be stored.

<table>
<thead>
<tr>
<th></th>
<th>BSSIM</th>
<th>BESIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-Gaussian assumption</td>
<td>Not required</td>
<td>Required</td>
</tr>
<tr>
<td>Histogram reproduction</td>
<td>Well reproduced</td>
<td>Not reproduced</td>
</tr>
<tr>
<td>Block data reproduction</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Variogram reproduction</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>CPU time</td>
<td>Slower (55 Seconds)</td>
<td>Faster (30 Sec. (first); 13 Sec. (subseq.))</td>
</tr>
<tr>
<td>Memory cost</td>
<td>Lower (59M)</td>
<td>Higher (73M)</td>
</tr>
</tbody>
</table>

Table 1: Table of comparing BSSIM with BESIM (2D data case).

4.2 Different types of conditioning data

The three algorithm programs, BKRIG, BSSIM and BESIM, are able to condition either to point data only, to block data only, or to point and block data simultaneously. In this section, these three different conditioning cases are studied with BSSIM using the 2D data set defined in Section 2.1. The resulting E-types and realizations are shown in Figure 40. The left three figures (Figures 40(b), 40(e) and 40(h)) give the E-type map and two realizations conditioned only to the well data. The left thin high-value area is identified, but not the other heterogeneities. The middle three figures (Figures 40(c), 40(f) and 40(i)) give the E-type map and two realizations conditioned only to the block data. The overall patterns seen on the reference model are reproduced, but the high frequency information that would have been provided by well data is missing. The right three figures (Figures 40(d), 40(g) and 40(j)) give the E-type map and two realizations conditioned to both well and block data.
Both well and block information are reflected in the results. The E-type map conditioned to both types of data is much closer to the reference model (Figure 46(a)) than the other two E-type maps.

4.3 Different block data

4.3.1 Different block sizes

In this section, two sets of blocks with different sizes are used for conditioning. Figure 41(a) gives the 9 coarse block data previously used for the 2D downscaling study (Section 2). Figure 41(c) gives the 25 fine scale block data retrieved from the same reference model. The fine block data have the same mean as the coarse block data, but their variance is higher, see Figures 41(b) and 41(d).

Figures 42(b) and 42(c) give the E-type maps averaged from 50 realizations conditioned to the coarse block data only and to the fine block data only. The fine-block E-type map shows much better reproduction of the patterns of the reference model (Figure 46(a)). For example, the two top high value areas are better reproduced and they are clearly separated in the fine-scale E-type map. Comparing the realizations conditioned to coarse-block data only (Figures 42(d) and 42(f)) with those conditioned to fine-block data only (Figures 42(e) and 42(g)), one observe that the high frequency features of the reference model are better captured by the realizations conditioned to fine-block data.

4.3.2 Different block shapes

As mentioned in Section 2.2, the shapes of blocks may also affect the simulation results. In this section, conditioning to blocks of different shapes is investigated. Figure 43(a) shows $3 \times 5$ rectangular blocks. The elongation of the rectangular blocks matches the longer axis of the variogram anisotropy. Figure 43(c) shows $5 \times 3$ rectangular blocks, whose longer size is perpendicular to the variogram longer axis. Because of the field anisotropy, the $3 \times 5$ block data have higher values and their variance is higher, which means that these $3 \times 5$ block data are more informative than the $5 \times 3$ blocks.

Figures 44(b) and 44(c) give the E-type maps averaged from 50 realizations conditioned
to $3 \times 5$ block data, then to the $5 \times 3$ fine block data. The $3 \times 5$ block E-type map provides better reproduction of the patterns of the reference model (Figure 46(a)). It is very close to the E-type map conditioned to 25 blocks (Figure 42(c)). In the $5 \times 3$ block E-type map, the high value areas tend to be smeared out. Comparing the realizations conditioned to $3 \times 5$ block data (Figures 44(d) and 44(f)) with those conditioned to $5 \times 3$ block data (Figures 44(e) and 44(g)), the reference heterogeneities are better reproduced in the realizations conditioned to $3 \times 5$ block data whose elongation matches the field anisotropy.

4.3.3 Different block coverage

In BGEOST, the conditioning block data can partially cover the field, also any single block can be made of different discontinuous parts.

Figure 45 gives block data covering only part of the field. The three blocks in Figure 45(a) are located in either high or low heterogeneous area of the reference field (Figure 46(a)). Thus, they are very informative data. Figure 45(b) gives their histogram. In Figure 45(c), the previous lower two blocks are averaged and treated as one single block with an average value 0.178 (Figure 45(d)). That mean (0.178) is close to the reference mean 0.168, thus this discontinuous large block data becomes uninformative. In Figure 45(e), all three previous blocks are pooled together as one single block with average value 0.242 (Figure 45(f)), a value higher than the reference mean (0.168).

The E-type maps averaged from 50 BSSIM realizations conditioned to these 3 types of blocks are shown in Figure 46(b), 46(c) and 46(d). The E-type map (Figure 46(b)) conditioned to the 3 separate block data is close the E-type conditioned to the 25 blocks, see Figure 42(c). The CPU cost of each BSSIM realization in this case is 24 seconds and the memory cost is 46 megabytes. This means that conditioning to a few but informative blocks can achieve the same good result as conditioning to dense block data, but with much faster speed and at lower memory cost. In the E-type map (Figure 46(c)) conditioned to only 2 blocks (Figure 45(c)), the middle left high-value area shrinks to the well location and the bottom low-value area is missing. This is because the large discontinuous block value is close to the reference mean and does not provide additional information. In the E-type map (Figure 46(d)) conditioned to the single block (Figure 45(e)), both the top...
high-value and the bottom low-value areas are missing because that single big block datum does not provide any resolution about where the high or low heterogeneity is. The middle left high-value heterogeneity comes from the conditioning well data.

The two realizations (Figures 46(e) and 46(h)) conditioned to the 3 blocks provide patterns close to the reference model and provide a measure of uncertainty. In the two realizations (Figures 46(i) and 46(l)) conditioned to the 2 blocks, a high-value heterogeneity appears in the bottom area which is actually low-value in the reference model (Figure 46(a)). Similarly, in the two realizations (Figure 46(g) and Figure 46(j)) conditioned to the single large block, a high-value area can appear on the bottom and a low-value area can appear on the top middle area, a significant difference from the reference model (Figure 46(a)).

4.4 Different correlation ranges

In this section, the impact of different correlation ranges is studied using BKRIG with the same reference 2D model defined in Section 2.1. Only the 9 block data are used for conditioning (Figure 1(e)). Three isotropic correlation ranges, 11000, 4000 and 1250, are used. An isotropic spherical variogram model is used for each case.

As the correlation range decreases, the kriging estimation changes from smooth maps (Figures 47(a) and 47(b)) to a map (Figure 47(c)) replicating the block data shape (Figure 1(e)).

The kriging variances increase as the correlation range decreases (Figures 47(d), 47(e) and 47(f)). The kriging variances are lower at the center of any one block and higher at its edges.

The number of conditioning blocks found at each node decreases as the correlation range decreases, see Figures 47(g) and 47(h) and 47(i). Since the variogram model is isotropic, the number of conditioning blocks map is symmetric.

The block data are well reproduced in all these three cases, see Figures 47(j), 47(k) and 47(l). With a longer range, the block data are better reproduced.
5 Conclusion and further work

The previous studies show that BGEOST can be used for integrating data of different scales (block and point) for both 2D and 3D applications. Point data are exactly reproduced and the block data are reasonably well reproduced without bias. BKRIG generates a unbiased but smooth kriging estimation. BSSIM and BESIM generates alternative equally probable simulated realizations with a good reproduction of the variogram model. BSSIM realizations can in addition reproduce a target reference histogram using the algorithm proposed by Soares. BESIM is much faster than BSSIM when many realizations are asked for but can not reproduce a target histogram. BGEOST is very flexible in handling blocks of any shape.

Additional work should address the following aspects.

- Extensively test BGEOST on real data set
- Optimize the program, make it even faster and more memory efficient and easier to use.
- Find new applications.

Downscaling cases have been studied. The tomography studies given in this report can be one step of an iterative tomography process. Iterative tomographic inversion can be performed by combining BGEOST with an external ray-tracing program.

Since BGEOST is a general program, it can be used for any application integrating fine-scale point-support data with coarse-scale linear average block-support data.
Figure 1: The reference field, the point and block data (2D downscaling case).
Figure 2: Parameter settings of BKRIG (2D downscaling case).
(a) **BKRIG** estimation

(b) Histogram

(c) Kriging variance

(d) Number of cond. blocks per node

(e) Block data reproduction check

Figure 3: Results from **BKRIG** (2D downscaling case).
Figure 4: Parameter settings of BSSIM (2D downscaling case).
Figure 5: Two realizations from BSSIM (2D downscaling case).
Figure 6: **BSSIM** E-type from 50 realizations vs. **BKRG** estimation result (2D downscaling case).

Figure 7: **BSSIM** block data reproduction check (2D downscaling case).
Figure 8: *BSSIM* variogram reproduction check (2D downscaling case).
Figure 9: Parameter settings of BESIM (2D downscaling case).
(a) Reference model

(b) Reference model histogram

(c) Realization 1

(d) Histogram

(e) Realization 2

(f) Histogram

Figure 10: Two realizations from $BESIM$ (2D downscaling case).
Figure 11: BESIM E-type from 50 realizations vs. BKRIG estimation result (2D downscaling case).

Figure 12: BESIM block data reproduction check (2D downscaling case).
Figure 13: *BESIM* variogram reproduction check (2D downscaling case).
Figure 14: The reference field, point and block data (3D downscaling case).
Figure 15: Parameter settings of BKRIG (3D downscaling case).
Figure 16: Results from BKRIIG (3D downscaling case).
Figure 17: Parameter settings of BSSIM (3D downscaling case).
Figure 18: Realizations from BSSIM (3D downscaling case).
Figure 19: Results of realization 1 from BSSIM (3D downscaling case).
Figure 20: BSSIM E-type from 20 realizations vs BKRIIG estimation (3D downsampling case).
Figure 21: Parameter settings of BESIM (3D downscaling case).
Figure 22: Realizations from BESIM (3D downscaling case).
Figure 23: Results of realization 1 from BESIM (3D downscaling case).
Figure 24: *BESIM* E-type from 20 realizations vs *BKRG* estimation (3D downscaling case).
Figure 25: The reference field, point and ray data (3D tomography case).
Figure 26: Parameter settings of BKRIG (3D tomography case).
Figure 27: *BK*IG estimation vs. reference model (3D tomography case).
Figure 28: Results from BKRI\textit{G} (3D tomography case).
Figure 29: Parameter settings of BSSIM (3D tomography case).
Figure 30: Realizations from *BSSIM* (3D tomography case).
Figure 31: Results of realization 1 from BSSIM (3D tomography case).
Figure 32: BSSIM E-type from 20 realizations vs BKRG estimation (3D tomography case).
Figure 33: Parameter settings of BESIM the 3D tomography case.
Figure 34: Realizations from *BESIM* (3D tomography case).
Figure 35: Results of realization 1 from BESIM (3D tomography case).
Figure 36: *BESIM* E-type from 20 realizations vs *BKRG* estimation (3D tomography case).
Figure 37: QQ plots of realization vs. the reference model for BSSIM and BESIM (2D downscaling case).
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Figure 43: The block data with different shapes (2D downscaling case).
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Figure 47: *BKRG* results with different correlation ranges (2D downscaling case).
Appendix

Acronyms

SGeMS  Stanford geostatistical modeling software

SGSIM  sequential Gaussian simulation

BGEOST  block geostatistics

BSSIM  block sequential simulation

BESIM  block error simulation

BKTRIG  block kriging estimation

BCOVAR  block covariance calculation

BDSIM  block direct simulation

E-type  conditional expectation estimate

SK   simple kriging

OK   ordinary kriging
Bibliography


