Abstract

Appraisal of new hydrocarbon fields is a high risk exercise, where large investments are based on very sparse exploration data. At such an early stage, quantifying the uncertainty on a global petrophysical parameter such as the net-to-gross (proportion of reservoir facies) cannot be done with traditional statistical approaches that, typically, assume data independence and cannot easily account for the contribution of seismic data and, most critically, geological interpretation.

We propose a framework to assess global uncertainty in early exploration. This framework accounts for the different alternative geological scenarios resulting from geological interpretation, and addresses the potential bias induced by the preferential location of appraisal wells in high-pay zones.

Application to a large synthetic 3D reservoir demonstrates the proposed methodology.

1 Introduction

All scientific measurements are affected by uncertainty. Expressing this uncertainty, even for common quantities like temperature or length, is not an easy task (Taylor and Kuyatt, 1994). This is even more true when geosciences data and their interpretation are combined to produce a global measure such as ore or hydrocarbon reserves, attributes that, by definition, have no replicate in space or time. This problem is particularly critical during early exploration, because extremely sparse data have to support significant investments.

In this paper, we consider the problem of assessing uncertainty about the global net-to-gross ratio (NTG) of a deep offshore reservoir at the appraisal stage. At such an early stage, the net-to-gross is, with the total rock volume, the main unknown controlling the original oil in place.

1.1 Related work

Subsurface data provide an incomplete view of the reservoir under study. Most uncertainty assessment methods model the error between a measure and the true value by statistically repeating observations while varying —i.e., randomizing— some relevant aspects affecting the estimate only (devices used, calibration, etc.) or both the estimate and the true value (time, location, temperature, etc.). The choice of which parameters to vary and the way...
these parameters are varied from one observation to another define the final uncertainty result, and thus need to be properly documented.

In geosciences, uncertainty assessment is made difficult by the complexity of the subsurface and by the uniqueness of the observed data: there are no alternative data available on the same reservoir. Geostatistical simulation techniques aim at assessing the local uncertainty at any unsampled location in the reservoir, but require global parameters such as the facies global proportions and variograms, themselves uncertain. For instance, sequential indicator simulation of two rock types produces a set of simulated images of the spatial distribution of these two rock types, that all globally honor a prior target net-to-gross value. The variations of net-to-gross observed from one realization to another are mere ergodic fluctuations around this target. Those ergodic fluctuations would vanish if the field being simulated was very large with regard to the range of the input variogram model. One should not rely entirely on such ergodic fluctuations to evaluate the uncertainty about the target input for NTG.

Most traditional approaches (see Biver et al., 1996) start by assuming unrealistic independent, identically distributed distributions that ignore geological structures, and consider non-robust statistics like the mean and the variance. Among such established statistical tools is the bootstrap method (Efron, 1979) which consists of drawing with replacement several synthetic wells from the actually observed well data, thus ignoring any spatial correlation between data.

Haas and Formery (2002) propose to derive analytically a probability distribution of the estimate from the observed data. The basic hypothesis is that the distribution $P_x(n)$ of the vector $n$ of facies samples observed along the well(s), given the vector of true facies proportions $x$, follows a multinomial law. Since the proportions $x$ are unknown in practice, their probability distribution $P_n(x)$, given the observed samples $n$, is obtained by Bayesian inversion. If the prior facies distribution is uniform or Dirichlet, the updated distribution $P_n(x)$ is also a Dirichlet distribution.

Bootstrap and Dirichlet approaches, both based on independent drawings from the data, make it difficult to incorporate local secondary information as provided by the seismic data. Moreover, both consider the well data values to be independent one from another. This hypothesis can never be correct in practice, since well data, sampled continuously along the well path, are spatially correlated. More importantly, this hypothesis is not safe: ignoring spatial dependency and data redundancy is likely to produce too narrow uncertainty ranges. Bitanov and Journel (2003) have shown that both bootstrap and Dirichlet could indeed lead to over-optimistic confidence intervals.

To overcome this limitation, Journel (1993); Norris et al. (1993) proposed the spatial bootstrap method to assess global uncertainty. The idea is to resample alternate sets of data from simulated fields. This method accounts for spatial dependency of the data, and can integrate secondary information. Recently, Bitanov and Journel (2003) have applied spatial bootstrap for assessing the uncertainty of NTG estimates from early exploration data.

This paper expands on the previous work by considering other sources of uncertainty, most notably that associated with the geological interpretation of the data.
1.2 Requirements of an uncertainty model

Because of the uniqueness (no replicate available) of a global parameter, there cannot be a “true” or objective model of uncertainty. Such a model is necessarily a mathematical and algorithmic construction which depends completely on the choice of the model parameters and how these parameters are made to vary.

Several points must be stressed when developing any model for uncertainty assessment:

1. Statement of the constitutive modeling decisions: which model parameter is frozen (deemed known and certain) and which is made variable, then how variable?

2. Ability to reproduce results: a model of uncertainty should lead to the same results when run again with the same parameters.

3. Physical and geological consistency: is the model built on acceptable physical and geological criteria and its parameters evaluated with the same considerations?

4. Usability: is the model reasonably easy for a geologist, a geophysicist or a reservoir engineer to use and are its parameters understandable?

5. Performance: is the model feasible for application to a real size 3D reservoir model within reasonable RAM demand and CPU time?

6. Heteroscedasticity: the uncertainty model should yield different confidence intervals when the data values change, everything else being constant.

7. Statistical consistency: by repeating this uncertainty model to, say, 100 different data sets taken from an exhaustively known model, the true value should lie about 90 times within the $[p_{10}, p_{90}]$ interval determined from each data set.

2 Parameters of the uncertainty model

These next two sections are retake of the previous paper by Journel and Caumon (2004): readers having understood the proposed workflow, as summarized by Figure 1, can move directly to the case study (Section 4).

In any uncertainty modeling task, the prior determination and ranking of the main sources of uncertainty is possibly as much significant as the implementation model itself.

In this work, the uncertainty model is built on a classical Bayes inversion approach, considering three jointly related sources of uncertainty and corresponding random variables: the geological scenario $S$, the global NTG value $A^\star$ estimated from the data available, and the true value $A$.

Notations

Random variables (RV’s) are denoted with capital letters. e.g., $A$ for the global NTG value. The $L$ realizations of a random variable are denoted by a set of indexed small letters, e.g., $a_l, l = 1, \ldots, L$. The subscript 0 is reserved to the true (unknown) value assumed to be one particular realization, e.g., $a_0$ for the true NTG value. Vectorial random variables and the associated vectorial outcomes are denoted in bold, e.g., $\mathbf{D}$ and $\mathbf{d}$ for the data, see hereafter.
2.1 Geological scenario

In early exploration, the main uncertainty is related to the geological settings of the domain under study. We propose to represent this unknown by a set of geological scenarios \( s_k, k = 1, \ldots, K \) associated with the data \( d_0 \) available. These are considered as the possible outcomes of a discrete random variable \( S \). Each scenario \( S = s_k \) is defined by:

- the vector of actually observed quantitative data \( d_0 \);
- a particular geological interpretation of these data, which could reduce to a mere variogram model, or consist of a prior conceptual image of spatial patterns depicted by a training image \( TI_k \) (Fig. 1). The uncertainty model presented hereafter is not specific to any particular representation of the spatial variability. We do, however, propose to use training images and the corresponding multiple point geostatistical formalism, since these can carry geological interpretation more comprehensively than variograms.

A scenario \( s_k \) can never be determined independently from the data \( d_0 \). For this reason, we define the following equivalence of notations:

\[
s_k \equiv \{d_0, TI_k\}
\]

The inference of a scenario from sparse exploration data is an expert, necessarily subjective, decision which has a significant influence over the uncertainty model. The consequent training images \( TI_k, k = 1, \ldots, K \) could be obtained from hand drawn sketches suggested by sedimentologists. In practice, drawing sediment bodies in 3D proves difficult and time-consuming: outcrops, aerial pictures used for sketching are inherently 2D. Moreover, drawing can only be done on one 2D section at a time, causing critical human interface issues in 3D drawing systems. Therefore, sedimentological interpretation is generally interpreted in term of generation processes and/or descriptions of geological bodies. The training images can then be obtained by runs of process-based forward simulations (Tetzlaff and Harbaugh, 1989; Bowman and Vail, 1999) or unconditional object-based simulations (Deutsch and Wang, 1996; Viseur, 2004; Scheepens, 1999).

2.2 Estimation algorithm

The observed data set \( d_0 \), is used to produce an estimate \( A^* = a^* \) of the global value \( A \). The estimation method can be represented by a function \( \varphi(d) \) mapping any particular set of data \( d \) into a “best” estimate \( A^* = a^* \). For NTG, this algorithm could for instance:

- average the observed sand / non-sand values at well locations;
- interpolate the sand / non-sand indicator values on the domain, using some indicator co-kriging of well data and seismic impedance \( ^1 \) (Goovaerts, 1997, p. 297);
- perform a co-located or a Bayesian calibration of impedance data based on well calibration (Caers and Ma, 2002; Bitanov and Journel, 2003);

\(^1\)In the case of co-kriging, the transfer function \( \varphi \) between the quantitative data \( d \) and the estimate \( a^* \) accounts for variograms and cross-variograms which are specific to each scenario \( s_k \).
• perform a canonical correlation between facies and various seismic attributes (Fournier
and Derain, 1995).

Note that robustness of these techniques generally relies on the number of calibration
data. Since so few data are available during early exploration, advanced methods (Caers
and Ma, 2002; Fournier and Derain, 1995) may not be more accurate than simpler and
faster estimation techniques (Bitanov and Journel, 2003).

The estimate $A^* = \hat{a} = \varphi(d)$ appears as a summary of the data set $d$. One way
to randomize the estimate $a^*$ is to randomize the data $d$ themselves. Therefore, we will
use a probabilistic notation for the data event. $D = d_0$ denotes the actual data values
observed. In our case, these data comprise both seismic impedance and well observations.
$D = d$ denotes any alternate data set, when some aspect of the data is changed, e.g., the
well locations are changed while freezing the number and configuration of well data and
the drilling strategy.

Ideally, randomizing the estimate $a^*$ into the random variable $A^*$ would call for ran-
domizing both the data $d_0$ into $D$ and the estimation algorithm $\varphi(\cdot)$. In this paper, only
the locations of wells hence the well data are varied, while the estimation algorithm $\varphi(\cdot)$,
the seismic data and the well drilling strategy are frozen (see Appendix A).

2.3 Unknown global value

Most error assessment algorithms, including bootstrap resampling (Efron, 1979), con-
sider only a randomization $A^*$ of the estimate $a^*$. Instead, we suggest that the unknow
global value $a$ should be also randomized into a random variable $A$, conditionally to each
geological scenario $S = s_k, k = 1, \ldots, K$.

Selecting a prior probability $P(A = a)$ for the random variable $A$ taking any value $a \in
[0, 1]$ is difficult, since that prior probability should almost never be a uniform distribution.
Moreover, it is not appropriate to dissociate this prior probability $P(A = a)$ from the the
observed data $d_0$: an expert can only judge the appropriateness of any value $a$ from
the data $d_0$ interpreted with his prior experience. The idea of our proposed conditional
randomization is to update this prior probability by a given geological scenario $s_k$.

3 Global uncertainty model

3.1 Problem statement

Our goal is to assess the uncertainty about any possible realization $A = a$ of the global
value, given the available exploration data $d_0$ and various geological interpretations
$s_1, \ldots, s_K$ based on the same data. The uncertainty about the global value $a$ is defined
conditionally to a specific estimate $a^* = \varphi(d_0)$ made from the data $d_0$. For a given
scenario $s_k$, our goal is then to obtain the probability distribution

$$P(A = a | A^* = \varphi(d_0), S = s_k).$$

In early exploration, the most reliable source of information is the observed quantita-
tive data $d_0$. Therefore, provided that the scenarios $s_1, \ldots, s_k$ do cover the range of all
possible geological interpretations of these data, it is useful to consider the uncertainty about the true value \( a \), conditional only to the data \( d_0 \):

\[
P(A = a | A^* = \varphi(d_0)) = \sum_{k=1}^{K} P(A = a | A^* = \varphi(d_0), S = s_k) \cdot P(S = s_k) \quad (1)
\]

The main hurdle in equation (1) is the definition of a prior probability \( P(S = s_k) \) for geological scenarios. This scenario probability is likely the most consequential for the final result. In a pure Bayesian mode, this prior scenario probability should be chosen independently of the observed quantitative data \( d_0 \). A convenient solution typically consists of taking a uniform distribution:

\[
P(S = s_k) = \frac{1}{K}, \quad \forall k = 1, \ldots, K
\]

Such a choice gives an equal importance to all scenarios, which is questionable. For instance, one should decrease the prior probability \( P(S = s_k) \) of scenarios deemed unlikely from qualitative information, such as provided by the regional context or analogs, see Appendix B for further discussion.

The second term needed to evaluate equation (1) is the distribution of the true value \( A \) conditional to the estimate \( A^* = \varphi(d_0) \) and a specific geological scenario \( S = s_k \).

### 3.2 Distribution of the true value \( A \) given \( d_0 \) and \( s_k \)

We propose to compute the probability for the global property \( A \) to take any value \( a \) given a specific geological scenario \( s_k = \{d_0, TI_k\} \) using Bayesian inversion (Spiegel 1968, p. 9):

\[
P(A = a | S = s_k, A^* = \varphi(d_0)) = \frac{P(A^* = \varphi(d_0) | S = s_k, A = a) \cdot P(A = a | S = s_k)}{P(A^* = \varphi(d_0) | S = s_k)} \quad (2)
\]

The evaluation of the right-hand side terms of formula (2) is now described, see also Figure [1].

### 3.3 Probability \( P(A = a | S = s_k) \) given a scenario \( s_k \)

The probability \( P(A = a | S = s_k) \) should be assessed from the available data \( d_0 \) and their interpretation under a given geological scenario \( S = s_k \equiv \{d_0, TI_k\} \), see second row of Figure [1]. Since the geological scenario \( s_k \) has been retained from analysis of the actually observed data \( d_0 \), the distribution \( P(A = a | S = s_k) \) depends on \( d_0 \), although the corresponding simulated realizations shown in the second row of Figure [1] need not be conditioned exactly to each observed datum value at its exact location. In the following, we name this distribution \( P(A = a | S = s_k) \) a prior distribution: it is a “prior” in that it is defined before any quantitative estimation \( a^* = \varphi(d_0) \) of the global value.

An automatic method can be considered to obtain this distribution \( P(A = a | S = s_k) \): for each geological scenario \( S = s_k \), one can simulate, conditional or not to the observed
Initial data \( D = d_0 \)

\[
\begin{array}{c}
+ \\
+ \\
+ \\
\end{array}
\]

\( K \) different training images

\( TI_i \)

\( TI_k \)

\( TI_K \)

Scenario \( S = s_k = \{ d_0, TI_k \} \)

\[ P(A \in a_m | S = s_k) \]

\[ P(\hat{A} = \phi(d) | A \in a_1, S = s_k) \]

\[ P(\hat{A} = \phi(d) | A \in a_M, S = s_k) \]

Figure 1: Flow chart of the simulations needed to evaluate Equation (2).

data values \( d_0 \) at their actual locations, a series of \( L \) fields and observe the resulting simulated distribution \( P(A = a | S = s_k) \) of the global property \( A \). These simulations should not start by freezing the global property to \( A = a^* = \phi(d_0) \) or to any other estimated value. If reproduction of multiple point statistics is called for, then a corresponding simulation algorithm (Strebelle, 2002) could be used. If the training images are reduced to variogram models, an algorithm such as sequential Gaussian or indicator simulation would suffice (Goovaerts, 1997). Since few data are available, an object-based algorithm as described by Deutsch and Wang (1996); Viseur (2004) could also be used.

Remarks

- The NTG ratios of the simulated realizations will all be somewhat different from the NTG of the training image \( TI_k \), because of conditioning to the observed data \( d_0 \) and fluctuations from realization to realization. The distribution of the simulated NTG ratios can then be taken as the prior probability \( P(A = a | S = s_k) \) in the Bayesian inversion expression (2), see second row of Figure 1.

- The resulting prior probability distribution \( P(A = a | S = s_k) \) depends upon the
multi-point simulation method chosen and its parameters. Such a subjective algorithmic decision is inherent to any uncertainty model. The point of our proposal is that it honors the data $\mathbf{d}_0$ and the description of spatial patterns $TI_k$ produced by each specific expert geological interpretation of these data.

The determination of this prior distribution $P(A = a|S = s_k)$ is a consequential yet difficult task. The proposed method above aims at alleviating the geological expertise required for an uncertainty study by leaving this task to an algorithm. This is of course debatable, and one may prefer to define explicitly this distribution, based for instance on fields deemed similar to the one under study.

### 3.4 Likelihood probability $P(A^* = \varphi(\mathbf{d}_0))|S = s_k, A = a)$

For any specific true value $A = a$, several methods can considered to assess the likelihood of any estimate $\varphi(\mathbf{d})$ based on an initial data set $\mathbf{d}$ (Fig. 1, third row); examples are:

- For any estimation algorithm $\varphi(\cdot)$, a random perturbation of the parameters of the estimation algorithm may provide a distribution of estimates.
- Alternative estimation algorithms $\varphi(\cdot)$ may also be considered (see Section 2.2).
- Most global estimation methods $\varphi(\cdot)$ start by computing some local value $v(\mathbf{u})$ at all locations $\mathbf{u}$ of the reservoir grid, then combine these values $v(\mathbf{u})$ to produce a global estimate $a^*$. For instance, a NTG estimation can be achieved by averaging the local probability of sand over all grid locations. Typically, some uncertainty can be associated to each local value $v(\mathbf{u})$, depending on the location $\mathbf{u}$ and on the available well-to-seismic calibration set. The combination of all these local uncertainty measures can provide a global uncertainty measure.

The three sources of uncertainty above relate to the impact of the estimation method $\varphi(\cdot)$. This is indeed relevant, but possibly of second order as compared to the uncertainty associated to the data themselves. Since data are so sparse at an early exploration stage, their location is a significant source of uncertainty: what if the exploration well(s) had been drilled elsewhere? We propose here to use the spatial bootstrap approach (Journel, 1993; Bitanov and Journel, 2003) to assess this critical aspect of the uncertainty. In this paper, the sources of uncertainty related to the estimation algorithm (as above), to seismic processing and to well log interpretation are not accounted for, although they should and our conceptual workflow would allow it.

Spatial bootstrap models the uncertainty associated with data locations: if the well(s) had been drilled elsewhere, different NTG results would have been obtained. Since drilling new wells in the real reservoir for mere bootstrap is not an option, the idea of spatial bootstrap is to perform that resampling from synthetic fields obtained by geostatistical simulation. This resampling can for instance be achieved by moving the data configuration over the simulated fields, while honoring some drilling strategy (e.g., drill only in the lowest impedance values, potentially the best reservoir locations).

In principle, the likelihood distributions $P(A^* = \varphi(\mathbf{d}_0))|S = s_k, A = a)$ are needed for all possible true values $A = a$. For practical purposes, only $M$ classes $a_1, \ldots, a_M$ of NTG
values can be considered within the range span by the prior distribution $P(A = a|S = s_k)$. For any class $a_m$, $m = 1, \ldots, M$, one or more representative simulated fields can be generated by a geostatistical simulation algorithm. Alternative data sets $D = d_i, i = 1, \ldots, N$, can then be resampled from each of these fields, see last row of Figure 1.

Applying the estimation method $\varphi$ on all resampled data $D$ provides the likelihood probability as called for by expression (2):

$$P(A^* = \varphi(d)|A \in a_m, S = s_k), \quad \forall m = 1, \ldots M$$

3.5 Summary

In sections 3.3 and 3.4, we have considered how to evaluate the prior probability distribution $P(A = a|S = s_k)$ and the likelihood $P(A^* = \varphi(d_0)|S = s_k, A = a)$ of observing the actual data $d_0$.

These two terms can now be combined to evaluate the probability $P(A^* = \varphi(d_0)|S = s_k)$ by integrating $P(A^* = \varphi(d_0)|S = s_k, A = a)$ over a number $M$ of possible classes of values for $A$. From Equation (2), the probability for the global property $A$ to take any value $a$, given a specific geological scenario $s_k = \{d_0, TI_k\}$, then becomes:

$$P(A = a|S = s_k, A^* = \varphi(d_0)) = \frac{P(A^* = \varphi(d_0)|S = s_k, A = a) \cdot P(A = a|S = s_k)}{\sum_{m=1}^{M} P(A^* = \varphi(d_0)|S = s_k, A \in a_m) \cdot P(A \in a_m|S = s_k)} \quad (3)$$
4 Application to Stanford V reservoir

4.1 The reference reservoir

The proposed uncertainty model has been applied to the exhaustively known Stanford V synthetic reservoir (Mao and Journel, 1999; Mao, 1999). This is a necessary first step to check the consistency of the obtained results before moving on to real cases where the “truth” is unknown.

Stanford V is a three layer reservoir created using an object-based simulation of fluvial bodies. Within each of the three facies (channel, crevasse, and floodplain), sequential Gaussian simulations were run to generate petrophysical properties. The resulting petrophysical model was then used to forward simulate an impedance cube and seismic travel times. This synthetic seismic signal was purposely degraded by applying moving average and Born filters to emulate limited vertical and areal seismic resolutions typically observed on actual seismic surveys.

For the sake of simplicity, only the synthetic impedance and the facies values are considered in this paper. The original 3 facies, channel, flood plain and crevasse have been lumped into two categories, reservoir (sand) or non reservoir (shale), see Figure 2. All impedance and facies values are available at the seismic resolution only. This would not be the case in a standard exploration situation where facies values observed along wells have a higher resolution than surrounding seismic-derived attributes. In this study, uncertainties due to the change of scale between wells, seismic, and flow simulation support volumes are not accounted for.

The Stanford V reservoir is represented as a stratigraphic grid of size $100 \times 110 \times 30$ cells. For convenience, all calculations have been performed on a Cartesian grid; the actual reservoir geometry could be accounted for using a geochronological coordinates transform (Mallet, 2004). The three layers have the same normalized thickness of 10 cells, but have varying channel orientation, thickness, width and sinuosity (Fig. 2). The net-to-gross values for the three layers are 0.34, 0.51 and 0.58, resulting into a global net-to-gross of 0.48 for the whole reservoir.

In an early exploration survey, the differences between these three layers may not be obvious, unless seen on the seismic cube. In this study, we have chosen to assess the net-to-gross uncertainty for the reservoir as a whole.

4.2 Uncertainty assessment with a single exploration well and a single geological scenario.

A single vertical initial exploration well has been selected, at the location of the lowest vertically averaged impedance, yielding 30 facies data sampled from the exhaustive grid (Fig. 3). This preferential location in low impedance (high porosity) values aims at mimicking an actual drilling strategy: the first exploration well may also be used later as a production well, especially in deep offshore settings where drilling is extremely expensive.

The facies values obtained along this initial well are classified into reservoir (sand, $R$) or non-reservoir (shale, $\bar{R}$) rock types. The proportion 0.60 of reservoir rocks along
Figure 3: Initial data set: one single exploration well at location of lowest vertically averaged impedance. The well displays the rock types lumped into sand (white) or non sand (black). The vertical impedance average is shown on the bottom section of the cube.

Figure 4: 3D training image for the scenario $s_1$. Three cross-sections show the stacking of channels; low sinuosity meandering style is visible on the horizontal section.
this well is suspected to be biased, higher than the true NTG of the reservoir due to the preferential location of that well.

4.2.1 Geological scenario

For this first study, one single scenario \( s_1 = \{d_0, TI_1\} \) has been considered. The data \( d_0 \) consist of the vertical well presented above and of the 3D cube of seismic impedance. The conceptual picture (training image) \( TI_1 \) corresponding to this scenario \( s_1 \) was obtained by an unconditional object-based simulation with the fluvsim program [Deutsch and Wang, 1996], see Figure 4. This 3D picture is conceptually similar to 2D sketches that could be drawn by sedimentologists to depict the style of meandering channels deemed realistic given the available data \( d_0 \). The training image contains multiple point statistics that are borrowed to simulate fields honoring the observed data \( d_0 \).

This first scenario is relatively close to the actual reservoir: the training image \( TI_1 \) was generated with the same program used to generate Stanford V, and the net-to-gross of \( TI_1 \) happens to be equal to the true value 0.48. This can be considered as a “lucky” geological interpretation, even though the channels in the training image \( TI_1 \) (Fig. 4) and in Stanford V (Fig. 2) have different shapes. The case where the training image NTG differs significantly from that (unknown) of the real reservoir is addressed in Section 4.4 where two significantly different geological scenarios are considered.

Choosing the prior distribution \( P(A = a|S = s_1) \)

The choice of a prior distribution is inherent to any Bayesian process. In our workflow, the prior distribution of the true value \( A \) is made conditional only to the geological scenario retained. That distribution should state to which extent the reservoir net-to-gross may depart from the net-to-gross carried by the training image. For this study, a triangular NTG prior distribution has been retained, with a minimum of 0.28, a maximum of 0.62 and a mode of 0.5. The mean of this distribution, 0.46, is close to the net-to-gross 0.48 carried by the training image \( TI_1 \).

This prior distribution \( P(A = a|S = s_k) \) is relatively conservative, because it allows for a large range of possible NTG values. The goal of the proposed method is to update this prior distribution by a specific estimate \( a^* = \varphi(d_0) \) made from the observed data \( d_0 \).

Best estimate \( a^* = \varphi(d_0) \)

The proposed workflow can accommodate any estimation method \( \varphi(\cdot) \). The simplest estimator of a global net-to-gross from an initial exploration well is the average of the observed sand data. This average here yields a 0.6 proportion of sand, a value expected to be overestimated due to the preferential location of that single well.

A better estimate should account for the preferential location of the initial well in the high-pay, low impedance zone, and could be obtained by using both the facies observed along the well and the seismic data available throughout the reservoir. Typically, the lack of calibration data does not allow to apply advanced techniques such as kriging with local prior means [Goovaerts, 1997, p. 308], principal component analysis [Scheevel and Payrazyan, 1999], canonical correlation techniques [Fournier and Derain, 1995] or multiple-point calibration [Caers and Ma, 2002].
Figure 5: Likelihoods of observing an impedance value $z$ given the co-located presence of reservoir facies ($R$, top) or non reservoir facies ($\bar{R}$, bottom). The discrete likelihoods (bars) are smoothed (lines). The upside-down dash lines give the reference likelihood probabilities computed on the whole Stanford V grid (note the significant difference with the sample curves).
Here, we have retained a simple estimation method based on the Bayesian calibration proposed by Bitanov and Journel (2003). This method first calibrates from the well data the likelihoods $P(Z = z|R)$ and $P(Z = z|\bar{R})$ of observing any impedance value $z$ given the co-located presence ($R$) or absence of sand ($\bar{R}$), see Figure 5.

Per Bayes’ inversion, the probability $P(R|Z = z)$ of observing sand at any location given the known impedance value $z$ at that same location is given by:

$$P(R|Z = z) = \frac{P(Z = z|R) \cdot P_0(R)}{P(Z = z|R) \cdot P_0(R) + P(Z = z|\bar{R}) \cdot (1 - P_0(R))} \tag{4}$$

where $P_0(R)$ is the sand probability prior to any seismic observation.

Since the seismic data calibration is done from only the 30 available well data, the results obtained are sensitive to the values observed along that single well. In order to account for the small number of the calibration data, a smoothing filter is applied to the sample likelihoods before evaluating expression (4), see Figure 5. This smoothing, implemented with an iterative Discrete smoothing algorithm (DSI, Mallet, 1989), ensures in particular that the denominator in expression (4) is not null.

The global net-to-gross estimate $a_{\star}^1 = \varphi(d_0)$ is obtained by averaging the local sand probability at all $N$ locations $u_\alpha$ of the reservoir grid:

$$a_{\star}^1 = \frac{1}{N} \sum_{\alpha=1}^{N} P(R(u_\alpha)|Z(u_\alpha) = z(u_\alpha)) = 0.31 \tag{5}$$

This results in a correction of the initial biased well sample proportion (0.60). That correction is, however, too strong, leading now to a severe underestimation: 0.31 is much lower than the true value 0.48. This overcorrection is due to the single well likelihood probabilities being non-representative of the actual true (but unknown) likelihoods given as dash curves in Figure 5. In particular, note the relatively poor discrimination of sand from non sand for impedance values lower that 8000 rayl.

**Remark**

The prior sand probability $P_0(R)$ appearing in expression (4) is consequential for the resulting estimate $a^\star$. Bitanov and Journel (2003) propose to use for $P_0(R)$ the average facies proportion along the well, that is 0.6. Because the final estimate $a^\star$ is conditioned by any retained scenario $s_k$, we prefer retaining for $P_0(R)$ the NTG value 0.48 carried by the training image $TI_1$ here retained.

**Uncertainty of the estimate $A^\star$ and spatial bootstrap**

The previous estimated value $a_{\star}^1 = 0.31$ obtained from the Bayesian calibration is itself highly uncertain. For instance, the likelihood probability $P(Z = z|R)$ could have been smoothed with the program `histsmth` (Deutsch and Journel, 1998, p. 214) instead of using our DSI implementation, yielding a slightly different estimate. Or, a different seismic calibration algorithm, other than (4), could have been used, see Section 2.2.
We consider (see Appendix A) that such algorithm-related uncertainty is of second order as compared to the uncertainty related to the data themselves: if the single exploration well had been drilled elsewhere, everything else being the same, the estimate could be drastically different from our 0.31 value. Therefore, to assess the likelihood probability 

$$P(A^* = \varphi(d_0)|S = s_1, A = a),$$

we have adopted a spatial bootstrap approach as described by Journel [1993], Bitanov and Journel [2003]. The seismic data have not been made variable, although this significant source of uncertainty should be accounted for.

Since the true value $A = a$ is unknown, all possible NTG values between the interval $[0.28, 0.62]$ of the prior distribution $P(A = a|S = s_1)$ have been classified into $M = 15$ classes $a_1, \ldots, a_M$. Fifteen synthetic fields $F_m(s_1), m = 1, \ldots, 15$ each honoring one of these fifteen NTG classes were generated using the multiple point simulation algorithm snesim [Strebelle, 2002]. The geological scenario $s_1 \equiv \{TI_1, d_0\}$ was used for these simulations, see Figure 4 for $TI_1$. The 30 facies data observed along the initial exploration well and the entire cube of seismic impedance were used as hard and soft conditioning data, respectively.

For each of these $M = 15$ realizations, $N = 500$ alternative vertical well locations were simulated within the 20% lowest vertically averaged impedance. This preferential resampling was done to mimick the drilling strategy applied to drill the actual unique exploratory well. Other criteria could have been be used, e.g., topography of the surface, neighborhood of faults, etc.

The estimation procedure [5] was then exactly repeated for each resampled well, providing a distribution of $N = 500$ estimated values $a^*$ for each of the $M = 15$ realizations. More precisely, each estimate $a^*_{m,n}, m = 1, \ldots, 15; n = 1, \ldots, 500$ was obtained by:

- Applying Equation (4) to get the local sand probability at every location $u_\alpha$, $
\alpha = 1, \ldots, N$ of the Stanford V reservoir grid; the prior sand proportion $P_0(R)$ in (4) is set equal to the actual NTG value $a_m$ constraining the resampled simulated field $F_m(s_1)$.

- Averaging the local sand probability at all $N$ locations of the reservoir grid, see Equation (5).

Figure 6 shows three of the realizations obtained for three NTG target values $a_m$, and the resulting three probability distributions $P(A^* = a^*|S = s_1, A = a_m)$ obtained by spatial bootstrap.

**Distribution obtained**

The combination [3] of the prior triangular distribution $P(A \in a_m|S = s_1)$ and the probabilities $P(A^* = \varphi(d_0) = 0.31|A \in a_m, S = s_1), l = 1, \ldots, 15$ provides the posterior distribution of the true value $P(A \in a_m|A^* = 0.31, S = s_1)$ shown as vertical bars in Figure 7.

The shift of the posterior distribution towards low values results from the initial biased estimate $a_1^* = 0.31$, a value on the low end of the prior distribution $P(A = a|S = s_1)$. Spatial bootstrap corrects this bias by considering alternate well locations on simulated
Figure 6: Three probability distributions for the estimate value $A^*$ as obtained by spatial bootstrap. In the Bayesian inversion [2], only the specific probability for $A^* = a^*_1 = 0.31$ is used.
fields. The mean of the posterior distribution is 0.37, a value closer to the true Stanford V NTG value 0.48 than the initial estimate $a^*_1 = 0.31$.

The confidence interval for the true value as derived from the probability distribution $P(A = a|A^* = \varphi(d_0), S = s_1)$ is $[p_{10}, p_{90}] = [0.30, 0.45]$. The fact that this interval $[0.30, 0.45]$ fails, although barely, to include the true value is troublesome. Could it be related to the original single well being located in one of the worst biased locations possible? To check this point, we repeat the entire uncertainty study many (100) times, each time starting with a different single initial well.

4.3 Uncertainty assessment with multiple alternative single exploration wells and a single geological scenario.

An uncertainty study is necessarily case-specific, which makes it difficult from one single experiment to judge if the approach used is appropriate. Therefore, the exact previous study has been repeated 100 times, starting each time with a different single initial exploration well drilled within the 20% lowest vertically averaged impedance, all other parameters being identical.

The 100 confidence intervals $[p_{10}, p_{90}]$ computed from these 100 runs are shown on
Figure 8. Out of these 100 intervals, the actual value 0.48 has been excluded only three times. This show that the results obtained are conservative. The reason for this good result is probably the choice of a relatively conservative prior NTG distribution \( P(A = a|S = s_k), k = 2, 3 \), the triangular distribution shown in Figure 7.

4.4 Uncertainty assessment with a single exploration well and several geological scenarios.

In the previous sections, we have retained a single geological scenario \( s_1 = \{d_0, TI_1\} \) relatively close to the actual reservoir: the training image \( TI_1 \) had a NTG of 0.48 equal to that of Stanford V. What would be the impact of considering significantly different scenarios? Two additional uncertainty runs were performed with two other scenarios \( s_2 \) and \( s_3 \) containing the same initial data set \( d_0 \), but two distinctly different training images \( TI_2 \) and \( TI_3 \) (see first row of Figure 9). For both scenarios, the prior probability distribution \( P(A = a|S = s_k), k = 2, 3 \) was chosen as previously, triangular spanning a relatively conservative range of NTG values, see Figure 9. The modes (0.30 for scenario \( s_2 \) and 0.55 for scenario \( s_3 \)) of these two triangular distributions were set so that their means be similar to the NTG of the corresponding training images.

The scenario \( s_2 = d_0, TI_2 \) can be considered as a conservative, very pessimistic one, because the NTG carried by the training image is 0.35, a value much lower than the observed well average 0.60. Using this value 0.35 as the prior sand proportion \( P_0(R) \) in
expression (1), we obtained a new NTG estimate $a_2^* = 0.24$. The prior NTG probability distribution $P(A = a|S = s_2)$ associated with the scenario $s_2$ and the corresponding posterior distribution $P(A = a|A^* = a_2^* = 0.24, S = s_2)$ are shown in Figure 9. As could be expected, the $[p_{10}, p_{90}]$ confidence interval $[0.27, 0.35]$ does not include the actual Stanford V value 0.48. This result is not surprising because the scenario $s_2$ was so pessimistic. This pessimistic scenario $s_2$ compounds with the already severely biased estimate $a_2^* = 0.24$.

Quite the reverse, the scenario $s_3 = d_0, TI_3$ conveys an initial guess NTG value of 0.55, higher that the actual NTG value 0.48 of the Stanford V reservoir. The scenario $s_3$ is an optimistic scenario: the training image NTG value, 0.55, is taken lower than the 0.60 well average to account for the suspected bias due to the preferential well location. Using this value 0.55 as the prior sand proportion $P_0(R)$ in expression (1), we obtained a new NTG estimate $a_3^* = 0.34$. The prior NTG probability distribution $P(A = a|S = s_3)$ associated with the scenario $s_3$ and the corresponding posterior distribution $P(A = a|A^* = \varphi(d_0) = 0.24, S = s_3)$ obtained are shown in Figure 9. As compared to our first study (Section 4.2), the $[p_{10}, p_{90}]$ confidence interval $[0.30, 0.48]$ now contains the actual value 0.48. This optimistic scenario $s_3$ has compensated for the biased estimate $a_2^* = 0.34$.

**Figure 9**: Uncertainty results obtained with t pessimistic scenario $s_1$ (left) and the optimistic scenario $s_3$ (right).
4.5 Performance

All multiple-point simulations were run with the unoptimized FORTRAN implementation of \texttt{snesim} described by Zhang (2003). Using a training image of size $75 \times 150 \times 15$, it takes about 2 minutes to compute a realization on the Stanford V $100 \times 110 \times 30$ grid on a 1.13 GHz, 512 RAM, laptop. Running 500 well-to seismic calibration and the estimation process took 90 seconds on the same platform. The time needed to run a complete uncertainty run as described in Section 4.2 is slightly less than one hour.

5 Concluding remarks

As opposed to bootstrap resampling (Efron, 1979) or the Dirichlet formalism (Haas and Formery, 2002), the proposed model of uncertainty accounts for spatial correlation and spatial patterns between data, as delivered by training images (Journel, 1993; Bitanov and Journel, 2003).

The novelty of the proposal is the Bayesian updating of the prior probability $P(A = a | S = s_k)$. As opposed to the methods presented by Biver et al. (1996), this updating does not rely on a Gaussian distribution for the global variable.

How does this framework answer the requirements stated in the introduction?

1. Statement of the constitutive modeling decisions: the proposed model of uncertainty does not call for data independence. Each reservoir model generated by the method honor a proposed geological scenario, be it based on a variogram or on a training image.

For the study presented in this paper, the geophysical data were not questioned, hence not randomized, and a single estimation algorithm $\varphi(\cdot)$ has been used. These two factors are potential significant sources of uncertainty. Therefore, the uncertainty model could be made more complete by their randomization.

2. Ability to reproduce results: each randomization algorithm used (spatial resampling and geostatistical simulation) takes a random seed as input parameter; the outcome of that algorithm is uniquely determined by the value of that seed. The use of this random seed paradigm guarantees the reproducibility of any obtained result.

3. Physical and geological consistency: the proposed use of a multiple point geostatistical simulation algorithm produces results that are compliant with the information-rich geological scenarios retained.

In this study, only vertical wells have been considered; more realism would call for resampling non-conventional wells during the spatial bootstrap procedure.

4. Usability: the proposed workflow reduces to the flowchart of Figure 1 to be adapted to each particular study.

5. Performance: for one scenario, a typical run of uncertainty takes about one hour to compute. This time may vary according to the richness of the geological scenario, i.e., the size and the pattern complexity of the training image.
6. Heteroscedasticity: both the true value $A$ and the global estimate $A^\star$ are randomized by the proposed model of uncertainty. This joint randomization provides dependence of the final uncertainty results on the data values.

7. Statistical consistency: the results obtained on our example (Section 4.3) are conservative, that is, the confidence intervals obtained are safe. This observation is not an intrinsic property of our proposed workflow, it is a consequence of the parameters used for this particular study.

As a final remark, one could object that the uncertainty results depend too much on the choice of geological scenarios and of the associated prior probability distributions $P(A = a | S = s_k)$. The scenarios retained have indeed a strong influence on the obtained results. This is actually a good point, since indeed the main source of uncertainty during early exploration is precisely the geological interpretation! The use of various scenarios is therefore strongly recommended when assessing early uncertainty of global reservoir parameters.

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References


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A Quality of the calibration

A good calibration of facies and seismic data is critical for determining a “best” estimate $\varphi(d_0)$ that utilizes seismic data. Why not then focus on improving this calibration? If there were enough data available to model a coregionalization or infer multiple point statistics between facies and seismic data, this question would indeed be appropriate, and advanced modeling techniques, including multiple-point geostatistics could be applied.

Yet, we claim it is difficult at an early exploration stage to expect much better than the simple calibration method described in this paper: so few data are available that any more sophisticated algorithm based on statistical inference would be necessarily highly sensitive to the small number of data values available hence to the well location.

The Bayesian approach described in Section 4.2.1 calls for a prior sand proportion $P_0(R)$. As always with Bayesian inversion, one can – and should – discuss how this prior probability is determined. Other methods less sensitive (or not sensitive) to such prior sand proportion $P_0(R)$ may be considered but will depend even more on the very few actual data available.

This remark should not preclude, however, using alternate estimation techniques.

B Assessing the probability distribution of geological scenarios

Determining the prior probability $P(S = s_k)$ of a given geological scenario $s_k$ is a difficult task, yet consequential if one wishes to account for that critical element of uncertainty, see Section 3.1.

One must accept to rely on geologists’ expert opinions when determining the priors $P(S = s_k)$. We could think of an algorithmic approach to determine this scenario probability. Consider for example the case of a training image $TI_k$ for the geological scenario $s_k$ obtained by running a process-based forward modeling program such as SEDSIM (Tetzlaff and Harbaugh, 1989) or PHILL (Bowman and Vail, 1999). If probability distributions for the input parameters (subsidence rate, distance to sediment source, duration of deposit, . . .), could be defined, multiple runs of the program could yield an alternative to the bland unrealistic uniform distribution for $P(S = s_k)$ suggested in Section 3. The problem is that such probability distributions for parameters of physical processes that have occurred millions of years ago are difficult to get.