A METROPOLIS SAMPLING METHOD TO ASSESS UNCERTAINTY OF SEISMIC IMPEDANCE INVERTED FROM SEISMIC AMPLITUDE DATA

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I certify that I have read this report and that in my opinion it is fully adequate, in scope and in quality, as partial fulfillment of the degree of Master of Science in Petroleum Engineering.

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Abstract

Inversion for geomodeling from data measurement is often ill-posed due to the large model dimension and the sparse, limited, and indirect data measurement, and the solution of the ill-posed inversion is a Posterior Probability Density (PPD) of the models. It is not feasible to represent the PPD due to its large number of dimensions. However one could sample the PPD to assess uncertainty of a modeling property. In this study, a Metropolis sampling method was applied to assess uncertainty of seismic impedance from seismic amplitude data for four synthetic case studies, one base case with single geological scenario and three cases with two geological scenarios, and results were compared to Rejection sampling and Sampling within tolerance.
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Chapter 1

1. Introduction

In order to better understand the Earth and predict reservoir performance, Earth models are built from geophysical data through the solution of an inverse problem. Earth properties usually vary a lot spatially thus the model dimension is large. Data measurements are often sparse, limited, and indirect. As a result, inversion for geomodeling is often an ill-posed problem.

The solution of the inversion problem is a Posterior Probability Density (PPD) of the models. It is not feasible to represent the PPD due to its large number of dimensions. However one could sample the PPD. For the ill-posed inversion problem under study, the solution of the inversion problem is non-unique and would not be a single image. Even a single image associated with uncertainty description is not adequate. The solution would be a set of images from a statistical formulation. In our study, we are not aiming directly at presenting a best matched image, or an average image. Instead, we seek to sample a series of posterior images which integrate as much information as possible. Finally the samples represent the uncertainty of a model property.

The systematic exploration of the model space is not possible due to the large number of dimensions. We resort to random exploration of the model space with Monte Carlo methods. In this study we studied Metropolis sampling, which is a Markov Chain Monte Carlo method and compared it with other two methods: Rejection sampling and Sampling within tolerance. Rejection sampling is considered as a perfect sampler in terms of honoring PPD and Sampling within tolerance is the most efficient in terms of selecting the best matched models.

In this work, we performed synthetic case studies which used seismic amplitude data to invert for seismic impedance where the measurement provides only indirect information
on the target property. A simple base case was performed, which demonstrated the workflow and framework of the sampling process. Then three more cases with two different geological scenarios were performed to further evaluate the sampling process, and demonstrate how to use posterior samples to handily estimate posterior probability of each geological scenario.
Chapter 2

2. Literature Review

Geophysical inversion can be broadly classified as direct inversion methods and model based inversion methods (Sen & Stoffa, 1995). Direct inversion methods are formulated based on the physics of forward problem and derive the model from the data directly. However, only a small portion of geophysical forward processes are reversible due to high model dimension and sparse, limited, and indirect data measurement. Furthermore most forward processes are too sensitive to small error in the data. So model based inversion methods have gained popularity. Model based inversion methods obtain solution via an optimization process as follows: iteratively predict data from an assumed model by the forward process and compare the mismatch between the observed data and the predicted data, then accept the model with a tolerable mismatch. When local optimization processes are applied, the solution may be trapped in local minima for highly nonlinear functions which may have multiple minima. We have to resort to a more general framework such as the statistical framework proposed by Tarantola (1987) to integrate the data observation, a priori and a forward relation, and resort to more general and more computational intensive methods, such as Monte Carlo methods.

Geostatistical inversion can generate models that already include some data and ideas of spatial variation (Caers, 2005). However, there are still data that are difficult to be integrated within a Geostatistical inversion process. Geostatistical methods are applied to generate a very large set of prior Earth models. Then the geophysical inversion framework is used to further constrain the models with complex data sources by sampling the posterior models out of this large prior set based on some acceptance rules of selected sampling technique.
2.1. Geostatistical seismic inversion

Geostatistical inversion aims to infer reservoir properties by integrating geology, rock physics, petrophysics, and geostatistics. Seismic forward modeling links seismic data to elastic properties, and rock physics links elastic properties with reservoir properties, and geostatistics handles the spatial correlation and data conditioning. There are mainly two types of workflows: sequential approach and simultaneous approach (Bosch et al, 2010; Doyen, 2007).

The sequential workflow mainly involves the following steps. First the seismic data is inverted to elastic properties. Then the rock physics models are used to transform the elastic properties to reservoir properties, which may then be used as soft constraints for geostatistical simulations to generate multiple reservoir properties realizations.

For the simultaneous approach, the elastic properties and reservoir properties are handled together. First prior reservoir properties models are generated with geostatistical methods. Rock physics models are used to get the likelihood relating reservoir properties to elastic properties and seismic forward modeling is used to get likelihood relating elastic properties to seismic data. Finally the posterior reservoir properties are obtained using Bayesian formulation.

The workflow in our study is a simultaneous approach workflow which inverts the elastic impedance property from the seismic amplitude data. A convolution model is used as the seismic forward modeling for relating seismic data to elastic properties.

2.2. Integrating data observation, a priori, and a forward relation

For an inverse problem, both the model and the data measurement could be uncertain. From an approach proposed by Tarantola (1987), the posterior joint probability density of model and data can be obtained by combination of their prior information, data observation and a forward relation. The posterior information about the model can be obtained from a marginal integration of the joint distribution, and expressed in a form of a
prior multiplied by a likelihood function. When data uncertainty is negligible, the posterior probability of model can be a further simplified form.

Tarantola (1987) proposed that the posterior joint probability of data and model \( \sigma(d,m) \) is given by the conjunction of two states of information: the prior joint probability density \( \rho(d,m) \), and the theoretical probability which represents the forward relation between data and model \( \Theta(d,m) \).

\[
\sigma(d,m) = \frac{k \rho(d,m) \Theta(d,m)}{\mu(d,m)}
\]  
where \( k \) is a normalization constant.

\[
\rho(d,m) = \rho_D(d) \rho_M(m)
\]  
where \( \rho(d,m), \rho_D(d) \) and \( \rho_M(m) \) are prior probabilities for data & model, data only and model only.

\[
\mu(d,m) = \mu_D(d) \mu_M(m)
\]  
where \( \mu(d,m), \mu_D(d) \) and \( \mu_M(m) \) are homogeneous probabilities for data & model, data only, and model only.

\[
\Theta(d,m) = \theta(d|m) \mu_M(m)
\]  
where \( \Theta(d,m) \) is theoretical probability which represents the forward relation between \( d \) and \( m \). \( \theta(d|m) \) is the probability density for data given the model.

**2.2.1. Posteriori probability of the model**

Posteriori probability of the model \( \sigma_M(m) \) is the marginal probability of \( m \) when the joint posteriori probability \( \sigma(d,m) \) is defined, and it can be obtained by integrating the joint probability over data space \( D \).

\[
\sigma_M(m) = \int_D d \sigma(d,m)
\]  
Using the conjunction of two states of information in Eq. 2-1, posteriori probability is:
\[
\sigma_m(m) = \int_{D} \frac{k \rho_D(d) \rho_M(m) \theta(d \mid m) \mu_M(m)}{\mu_D(d) \mu_M(m)} = k \rho_M(m) \int_{D} \frac{\rho_D(d) \theta(d \mid m)}{\mu_D(d)} \quad (2-6)
\]

The Eq. 2-6 can be written as:

\[
\sigma_m(m) = k \rho_M(m) L(m) \quad (2-7)
\]

Where \( k \) is a normalization constant and \( L(m) \) is the likelihood function, which gives a measure of how good a model \( m \) is in explaining the data,

\[
L(m) = \int_{D} \frac{\rho_D(d) \theta(d \mid m)}{\mu_D(d)} \quad (2-8)
\]

**Negligible observational uncertainty**

When the observational uncertainty is negligible, we have the following equation

\[
\rho_D(d) = \delta(d - d_{\text{obs}}) \quad (2-9)
\]

The marginal probability \( \sigma_m(m) \) can be further simplified from Eq. 2-6 as:

\[
\sigma_m(m) = k \rho_M(m) \theta(d_{\text{obs}} \mid m) \quad (2-10)
\]

The likelihood expression in Eq. 2-8 will be simplified to:

\[
L(m) = \theta(d_{\text{obs}} \mid m) \quad (2-11)
\]

Our work is based on the formulation of negligible observational uncertainty.

**2.2.2. Bayesian formulation**

Another way to look at the same formulation in Eq. 2-10 is the Bayesian formulation.

\[
\theta(m \mid d_{\text{obs}}) \rho_D(d_{\text{obs}}) = \theta(d_{\text{obs}} \mid m) \rho_M(m) \quad (2-12)
\]

\[
\theta(m \mid d_{\text{obs}}) = \frac{\theta(d_{\text{obs}} \mid m) \rho_M(m)}{\rho_D(d_{\text{obs}})} \quad (2-13)
\]

where

\( \rho_M(m) \) is prior probability of a model.
\( \theta(m \mid d_{\text{obs}}) \) is posterior probability of a model given data observation, identical to \( \sigma_M(m) \) in Eq. 2-5.

\( \theta(d_{\text{obs}} \mid m) \) is conditional probability of data given model, identical to \( L(m) \) in Eq. 2-11.

From 2-13, the posterior probability \( \theta(m \mid d_{\text{obs}}) \) is proportional to the product of \( \rho_M(m) \) and \( \theta(d_{\text{obs}} \mid m) \).

\[
\theta(m \mid d_{\text{obs}}) \propto k \rho_M(m) \theta(d_{\text{obs}} \mid m)
\] (2-14)

### 2.2.3. Usage of the solution

The solution of the inverse problem can be used to model parameter uncertainty. For example, the mean of the models \( \langle m \rangle \), and the probability of models of a subset scenario \( A \) in the model space \( P(A) \) can be obtained from:

\[
\langle m \rangle = \int_M m \sigma_M(m) \, dm
\] (2-15)

\[
P(A) = \int_A \sigma_M(m) \, dm
\] (2-16)

Eq. 2-16 can be used to characterize the geological scenario uncertainty. The integration is just as simple as counting the sample numbers belonging to a given geological scenario.

### 2.2.4. Sampling the prior probability

Prior models are the starting point for the inverse problem, and the posterior samples are further constrained by data measurement. In this study, we used geostatistical methods to generate realizations as prior models that already include some data and some idea of spatial variation (Caers, 2005).

\[
\theta(m \mid d_{\text{grt}}, d_{\text{obs}}) \propto k \theta(d_{\text{obs}} \mid m) \theta(m \mid d_{\text{grt}})
\] (2-17)

In Eq. 2-17, the data \( d_{\text{grt}} \) is included when generating Geostatistical realizations and they are part of our prior for the inversion process.
2.2.5. Sampling the posterior probability

For large dimensional and non-linear problems, it is impossible to represent the posterior probability density (PPD), but we could sample the PPD. When the problem dimension is large, it is impossible to systematically explore the model space for all the possible values of the parameters. It is necessary to resort to more general methods such as Monte Carlo methods.

2.3. Monte Carlo sampling techniques

The sampling philosophy of Monte Carlo methods is to propose a model randomly, and then decide to accept or reject it based on some acceptance rules. In our study, the proposed models honor the prior distribution and the acceptance rules honor the likelihood function. Rejection method and Metropolis method are widely used among the Monte Carlo methods.

2.3.1. The Rejection method

For Rejection method, models are generated randomly, and each model is accepted with the following probability

\[ P_k = \frac{P(m_k)}{P(m)_{\text{max}}} \]  \hspace{1cm} (2-18)

where

\[ P_k \] is the probability for accepting \( m_k \).

\[ P(m_k) \] is the probability of \( m_k \).

\[ P(m)_{\text{max}} \] is the maximum probability among all models in the model space.

Rejection sampling is a perfect sampling method in a sense that it perfectly honors the PPD. However, it is not efficient due to two reasons. Firstly the high dimensional model space is very empty and the average chance for a model to be accepted is small. Secondly,
each model is generated randomly from scratch without any memory, so there is no path for incrementally approximating the denser area.

2.3.2. The Metropolis sampling method

Metropolis sampling is a Markov Chain Monte Carlo process, as it is a random (Monte Carlo) process which only has memory of its previous step (Markov Chain).

In this study, we used a special version of Metropolis sampling, where we generated the samples of the prior probability density \( \rho_{s_i} (m) \), and then applied the Metropolis rule to the likelihood function to draw the samples of posterior probability density \( \sigma_{s_i} (m) \).

The Metropolis sampling process starts with an initial model or the previous sample if there is one \( m_i \), and then a new model \( m_j \) is proposed. Whether the new model should be rejected or accepted depends on the following rule applied to the likelihood function, which is called the Metropolis rule:

\[
\text{If } L(m_j) \geq L(m_i) \text{ then accept the proposed new model } m_j, \text{ otherwise }
\]

\[
\text{If } L(m_j) < L(m_i) \text{ accept } m_j \text{ with the probability } L(m_j) / L(m_i)
\]

The accepted model will be remembered as the old model for next sampling step.

Metropolis sampling will be more effective when appropriate transition process is applied. The simplest way to propose a new model \( m_j \) is to propose it as a totally new random prior model, however, when doing so the Metropolis sampler has no significant advantage of efficiency compared to reject sampling.

In the Metropolis sampling method, the samples are not mutually independent. So some iterative random walk is needed to force the process to “forget” the initial model and to decrease the dependence amongst samples.
2.4. Forward modeling

Forward relationship defines the physical relation between a model and data. Forward modeling is applied to predict data response for given models, and it is an indispensable component for solving the inverse problem. In our study, the forward model is the normal incidence synthetic seismic calculation based on a convolution model. The algorithm calculates and displays seismic amplitude from density and velocity data, or equivalently, from seismic impedance data and density data.

2.5. Distance and error distribution

A distance is a single scalar value that measures the difference between any two models (Scheidt and Caers, 2008). One of the most studied distances is the Euclidean distance which is defined as:

\[ d(m_i, m_j) = |m_i - m_j| = \sqrt{(m_i - m_j)^T (m_i - m_j)} \] (2-19)

In order to visualize and compare the prior and posterior samples distribution, we calculated and displayed the distance, named alternatively as error in this study, amongst the reference and samples.

2.6. MDS image to visualize samples of an ensemble

Multi-Dimensional Scaling (MDS) is used to visualize the ensemble of prior samples and posterior samples. Given the pair-wise distance between a set of L Earth models, MDS uniquely (up to rotation, translation and reflection) maps these models in any dimension less or equal to L. MDS embeds an ensemble of models to points in the plot, which conveniently represent the difference amongst the samples.
Chapter 3

3. Methodology

The statistical framework proposed by Tarantola in Eq. 2-7 and Eq. 2-10 is very general and can be used to many cases as long as new model realizations can be proposed, the forward relation and likelihood function can be defined. In our methodology section, we will discuss how to generate the prior models, how to define the error between the synthetic seismic amplitude observation and the predicted seismic amplitude of a proposed model, and how to customize the likelihood function. Then we will outline the detailed workflow for implementing the three sampling techniques: Rejection sampling, Metropolis sampling, and Sampling within tolerance.

3.1. Prior models generation

The prior models are generated using widely used Geostatistical techniques including variogram-based, training image based and object-based methods, and these prior models already include information of some data which can be handled conveniently within the Geostatistical inversion framework.

When more than one geological scenario exists, random models honor the prior probability of each scenario.

3.2. Error function

The error function we use is the Euclidean distance between the observed data and the predicted data:

$$E(m) = |G(m) - d_{obs}| = \sqrt{(G(m) - d_{obs})^T (G(m) - d_{obs})} \quad (3-1)$$

where
\( d_{\text{obs}} \) is the observed data.

\( G(m) \) is the predicted data from the model by the forward relation.

\( E(m) \) is the error between the observed data and the predicted data.

### 3.3. Likelihood function

A likelihood function quantifies how well a model does in explaining the data. We use a Gaussian error form, which is widely used for error PDF. A normalization factor and a shift value are introduced to customize the likelihood function for different cases. The role of the normalization factor is to represent the spread of the error distribution, and the shift value is accounting for the fact that prior error for models has negligible probability density below certain cutoff values.

\[
L(m) = \exp\left(-\left(\frac{\min[0, E(m) - a]}{b}\right)^2\right) \tag{3-2}
\]

where

- \( a \) - The Shift value as a cutoff for maximum likelihood value.
- \( b \) - The Normalization factor which represents the spread of the error distribution.

### 3.4. Sampling workflow

In addition to a general review of the three samplers in the previous chapter, here we outlined the detailed implementation workflow for each sampling method.

#### 3.4.1. The Rejection method

In Rejection sampling, we propose prior models and apply rejection rule to decide to accept or reject a proposed model.

1. Propose a prior model \( m_k \).
2. Calculate the accept probability \( P_k = P(m_k)/P(m)_{\text{max}} \).
3. Generate a random number \( \alpha \).
4 If $\alpha \leq P_k$, then accept the model $m_k$, otherwise reject it.

3.4.2. The Metropolis sampling method

In Metropolis sampler, we start with an initial model or the previous sample if there is one, and then propose a new model and apply the Metropolis rule to the likelihood function for the old and new models. If the new model is accepted, it will be recorded for sampling the next.

1. Take $m_k$ from previous step (Propose an initial prior model $m_k$ only at the beginning of the algorithm).

2. Propose a prior model $m_{k+1}$.

3. If $L(m_{k+1}) \geq L(m_k)$, accept the proposed new model $m_{k+1}$.

4. If $L(m_{k+1}) < L(m_k)$, generate a random number $\alpha$, If $\alpha \leq L(m_{k+1})/L(m_k)$, then accept $m_{k+1}$, otherwise reject $m_{k+1}$.

5. If $m_{k+1}$ is accepted, assign $m_k = m_{k+1}$ for the next sampling, otherwise keep $m_k$.

Random walk

For the Metropolis method, it is not appropriate to take all the samples because the process has an initial model and there is dependence amongst samples. The following two guidelines are used to design the random walk:

- At the beginning, a sufficient number of samples are discarded, so that the initial model is ‘forgotten’.

- For a succession of $n_k$ samples, keep only one sample and discard all the other samples so that the dependence amongst samples is reduced.

Cut the end tail of the likelihood function

At the tail of the Gaussian error function, the likelihood is very small and decreasing but never reaches zero. Metropolis sampler may pick many samples with increasing likelihood but still low likelihood value. This would cause Metropolis sampler to get stuck in low likelihood regions and render the performance of the sampler unstable.
In order to avoid this, the tail is truncated. This improves stability and efficiency of the Metropolis sampler.

### 3.4.3. Sampling within given error tolerance

Another approach is just to sample the models which have error within given error tolerance. It can be considered as a special case of Rejection sampling with a uniform likelihood curve, which has the constant maximum likelihood within the given tolerance, and zero beyond the given tolerance.

1. Give an error tolerance $\varepsilon$.
2. Propose a prior model $m_k$.
3. Calculate the error $E(m_k)$.
4. If $E(m_k) \leq \varepsilon$, then accept the model $m_k$, otherwise reject it.

### 3.4.4. Uncertainty of geological scenario

The geological scenario could be considered as a categorical parameter in the inverse problem. Uncertainty of the geological scenario is a high level form of uncertainty we would like to address before further detailed study. After we sample the posterior models, the posterior probability of each geological scenario is obtained by simply counting the samples number and gets its corresponding portion.
Chapter 4

4. Results

The synthetic case study starts with a base case. The purpose of the base case is to build the sampling workflow, compare Metropolis sampling with Rejection sampling and Sampling within tolerance.

Then the work was extended to three more cases each having two different geological scenarios of spatial variation, including two variograms cases, two alternative Boolean models and a case with two alternative training images. The impacts for the burn-in numbers and prior PDF were studied in variogram case.

4.1. Base case - SGSIM with hard data

Sequential Gaussian Simulation (SGSIM) was used for the base-case study. Realizations of seismic impedance on a 100*100 grid were generated for a given variogram and histogram, constrained with 16 well data points. 32 samples were drawn from each of the three different samplers with total evaluation of 15218 prior models.

4.1.1. Workflow

The design of the base-case workflow took several aspects into consideration: apply the sampling algorithms; provide guidelines for analyzing a prior pool; customize the coefficients in the likelihood function and provide guidelines to evaluate how well each sampler performs.

**Step 1.** Generate a prior pool with enough number of prior samples and pick one as the reference impedance model. Then compute the seismic amplitude data measurement for the selected reference impedance model. In this study, a constant
density was used for the forward modeling and seismic amplitude responses were calculated for different impedance models and the given density.

**Step 2.** Analyze the reference model and the prior pool, and define the likelihood function.

Check whether the reference was a common realization instead of an outlier, and then define the likelihood function in Eq. 3-2 by selecting the shift value and the normalization factor.

**Step 3.** Predict posterior error distribution.

According to Eq. 2-7 the PPD is the prior multiplied by the likelihood. In the study we had the prior pool, prior error distribution and the defined likelihood function, so we could predict the posterior error distribution.

**Step 4.** Run the samplers: Metropolis sampler, Rejection sampler and Sampling within tolerance.

The prior samples were proposed using Geostatistical tools which already integrated the hard data and the given spatial variation model. A forward relation was then applied to predict the response for the proposed models. Evaluation was done by comparing the predicted data response to the data observation. Finally the PPD samples were picked according to different acceptance rules for different algorithms.

**Step 5.** Compare results to see how much error was reduced from prior to posterior.

Compare the results by presenting several model sample images randomly picked in the posterior samples, and evaluate the posterior error distribution from the error curves and the points in the MDS plot.

**Step 6.** Get the statistics of different sampling methods and see whether they were informative.

Get the average images of the posterior samples, and see whether average images spotted the high and low value patches.
Step 7. Get the parameter of posterior probability for different geological scenarios.

For cases with more than one geological scenario, get the high level uncertainty of the geological scenario by counting the sample numbers for each geological scenario.

4.1.2. Reference and prior models generation

400 seismic impedance realizations were generated using SGSIM with the same variogram and hard data constraints. The settings for SGSIM are shown in Table 4-1. They were used as the prior impedance models pool, and one realization was picked as the reference model. The seismic amplitude response from the reference model was used as the data measurement.

Table 4-1: SGeMS settings for the base case

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SGSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid</td>
<td>100x100</td>
</tr>
<tr>
<td>Variogram Type</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Variogram Range</td>
<td>60</td>
</tr>
<tr>
<td>Variogram Angel</td>
<td>75</td>
</tr>
<tr>
<td>Nugget Effect</td>
<td>0.01</td>
</tr>
<tr>
<td>Well data</td>
<td>16 points</td>
</tr>
</tbody>
</table>

In our synthetic study, we had the reference seismic impedance model and its seismic amplitude response. But in reality we will only have the amplitude observation. The true reference impedance model is unknown. Figure 4-1 shows the MDS plot for pair-wise Euclidean distances amongst the prior models and the reference model (red). The reference was located in a dense area with neighbor points which indicates that the reference was not an outlier.
4.1.3. Prior error distribution analysis

Study of the prior samples and their error distribution helps us to define the likelihood function if we know roughly how much uncertainty we would like to reduce for the posterior. The study also helps to verify questions such as: whether there are neighbor models for the reference model, whether enough prior samples are used for analyzing the prior error distribution. The prior error distribution analysis for our base-case study is summarized as:

- PDF of all the distances (400*400) amongst the reference and all the prior models are drawn (dashed line) as the average error distribution.

- PDF of 400 (399 to be exact since one was picked out as the reference) distances between the reference and the rest prior sample models are drawn (solid line). The values were smaller than the average error distribution which indicates that the reference had more close neighbors than average.

- 15218 models were evaluated during the sampling. Their error PDF (dots) to the reference was nearly the same as the case of 400 prior models. This verified that enough number (400) of prior samples were used for analyzing the error distribution.
• The space of the grid (100*100) was quite sparse. So we see from the 170,000 plus distances, none was close to zero. The min error was 0.47e-3, and max was 2.2e-3. This justified that a shift value in Eq.3-2 for the likelihood function was necessary.

![Figure 4-2: Prior error distribution analysis](image)

**4.1.4. Posterior error distribution prediction**

From the prior error of a large enough number of prior models and the likelihood function, we could predict what the posterior samples error would look like. Base on Eq.2-7, we have:

\[
\sigma(E(m)) = k \rho(E(m))L(E(m))
\]  

(4-1)

Figure 4.3 shows the posterior error distribution prediction for our base case.

The models with smaller error have higher likelihood, which means we would more likely accept models with smaller error. However, it is possible that the prior probability for that model could be low so the final PDF for models with very small error could still be low.

This prediction is helpful to evaluate whether a sampling technique worked well or not, and whether the set of samples of the PPD is a reasonable solution of the inverse problem.
Figure 4-3: Posterior error distribution predicted from the prior error distribution (cyan) and the likelihood function (pink). Predicted posterior error distribution are shown in two black curves, one used the whole likelihood curve, the other one was with zero likelihood values at its tail.

4.1.5. *Base-case results*

**Results and sample images**

Figure 4-4 shows eight impedance models along with the reference model for four groups: the prior models and the samples from Metropolis sampling, Sampling within tolerance, and Rejection sampling. In general, the posterior samples from each sampler match the reference better than the prior models.
Figure 4-4: Seismic impedance model samples shown along with the reference for: prior, Metropolis sampling, Sampling within tolerance, and Rejection sampling.

Figure 4-5 shows the seismic amplitude responses for the models in Figure 4-4. We also see better match to the reference measurement for posterior samples’ amplitude responses than for the prior models in general.

The prior, predicted and posterior error distributions are shown in Figure 4-6 to evaluate the performance of each sampler. Two black curves are the predicted posterior error distribution with and without truncating the tail. We see that the posterior error distribution sampled from Metropolis sampling method (red dots) and from Rejection sampling method (green dots) match the predicted curve very well. Sampling within tolerance has a posterior error distribution in the smaller error range, which means it narrowed the uncertainty more than predicted.
Figure 4-5: Predicted seismic amplitude response shown along with the reference measurement for: prior, Metropolis sampling, Sampling within tolerance, and Rejection sampling.

The error distribution plot also provides information about the prior and reference. The prior pool has 400 prior samples. It was based on this pool that we customize the likelihood function. After running the samplers, 15218 prior samples were evaluated. We also plotted the error distribution of this huge number of prior samples. The close overlay of these two curves justified that 400 is a large enough number of prior model pool. Similar conclusion could be drawn from the prior and posterior CDF curves in Figure 4-7.
Figure 4-6: The prior, predicted and posterior error curves a) Prior errors between the reference and 400 prior pool models (cyan solid). b) Prior errors between the reference and 15218 prior models evaluated during the execution of the sampling processes (cyan dots). c) All the distances (400*400) amongst all the prior models (cyan dashed). d) Posterior error of Metropolis sampling (red dots). e) Posterior error of Sampling within tolerance (blue dots). f) Posterior error of Rejection sampling (red dots). g) Predicted posterior error without the far end tail (black dashed). h) Predicted posterior error with the tail (black solid). i) Likelihood function (pink solid).

Figure 4-7: Prior and posterior Error CDF curves: Prior error for the 400 prior pool models and for the 15218 prior models evaluated when running the process; Posterior error for Metropolis sampler, Rejection sampler and Sampling within tolerance.

In the MDS plot shown in Figure 4.8, the distributions of the posterior samples (blue star) were shrunk to a much smaller area around the reference (red star) compared to the spread of the prior models (green circle). This means that uncertainty of the posterior impedance models were narrowed, and this is observed for all the three samplers.
Figure 4-8: MDS plots for seismic impedance models along with the reference, the first one was for prior models. The others show the posterior sample points obtained from Metropolis sampling, Sampling within tolerance, and Rejection sampling.

The following plot shows the average of 32 samples from different samplers. The mean model gives a very good idea of the reference. It clearly spotted the areas for high values and areas for low values.
4.1.6. **Base-case conclusions**

- Metropolis sampling, Sampling within tolerance, and Rejection sampling reduced the error and provided samples that better matched the reference.

- The 10000 dimension space for a 2D 100*100 grid is very empty. Prior models proposed without perturbation rarely had near zero error to the reference. The min error was 0.47e-3, and max error was 2.2 e-3 amongst 17000 models.

- If we want to match the reference data, either perturbation for matching purpose is needed, or we should narrow the likelihood function which requires very large number of evaluations. (In this example 15218 evaluation were performed for 32 MH samples )

- Metropolis sampling and Rejection sampling matched the predicted target posterior PDF, while Sampling within tolerance narrowed the error more than predicted.

- Average values from the samples are informative to spot areas for high values and areas for low values.
4.2. Case #2 – SGSIM with two variograms

Next, the base-case study was expanded to address cases with two geological scenarios. The methodology and workflow are roughly the same, except that when proposing a prior model, the prior probability of each geological scenario is honored. For example, if the prior is 50% for each of the two scenarios, then there would be 50% chance of proposing a realization from scenario one and 50% chance of proposing a realization from scenario two.

The reference was selected from one of the scenarios, and the samples could be from either scenario. The posterior probability in case 2 was between 18/24~22/24 for different samplers which was quite different from the prior 50%. With the contribution of the forward relation and the data measurement, the uncertainty of geological scenario was significantly narrowed.

In this case study, we also investigated the effect of the prior PDF. Theoretically it is correct not to consider the prior probability density when applying the Metropolis rule if the proposed prior samples already honor the prior PDF. Here, we would like to see the effect of the prior PDF, so we also studied its impact by applying the Metropolis rule with prior PDF multiplied by the likelihood function.

The effect of burn-in number for Metropolis sampling was also studied. Since the Metropolis method starts with an initial model, we would like to know how many burn-ins were necessary for the algorithm to ‘forget’ the initial and achieve a stabilized spread in the MDS plot.

4.2.1. Case #2 the reference and prior models generation

The settings of the case 2 are shown in table 4-2. It has two geological scenarios, one with variogram having maximum range along -45 deg, and the other along 45 deg.
Table 4-2: SGeMS Settings for case 2

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SGSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid</td>
<td>100x100</td>
</tr>
<tr>
<td>Variogram</td>
<td></td>
</tr>
<tr>
<td>Range</td>
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</tr>
<tr>
<td></td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Variogram</td>
<td></td>
</tr>
<tr>
<td>Angel (1)</td>
<td>-45</td>
</tr>
<tr>
<td></td>
<td>0</td>
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<td>45</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Nugget Effect</td>
<td>0.01</td>
</tr>
<tr>
<td>Well data</td>
<td>28 points</td>
</tr>
</tbody>
</table>

400 prior models were generated, 200 from each scenario. The reference was a realization selected from scenario two. The reference impedance model and its amplitude data from the forward modeling are shown in Figure 4-10.

![Reference impedance and amplitude](image)

Figure 4-10: Reference impedance model and the amplitude calculated form forward modeling as the amplitude observation.

4.2.2. Prior PDF calculation

The prior PDF within a given geological scenario was calculated using MDS. MDS for the prior pool samples and the proposed prior model were calculated, and the 2D kernel
smoothed density for the model locations in the MDS plot was estimated. Figure 4-11 shows an example.

Figure 4-11: Left) MDS map of the prior pool and the proposed prior model. Right) A smoothed density estimate of the locations in that MDS map.

4.2.3. Case #2 results

The reference and realizations from each variogram scenario are shown in Figure 4-12.

Figure 4-12: The reference seismic impedance and four realizations from each variogram scenario.

The reference and models sampled using Metropolis Sampling are shown in Figure 4-13. Overall, the posterior samples better matched the reference compared with the prior samples. All the randomly selected eight samples had the same variogram as the
reference, which indicates that the amplitude data in this case helped to narrow the uncertainty of geological scenarios.

**Figure 4-13:** The reference and posterior samples of Metropolis Sampling. The first 8 samples had the same variogram as the reference

The sampling results and the prior samples of seismic amplitude are shown in Figure 4-14. Overall, the posterior amplitude images better matched the reference amplitude observation.

**Figure 4-14:** Seismic amplitude prediction of posterior samples shown along with the reference amplitude measurement for: prior, Metropolis sampling, Sampling within tolerance

**Figure 4-15 and Figure 4-16** show the MDS results for Metropolis sampling with and without prior PDF. We observe that the results are similar. For each scenario, results for
different burn-in numbers are also shown. When the burn-in number was smaller, say less than five, the spread was wider, which means that some models further from the reference were accepted. When the burn-in number was larger, say 15 or more, the spread has become stabilized, which means after 15 burn-in iterations, the initial model was ‘forgotten’. In this example, it was appropriate to use a burn-in number of 15 or larger.

Figure 4-15: Metropolis sampling MDS results considering the prior in the Metropolis rule for different burn-in numbers.

Figure 4-16: Metropolis sampling MDS results without considering the prior in the Metropolis rule for different burn-in numbers.
In Figure 4-17 and Figure 4-18, the results for Sampling within tolerance are displayed for cases with and without considering the prior PDF. In each figure, results for different tolerance values are shown. The two figures show that for smaller tolerance value, more prior model evaluation is needed therefore takes more computation time, and the results are closer to the reference point.

Figure 4-17: Sampling within tolerance results considering the prior in the Rejection rule for different tolerance value.
4.2.4. Case #2 conclusion

- With the contribution of the amplitude data measurement and the forward relation, the uncertainty of the impedance models was narrowed as shown by the posterior points spread in the MDS plot. The uncertainty of geological scenario was also narrowed with the posterior probability of geological scenario 18/24~22/24 compared to the prior 50%.

- The results were not very different for the case with or without prior probability density in the Metropolis rule and the Rejection rule.

- When the burn-in number was less than five, the spread of posterior sample points on the MDS plot was larger than expected, when the burn-in number was 15 or more, the posterior spread was stabilized, and the initial model was ‘forgotten’.
• The Sampling within tolerance method depends on the given tolerance. The smaller the tolerance, the smaller the posterior probability density spread in the MDS plot.

4.3. Case #3 – Object modeling using tiGenerator

In the third case study, the base-case study was expanded to address cases with two Boolean model scenarios. tiGenerator was used to generate two types of objects: ellipsoids and squares.

The reference model is an ellipsoids realization. The posterior probability of the 30 posterior samples to be ellipsoids is between 25/30~30/30 compared to the prior 50%

4.3.1. Case #3 the reference and prior models generation

For both the ellipsoids and squares models, 200 realizations were generated for each geological scenario. One realization of ellipsoid was selected as the reference seismic impedance model.

One realization from each object type is shown in Figure 4-19. MDS plot in Figure 4-20 shows that the reference has neighbors from both geological scenarios.

Figure 4-19: Left) A realization of ellipsoid objects scenario. Right) A realization of squares objects scenario.
Figure 4-20: Left) The synthetic reference seismic impedance model, which was an ellipsoids realization. Middle) Synthetic seismic observation. Right) MDS plot for all the prior models of two geological scenarios and the reference model (red star).

4.3.2. Case #3 results

The prior, predicted and posterior error distributions for case 3 are shown in Figure 4-21. Posterior error distribution curves of the Rejection sampler and the Metropolis sampler are close to the predicted posterior error distribution curve, which means the samplers performed well. The Sampling within tolerance has posterior error distribution slightly lower than the prediction error, which means it narrowed the uncertainty more than expected.

Figure 4-21: The prior, predicted and posterior error curves.
Figure 4-22. Prior and posterior Error CDF curves.

Figure 4-23 and figure 4-24 show the reference along with the prior and three sampler’s results. The prior samples contain 50% samples from each Boolean model scenario. Overall the posterior samples from the three samplers better matched the prior samples. For the three samplers, almost all the picked samples are from the same scenario as the reference. The geological scenario uncertainty was dramatically narrowed. For detailed comparison of each image to the reference, we see that not every sample is nearly the same as the reference since the model space has very high dimension and it is hard to find a perfect match without special optimization process for reducing the error.
Figure 4-23: Seismic impedance model samples shown along with the reference for: prior, Metropolis sampling, Sampling within tolerance, and Rejection sampling.
Figure 4-24: Seismic amplitude data shown along with the reference for: prior, Metropolis sampling, Sampling within tolerance, and Rejection sampling.

In the MDS plot shown in Figure 4-25, we see that the posterior points distribution was narrowed compared to the prior points distribution. The posterior geological scenario uncertainty is 30/30, 29/30, and 25/30 for the Metropolis sampling, Sampling within tolerance and Rejection sampling respectively.
Figure 4-25: MDS plots for seismic impedance models along with the reference, the first one was for prior models. The other three overlaid the posterior sample points with prior for Metropolis sampling, Sampling within tolerance, and Rejection sampling.

Figure 4-26 shows that the sample average of Metropolis sampling, Sampling within tolerance and Rejection sampling well matched the reference, and the sample average images identified the shape to be ellipsoids and spotted their locations: four out of five ellipsoids were clearly spotted, and the fifth one was vaguely spotted.
4.3.3. Case #3 conclusion

- Although the two groups of ellipsoids and squares models commingled in MDS plot, the samplers picked the right geological scenario as the reference. The posterior probabilities for geological scenario are 30/30 for Metropolis sampling, 29/30 for Sampling within tolerance, and 25/30 for Rejection sampling.

- Samples obtained by Metropolis sampling and Rejection sampling have posterior error distribution similar to the predicted posterior error distribution. The samplers from Sampling within tolerance narrowed the uncertainty more than expected.

- Average models are informative to locate the objects. Four out of five ellipsoids were clearly spotted, and the fifth one was vaguely spotted.

4.4. Case #4 – SNESIM with hard data

Case four was a two geological scenario case with two training images to mimic the vertical section of river channels. The first training image contains cups shape, and the second training image contains squares shape.
The reference is a realization generated from the cups training image with the same hard data constraints. After running three samplers, the posterior probability for each geological scenario is not very different from the prior 50%. The differences between the cups and the squares is not enough for the seismic data to distinguish between them. Additionally the patterns in the training image were not reproduced exactly in the realizations, so the two groups of realizations and their forwarded responses are quite similar.

4.4.1. Case #4 the reference and prior models generation

From both of the cups and squares training images, 200 realizations were generated with the same hard data constraints. One realization of cups shape was selected as the reference. Figure 4-27 shows the two training images used in this case.

Figure 4-27: Left) The training image of cups shape. Right) The training image of squares shape.

Figure 4-28 shows the reference impedance model, the reference amplitude observation, and the MDS plot for the prior model pool. We see that the two groups of realizations are commingled.
4.4.2. Case #4 results

From the error distribution in Figure 4-29, we see that Metropolis sampler and Rejection sampler posterior errors roughly match the predicted posterior error. The sampling within given error has smaller errors compared to the prediction. The posterior error distribution for Metropolis sampler is nearly overlaid with that of the Rejection sampler.
In Figure 4-31 and Figure 4-32 we observe that the prior samplers are drawn with equal probability from each Boolean model scenario. For the three samplers, 4, 5, and 2 out of 8 picked samples are from the same scenario as the reference. The geological scenario uncertainty did not narrow. Overall the posterior samples from the samplers better match the reference than the prior samples.

Figure 4-31: Seismic impedance model samples shown along with the reference for: prior, Metropolis sampling, Sampling within tolerance, and Rejection sampling.
Figure 4-32: Seismic amplitude data shown along with the reference for: prior, Metropolis sampling, Sampling within tolerance, and Rejection sampling.

In the MDS plots shown in figure 4-33, the posterior errors did narrow as compared to the prior error which did not incorporating amplitude observation. The geological scenario uncertainties from three samplers are still very close to the prior 50%. In this particular case we see that our methodology did not contribute much in distinguishing the two geological scenarios for cups shape and squares shape.

Average of Metropolis samples, optimization samples and Rejection samples are shown in Figure 4-34. The average images match the reference better and identified the shape to be close to cups.
Figure 4-33: MDS plots for seismic impedance models along with the reference, the first one is for prior models. The other three overlaid the posterior sample points with prior for Metropolis sampling, Sampling within tolerance, and Rejection sampling.

Figure 4-34: Impedance and amplitude images for the reference and for the average of posterior samples from Metropolis sampling, Sampling within tolerance and Rejection sampling.
4.4.3. Case #4 conclusion

- The two groups of cups and squares models are commingled in the MDS plot. The posterior probabilities for objects type are as follows: Metropolis sampling (11/18), Sampling within tolerance (7/18), and Rejection sampling (5/18).

- Posterior errors are smaller as shown by the error curves and the MDS plots. Posterior samples error distribution curves are similar to the predicted posterior error distribution curves for three samplers.

- The reason of not reducing geological scenario uncertainty in this case could be that the differences of the two types of training images are at a small scale which is not obviously identified by the forward modeling. Also the patterns in training images were not perfectly reproduced.

- The average is informative about locating the objects and discerning they shapes. Four out of five ellipsoids were clearly spotted, and the shape is much close to cups than squares.
Chapter 5

5. Conclusion and future work

Conclusion

• In our study, Geostatistical inversion provided a starting prior pool with some idea of data and spatial variation. Then a general inversion framework was used to make seismic impedance models further constrained by seismic amplitude data. We assessed the posterior impedance uncertainty by sampling its PPD with Metropolis sampling, Rejection sampling, and Sampling within tolerance.

• Formulation of Tarantola which incorporates prior, data measurement, and a forward relation to obtain the PPD is a general framework which works for many situations.

• Workflow and process were studied in a base case, and then the work was extended to three more cases each containing two geological scenarios.

• Metropolis sampler performed well by comparison with Rejection sampler which is a perfect sampler in terms of honoring the PPD.

• Drawing the posterior samples of a seismic impedance inversion problem help to know the posterior probability of high level uncertainty associated with geological scenarios. The sample average images are informative.

• The sampling results are similar in this study between applying the acceptance rules with or without prior PDF.
Future work

There are several aspects to consider for the future work:

- Use some optimization technique and perturbation so that the models can approach denser area quicker and be sampled more efficiently.

- Instead of using likelihood function in the acceptance rules and propose models honoring prior PPD, try to use different formulation for acceptance rules and propose the models in a more efficient way to avoid evaluation of a huge number of prior models.

- Use the same framework to integrate both seismic measurement and production data.
References


