The Rock Physics of AVO
AVO

Water-saturated 40 MPa

Water-saturated 10-40 MPa

Gas and Water-saturated 10-40 MPa
More than 400 sandstone data points, with porosities ranging over 4-39%, clay content 0-55%, effective pressure 5-40 MPa - all water saturated.

When Vp is plotted vs. Vs, they follow a remarkably narrow trend. Variations in porosity, clay, and pressure simply move the points up and down the trend.
Variations in porosity, pore pressure, and shaliness move data along trends. Changing the pore fluid causes the trend to change.
Different shear-related attributes.
In an isotropic medium, a wave that is incident on a boundary will generally create two reflected waves (one P and one S) and two transmitted waves. The total shear traction acting on the boundary in medium 1 (due to the summed effects of the incident and reflected waves) must be equal to the total shear traction acting on the boundary in medium 2 (due to the summed effects of the transmitted waves). Also the displacement of a point in medium 1 at the boundary must be equal to the displacement of a point in medium 2 at the boundary.
By matching the traction and displacement boundary conditions, Zoeppritz (1919) derived the equations relating the amplitudes of the P and S waves:

\[
\begin{pmatrix}
\sin(\theta_1) & \cos(\phi_1) & -\sin(\theta_2) & \cos(\phi_2) \\
-\cos(\theta_1) & \sin(\phi_1) & -\cos(\theta_2) & -\sin(\phi_2) \\
\sin(2\theta_1) & \frac{V_{p1}}{V_{s1}}\cos(2\phi_1) & \frac{\rho_2 V_{s2}^2 V_{p1}}{\rho_1 V_{s1}^2 V_{p2}}\sin(2\theta_2) & -\frac{\rho_2 V_{s2} V_{p1}^2}{\rho_1 V_{s1}^2} \cos(2\phi_2) \\
-\cos(2\phi_1) & -\frac{V_{s1}}{V_{p1}}\sin(2\phi_1) & -\frac{\rho_2 V_{p2}}{\rho_1 V_{p1}} \cos(2\phi_2) & -\frac{\rho_2 V_{s2}}{\rho_1 V_{p1}} \cos(2\phi_2)
\end{pmatrix}
\begin{pmatrix}
R_{pp} \\
R_{ps} \\
T_{pp} \\
T_{ps}
\end{pmatrix}
= 
\begin{pmatrix}
-\sin(\theta_1) \\
-\cos(\theta_1) \\
\sin(2\theta_1) \\
-\cos(2\phi_1)
\end{pmatrix}
AVO - Shuey's Approximation

P-wave reflectivity versus angle:

\[ R(\theta) = R_0 + \left[ ER_0 + \frac{\Delta v}{(1 - \nu)^2} \right] \sin^2 \theta + \frac{1}{2} \frac{\Delta V_P}{V_P} \left[ \tan^2 \theta - \sin^2 \theta \right] \]

\[ R_0 \approx \frac{1}{2} \left( \frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\rho} \right) \]

\[ E = F - 2(1 + F) \frac{1 - 2\nu}{1 - \nu} \]

\[ F = \frac{\Delta V_P / V_P}{\Delta V_P / V_P + \Delta \rho / \rho} \]

\[ \Delta V_P = (V_{P2} - V_{P1}) \quad V_P = (V_{P2} + V_{P1}) / 2 \]
\[ \Delta V_S = (V_{S2} - V_{S1}) \quad V_S = (V_{S2} + V_{S1}) / 2 \]
\[ \Delta \rho = (\rho_2 - \rho_1) \quad \rho = (\rho_2 + \rho_1) / 2 \]
AVO - Aki-Richard's approximation:

P-wave reflectivity versus angle:

\[ R(\theta) = R_0 + \left[ \frac{1}{2} \frac{\Delta V_P}{V_P} - 2 \frac{V_S^2}{V_P^2} \left( \frac{\Delta \rho}{\rho} + 2 \frac{\Delta V_S}{V_S} \right) \right] \sin^2 \theta \]

\[ + \frac{1}{2} \frac{\Delta V_P}{V_P} \left[ \tan^2 \theta - \sin^2 \theta \right] \]

\[ R_0 \approx \frac{1}{2} \left( \frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\rho} \right) \]

\[ \Delta V_P = \left( V_{P2} - V_{P1} \right) \quad V_P = \left( V_{P2} + V_{P1} \right) / 2 \]

\[ \Delta V_S = \left( V_{S2} - V_{S1} \right) \quad V_S = \left( V_{S2} + V_{S1} \right) / 2 \]

\[ \Delta \rho = \left( \rho_2 - \rho_1 \right) \quad \rho = \left( \rho_2 + \rho_1 \right) / 2 \]
## AVO Response

<table>
<thead>
<tr>
<th>P-Velocity contrast</th>
<th>Poisson ratio contrast</th>
<th>AVO response</th>
</tr>
</thead>
<tbody>
<tr>
<td>negative</td>
<td>negative</td>
<td>increase</td>
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<tr>
<td>negative</td>
<td>positive</td>
<td>decrease</td>
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<tr>
<td>positive</td>
<td>negative</td>
<td>decrease</td>
</tr>
<tr>
<td>positive</td>
<td>positive</td>
<td>increase</td>
</tr>
</tbody>
</table>
Vp-Vs Relations

There is a wide, and sometimes confusing, variety of published Vp-Vs relations and Vs prediction techniques, which at first appear to be quite distinct. However, most reduce to the same two simple steps:

1. Establish empirical relations among Vp, Vs, and porosity for one reference pore fluid--most often water saturated or dry.

2. Use Gassmann’s (1951) relations to map these empirical relations to other pore fluid states.

Although some of the effective medium models predict both P and S velocities assuming idealized pore geometries, the fact remains that the most reliable and most often used Vp-Vs relations are empirical fits to laboratory and/or log data. The most useful role of theoretical methods is extending these empirical relations to different pore fluids or measurement frequencies. Hence, the two steps listed above.
Sandstones Water Saturated

Mudrock
Vs = .86 Vp - 1.17

Han (1986)
Vs = .79 Vp - 0.79

Castagna et al. (1993)
Vs = .80 Vp - 0.86

(after Castagna et al., 1993)

Shales Water Saturated

Mudrock
Vs = .86 Vp - 1.17

Han (1986)
Vs = .79 Vp - 0.79

Castagna et al. (1993)
Vs = .80 Vp - 0.86

(after Castagna et al., 1993)
Limestones Water Saturated

Castagna et al. (1993)
\[ V_s = -0.055 V_p^2 + 1.02 V_p - 1.03 \]

Pickett (1963)
\[ V_s = V_p / 1.9 \]

Dolomite Water Saturated

Castagna et al. (1993)
\[ V_s = 0.58 V_p - 0.078 \]

Pickett (1963)
\[ V_s = V_p / 1.8 \]
Shaly Sandstones
Water Saturated

Vp (km/s)

Vs (km/s)

Mudrock
Vs = 0.86 Vp - 1.17

Clay > 25%
Vs = 0.84 Vp - 1.10

Clay < 25%
Vs = 0.75 Vp - 0.66

(Data from Han, 1986)

Shaly Sandstones
Water Saturated (hf)

Vp (km/s)

Vs (km/s)

Mudrock
Vs = 0.86 Vp - 1.17

porosity < 15%
Vs = 0.85 Vp - 1.14

porosity > 15%
Vs = 0.76 Vp - 0.66

(Data from Han, 1986)
Dry Poisson’s Ratio Assumption

The modified Voigt Average Predicts linear moduli-porosity relations. This is a convenient relation for use with the critical porosity model.

\[
K_{dry} = K_0 \left( 1 - \frac{\phi}{\phi_c} \right) \quad \mu_{dry} = \mu_0 \left( 1 - \frac{\phi}{\phi_c} \right)
\]

These are equivalent to the dry rock Vs/Vp relation and the dry rock Poisson’s ratio equal to their values for pure mineral.

\[
\left( \frac{V_S}{V_P} \right)_{dry \, rock} \approx \left( \frac{V_S}{V_P} \right)_{mineral}
\]

\[
\nu_{dry \, rock} \approx \nu_{mineral}
\]

The plot below illustrates the approximately constant dry rock Poisson’s ratio observed for a large set of ultrasonic sandstone velocities (from Han, 1986) over a large range of effective pressures (5 \( < \) \( P_{eff} < \) 40 MPa) and clay contents (0 \( < \) \( C < \) 55\% by volume).
Krief’s Relation (1990)

The model combines the same two elements:

1. An empirical Vp-Vs-φ relation for water-saturated rocks, which is approximately the same as the critical porosity model.
2. Gassmann’s relation to extend the empirical relation to other pore fluids.

Dry rock Vp-Vs-φ relation:

\[ K_{dry} = K_{mineral} (1 - \beta) \]

where \( \beta \) is Biot’s coefficient. This is equivalent to:

\[ \frac{1}{K_{dry}} = \frac{1}{K_0} + \frac{\phi}{K_\phi} \]

where

\[ \frac{1}{K_\phi} = \frac{1}{v_p} \frac{dv_p}{d\sigma} \bigg|_{P_P = constant} \]

\[ \beta = \frac{dv_p}{dV} \bigg|_{P_P = constant} = \frac{\phi K_{dry}}{K_\phi} \]

\( \beta \) and \( K_\phi \) are two equivalent descriptions of the pore space stiffness. Determining \( \beta \) vs. \( \phi \) or \( K_\phi \) vs \( \phi \) determines the rock bulk modulus \( K_{dry} \) vs \( \phi \).

Krief et al. (1990) used the data of Raymer et al. (1980) to empirically find a relation for \( \beta \) vs \( \phi \):

\[ (1 - \beta) = (1 - \phi)^{m(\phi)} \]

\[ m(\phi) = 3 / (1 - \phi) \]
Assuming dry rock Poisson’s ratio is equal to the mineral Poisson’s ratio gives

\[ K_{dry} = K_0 \left(1 - \phi\right)^{m(\phi)} \]

\[ \mu_{dry} = \mu_0 \left(1 - \phi\right)^{m(\phi)} \]
Expressions for any other pore fluids are obtained from Gassmann’s equations. While these are nonlinear, they suggest a simple approximation:

\[
\frac{V_{P-sat}^2 - V_{fl}^2}{V_{S-sat}^2} = \frac{V_{P0}^2 - V_{fl}^2}{V_{S0}^2}
\]

where \(V_{P-sat}\), \(V_{P0}\), and \(V_{fl}\) are the P-wave velocities of the saturated rock, the mineral, and the pore fluid; and \(V_{S-sat}\) and \(V_{S0}\) are the S-wave velocities in the saturated rock and mineral. Rewriting slightly gives

\[
V_{P-sat}^2 = V_{fl}^2 + V_{S-sat}^2 \left( \frac{V_{P0}^2 - V_{fl}^2}{V_{S0}^2} \right)
\]

which is a straight line (in velocity-squared) connecting the mineral point \((V_{P0}^2, V_{S0}^2)\) and the fluid point \((V_{fl}^2, 0)\). A more accurate (and nearly identical) model is to recognize that velocities tend toward those of a suspension at high porosity, rather than toward a fluid, which yields the modified form

\[
\frac{V_{P-sat}^2 - V_R^2}{V_{S-sat}^2} = \frac{V_{P0}^2 - V_R^2}{V_{S0}^2}
\]

where \(V_R\) is the velocity of a suspension of minerals in a fluid, given by the Reuss average at the critical porosity.

This modified form of Krief’s expression is exactly equivalent to the linear (modified Voigt) \(K\) vs \(\phi\) and \(\mu\) vs \(\phi\) relations in the critical porosity model, with the fluid effects given by Gassmann.
Vp-Vs Relation in Dry and Saturated Rocks

\[ V_p^2 (\text{km/s})^2 \quad \text{vs} \quad V_s^2 (\text{km/s})^2 \]

Sandstones

mineral point

saturated

dry

fluid point
Vp-Vs Relation in Sandstone and Dolomite

\[ V_p^2 \text{ (km/s)}^2 \]

\[ V_s^2 \text{ (km/s)}^2 \]

- Sandstone
- Dolomite
- Mineral points
- Fluid points
Greenberg and Castagna (1992) have given empirical relations for estimating $V_s$ from $V_p$ in multimineralic, brine-saturated rocks based on empirical, polynomial $V_p$-$V_s$ relations in pure monomineralic lithologies (Castagna et al., 1992). The shear wave velocity in brine-saturated composite lithologies is approximated by a simple average of the arithmetic and harmonic means of the constituent pure lithology shear velocities:

$$V_s = \frac{1}{2} \left\{ \left[ \sum_{i=1}^{L} X_i \sum_{j=0}^{N_i} a_{ij} V_p^j \right] + \left[ \sum_{i=1}^{L} X_i \left( \sum_{j=0}^{N_i} a_{ij} V_p^j \right)^{-1} \right]^{-1} \right\}$$

$$\sum_{i=1}^{L} X_i = 1$$

where
- $L$ number of monomineralic lithologic constituent
- $X_i$ volume fractions of lithological constituents
- $a_{ij}$ empirical regression coefficients
- $N_i$ order of polynomial for constituent $i$
- $V_p$, $V_s$ P and S wave velocities (km/s) in composite brine-saturated, multimineralic rock

Castagna et al. (1992) gave representative polynomial regression coefficients for pure monomineralic lithologies:

<table>
<thead>
<tr>
<th>Lithology</th>
<th>$a_{i2}$</th>
<th>$a_{i1}$</th>
<th>$a_{i0}$</th>
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</thead>
<tbody>
<tr>
<td>Sandstone</td>
<td>0</td>
<td>0.80416</td>
<td>-0.85588</td>
</tr>
<tr>
<td>Limestone</td>
<td>-0.05508</td>
<td>1.01677</td>
<td>-1.03049</td>
</tr>
<tr>
<td>Dolomite</td>
<td>0</td>
<td>0.58321</td>
<td>-0.07775</td>
</tr>
<tr>
<td>Shale</td>
<td>0</td>
<td>0.76969</td>
<td>-0.86735</td>
</tr>
</tbody>
</table>

Regression coefficients for pure lithologies with $V_p$ and $V_s$ in km/s:

$$V_s = a_{i2} V_p^2 + a_{i1} V_p + a_{i0} \quad \text{(Castagna et al. 1992)}$$
Note that the above relation is for 100% brine-saturated rocks. To estimate $V_s$ from measured $V_p$ for other fluid saturations, Gassmann’s equation has to be used in an iterative manner. In the following, the subscript $b$ denotes velocities at 100% brine saturation and the subscript $f$ denotes velocities at any other fluid saturation (e.g. this could be oil or a mixture of oil, brine, and gas). The method consists of iteratively finding a $(V_p, V_s)$ point on the brine relation that transforms, with Gassmann’s relation, to the measured $V_p$ and the unknown $V_s$ for the new fluid saturation. The steps are as follows:

1. Start with an initial guess for $V_{pb}$.
2. Calculate $V_{sb}$ corresponding to $V_{pb}$ from the empirical regression.
3. Do fluid substitution using $V_{pb}$ and $V_{sb}$ in the Gassmann equation to get $V_{sf}$.
4. With the calculated $V_{sf}$ and the measured $V_{pf}$, use the Gassmann relation to get a new estimate of $V_{pb}$. Check with previous value of $V_{pb}$ for convergence. If convergence criterion is met, stop; if not go back to step 2 and continue.
Polynomial and powerlaw forms of the Gardner et al. (1974) velocity-density relationships presented by Castagna et al. (1993). Units are km/s and g/cm³.

<table>
<thead>
<tr>
<th>Lithology</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Vp Range (Km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shale</td>
<td>-0.0261</td>
<td>.373</td>
<td>1.458</td>
<td>1.5-5.0</td>
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<tr>
<td>Sandstone</td>
<td>-0.0115</td>
<td>.261</td>
<td>1.515</td>
<td>1.5-6.0</td>
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<tr>
<td>Limestone</td>
<td>-0.0296</td>
<td>.461</td>
<td>0.963</td>
<td>3.5-6.4</td>
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<tr>
<td>Dolomite</td>
<td>-0.0235</td>
<td>.390</td>
<td>1.242</td>
<td>4.5-7.1</td>
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<tr>
<td>Anhydrite</td>
<td>-0.0203</td>
<td>.321</td>
<td>1.732</td>
<td>4.6-7.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lithology</th>
<th>d</th>
<th>f</th>
<th>Vp Range (Km/s)</th>
</tr>
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<tbody>
<tr>
<td>Shale</td>
<td>1.75</td>
<td>.265</td>
<td>1.5-5.0</td>
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<tr>
<td>Sandstone</td>
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<tr>
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<tr>
<td>Anhydrite</td>
<td>2.19</td>
<td>.160</td>
<td>4.6-7.4</td>
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</table>
Both forms of Gardner’s relations applied to laboratory dolomite data.
Both forms of Gardner’s relations applied to laboratory limestone data. Note that the published powerlaw form does not fit as well as the polynomial. We also show a powerlaw form fit to these data, which agrees very well with the polynomial.