Shaly Sands

P-wave Velocity (km/s) vs. Porosity

- Jizba 9,000 ft (75 MPa)
- Han 15,000 ft (40 MPa), $R = 0.70$
- Han 12,000 ft (40 MPa), $R = 0.80$
- Han 10,000 ft (40 MPa), $R = 0.96$
- Blangy 5,000 ft, Troll (30 MPa), $R = 0.76$
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Xu and White (1995) developed a theoretical model for velocities in shaly sandstones. The formulation uses the Kuster-Toksöz and Differential Effective Medium theories to estimate the dry rock P and S velocities, and the low frequency saturated velocities are obtained from Gassmann’s equation. The sand-clay mixture is modeled with ellipsoidal inclusions of two different aspect ratios. The sand fraction has stiffer pores with aspect ratio $\alpha \approx 0.1 - 0.15$, while the clay-related pores are more compliant with $\alpha \approx 0.02-0.05$. The velocity model simulates the “V” shaped velocity-porosity relation of Marion et al. (1992) for sand-clay mixtures. The total porosity $\phi = \phi_{\text{sand}} + \phi_{\text{clay}}$ where $\phi_{\text{sand}}$ and $\phi_{\text{clay}}$ are the porosities associated with the sand and clay fractions respectively. These are approximated by

$$\phi_{\text{sand}} = \frac{(1 - \phi - V_{\text{clay}}) \phi}{1 - \phi} = V_{\text{sand}} \frac{\phi}{1 - \phi}$$
$$\phi_{\text{clay}} = V_{\text{clay}} \frac{\phi}{1 - \phi}$$

where $V_{\text{sand}}$ and $V_{\text{clay}}$ denote the volumetric sand and clay content respectively. Shale volume from logs may be used as an estimate of $V_{\text{clay}}$. Through the log derived shale volume includes silts, and overestimates clay content, results obtained by Xu and White justify its use. The properties of the solid mineral mixture are estimated by a Wyllie time average of the quartz and clay mineral velocities, and arithmetic average of their densities:

$$\frac{1}{V_{P_0}} = \left( \frac{1 - \phi - V_{\text{clay}}}{1 - \phi} \right) \frac{1}{V_{P_{\text{quartz}}}} + \frac{V_{\text{clay}}}{1 - \phi} \frac{1}{V_{P_{\text{clay}}}}$$
$$\frac{1}{V_{S_0}} = \left( \frac{1 - \phi - V_{\text{clay}}}{1 - \phi} \right) \frac{1}{V_{S_{\text{quartz}}}} + \frac{V_{\text{clay}}}{1 - \phi} \frac{1}{V_{S_{\text{clay}}}}$$

$$\rho_0 = \left( \frac{1 - \phi - V_{\text{clay}}}{1 - \phi} \right) \rho_{\text{quartz}} + \frac{V_{\text{clay}}}{1 - \phi} \rho_{\text{clay}}$$

where subscript 0 denotes the mineral properties. These mineral properties are then used in the Kuster-Toksöz formulation along with the porosity and clay content, to calculate dry rock moduli and velocities. The limitation of small pore concentration of the Kuster-Toksöz model is handled by incrementally adding the pores in small steps such that the non-interaction criterion is satisfied in each step. Gassmann’s equations are used to obtain low frequency saturated velocities. High frequency saturated velocities are calculated by using fluid-filled ellipsoidal inclusions in the Kuster-Toksöz model.

The model can be used to predict shear wave velocities (Xu and White, 1994). Estimates of Vs may be obtained from known mineral matrix properties and measured porosity and clay content, or from measured Vp and either porosity or clay content. Su and White recommend using measurements of P-wave sonic log since it is more reliable than estimates of shale volume and porosity.
Shaly Sands

Dvorkin’s Cement Model

Jack Dvorkin introduced a cement model that predicts the bulk and shear moduli of dry sand when cement is deposited at grain contacts. The model assumes that the cement is elastic and its properties may differ from those of the grains. It assumes that the starting framework of cemented sand is a dense random pack of identical spherical grains with porosity $\phi_0 \approx 0.36$, and the average number of contacts per grain $C = 9$. Adding cement reduces porosity and increases the effective elastic moduli of the aggregate. The effective dry-rock bulk and shear moduli are (Dvorkin and Nur, 1996)

$$K_{\text{eff}} = \frac{1}{6} C (1 - \phi_0) M_c \hat{S}_n$$
$$G_{\text{eff}} = \frac{3}{5} K_{\text{eff}} + \frac{3}{20} C (1 - \phi_0) G_c \hat{S}_\tau$$

where

$$M_c = \rho_c V_{Pc}^2$$
$$G_c = \rho_c V_{Sc}^2$$

$\rho_c$ is the cement's density; and $V_{Pc}$ and $V_{Sc}$ are its P- and S-wave velocities. Parameters $\hat{S}_n$ and $\hat{S}_\tau$ are proportional to the normal and shear stiffness, respectively, of a cemented two-grain combination. They depend on the amount of the contact cement and on the properties of the cement and the grains. (see next page)
Shaly Sands

Dvorkin’s Cement Model

Constants in the cement model:

\[ \hat{S}_n = A_n \alpha^2 + B_n \alpha + C_n \]

\[ A_n = -0.024153 \Lambda_n^{-1.3646} \]
\[ B_n = 0.20405 \Lambda_n^{-0.89008} \]
\[ C_n = 0.00024649 \Lambda_n^{-1.9864} \]

\[ \hat{S}_\tau = A_\tau \alpha^2 + B_\tau \alpha + C_\tau \]

\[ A_\tau = -10^{-2} (2.26\nu^2 + 2.07\nu + 2.3) \Lambda_\tau^{0.079\nu^2 + 0.175\nu - 1.342} \]
\[ B_\tau = (0.0573\nu^2 + 0.0937\nu + 0.202) \Lambda_\tau^{0.0274\nu^2 + 0.0529\nu - 0.8765} \]
\[ C_\tau = -10^{-4} (9.654\nu^2 + 4.945\nu + 3.1) \Lambda_\tau^{0.01867\nu^2 + 0.4011\nu - 1.8186} \]

\[ \Lambda_n = \frac{2G_c}{\pi G} \frac{(1 - \nu)(1 - \nu_c)}{(1 - 2\nu_c)} \]
\[ \Lambda_\tau = \frac{G_c}{\pi G} \]
\[ \alpha = \frac{a}{R} \]

where G and \( \nu \) are the shear modulus and the Poisson's ratio of the grains, respectively; \( G_c \) and \( \nu_c \) are the shear modulus and the Poisson's ratio of the cement; \( a \) is the radius of the contact cement layer; \( R \) is the grain radius.
The amount of the contact cement can be expressed through the ratio $\alpha$ of the radius of the cement layer $a$ to the grain radius $R$:

$$\alpha = \frac{a}{R}$$

The radius of the contact cement layer $a$ is not necessarily directly related to the total amount of cement: part of the cement may be deposited away from the intergranular contacts. However, by assuming that porosity reduction in sands is due to cementation only, and by adopting certain schemes of cement deposition we can relate parameter $\alpha$ to the current porosity of cemented sand $\phi$. For example, we can use Scheme 1 (see figure above) where all cement is deposited at grain contacts:

$$\alpha = 2 \left( \frac{\phi_0 - \phi}{3C (1 - \phi_0)} \right)^{0.25} = 2 \left( \frac{S\phi_0}{3C (1 - \phi_0)} \right)^{0.25}$$

or we can use Scheme 2 where cement is evenly deposited on the grain surface:

$$\alpha = \left( \frac{2(\phi_0 - \phi)}{3(1 - \phi_0)} \right)^{0.5} = \left( \frac{2S\phi_0}{3(1 - \phi_0)} \right)^{0.5}$$

In these formulas $S$ is the cement saturation of the pore space - the fraction of the pore space occupied by cement.
If the cement's properties are identical to those of the grains, the cementation theory gives results which are very close to those of the Digby model. The cementation theory allows one to diagnose a rock by determining what type of cement prevails. For example, it helps distinguish between quartz and clay cement. Generally, Vp predictions are much better than Vs predictions.

Predictions of Vp and Vs using the Scheme 2 model for quartz and clay cement, compared with data from quartz and clay cemented rocks from the North Sea.
Sand models can be used to “Diagnose” sands
**Shaly Sands**

**Dvorkin’s Uncemented Sand Model**

This model predicts the bulk and shear moduli of dry sand when cement is deposited away from grain contacts. The model assumes that the starting framework of uncemented sand is a dense random pack of identical spherical grains with porosity $\phi_0 = 0.36$, and the average number of contacts per grain $C = 9$. The contact Hertz-Mindlin theory gives the following expressions for the effective bulk ($K_{HM}$) and shear ($G_{HM}$) moduli of a dry dense random pack of identical spherical grains subject to a hydrostatic pressure $P$:

$$K_{HM} = \left[ \frac{C^2 (1 - \phi_0)^2 G^2}{18 \pi^2 (1 - \nu)^2} P \right]^{1/3}$$

$$G_{HM} = \frac{5 - 4\nu}{5 (2 - \nu)} \left[ \frac{3C^2 (1 - \phi_0)^2 G^2}{2\pi^2 (1 - \nu)^2} P \right]^{1/3}$$

where $\nu$ is the grain Poisson's ratio and $G$ is the grain shear modulus.
Shaly Sands

Dvorkin’s Uncemented Sand Model

In order to find the effective moduli at a different porosity, a heuristic modified Hashin-Strikman lower bound is used:

\[
K_{\text{eff}} = \left[ \frac{\phi / \phi_0}{K_{\text{HM}} + \frac{4}{3} G_{\text{HM}}} + \frac{1 - \phi / \phi_0}{K + \frac{4}{3} G_{\text{HM}}} \right]^{-1} - \frac{4}{3} G_{\text{HM}}
\]

\[
G_{\text{eff}} = \left[ \frac{\phi / \phi_0}{G_{\text{HM}} + \frac{6}{G_{\text{HM}}}} \left( \frac{9K_{\text{HM}} + 8G_{\text{HM}}}{K_{\text{HM}} + 2G_{\text{HM}}} \right) \right. \\
\left. + \frac{1 - \phi / \phi_0}{G + \frac{6}{G_{\text{HM}}}} \left( \frac{9K_{\text{HM}} + 8G_{\text{HM}}}{K_{\text{HM}} + 2G_{\text{HM}}} \right) \right]^{-1} - \frac{G_{\text{HM}}}{6} \left( \frac{9K_{\text{HM}} + 8G_{\text{HM}}}{K_{\text{HM}} + 2G_{\text{HM}}} \right)
\]

Illustration of the modified lower Hashin-Shtrikman bound for various effective pressures. The pressure dependence follows from the Hertz-Mindlin theory incorporated into the right end member.

\[
M = \rho V_p^2
\]
**Shaly Sands**

**Dvorkin’s Uncemented Sand Model**

This model connects two end members: one has zero porosity and the modulus of the solid phase and the other has high porosity and a pressure-dependent modulus as given by the Hertz-Mindlin theory. This contact theory allows one to describe the noticeable pressure dependence normally observed in sands.

The high-porosity end member does not necessarily have to be calculated from the Hertz-Mindlin theory. It can be measured experimentally on high-porosity sands from a given reservoir. Then, to estimate the moduli of sands of different porosities, the modified Hashin-Strikman lower bound formulas can be used where $K_{HM}$ and $G_{HM}$ are set at the measured values. This method provides accurate estimates for velocities in uncemented sands. In the figures below the curves are from the theory.

Prediction of $V_p$ and $V_s$ using the lower Hashin-Shtrikman bound, compared with measured velocities from unconsolidated North Sea samples.
This method can also be used for estimating velocities in sands of porosities exceeding 0.36.
Study by Per Avseth, along with J. Dvorkin, G. Mavko, and J. Rykkje
Shaly Sands

Sorting Analysis of Thin-Sections

[Histograms and images of thin-sections showing grain size distribution]
Shaly Sands

Thin-Section and SEM Analyses

Well #1 Uncemented

Well #2 Cemented

SEM back-scatter image, Well #2

SEM cathode-luminescent image, Well #2

Unconsolidated (Facies IIb)

Cemented (Facies IIa)