Lecture 8: Western intensification

Atmosphere, Ocean, Climate Dynamics

EESS 146B/246B
Calculation of the depth integrated wind-driven circulation

• The depth integrated flow can be expressed in terms of a streamfunction since it is horizontally non-divergent.

\[ U = -\frac{\partial \psi}{\partial y}, \quad V = \frac{\partial \psi}{\partial x}, \quad U = \int_{z=-h}^{z=0} u dz \]

• The Sverdrup relation becomes

\[ \beta \frac{\partial \psi}{\partial x} = \frac{1}{\rho_{ref}} \hat{z} \cdot \nabla \times \tau_{wind} \]

• Which can be integrated in x to yield the streamfunction and hence flow everywhere. But what is the boundary condition?
Calculation of the depth integrated wind-driven circulation

• What is the boundary condition on the streamfunction?

• At the zonal boundaries of the basin, the streamfunction must equal a constant so that no flow goes into the boundary → \[ \psi = 0 \] at zonal boundaries.

\[
\begin{align*}
\psi &= 0 \\
-x &= x_w \\
\psi &= 0 \\
x &= x_e
\end{align*}
\]

• Which direction should the integral be made?
Western boundary currents

- Currents are much stronger on the western than eastern side of ocean basins → western intensification.
- The integral should start at the eastern boundary so that the strong boundary current is in the west rather than the west.
Depth averaged circulation in the Pacific

Transport in units of Sverdrups = 1x10^6 m^3 s^{-1}

The strength of the transport in the gyres increases with basin width → the transport is stronger in the gyres of the Pacific than the Atlantic.
Closing the circulation in the western boundary

\[ \psi = 0 \]

\[ \int_{x_w}^{x_w + \delta_w} V \, dx = - \int_{x_w}^{x_{e}} V \, dx = \psi \bigg|_{x_w + \delta_w} \]

Transport in western boundary current

-Transport in the interior
Closing the circulation in the west versus the east

- Both of these scenarios satisfy the Sverdrup relation in the interior, what is the physics that selects a western boundary current?
Western intensification

• Energetics
• Closing the vorticity budget
• The time-dependent acceleration of the gyres $\rightarrow$ Rossby waves
Energetics argument for western intensification

- The winds input energy to the circulation at a rate given by the wind-work

\[ \tau_{wind} \cdot u_s \]

- Geostrophic velocity at the surface

- Winds remove energy from a gyre with an eastern boundary current, therefore such a gyre will not be generated spontaneously.
Closing the vorticity budget

\[ 0 = -\beta v + f \frac{\partial w}{\partial z} + \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \]

• In a steady state, the sources and sinks of vorticity summed across the depth and width of the ocean basins must add up to zero.

• Since there is no net transport across a latitude circle:

\[ \beta \int_{x_w}^{x_e} V \, dx = 0 \]

• And the vertical velocity at the top and bottom of the ocean is zero,

\[ \int_{z=-h}^{z=0} \int_{x_w}^{x_e} \left( \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \, dx \, dz = 0 \]

• There must be no net frictional torque \( \rightarrow \) the frictional torque associated with the wind-stress must be compensated.
Closing the vorticity budget with bottom friction

\[
\int_{z=-h}^{z=0} \int_{x_w}^{x_e} \left( \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) dx dz = 0
\]

\[
\int_{x_w}^{x_e} \hat{z} \cdot (\nabla \times \tau_{\text{wind}} - \nabla \times \tau_{\text{bot}}) \, dx = 0
\]

• A bottom stress is necessary to close the vorticity budget.

• The simplest parameterization for the bottom stress is using a linear drag law:

\[
\tau_{\text{bot}} = \rho_{\text{ref}} (rU, rV) \quad r \approx 1 \times 10^{-6} \text{ s}^{-1}
\]

\[
\int_{x_w}^{x_e} \hat{z} \cdot \nabla \times \tau_{\text{wind}} \, dx = \rho_{\text{ref}} r \int_{x_w}^{x_e} \hat{z} \cdot \nabla \times \mathbf{U} \, dx
\]
Which scenario closes the vorticity budget?

**WESTERN BOUNDARY CURRENT**

**EASTERN BOUNDARY CURRENT**

- To close the vorticity budget:

\[
\int_{x_w}^{x_e} \hat{z} \cdot \nabla \times \tau_{\text{wind}} \, dx = \rho_{\text{ref}} \int_{x_w}^{x_e} \hat{z} \cdot \nabla \times \mathbf{U} \, dx
\]
Time dependent acceleration of the gyres

• Consider the spin-up of the circulation by allowing the geostrophic flow to be time dependent:
  \[ \mathbf{U}(x, y, t) = h \hat{z} \times g \nabla \eta(x, y, t) \]

• Imagine if the transient response of the sea surface had the structure:

Consider two cases:
1. The SSH anomaly propagates to the WEST
2. The SSH anomaly propagates to the EAST

Which case satisfies the Taylor-Proudman theorem?
The Taylor-Proudman effect on a sphere

The anomalies that propagate to the west satisfy the Taylor-Proudman theorem.

\[ \beta \frac{V}{h} = f \frac{w_{\text{top}}}{h} \]

\[ w_{\text{top}} = \frac{\partial \eta}{\partial t} \quad V = \frac{g}{f} \frac{\partial \eta}{\partial x} \]

Waves that satisfy this equation are known as ROSSBY WAVES, and play a key role in the transient acceleration of the ocean circulation.
Observations of Rossby waves

- Rossby waves transmit energy to the west, piling it up in the western boundary.
- This energy goes into the mean circulation and thus generates strong currents in the western sides of ocean basins.