Lecture 3: the equations of motion, the Coriolis force, and the geostrophic balance

Atmosphere, Ocean, Climate Dynamics

EESS 146B/246B
Equations of motion and the basic force balances

• Review of the equations of motion
• The physics behind the Coriolis force
• The geostrophic balance
The equations of motion

- Horizontal momentum equations in a frame rotating about the vertical at angular velocity $f/2$

  Acceleration  =  \( \frac{\text{Sum of forces}}{\text{mass}} \)

  \[
  \frac{Du}{Dt} = fu - \frac{1}{\rho_{ref}} \frac{\partial p}{\partial x} + F_x
  \]

  \[
  \frac{Du}{Dt} = -fu - \frac{1}{\rho_{ref}} \frac{\partial p}{\partial y} + F_y
  \]

  Coriolis force  Pressure gradient force  Friction

- Vertical momentum equations: **hydrostatic balance**

  \[
  0 = -\frac{\partial p}{\partial z} - \rho g
  \]

- Flow in the ocean is nearly incompressible

  \[ \nabla \cdot \mathbf{u} = 0 \]  
  **CONTINUITY EQUATION**

- \( f = 2\Omega \sin \varphi \)

- \( \Omega = \frac{2\pi}{\text{day}} = 7.27 \times 10^{-5} \text{ rad s}^{-1} \)

- \( \varphi \)  latitude
The Coriolis Force

- Northern Hemisphere

\[ \text{COR}^y = -fu \]

- Southern Hemisphere

\[ \text{COR}^y = -fu \]

The Coriolis force causes moving objects to veer to the right (left) in the Northern (Southern) Hemisphere.
The physics behind the Coriolis force

Viewed in non-rotating frame, water is moving in solid body rotation

\[ \bar{u} = \frac{f}{2r} \]
Surface circulation in the Pacific

$L = 3500 \text{ km}$

$T_{adv} = \frac{L}{U}$

$\sim 400 \text{ days}$

(Data courtesy of Mazzurco and Niiler (personal communication, 2003).)
Geostrophic balance

\[ -f \hat{z} \times u \]

\[ -\frac{1}{\rho} \nabla p \]
The effect of the Earth’s rotation: the generation of geostrophic currents

Northern Hemisphere

Floats on the sea-surface
The effect of the Earth’s rotation: the generation of geostrophic currents

- In the Northern Hemisphere, water moves in a clockwise direction around highs in the free surface.

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Northern Hemisphere

Floats on the sea-surface

In the Northern Hemisphere, water moves in a clockwise direction around highs in the free surface.
The effect of the Earth’s rotation: the generation of geostrophic currents

Northern Hemisphere

- In the Northern Hemisphere, water moves in a counter-clockwise direction around lows in the free surface.
The effect of the Earth’s rotation: the generation of geostrophic currents

In the Southern Hemisphere, water moves in a counter-clockwise direction around highs and clockwise around lows in the free surface.
Thermal wind balance at the subpolar front of the Japan/East Sea
Thermal wind balance at the subpolar front of the Japan/East Sea

\[
f \frac{\partial u_g}{\partial z} = \frac{g}{\rho_{ref}} \frac{\partial \rho}{\partial y}
\]
Thermal wind balance at the subpolar front of the Japan/East Sea