When is the strain in the meter the same as the strain in the rock?

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Received 17 June 2003; revised 31 July 2003; accepted 25 August 2003; published 8 October 2003.

[1] Borehole dilatometers are emplaced in porous fluid saturated rock. Pore-fluid flow induces strain, however there is no fluid exchange with the dilatometer. Thus, the strainmeter response is the same as the strain in the rock only when the rock remains undrained. Otherwise the instrumental strain $\Delta_{\text{inst}}$ is given by $\Delta_{\text{inst}} = C_1 (\Delta^\infty - C_2 p^\infty)$, where $\Delta^\infty$ and $p^\infty$ are strain and pore pressure far from the borehole, and $C_1$ and $C_2$ depend on poroelastic rock properties. Postseismic strain in the rock is expected to increase as the induced pore pressure gradients relax. However, a dilatometer $\sim 3$ km from a $M_w$ 6.5 earthquake in south Iceland shows a postseismic strain change opposite in sign to the coseismic response. The theory developed here for a homogeneous, isotropic medium can only partly explain this discrepancy. Fracture dominated poroelastic response may yield an improved fit to the data. INDEX TERMS: 7294 Seismology: Instruments and techniques; 8159 Tectonophysics: Rheology—crust and lithosphere; 8194 Tectonophysics: Instruments and techniques. Citation: Segall, P., S. Jónsson, and K. Ágústsson, When is the strain in the meter the same as the strain in the rock?, Geophys. Res. Lett., 30(19), 1990, doi:10.1029/2003GL017995, 2003.

1. Introduction

[2] Borehole strainmeters have become an important tool for monitoring crustal deformation. The most widely deployed borehole instrument is the dilatometer, developed by Sacks and Evertson [Sacks et al., 1971], which measures the dilatational strain, or some related component of the strain tensor. In detail, the dilatometer may be more sensitive to the areal strain in the plane perpendicular to the borehole than it is to the strain parallel to the borehole axis [Agnew, 1986]. Borehole dilatometers have been deployed along the San Andreas Fault, in Japan, China, and Iceland, and in volcanically active regions including Long Valley Caldera and Hawaii.

[3]Crudely speaking dilatometers are steel cylinders filled with hydraulic fluid. As the instrument is compressed fluid flows from a sensing volume into a secondary compartment that is partly filled with inert gas, through a small tube. The tube connects to a bellows within the secondary reservoir, and deflection of the bellows is monitored by a displacement transducer [e.g., Agnew, 1986].

[4] Dilatometers are extremely sensitive at periods of several weeks or less and are thus the instrument of choice for many monitoring applications. Strainmeters have recorded coseismic strain steps [Johnston and Linde, 2002], slow and silent earthquakes [Linde et al., 1996], deformation related to volcanic intrusions and eruptions [Linde et al., 1993], as well as transient strain events associated with remotely triggered earthquakes [Hill et al., 1993]. Borehole strainmeters are also an important component of the Plate Boundary Observatory.

[5] The strain recorded by the dilatometer may, however, not be the same as the strain experienced by rock surrounding the instrument. The reason for this is that crustal rock is porous and fluid saturated. An increase in pore-fluid pressure causes rock to expand; decreases lead to contraction. The dilatometer is, on the other hand, a steel cylinder; pore-fluid does not flow into or out of the instrument. Thus, any time fluid flow occurs the strain in the rock cannot be the same as the strain in the strainmeter.

[6] Consider a cylindrical hole in a block of poroelastic material (Figure 1). If the block is compressed rapidly the pore pressure will increase and the hole will contract elastically. If the exterior boundaries of the block are held fixed and the fluid is allowed to drain the rock will contract. However, as the rock shrinks the borehole must expand, since the exterior boundary is fixed. Thus, the far-field strain is zero. The rock locally undergoes a volume decrease, but the borehole undergoes a volume increase.

[7] Here we derive the response of a dilatometer to strain and pore pressure changes in the surrounding poroelastic medium.

2. Method

[8] We assume that the rock is isotropic, and the pores are small relative to the size of the strainmeter. Because dilatometers are insensitive to shear strain we need only consider the isotropic component of the stress. Furthermore for simplicity, we will consider that the instrument is emplaced at shallow depth, that the vertical stress vanishes, and the instrument is in a state of plane stress. We ignore three dimensional effects associated with the bottom of the borehole.

[9] Since the output of the displacement transducer in the dilatometer is converted to strain by calibration with the solid earth tides, we simply model the strainmeter as a cylindrical elastic inclusion embedded in a poroelastic earth (Figure 2).
Figure 1. Thought experiment to illustrate the effect of pore-fluid flow on a dilatometer response. A block of fluid saturated rock with a cylindrical hole is compressed. With the external displacements fixed the fluid is allowed to drain. As the fluid drains the rock contracts and the hole expands.

[19] Assuming plane stress conditions with the z-axis vertical ($\sigma_{zz} = 0$), and aligned with the borehole, the constitutive equations for the rock (region-2), relating stress $\sigma_{ij}$ to strain $\varepsilon_{ij}$, and pore-pressure $p$, following Rice and Cleary [1976] are

$$\sigma_{ij} = 2\mu(2)\varepsilon_{ij} + \frac{\gamma}{1-\nu(2)} \varepsilon_{kk} \delta_{ij} - \frac{\gamma}{1-\nu(2)\nu(1)} \varepsilon_{rr} p \delta_{ij},$$

(1)

where $i, j, k = 1, 2$, $\mu(2)$ is shear modulus, $\nu(2)$ and $\nu(2)$ the drained and undrained Poisson’s ratios, $B$ is Skempton’s pore pressure coefficient, and $\gamma = 3(\nu(2) - \nu(2))B(1 + \nu(2))$. The corresponding vertical strain ($\varepsilon_{zz} = \varepsilon_{33}$) is

$$\varepsilon_{zz} = -\frac{\nu(2)}{1-\nu(2)} \varepsilon_{kk} + \frac{\gamma}{2\mu(2)(1-\nu(2))} p,$$

(2)

where $k = 1, 2$. Hooke’s law for the instrument (region 1) is

$$\sigma_{ij} = 2\mu(1)\varepsilon_{ij} + \frac{\gamma}{1-\nu(1)} \varepsilon_{kk} \delta_{ij}, \quad i, j = 1, 2.$$  

(3)

Assuming cylindrical symmetry about the borehole axis, the only relevant equilibrium equation for both regions is

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} (\sigma_{rr} - \sigma_{rr}) = 0$$

(4)

[Malvern, 1969, p. 668]. Substituting the constitutive equations, and the strain-displacement relations [Malvern, 1969, p. 668], into (4), we find that pore pressure gradients enter the Navier form of the equilibrium equations equivalent to body forces [Segall, 1992]

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (ru^{(1)}_r) \right] = 0$$

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (ru^{(2)}_r) \right] = \gamma \frac{dp}{2\mu(2) dr}.$$  

(5)

[Geerstma, 1966]. Note that the sum of the in-plane stresses or strains are independent of coordinate system ($\varepsilon_{rr} + \varepsilon_{00} = \varepsilon_{xx} + \varepsilon_{yy}$). Integrating, the displacements are given by

$$u^{(1)}_r = Ar + \frac{E}{r}$$

$$u^{(2)} = Cr + \frac{D}{r} + \frac{\gamma}{2\mu(2)} \frac{1}{r} \int_s^r p(r') dr'.$$

(6)

The requirement that the displacement remain finite at $r = 0$ implies $E = 0$. The strains are found from the displacements as

$$\varepsilon^{(1)}_{rr} = \frac{\partial u^{(1)}_r}{\partial r} = A$$

$$\varepsilon^{(1)}_{00} = \frac{u^{(1)}_r}{r} = A$$

$$\varepsilon^{(2)}_{rr} = \frac{\partial u^{(2)}_r}{\partial r} = C - \frac{D}{r^2} + \frac{2\gamma}{\mu(2)} \frac{1}{r} \int_s^r p(r') dr'$$

(7)

$$\varepsilon^{(2)}_{00} = \frac{u^{(2)}_r}{r} = C + \frac{D}{r^2} + \frac{\gamma}{2\mu(2)} \frac{1}{r} \int_s^r p(r') dr',$$

and

$$\varepsilon^{(2)}_{rr} + \varepsilon^{(2)}_{00} = 2C + \frac{\gamma p(r)}{2\mu(2)}.$$  

(8)

The stresses are found by substituting the strains into the appropriate constitutive equations (1) and (3), yielding

$$\sigma^{(1)}_{rr} = 2\mu^{(1)} \frac{(1 + \nu^{(1)})}{(1-\nu^{(1)})} (ru^{(1)}_r - \frac{2\gamma}{2\mu(2)} (ru^{(1)}_r - \frac{1}{r} \int_s^r p(r') dr'))$$

(9)

Where in (9), and from here on, the drained and undrained Poisson’s ratios without superscripts refer to the poroelastic rock. The superscript (1) always refers to the elastic inclusion.

[11] The three integration constants $A, C, D$ are determined by three boundary conditions

$$u^{(1)}(r = a) = u^{(2)}(r = a)$$

(10a)

$$\sigma^{(1)}(r = a) = \sigma^{(2)}(r = a)$$

(10b)

$$\sigma^{(2)}(r \to \infty) = \sigma^{(2)}_\infty.$$  

(10c)

Figure 2. The strain meter is modeled as a cylindrical elastic inclusion of radius $a$ in an otherwise poroelastic earth. Region-1 refers to the elastic inclusion. Region-2 to the surrounding poroelastic rock.
A similar far-field boundary condition exists for the pore-pressure. It is important to note that the limit $r \to \infty$ means far from the effects of the borehole, or $r \gg a$, however $r$ will generally be small in comparison to the characteristic dimensions of the deformation source (fault plane or magma chamber). Notice that the areal strain in the inclusion is $\varepsilon^{(1)}_r + \varepsilon^{(1)}_\theta = 2\lambda$. Boundary conditions (a) and (b) lead to

$$\varepsilon^{(1)}_r + \varepsilon^{(1)}_\theta = \frac{2\Psi C}{1 - v} \Psi \equiv 2\left(\frac{\mu(1 + \nu)(1 - 2\nu)}{E(1 - \nu)} + 1\right)^{-1}$$

where $C$ is determined by the far-field boundary condition. Note that (8) gives an expression for $C$ in terms of the far-field areal strain $(\varepsilon^{(2)}_r + \varepsilon^{(2)}_\theta)^\infty$ and pore pressure change $p^\infty$

$$\varepsilon^{(1)}_r + \varepsilon^{(1)}_\theta = \frac{\Psi}{1 - v} \left(\frac{\varepsilon^{(2)}_r + \varepsilon^{(2)}_\theta}{1 - \nu}\right)^\infty - \frac{\gamma}{2\mu c^2} p^\infty.$$  

This demonstrates that when the fluid pressure changes the strain measured by the strainmeter will not be the same as that in the rock around the borehole.

There is some uncertainty as to the sensitivity of the dilometer to strain parallel to the borehole axis. For this discussion we will assume that the instrument measures $\Delta \varepsilon^{\text{incl}} \equiv \varepsilon^{(2)}_r + \varepsilon^{(2)}_\theta + \alpha \varepsilon^{(2)}_\theta$, where $0 \leq \alpha \leq \frac{1}{2}$. At one extreme the instrument is completely insensitive to the vertical strain, in the other the instrument measures the true volume strain. Equation (12) leads to

$$\Delta \varepsilon^{\text{incl}} = \Psi \frac{1 - (1 + \alpha)\nu\varepsilon(1)}{1 - (1 + \alpha)\nu} \left[\frac{\varepsilon^{(2)}_r + \varepsilon^{(2)}_\theta}{1 - \nu}\right]^\infty - \frac{\gamma}{2\mu c^2} p^\infty.$$  

Defining the rock response as $\Delta \varepsilon^{\infty} \equiv \varepsilon^{(2)}_r + \varepsilon^{(2)}_\theta + \alpha \varepsilon^{(2)}_\theta$, we find that

$$\Delta \varepsilon^{\text{incl}} = \frac{\Psi}{1 - \nu}\left[1 - (1 + \alpha)\nu\varepsilon(1)\right] \Delta \varepsilon^{\infty} - \frac{\gamma(1 + \alpha)}{2\mu c^2} p^\infty.$$  

The constitutive equations (1) yield

$$\sigma^{(2)}_r + \sigma^{(2)}_\theta = 2\mu(1 + \nu)(1 - 2\nu)\varepsilon^{(2)}_r + \varepsilon^{(2)}_\theta) = \varepsilon^{\infty} - \frac{2\gamma}{(1 - \nu)} p^\infty.$$  

Equation (15) when combined with the undrained pore pressure change $p = -B\sigma^{(2)}_r$ can be solved for the rock strain in terms of the undrained pore pressure response. This result substituted into (13) yields

$$\Delta \varepsilon^{\text{incl}}(t = 0) = \left[\frac{\Psi}{1 - \nu}\left[1 - (1 + \alpha)\nu\varepsilon(1)\right]\right] \Delta \varepsilon^{\infty}(t = 0).$$  

The output of the strainmeter is calibrated against the solid earth tides. Denote the instrument response as $\Delta \varepsilon^{\text{inst}} \equiv C\Delta \varepsilon^{\text{incl}}$, where $C$ is a calibration factor. The calibration is chosen such that $\Delta \varepsilon^{\text{inst}} = \Delta \varepsilon^{\infty}$ for undrained conditions. This implies that $C = \left[\right]^{-1}$, where $\left[\right]$ denotes the terms in brackets in (16). With this definition (14) becomes

$$\Delta \varepsilon^{\text{inst}} = \left[1 - (1 + \alpha)\varepsilon(1)\right] \left[\Delta \varepsilon^{\infty} - \frac{3(1 + \alpha)(\varepsilon_\nu - \varepsilon)}{2\mu c^2 B(1 + \varepsilon) p^\infty}\right].$$  

which is the principal result of this paper. Notice that for $\alpha = 0$ the fully drained response is $\Delta \varepsilon^{\text{inst}}/\Delta \varepsilon^{\infty} = (1 - \varepsilon_\nu)/(1 - \nu) \leq 1$. For $\alpha = 1$ the fully drained response is $\Delta \varepsilon^{\text{inst}}/\Delta \varepsilon^{\infty} = (1 - 2\varepsilon)/(1 - 2\nu) \leq 1$. Note also that if $\Delta \varepsilon^{\infty} = 0$ then $\Delta \varepsilon^{\text{inst}} > 0$ for a pore pressure decrease, as was deduced from the thought experiment in Figure 1.

2.1. An Example: Plane Strain Edge Dislocation

Rice and Cleary [1976] give the plane-strain solution for a two dimensional edge dislocation in a poroelastic medium. Superposition of two edge dislocations of opposite sign approximates a strike-slip in a full space, with depth much greater than the fault length. From their results

$$\sigma(t) = \frac{\mu(b(v_\nu - v) - \varepsilon^\infty_\nu)}{1 - \nu} \left[\frac{e^{-\rho t/\nu} - 1 - \nu}{v_\nu - v}\right],$$

$$\Delta \varepsilon(t) = \frac{-b(1 - 2v_\nu) \sin(\phi)}{2\pi(1 - v_\nu) \rho} \left[1 + \frac{(v_\nu - v)e^{-\rho t/\nu}}{(1 - v_\nu)(1 - 2v_\nu)}\right],$$

$$p^\infty(t) = \frac{\mu b(1 + v_\nu) \sin(\phi)}{3\pi(1 - v_\nu) \rho} \left[1 - e^{-\rho t/\nu}\right].$$

where $\sigma$ is the sum of the inplane normal stresses, $b$ is the fault slip, $\rho$, $\phi$ are polar coordinate system centered at the dislocation end, and $c$ is the hydraulic diffusivity, proportional to permeability. The plane-strain result corresponding to equation (17), following a parallel derivation, is

$$\Delta \varepsilon^{\text{inst}} = \frac{(1 - \nu)}{2\pi(1 - v_\nu)} \left[\frac{1 - 2v_\nu}{1 - v_\nu}\right] \left[\Delta \varepsilon^{\infty} - \frac{\gamma}{2\mu c^2(1 - v_\nu)} p^\infty\right].$$

There is no dependence on $\alpha$ because there is no vertical strain in plane-strain. Combining (18) and (19) yields

$$\Delta \varepsilon^{\text{inst}} = \frac{-b(1 - 2v_\nu) \sin(\phi)}{2\pi(1 - v_\nu) \rho}. $$

Surprisingly, while the mean stress, pore pressure, and dilatational strain in the rock all are time dependent, the strain within the elastic inclusion is time invariant! Rice et al. [1978] showed that for a spherical cavity subject to radial traction, the cavity wall deforms as if the interior had undergone a uniform strain and is independent of the pore pressure at the cavity boundary. This is analogous to the result obtained here where the displacement in the inclusion does not change with time as the fluid pressure drains. Note that analytical dislocation solutions are limited to two dimensions, and the behavior in three dimensions will require numerical analysis.

3. Observations

Two $M_w$ 6.5 earthquakes occurred in the South Iceland Seismic Zone (SISZ) on June 17 and 21, 2000. Transient postseismic deformation following these earth- quakes was observed in SAR interferograms of the region. Jönsson et al. [2003] showed that these signals are well explained by poroelastic relaxation, a conclusion strongly supported by changes in water level in numerous geothermal wells within the SISZ. The duration of the water level recoveries (4–6 weeks) agreed very well with the duration of the deformation transient.
There are several dilatometers within the SISZ, with one (SAU) located only a few kilometers west of the June 17 rupture, within a compressional quadrant. The coseismic strain change (Figure 3) was roughly −12 μstrain, consistent with some dislocation models of the June 17 earthquake. The coseismic step change was followed by a roughly one month long transient strain recovery of ∼7 μstrain (Figure 3).

The duration of the strainmeter transient is comparable to that observed in the InSAR and water level data, however the sign is opposite to the expected poroelastic response. The fully (un)drained elastic response of the rock can be computed from a three dimensional elastic dislocation solution using the (un)drained Poisson’s ratios. This predicts that compressional strain should have further increased as the rock drained. For example, assuming β = 0.2 and νu = 0.31, the undrained response (for α = 0 and one slip model) is predicted to be −9.4 μstrain, whereas the drained (t → ∞) response is −12.6 μstrain. For a dislocation in a two-dimensional poroelastic medium the postseismic strain in the meter should be independent of time following the coseismic step (20), however we have no corresponding result in three dimensions.

We consider the possibility that permeability decreases with depth in the crust so that only the shallow crust drains over short time scales. Jónsson et al. [2003] showed that the InSAR data could be equally well fit with either drainage of the entire half-space or only the upper few kilometers of crust. Following the same procedure, the dilatational strain due to a specified pore-pressure change is

\[
\Delta^\infty(x) = \frac{\gamma}{2\pi\mu} \int p(\xi) \left( \frac{3 + \alpha + \alpha^3}{R^5} \right) dV
\]

where \(R^2 = (x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 - \xi_3)^2\) and the integration is taken over the entire domain where \(p(\xi) \neq 0\).

If a layer \(0 < \xi_3 < D\) drains completely (corresponding to infinite permeability) and the underlying half-space remains undrained (zero permeability) computed from (21). The dashed curve shows the strain in the rock, solid curve strain measured by a strainmeter. Strains are normalized by the coseismic strain. Strain recovery is less than 30% of the coseismic strain change, whereas the observed is ∼70%.

More complex models may be required to explain the observed postseismic strain change. For example, lateral variations in permeability could lead to significant horizontal strains [e.g., Segall and Fitzgerald, 1998]. Alternatively, if the entire region containing the strainmeter is partially drained by fluid filled fractures, the fractures would instantaneously transmit normal stresses to the strainmeters, however those stresses (and strains) might relax over time as fluid flowed out of the fractures.

4. Discussion and Conclusion

As long as the rock surrounding the strainmeter remains undrained at tidal periods the response to rapid deformations, such as coseismic strain changes, is not complicated by fluid flow. The strainmeter response to more slowly developing processes, such as slow earthquakes or many volcanic events, will require a convolution of the source time function with the poroelastic response of the rock surrounding the strainmeter. Finally, if significant

Figure 4. Postseismic strain at SAU when an upper layer drains completely (corresponding to infinite permeability) and the underlying half-space remains undrained (zero permeability) computed from (21). The dashed curve shows the strain in the rock, solid curve strain measured by a strainmeter. Strains are normalized by the coseismic strain when an upper layer drains completely (corresponding to infinite permeability) and the underlying half-space remains undrained (zero permeability) computed from (21). The dashed curve shows the strain in the rock, solid curve strain measured by a strainmeter. Strains are normalized by the coseismic strain.

Figure 3. Volumetric strain observed at station SAU between June 11 and July 19, 2000. Vertical axis is the dilatational strain (microstrain) with compression negative. The times of the two mainshocks are indicated as vertical lines. The June 17 earthquake exceeded the dilatometer’s dynamic range. The strain change from the time of the earthquake to June 20 when the instrument was reset is ∼−10 μstrain, indicating a coseismic strain of ∼−12 μstrain.
porefluid flow occurs at tidal periods, the strainmeter response will be significantly more complex than outlined here.

[21] The strain relaxation observed by a borehole strainmeter close to an earthquake in the South Iceland seismic zone can only be partially explained by fluid drainage in an isotropic, poroelastic medium. The observations may be better explained by a fracture dominated poroelastic response; the subject of ongoing work.

[22] Acknowledgments. We thank E. Roeloffs, D. Agnew, and N. Sleep for discussions, J. Rudnicki and an anonymous reviewer for extremely valuable comments. The latter pointed out an algebraic error in an earlier draft of the paper. This research was supported by the NSF and the USGS.

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