STANFORD GEOTHERMAL PROGRAM
STANFORD UNIVERSITY

Stanford Geothermal Program
Interdisciplinary Research in
Engineering and Earth Sciences
STANFORD UNIVERSITY
Stanford, California

SGP-TR-70

SLUG TEST DATA ANALYSIS

IN RESERVOIRS WITH DOUBLE POROSITY BEHAVIOR

By

Khalid Mateen

September, 1983

Financial support was provided through the Stanford Geothermal Program under Department of Energy Contract No. DE-AT03-80SF11459 and by the Department of Civil Engineering, Stanford University.

Abstract

Pressure analysis for a slug test which corresponds to the flow period of a Drill Stem test is extended to wells in reservoirs with double porosity behavior.

The modelling of fluid flow within a reservoir is achieved by forming two partial differential equations for the matrix and fissures. The matrix acts as a source, and fissures connect with the well. Conditions involving wellbore storage and skin effect are included.

Solutions are obtained with the assumptions of pseudo steady state or transient interporosity flow. The resulting distinctive specific features of each solution are identified.

An interpretation method based on type curve matching is proposed. Application of type curve matching techniques can provide the interporosity flow parameter λ, and storativity ratio ω, in addition to transmissivity, $\frac{kh}{\mu}$, and skin effect, S.

TABLE OF CONTENTS

1. Introduction

The slug test was introduced in ground water hydrology by Ferris and Knowles in 1954. An analogous heat conduction test was reported by Jaeger (1956). In 1967, Cooper et al. presented a solution for a well of a finite diameter and showed the line source approximation of Ferris and Knowles to be valid for large times. This solution is a special case of a problem presented by Jaeger in 1956. Meir presented an approximate analysis of the equivalent DST problem in 1970.

In a slug test, a batch of fluid is suddenly removed from (or added to) the static column of a wellbore. The tool is attached to the drill stem and is lowered to the test interval where the packer is set, and valves are opened by manipulation of drill pipe.

The slug test problem is posed by a formation with a pressure which is, at most, great enough to lift a column of reservoir fluid just to the surface of the earth. The initial production rate is high, and gradually declines as the accumulating fluid in the drill string increases the back pressure.

Neglecting inertial and frictional effects, the liquid column is assumed to be in static equibilibrium with well bore pressure. Since the pressure-time response is dependent od wellbore characteristics, interpretation yields reservoir parameters such as transmissivity ($\frac{kh}{\mu}$) and storativity ($\phi h c_t$), or skin effect. Slug test formulations based on liquid column height and those based on bottom hole pressure were separately proposed. Liquid column height is measured in ground water hydrology, whereas bottom hole pressure is measured in the petroleum industry. Pressure and liquid column height can be changed through stationary column calculations (Saldana, 1983).

In 1972, Ramey and Agarwal presented a detailed derivation for the DST problem with skin effect. These authors obtained a solution to the problem in the form of an inversion integral by Laplace transformation. This solution was obtained by drawing an analogy with a heat transfer problem of a cylinder with a heat resistance (skin effect), presented by Jaeger in 1956, Applications were presented by van Poollen, and Weber (1970), and Kohlhaas (1972) for the DST problem, but without a skin effect.

Ramey, Agarwal and Martin (1975) correlated the solution for the DST problem using a wellbore storage coefficient and dimensionless time based on effective wellbore radius, a technique used by Earlougher and Kersh (1974) in conventional well testing. Although the correlation was not mathematically rigorous, it was found to hold well for high values of C_De^{2s}. However, results were poor for small values of C_De^{2s}.

Extensive work has been done in recent years to explain the transient pressure behavior of naturally-fractured or fissured reservoirs. Such reservoirs have homogenously distributed regions of primary and secondary porosity. Primary porosity is synonymous with the matrix block whose properties are governed by sedimentation, cementation and lithification of deposits., Secondary porosity, or fracture network, is supposed to develop after sedimentation through mechanical distortion or dolomatization.

The present study is concerned with two such mediums, one medium presents a high conductivity and drains the reservoir fluid to the well, the other presents a much lower conductivity and feeds only to the fissure medium. They are termed two-t porosity reservoirs. Naturally-fractured reservoirs, and layered reservoirs with only the more permeable layer conducting to a wellbore exhibit the same double porosity behavior.

Warren and Root (1963) posed the problem for such reservoirs. They assumed pseudo-steady interporosity flow and showed that the interporosity flow parameter, λ, and storativity ratio, ω, are sufficient to characterize the flow model. Equations for the matrix and fissures were formulated and combined assuming either pseudo-steady state or transient interporosity flow. Fluid flow from the matrix to fissures under pseudo steady state is given by:

$$q = \frac{\alpha \; k_m}{\mu} (p_m - p_f) \quad \text{....................(1-1)}$$

Transient flow from the matrix to fissures is assumed to be governed by the equation:

$$\nabla^2 p_m = \frac{1}{\eta_m} \frac{\partial p_m}{\partial t} \quad \text{................(1-21}$$

This approach was first presented by de Swaan (1976).

Various matrix geometries (slab and sphere) were considered. A detailed derivation for a slab shape is given in Appendix A. Results presented can be used both for slab and spherically shaped matrix blocks.

The characteristic double-porosity diffusion equation was then coupled in Laplace space with the wellbore pressure through inner boundary conditions of a steady-state pressure drop occuring at the sand face (skin effect), and a wellbore storage effect.

A formal solution showing wellbore pressure as a function of two functional groups C_De^{2S} and t_D/C_D for a homogenous case is presented in this study. Type curves based on either pseudo--steady state or transient interporosity flow are presented separately. The pressure behavior of a double-porosity reservoir is characterized and pertinent parameters evaluated.

2. Statement Of Problem

In this section we consider the problem assumptions, descriptions, and formulation in dimensionless groups.

2.1. Assumptions

In Appendix A, a mass balance is made considering the reservoir as a continuum. The smallest incremental volume is of an extent large enough to include both secondary and primary porosity.

The following assumptions are made in forming the mathematical model:

(1) All flow is single phase and described by Darcy's law.

(2) Gravitational forces are negligible and pressure gradients are small.

(3) The fissure permeability is constant and is at least an order of magnitude greater than the permeability of the matrix blocks.

(4) Rock compressibilities are constant and independent of pressure in the other medium.

(5) The reservoir is of infinite lateral extent with closed top and bottom.

2.2. Problem Description

A mass balance results in the following equation for the fissure medium:

$$\frac{k_f}{\mu} \nabla^2 p_f = (\phi V c_t)\frac{\partial p_f}{\partial t} - q \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-1)$$

where:

ϕ is the true porosity (ratio of pore volume in a given medium to the volume of this medium),

V is the ratio of the volume of a given medium to the total volume,

q is the volume of fluid flowing from the matrix into the fissures per unit block volume per unit time.

For a Slug test, there is no flow at the surface and the flow coming from the sand face changes the liquid level in the wellbore, given by:

$$C\frac{d(p_i - p_{wf})}{dt} = \frac{2\pi k_f h}{\mu}\left(\frac{r\,\partial p_f}{\partial r}\right)_{r=r_w} \quad \ldots\ldots\ldots\ldots (2-2)$$

where:

$$C = \frac{\pi r_p^2}{\rho \frac{g}{g_c}} \quad .... \quad ....(2-3)$$

Pressure drop at the sand face is considered by using the Hurst **(1953),** and van Everdingen **(1953)** dimensionless skin factor:

$$P''f = [\; p_f - \frac{rS\, \partial p_f}{\partial r}]_{r=r_w} \quad ....(2-4)$$

As an initial condition, the reservoir is assumed to have a uniform pressure distribution:

$$p_f\;(r,t=0) = p_i \quad .... \quad (2-5)$$

The reservoir is considered as infinite acting throughout the test duration, a suitable condition for a short duration test.

$$\lim_{r\to\infty} p_f\;(r,t) = p_i \quad ....(2-6)$$

Wellbore pressure is assumed initially to be equal to gas column pressure or cushion pressure:

$$p_{wf} = p_o \quad \text{.......................................}(2\text{-}7)$$

2.3. Formulation In Dimensionless Groups

Equations 2-1 to 2-7 completely describe the slug test, neglecting frictional and inertial effects,

This problem can be posed in terms of dimensionless variables as follows:

Equation **for** fissures:

$$\frac{\partial^2 p_{fD}}{\partial r_D} + \frac{\partial p_{fD}}{r_D \partial r_D} = \frac{\partial p_{fd}}{\partial t_D} + \frac{\mu r_w^2}{k_f} \frac{q}{(p_i - p_o)} \quad \text{...........}(2\text{-}8)$$

Initial conditions:

$$p_{fD}\ (r_D, t_D = 0) = 0 \quad \text{..............................}(2\text{-}9)$$

$$p_{wfD}\ (t_D=0) = 1 \qquad \text{.............................(2-10)}$$

outer boundary condition:

$$\lim_{r_D\to\infty}\ p_{fD}\ (r_D, t_D) = 0 \qquad \text{..................(2-11)}$$

Inner boundary conditions:

$$\frac{C_D\ \partial p_{wfD}}{} = \left(\frac{\partial p_{fD}}{\partial r_D}\right)_{r_D=1} \qquad \text{.............................(2-12)}$$

$$p_{wfD} = \left[p_{fD} - S\left(\frac{\partial p_{fD}}{\partial r_D}\right)\right]_{r_D=1} \qquad \text{...................} \qquad (2\text{-}13)$$

where dimensionless quantities are defined as:

$$p_{fD} = \frac{p_i - p_f}{p_i - p_o} \qquad \text{.............................(2-14)}$$

$$p_{mD} = \frac{p_i - p_m}{p_i - p_o} \qquad \text{(2-15)}$$

$$p_{wfD} = \frac{p_i - p_{wf}}{p_i - p_o} \qquad \text{(2-16)}$$

$$t_D = \frac{k_f t}{[(\phi V c_t)_f + (\phi V c_t)_m] \mu r_w^2} \qquad \text{(2-17)}$$

$$r_D = \frac{r}{r_w} \qquad \text{(2-18)}$$

$$C_D = \frac{C}{2\pi h[(\phi V c_t)_f + (\phi V c_t)_m] r_w^2} \qquad \text{(2-19)}$$

$$\omega = \frac{(\phi V c_t)_f}{[(\phi V c_t)_f + (\phi V c_t)_m]} \qquad \text{(2-20)}$$

$$\lambda = \frac{\alpha k_f r_w^2}{k_m} \qquad \text{(2-21)}$$

where "a" is the interporosity shape factor, defined by Warren and Root (1956) as[1].

$$a = \frac{4\ n\ (n+2)}{l} \qquad \text{.........(2-22)}$$

where "n" is the number of normal subsets of fissures, and "l" is a characteristic dimension of the matrix.

The expression of q in Eq.(2-8) is dependent upon the natune of the interporosity flow regime. For the pseudo-steady state interporosity flow assumption, in Laplace space:

$$\bar{q} = \frac{\alpha\ k_m\ (p_i - p_o)}{\mu}\left[\frac{(1-\omega)\ u}{(1-\omega)\ u + XC_D}\right]\ \bar{p}_{fD} \qquad \text{.........(2-23)}$$

where u is the Laplace transform argument with respect do dimensionless time based on total storativity.

Under the transient flow assumption, for slab- shaped matrix blocks:

$$\bar{q} = \frac{4\ k_m\ \bar{p}_{fD}(p_i - p\ )}{h_m^2\ \mu}\ \frac{3\ (1-\omega)\ u}{\lambda\ C_D}\ \tan\ \frac{h_m}{2}\ \frac{12(1-\omega)\ u}{h_m^2\ \lambda C_D} \qquad \text{(2-24)}$$

A detailed derivation of all previous equations is presented in Appendix A. Solution of the problem is achieved by using Laplace transformation with numerical inversion. Details are given in Appendix B.

3. Type curve for Slug testing in a Reservoir With Double-Porosity Behavior

In the following section we consider useful limiting forms for the pseudo-steady and transient interporosity flows.

3.1 Limiting forms

As shown in Appendix B, the Laplace transform of the pressure response of a well in double-porosity reservoir is givep by Eq.(B-11).

$$\bar{p}_{wfD}=\frac{K_o\left(\frac{uf(u)}{C_D}\right)+S\frac{uf(u)}{C_D}K_1\left(\frac{uf(u)}{C_D}\right)}{\frac{uf(u)}{C_D}K_1\left(\frac{uf(u)}{C_D}\right)+u\left\{K_o\left(\frac{uf(u)}{C_D}\right)+S\frac{uf(u)}{C_D}K_1\left(\frac{uf(u)}{C_D}\right)\right\}}$$

................(3-1)

where "u" is the Laplace variable based upon the dimensionlesg time-wellbore storage ratio, which is independent of storativity, and is given by:

$$\frac{t_D}{C_D} = \frac{2\pi k_f h \Delta t}{\mu C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}2)$$

The function f(u) for pseudo-steady interporosity flow in Laplace space is given by:

$$f(u) = \frac{\omega (1-\omega) u + \lambda (C_{+m}}{(1-\omega) u + \lambda (C_n)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}$$

For slab-shaped matrix blocks with transient interporosity flow, the function f(u) is given by:

$$f(u) = [\ \omega + \frac{C_D \lambda (1-\omega)}{3u} \tanh \frac{3(1-\omega) u}{\lambda C_D}\ ] \ldots\ldots\ldots\ldots(3\text{-}4)$$

As indicated in Appendix B, dimensionless wellbore pressure, for all practical purposes, can be represented by:

$$\bar{p}_{wfD} = \frac{1}{u + \ell n \frac{2}{e^{\gamma}} \frac{(C_D)_{f+m} e^{2s}}{u\, f(u)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots^{-1} \quad \ldots(3\text{-}5)$$

where γ=0.5778......, Euler's constant

The function $f(u)$ for pseudo-steady state and transient interporosity flow have the same limits for early and late times (Deruyck et al., 1982). Hence, the same set of equations can represent the early and late time behavior for both interporosity flow regimes.

The limiting forms of Eq. 3-5 are obtained from the approximation of function $f(u)$ at early, intermediate and late times.

3.1.1 Early Times

$$\frac{t_D}{C_D} \to 0, \quad \text{or} \quad u \to \infty \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}6)$$

The function $f(u)$ both for pseudo-steady state and transient interporosity flow approaches ω:

$$f(u) \to \omega$$

Thus:

$$\bar{P}_{wfD} = \frac{1}{u + \left[\ln\frac{2}{e^{\gamma}}\,\frac{(C_D)_{f+m}e^{2S}}{u\,\omega}\right]^{-1}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}7)$$

which can be expressed *as:*

$$\bar{p}_{wfD} = \frac{1}{u + \ln \frac{2}{e^{\gamma}} \frac{(C_D)_f e^{2S}}{u} - 1} \quad \text{...............(3-8)}$$

where C_{Df} represents the dimensionless storage coefficient based on the storativity of fissures only.

3.1.2 Late Times

$$\frac{t_D}{C_D} \rightarrow \infty, \text{ or } \quad u \rightarrow 0$$

The function f(u) for the two flow regimes approaches unity:

$$f(u) \rightarrow 1$$

and Eq. 3-5 becomes:

$$\bar{p}_{wfd} = \frac{1}{u + \ln \frac{2}{e^{\gamma}} \frac{(C_D)_{f+m} e^{2S}}{u} - 1} \quad \text{.................(3-9)}$$

In fact, Eqs. 3-8 and 3-9 are the same, except that for early times, the wellbore storage coefficient is based on fissure storativity, whereas at late times, it is based on the total fissure and matrix storativity.

The behavior at early and late times is that of a homogeneous reservoir, and corresponds to two different curves on the type curves introduced by Ramey et al. (1975).

The pressure solution in Laplace space, Eqs. 3-7 and 3-8, indicates that for a homogeneous medium, wellbore pressure is a function of only two groups: C_De^{2S}, and t_D/C_D, as suggested by Ramey et al. (1975). However, as shown in Appendix B, this approximation holds only for small arguments of the Bessel functions, and does not neccesarily hold for small values of C_D. When the approximate form is inverted using the Stehfest algorithm (1970), for N=16, in double precision arithmetic, deviations were noticed below a value of $C_De^{2S} \leq 10^4$. The exact form of the pressure response, Eq. 3-1, was also tested for small values of C_De^{2S} by taking separately large values of C_D and negative skins. Unrealistic values of dimensionless pressures were found. Gringarten, et al. (1979), explained that a negative skin effect generates energy in the porous medium, and makes the flow equation unstable. All curves corresponding to $C_De^{2S} \leq 10^4$ were therefore evaluated using dimensionless wellbore storage coefficients and dimensionless times based on an effective wellbore radius.

3.1.3 Intermediate Times (Pseudo-Steady State Model)

$$\left[\lambda \ll u \ll \frac{1}{\omega} \; , \; \omega \ll 1 \right]$$

For this range, the function f(u) becomes:

$$f(u) = \frac{\lambda \, C_D}{u}$$

Thus:

$$\bar{p}_{wfD} = \frac{1}{u + \left[\ln \frac{2}{e^{\gamma}} \frac{e^{2S}}{\lambda} \right]^{-1}} \qquad \text{................(3-10)}$$

which can be inverted to real space:

$$p_{wfD} = e^{-\frac{t_D}{C_D} \left[\ln \frac{2}{e^{\gamma} \left(\lambda e^{-2S} \right)} \right]^{-1}} \qquad \text{................(3-11)}$$

Equation 3-11 suggests that the pressure response depends on λe^{-2S} during intermediate times.

3.1.4 Intermediate Times (Transient Flow)

$$\left[\lambda \ll u \ll \frac{1}{\omega} , \quad \omega \ll 1 \right]$$

for this range, function f(u) becomes:

$$f(u) \longrightarrow \frac{\lambda C_D}{3u}$$

Equation 3–5 becomes:

$$\bar{p}_{wfD} = \frac{1}{\ln \left(\dfrac{C_D e^{2S}}{u \left(\dfrac{C_D \lambda}{3u} \right)} \right)^{0.5}} \quad \text{......(3–12)}$$

$$\bar{p}_{wfD} = \frac{1}{u + \ln \dfrac{2}{e^{\gamma}} \left(\dfrac{3C_D e^{2S}}{u \lambda e^{-2s}} \right)^{1/4}} - 1 \quad \text{.......(3–13)}$$

$$\bar{p}_{wfD} = \frac{1}{u + \left[\ln \dfrac{2}{e^{\gamma}} \left(\dfrac{\beta}{u} \right)^{1/4} \right]^{-1}} \quad \text{.........(3–14)}$$

where β is defined as a transient interporosity flow group. For a slab-shaped matrix, it is given by:

$$\beta = \frac{3\ C_D\ e^{2s}}{\lambda\ e^{-2s}} \qquad \text{.............................(3-15)}$$

For spherically-shaped matrix blocks, it is equal to:

$$\beta = \frac{3\ C_D\ e^{2s}}{5\ A\ e^{-2s}} \qquad \text{..........................(3-16)}$$

3.2 Type Curve Analysis (Pseudo-Steady State Interporosity Flow Model)

The problem of a Slug test in a two-porosity system can therefore be classified into:

(1) a slug test in a homogeneous reservoir as presented by Ramey, et al. in 1975, with solutions given by Eqs. 3-7 and 3-8 and as shown in Figs. 3.1, 3.2 and 3.3;

(2) the interporosity flow effect for a double-porosity reservoir with λe^{-2S} as the governing parameter. Solution is given by Eq. 3-11 and is shown in Figs. 3.4, 3.5 and 3.6.

The corresponding pressure response in a slug test for a well in double-porosity reservoir is therefore simply obtained by

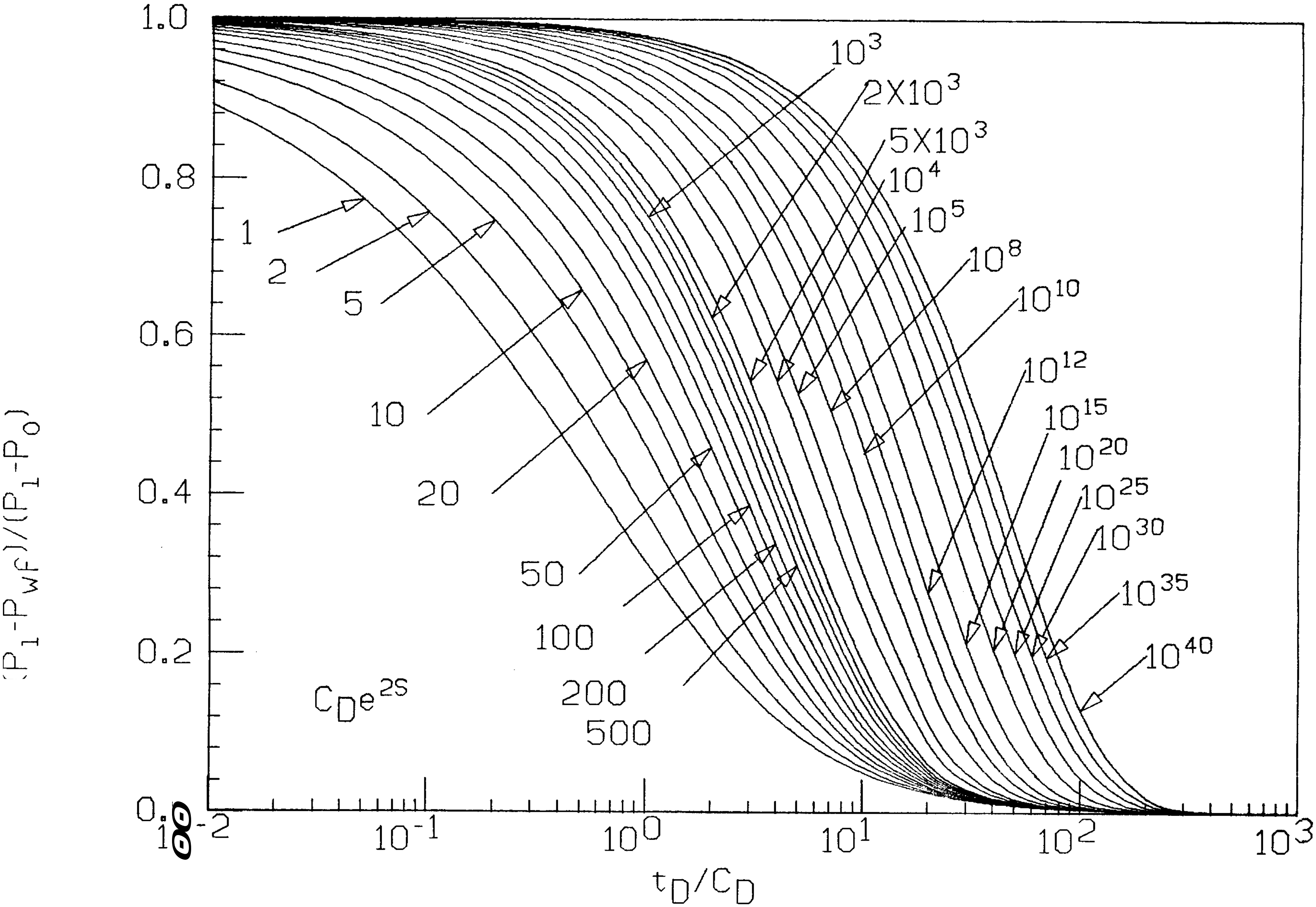

FIG. 3.1- Semi-log early and late time behaviour

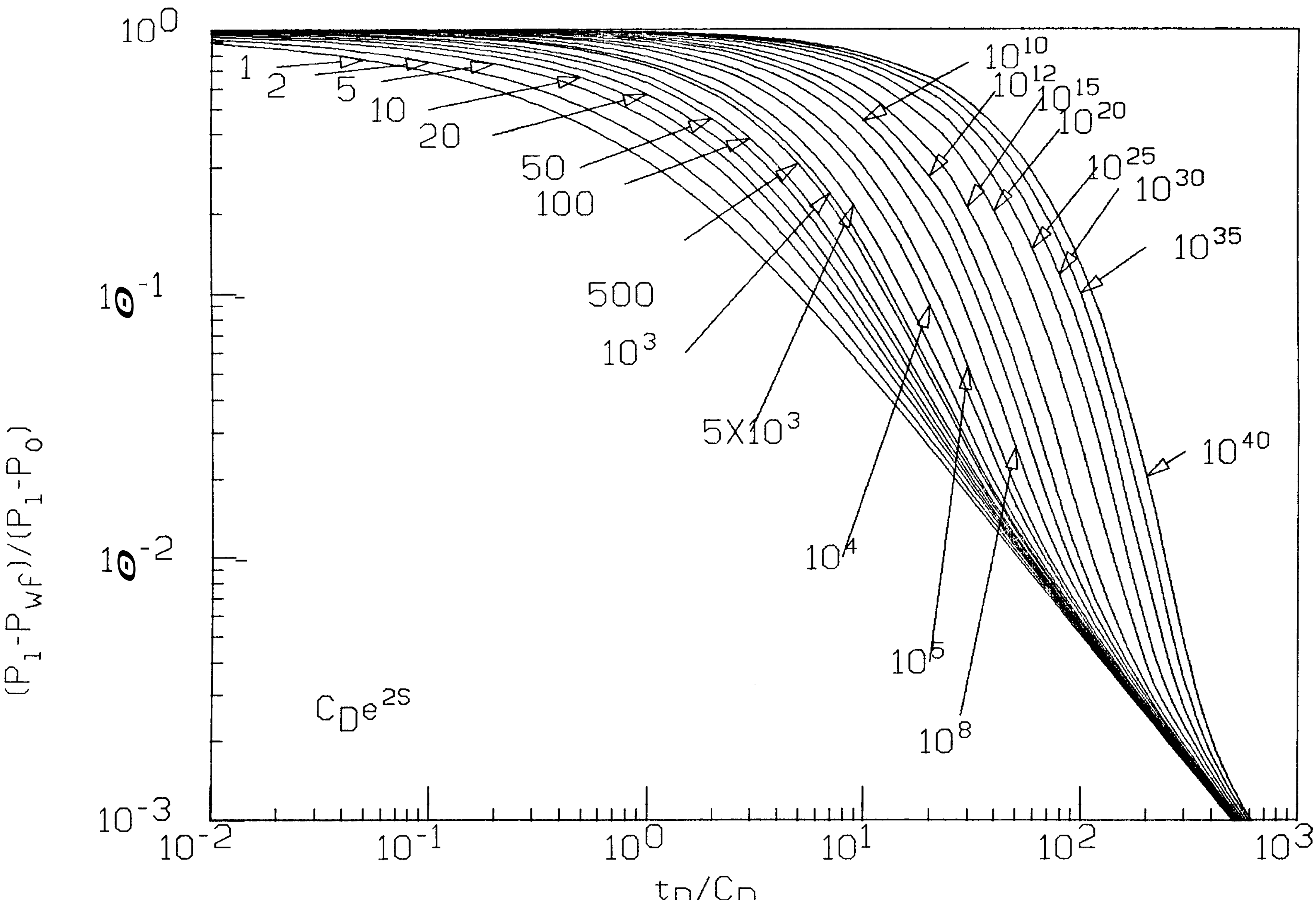

FIG. 3.2- Log-Log early and late time (homogenous) behaviour

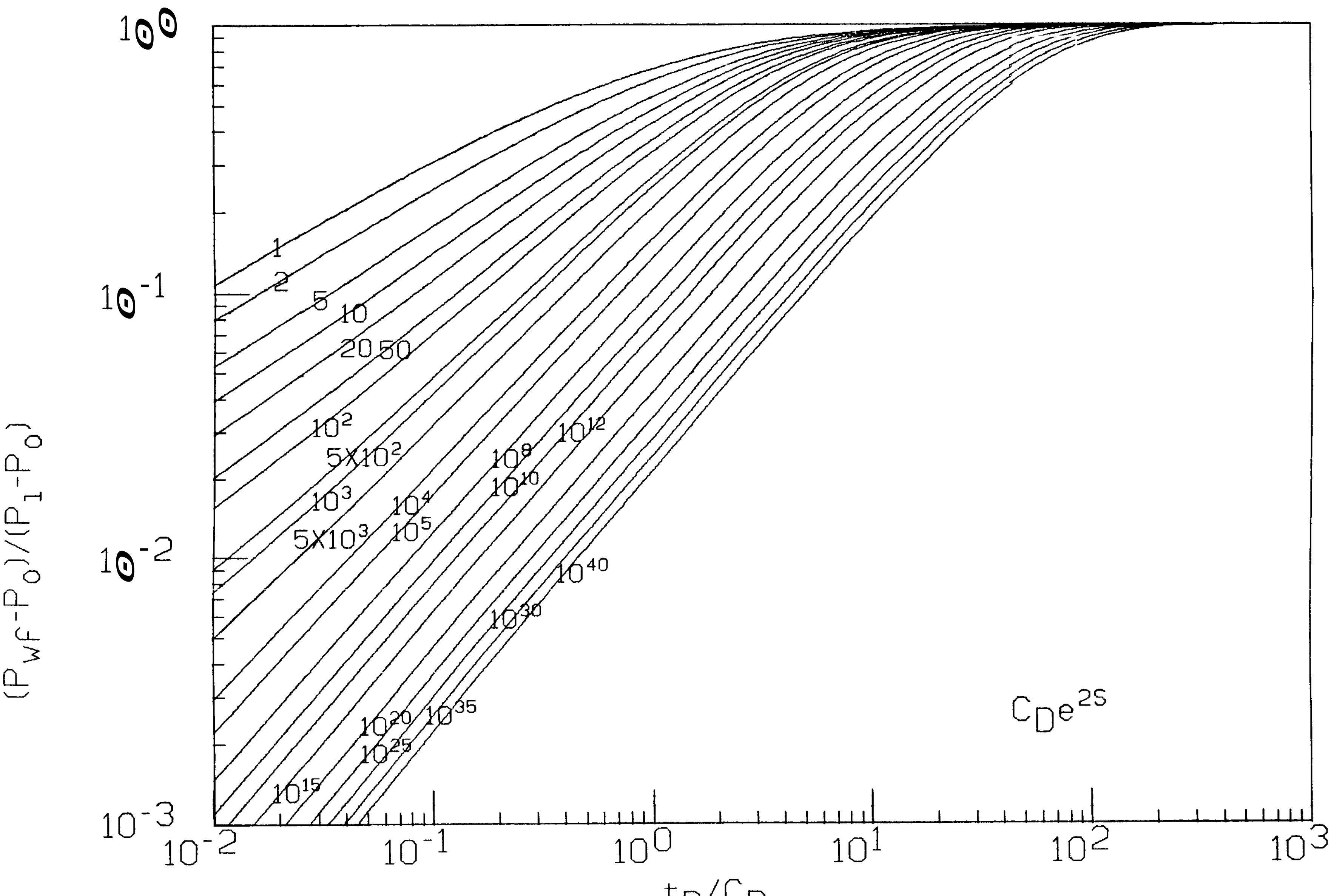

FIG. 3.3- Log-log early and late time(homogenous) behaviour

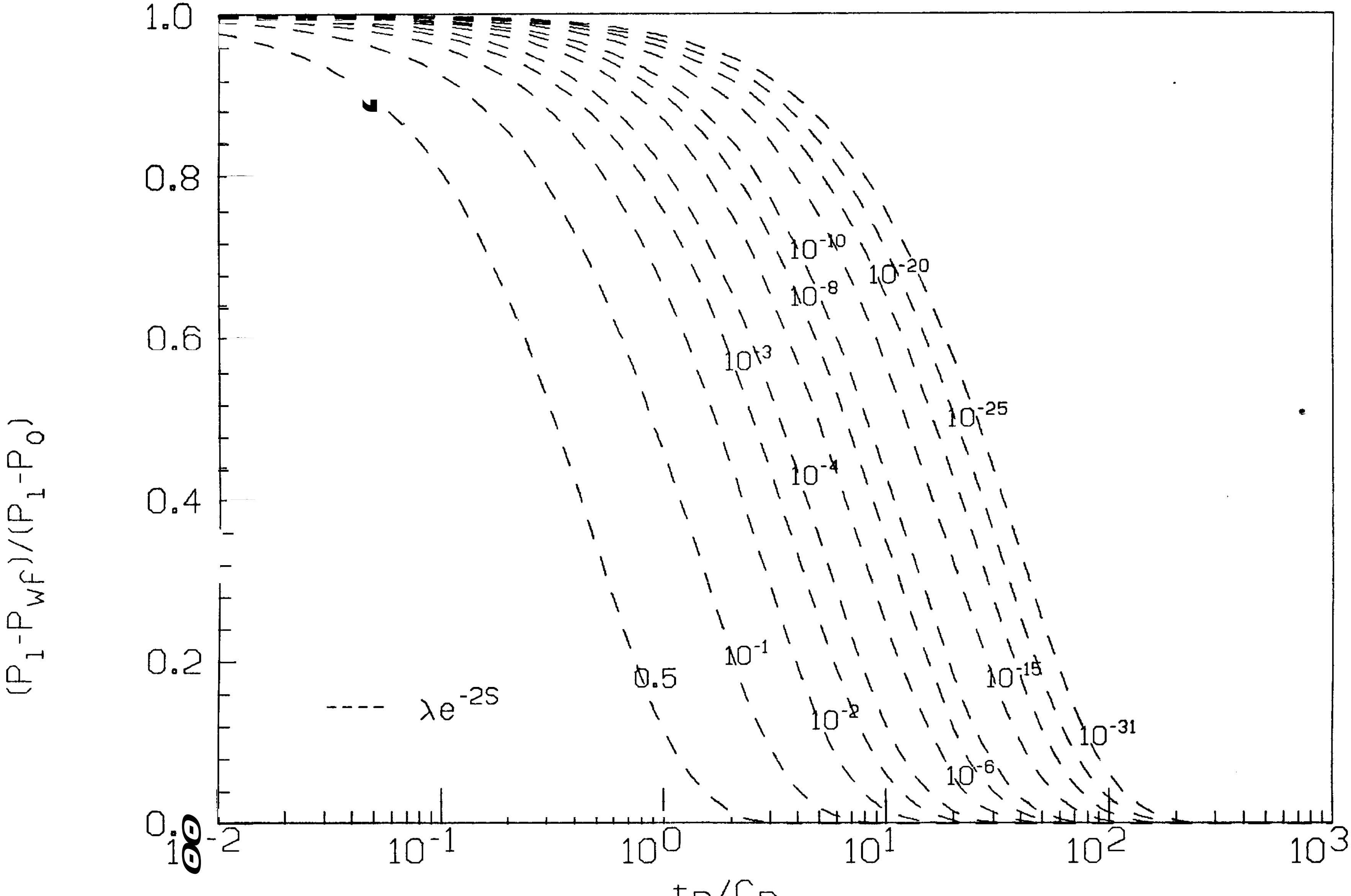

FIG. 3.4-Transition pressure in double porosity reservoir(PSS)
(Semi-log)

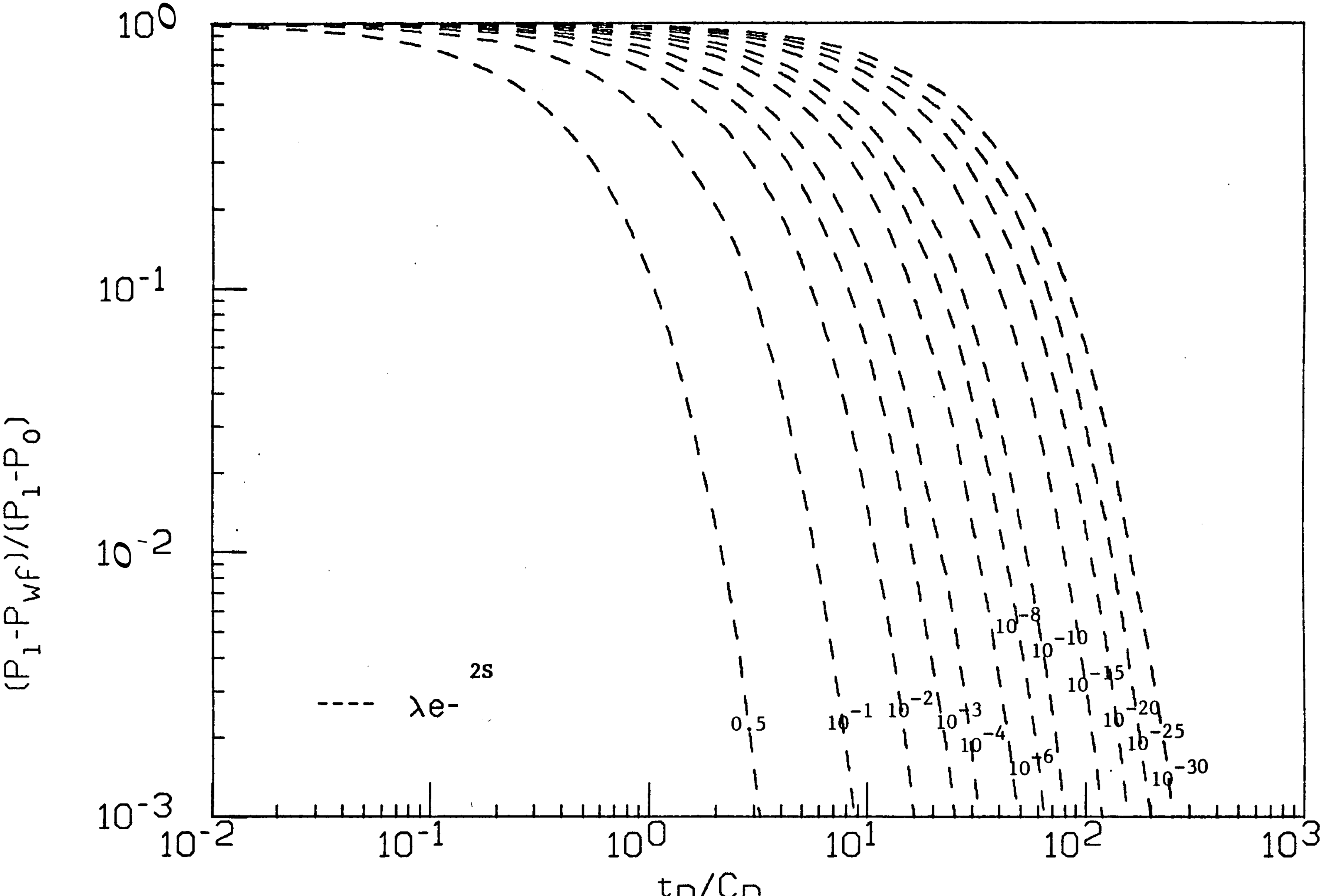

FIG. 3.5-Transition pressure in double porosity reservoir(PSS)
(Log-log)

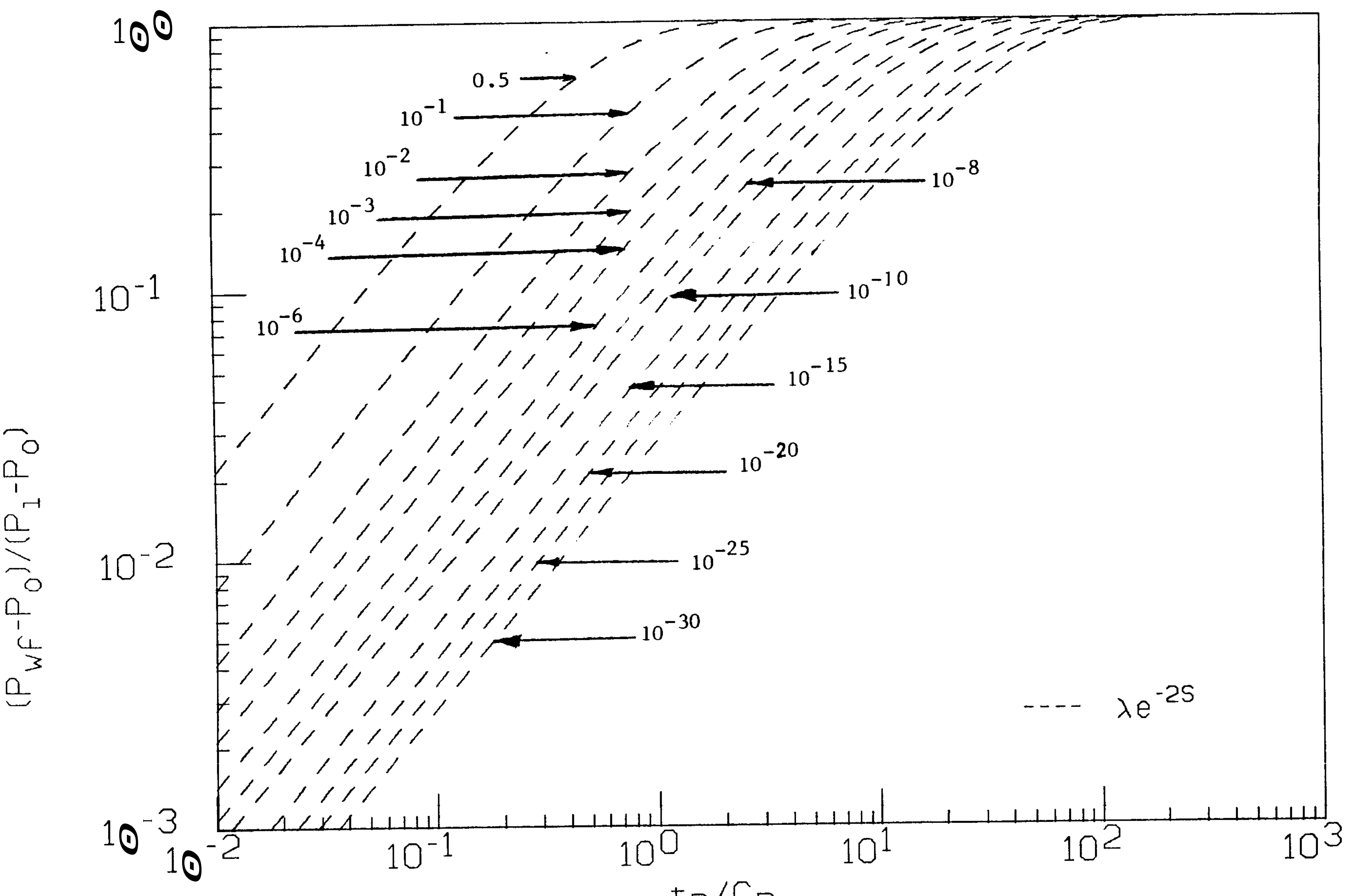

FIG. 3.6-Transition pressure in double porosity reservoir(PSS) Log-Log)

superposing the curves of early and late (homogeneous) behavior and transient (two-porosity) behavior and is presented in Figs. 3.7, 3.8 and 3.9

A typical result for wellbore pressure in a two-porosity reservoir is given in Figs. 3.10 and 3.11. At early times, production comes from the fissure system with $C_D=C_{Df}$. As the matrix starts feeding into the fissures, pressure leaves the C_De^{2S} curve and follows λe^{-2S} curve, until producion comes from the entire system. At this stage, pressure follows a new C_De2S with $C_D=C_{Df+m}$, below the first one.

Assuming (ϕhc_t) and the wellbore storage coefficient (C) are known, a semilog or a log-log type curve analysis of pressure response can yield all the system parameters. Transmissivity can be obtained from the time match, skin factor(S) from C_De^{2S} match, λ from λe^{-2S} match, and storativity ratio ω from the ratio of the C_De^{2S} value of last curve to the C_De^{2S} value of the first curve. Determination of ω is not always possible, specially when pressure follows a transition curve from very early times.

3.3 Type Curve Analysis (Transient Model Interporosty Flow Model)

As for the pseudo-steady state interporosity model, the mechanism of fluid flow in double-porosity reservoirs with transient interporosity flow involves: three successive flow regimes: at early times, there is a homogeneous system behavior

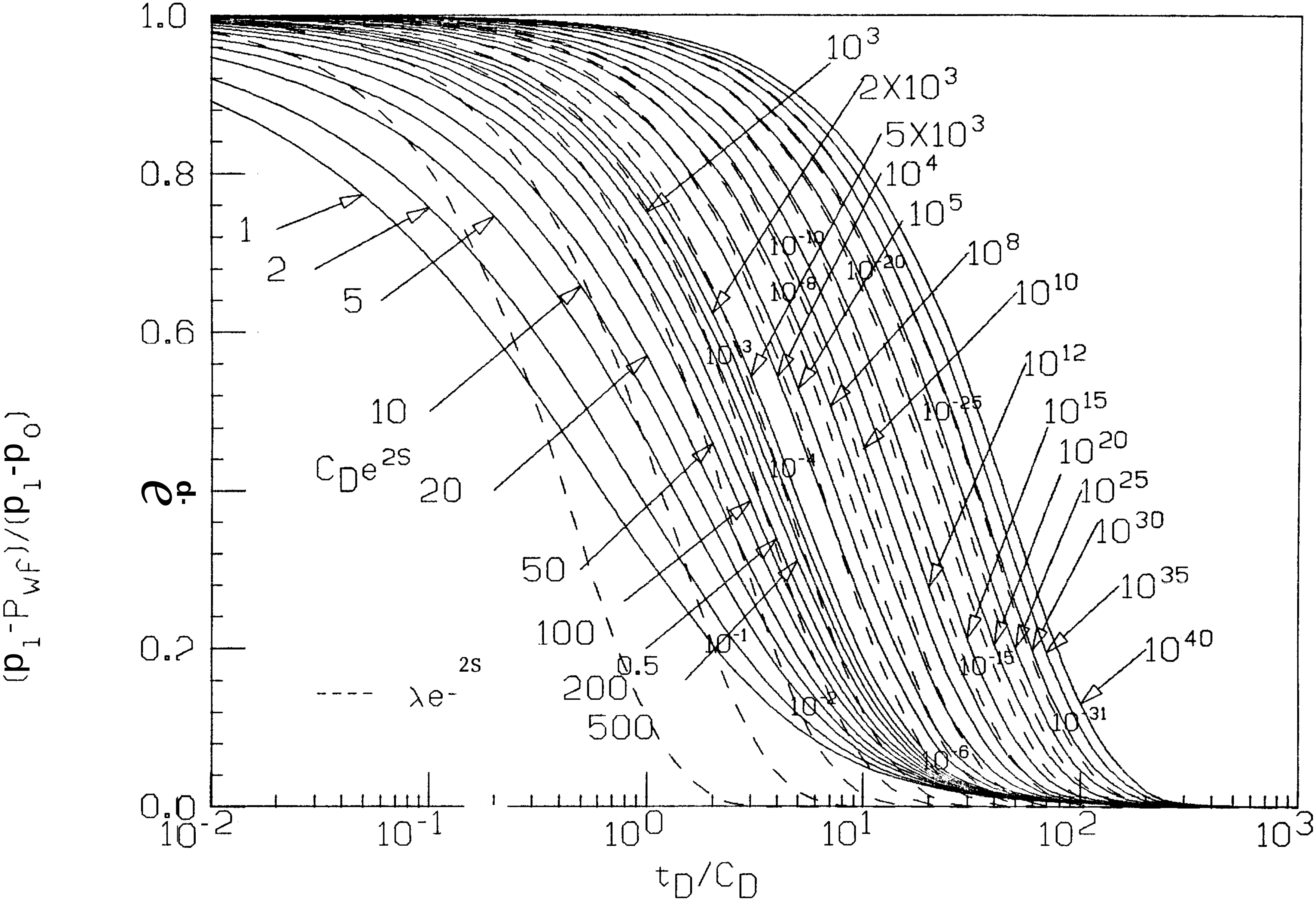

FIG. 3.7- Semi-log double porosity type curve(PSS)

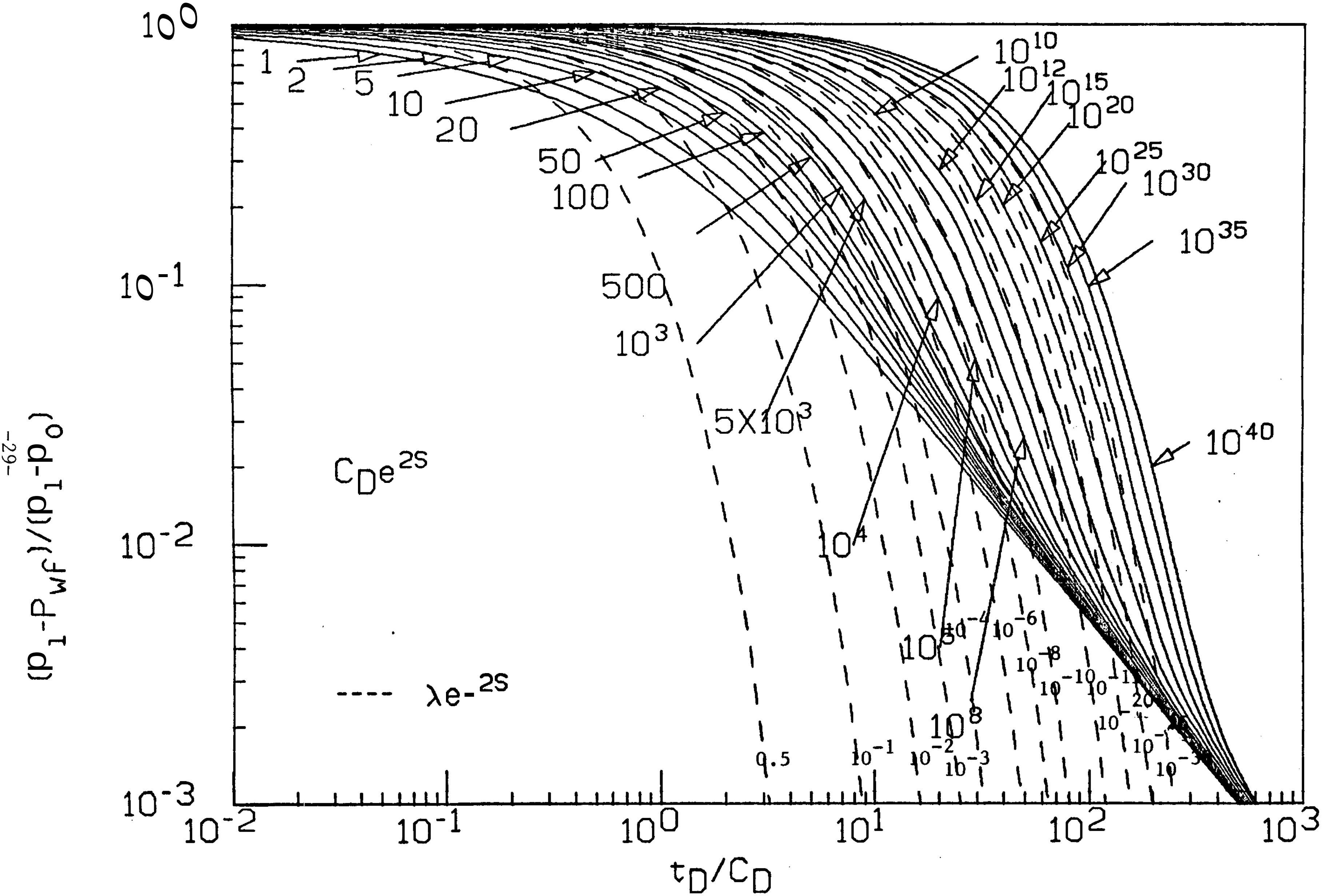

FIG. 3.8- Log-Log double porosity type curve for Slug testing (PSS)

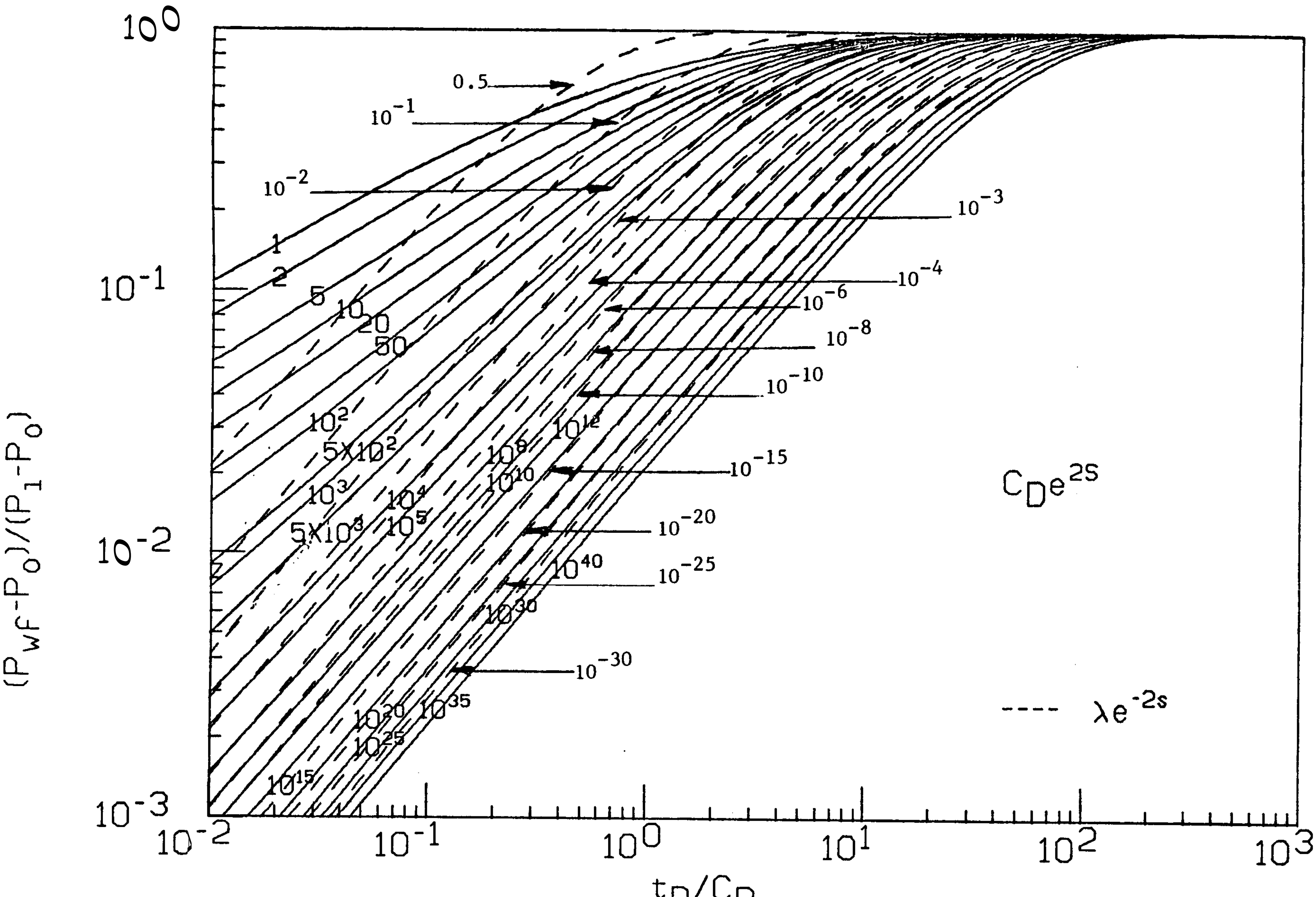

FIG. 3.9- Log-log double porosity type curve for Slug testing (PSS)

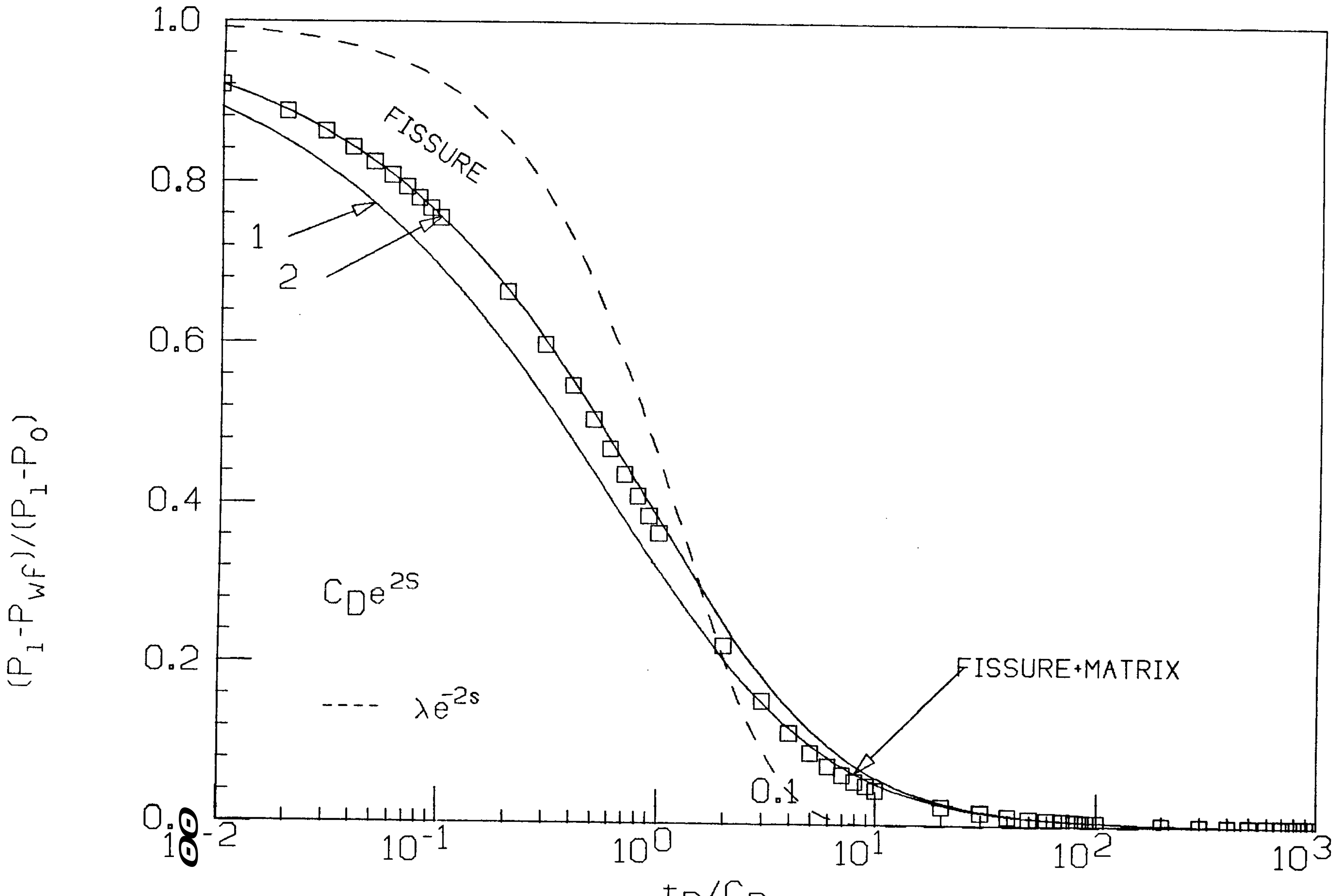

FIG. 3.10 - Example of semi-log double porosity behaviour(PSS)
(ω=0.5,$\lambda * e^{-2S}$=0.1)

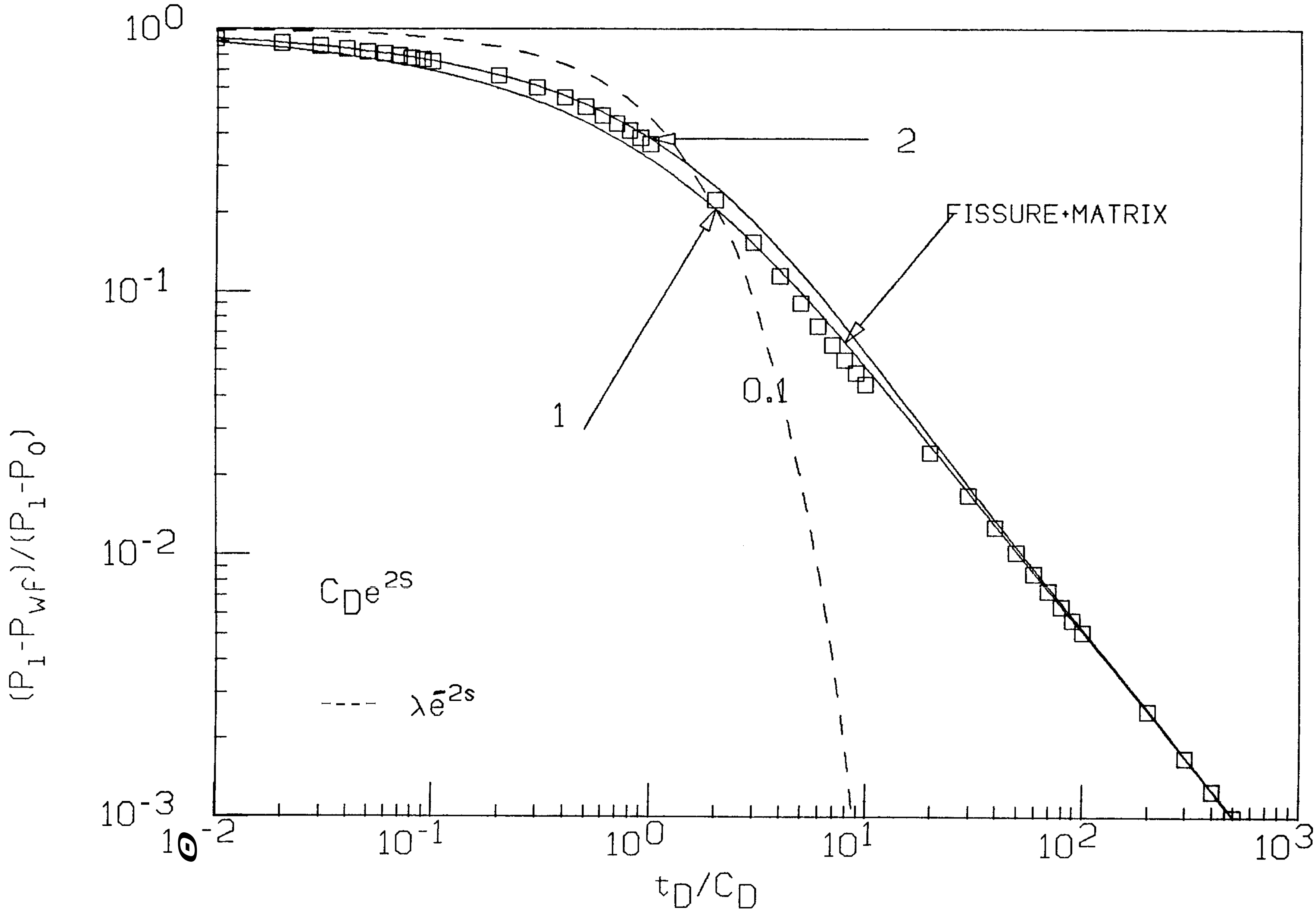

FIG. 3.11 - Example of log-log double porosity behaviour(PSS),
$(\omega=0.5, \lambda * e^{-2S}=0.1)$

with only fissures contributing to the system as represented by Eq. 3-8. At intermediate times, a transition occurs and the matrix contributes progressively. Response is described by one of the β curves in Figs. 3.12, 3.13 and 3.14. At late times, the behavior is again homogeneous with both matrix and fissures contributing. A typical two-porosity case with the transient flow assumption is portrayed in Fig. 3.15.

Type curves for the transient interporosity flow model are obtained by superposing curves of Figs. 3.1, 3.2 and 3.3 and Figs. 3.12, 3.13 and 3.14, respectively. They are presented ih Figs. 3.16, 3.17 and 3.18.

A semi-log or a log-log type curve analysis based on curves given in Figs 3.16,3.17 and 3.18 can be used to evaluate reservoir parameters in the same way as for the pseudo-steady state model. However, during the transition period, pressure follows a curve corresponding to some β value. Determination of the interporosity flow parameter λ, therefore, requires additional knowledge about the geometry of the matrix blocksc Knowledge that either the reservoir is fissured or layered can be obtained through logging, seismic or geological evidences (Deruyuck et al., 1982).

3.4 Pseudo-Steady State Model as compared to the Transient Interporosity Flow Model

Both the pseudo-steady state and transient models yields the same flow pattern: fissure-homogeneous at early times, transition at intermediate times, and total-homogeneous at late times.

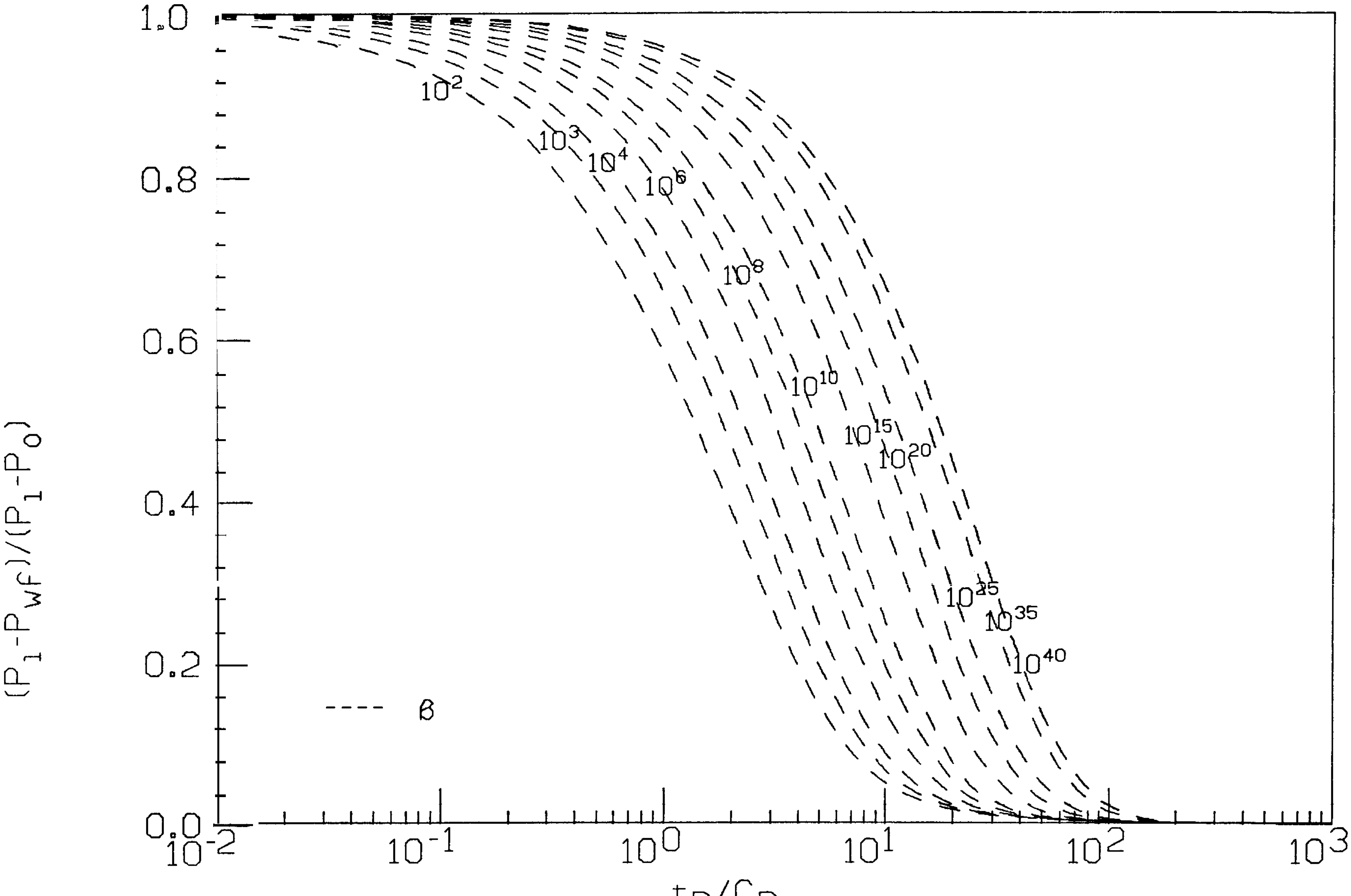

FIG. 3.12- Transition pressure in double porosity reservoir
Transient Model

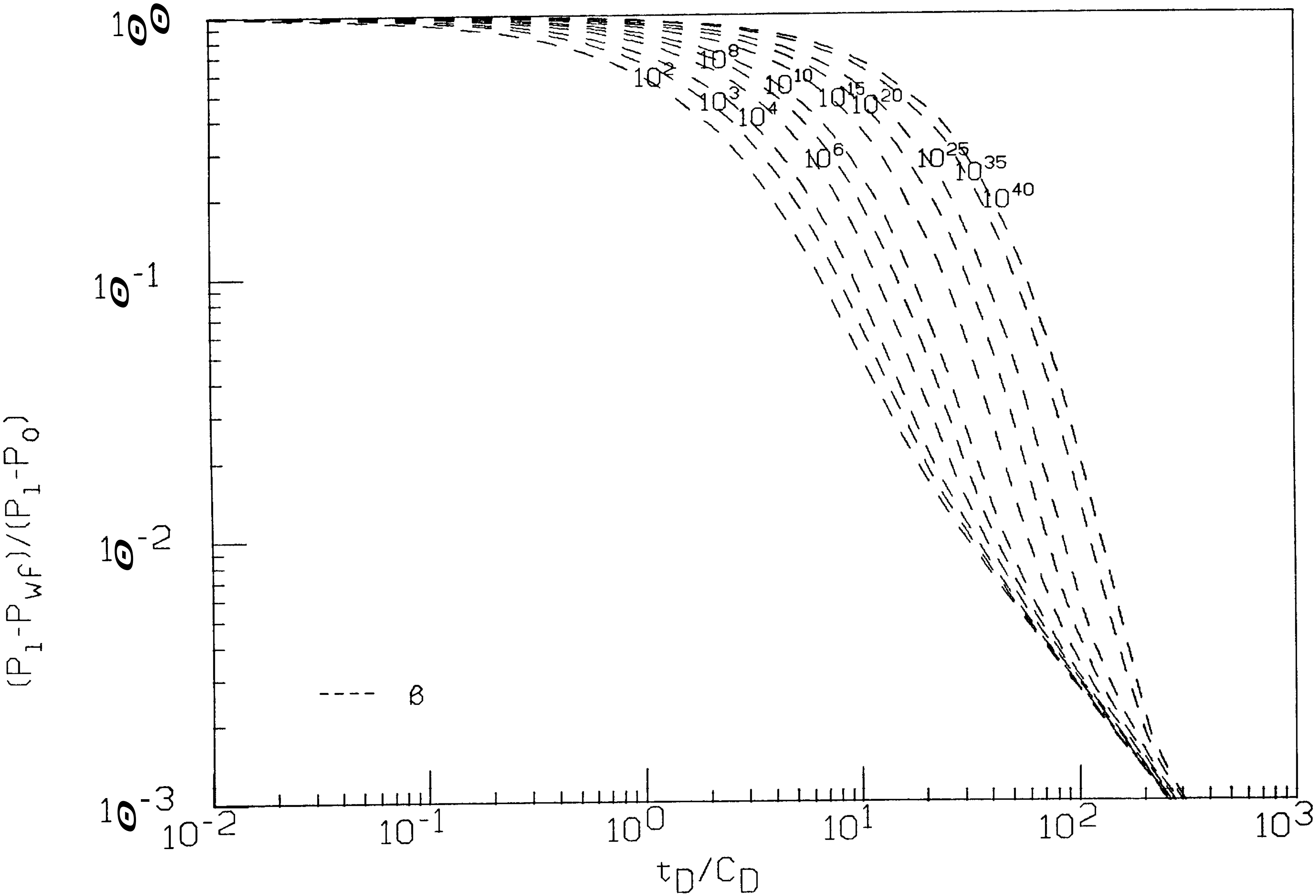

FIG. 3.13 - Transition pressure in double porosity reservoir

Log-Log - Transient Model

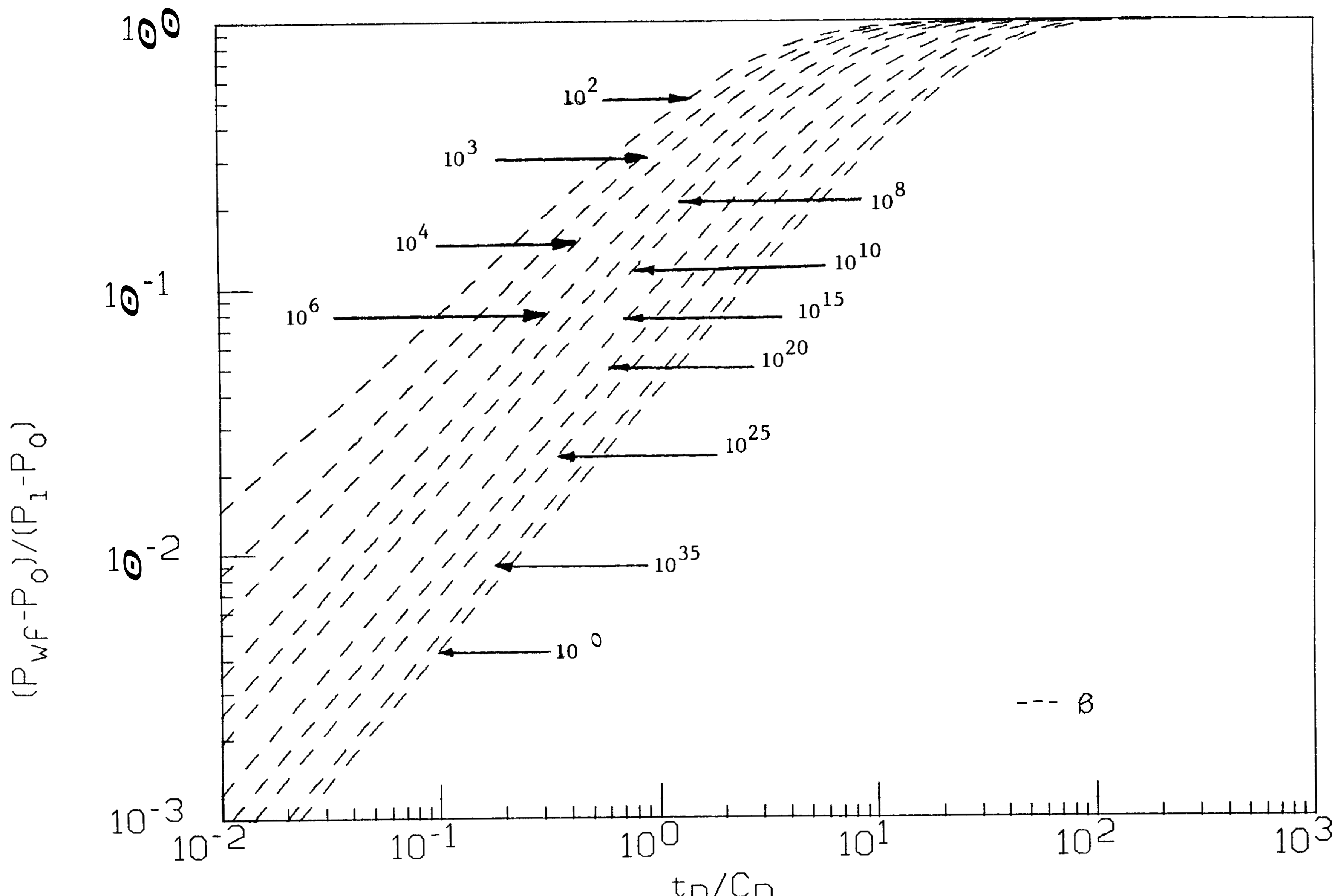

FIG. 3.14 - Transition pressure in double porosity reservoir
Log-Log - Transient Model

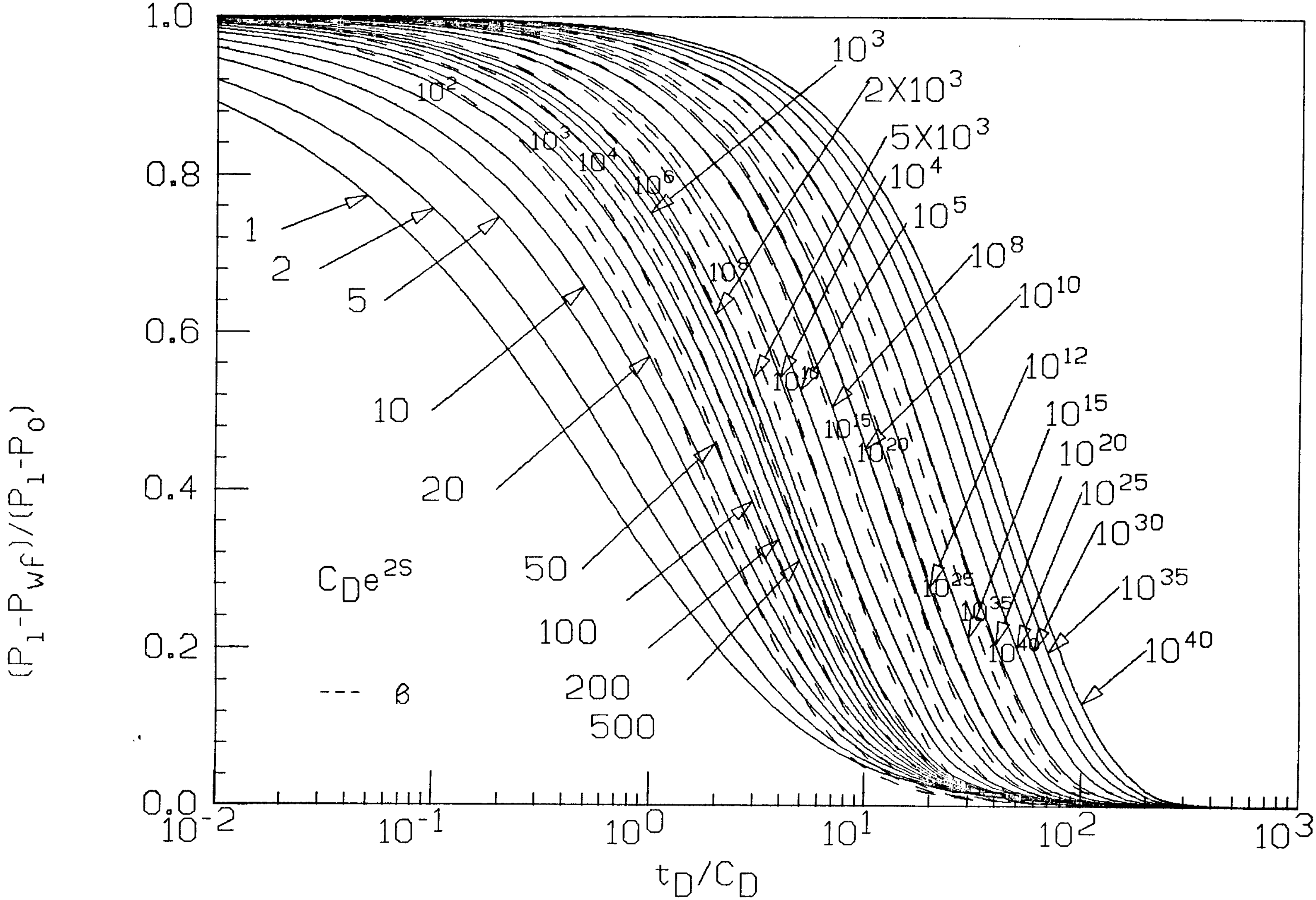

FIG. 3.16 - Semi-log double porosity type curve for Slug testing
Transient Model

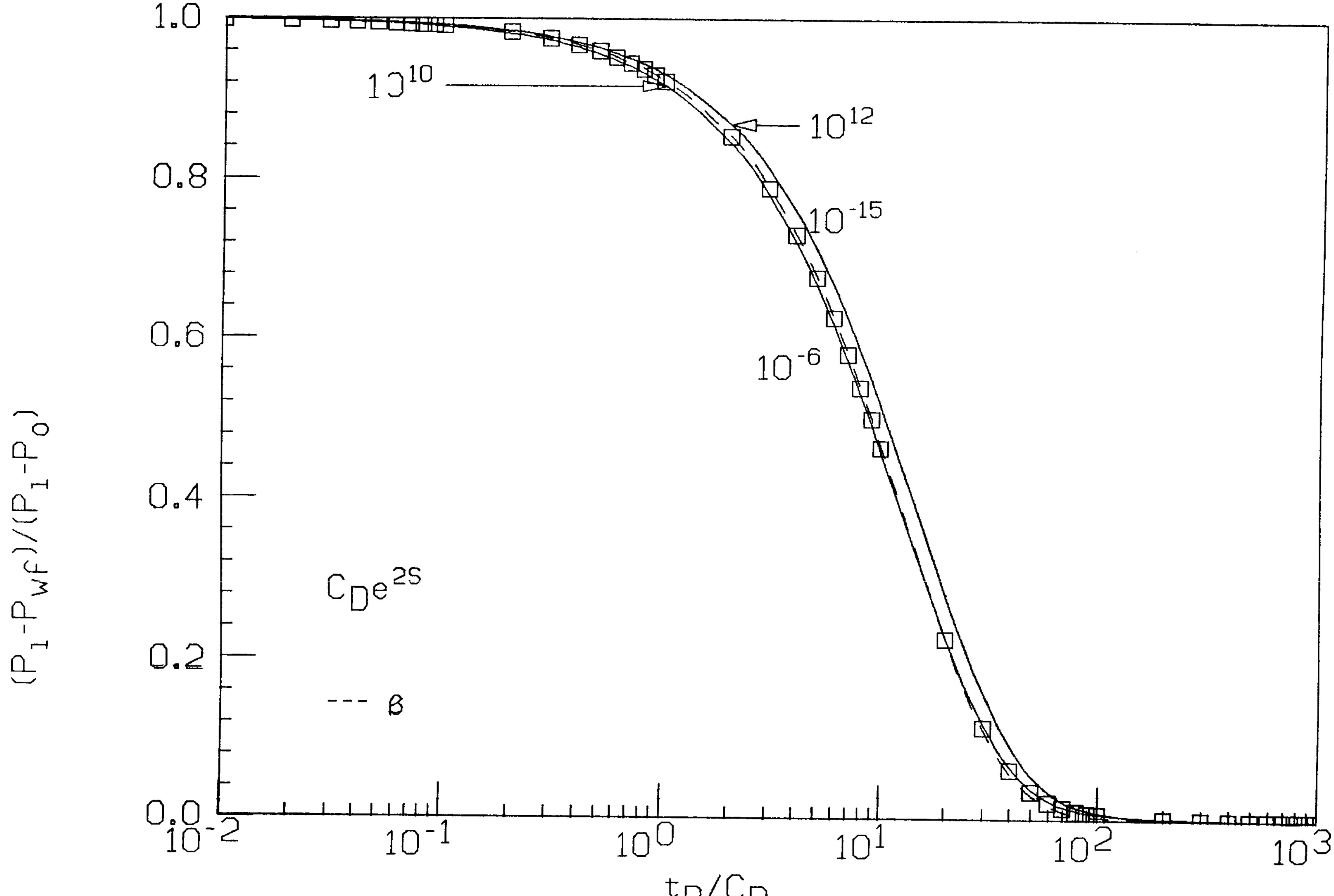

FIG. 3.15 - Example of semilog double porosity behaviour(Transient
(ω=0.01,β=3.0D+21)

510

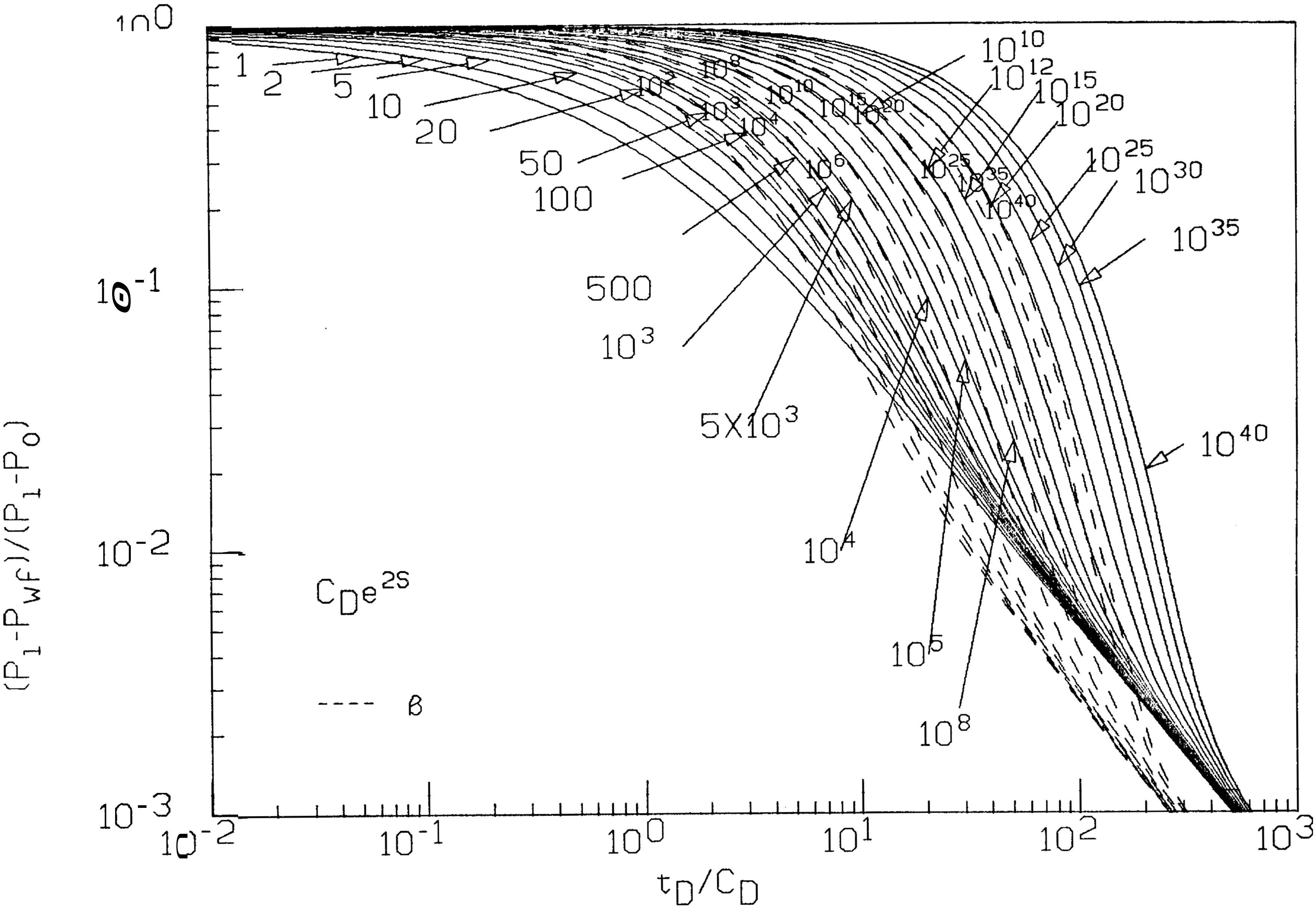

FIG. 3.17 - Log-Log double porosity type curve for Slug testing
TRANSIENT MØDEL

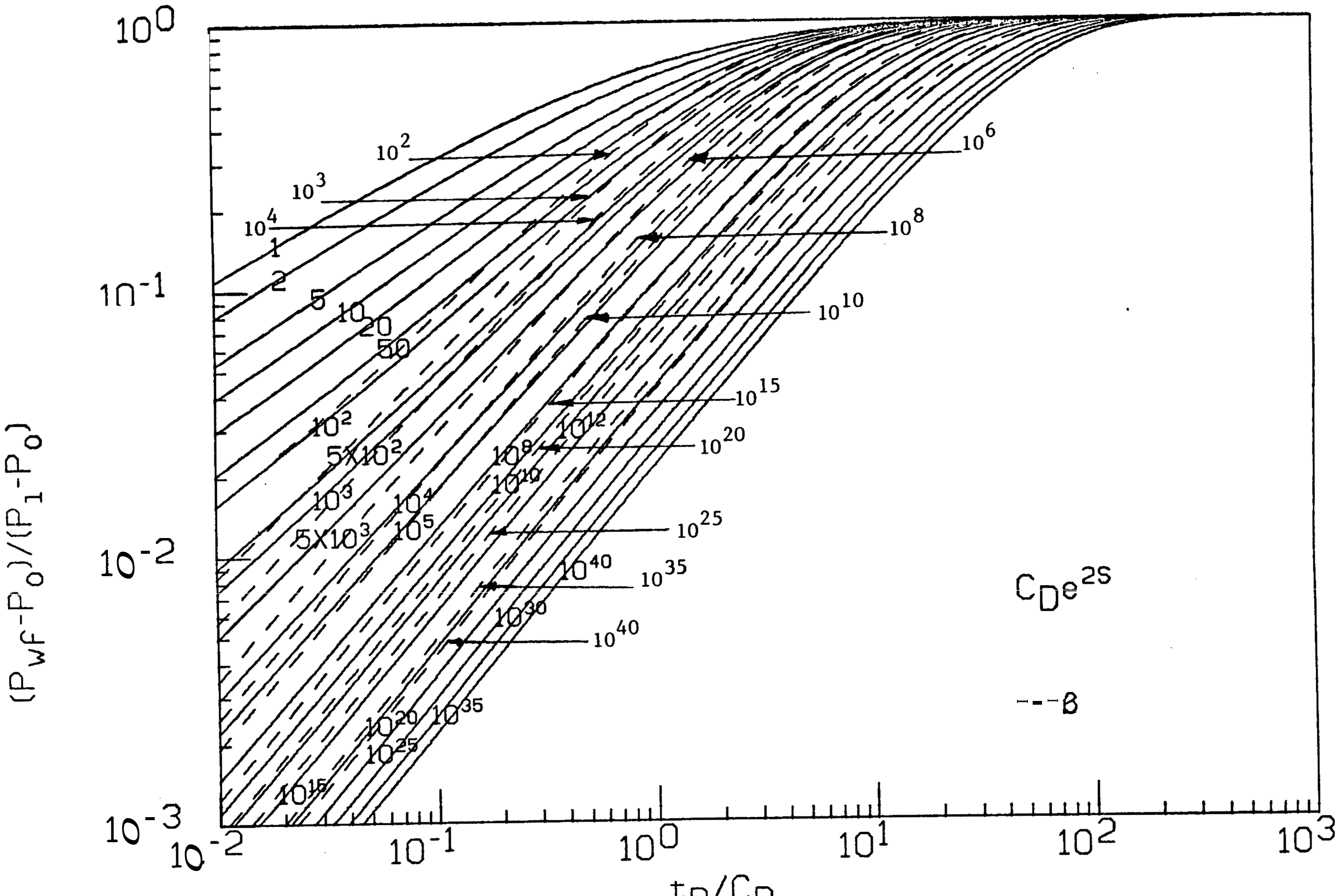

FIG. 3.18 - Log-log double porosity type curve for Slug testing
Transient model

The principal difference between the two models is the shape and duration of the transition curve. The difference is more pronounced on the left side of the type curve. That is, for large values of λe^{-2S}. Transition curves for the pseudo-steady state model in this range of time are steeper than for the transient interporosity flow model. This fact causes the transition duration to be shorter than for the transient inter-porosity flow model. Also, the transition from the early and late time curves is less smooth than that for the transient model.

For smaller values of λe^{-2S}, the pressure response of these two models becomes almost identical, as is shown in Fig. 3.19

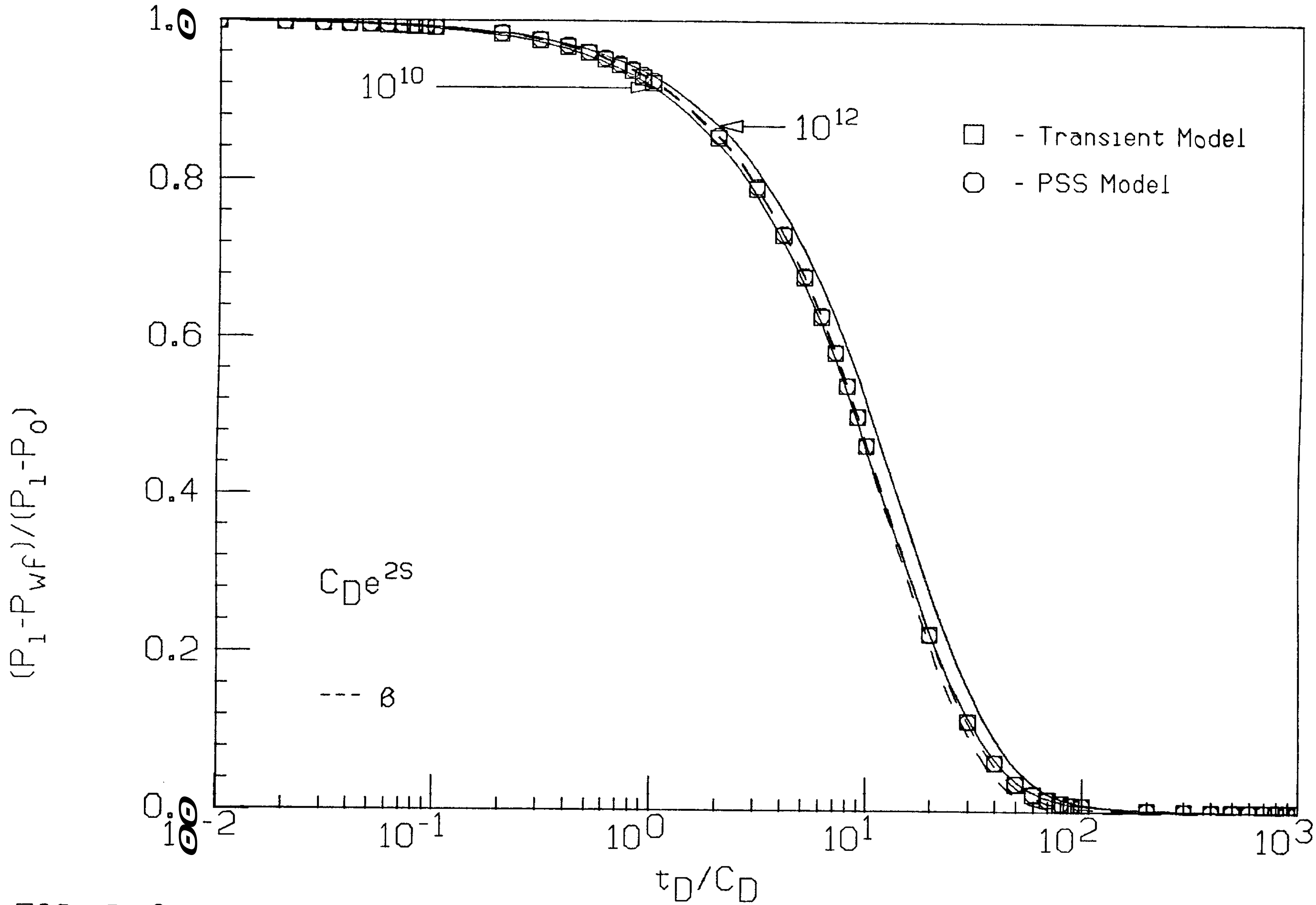

FIG. 3.19 - Example of semilog double porosity behaviour(Transient (ω=0.01,β=3.0D+21,λe^{-2S}= 1.0E-11)

Results And Interpretation

The pressure response of a well in a double-porosity reservoir during a slug test is given by Eq. B-11. Pressure response is dependent upon four parameters: C_D, S, λ, and ω. Several runs were made in order to ascertain the effect of each of these parameters. Results indicate that under certain ranges of values of ω and A, the double-porosity effects are not distinguishable from those of a homogeneous reservoir. Pressure in both pseudo-steady state and transient models responds in a similar way to a given change in parameters ω and A. The effect of a change of each parameter on pressure response is therefore shown on the pseudo-steady state model.

4.1 Effect of C_D

Effect of the group C_De^{2S} can be analyzed, instead of C_D, because the wellbore storage coefficient is based on the effective wellbore radius for low values of C_D. For values of $C_De^{2S} \geq 10^4$ the dimensionless wellbore storage coefficient and skin affect the solutions as one group, as shown by the approximate form of the pressure solution, Eq. 3-9.

When inverted using the Stehfest algorithm (1970) for N=16 in double precision arithmetic, Eq. 3-9 gives the same result as the exact solution to six decimal places for $C_De^{2S} > 10^4$, thereby establishing C_De^{2S} as the governing group. Using N=12, the pressure solution for $C_De^{2S} \geq 100$ can be obtained. For values of $C_De^{2S} \leq 100$, the approximate form provides values of dimensionless pressures greater than one, indicating perhaps that C_De^{2S} is not the sole governing group for values of $C_De^{2S} \leq 100$. Curves obtained with quantities based on effective wellbore radius, therefore, should be used with caution.

Two-porosity effects are less likely to be noticeable for values of $C_De^{2S} \geq 10^{15}$. Curves for different C_De^{2S} values are close, and small values of $\omega \leq 10^{-3}$ and $\lambda e^{-2S} \leq 10^{-16}$ are required to have the two-porosity effects in the time range of interest. Values for $\omega \leq 10^{-3}$ can be found in practice. However, the range of values for $\lambda e^{-2S} \leq 10^{-15}$ are found less frequently.

4.2 Effect of S

Analysis of results show that the presence of two-porosity effects is dependent upon a suitable combination of C_D, λ and ω, and is independent of S. This is supported by the fact that the two-porosity effects are observed when functions f(u) take values from ω to 1 in the time range of interest. Since the function f(u) for both pseudo-steady state and the transient interporosity flow model do not involve skin, two-porosity behavior is not affected by S (skin factor). As derived in Appendix A, the function f(u) for the two interporosity models are:

for pseudo-steady state model:

$$f(u) = \frac{\omega(1-\omega) + \frac{\lambda C_D}{u}}{(1-\omega) + \frac{\lambda C_D}{u}} \quad \text{..............(4-1)}$$

for transient model

$$f(u) = \omega + \frac{C_D\lambda(1-\omega)}{3u} \tanh \frac{3(1-\omega)u}{\lambda C_D} \quad \text{..............(4-2)}$$

For values of $C_De^{2S} \leqslant 10^4$, solutions were obtained using C_D and t_D based on effective wellbore radius. The interporosity flow parameter λ may also be based on effective wellbore radius giving λe^{-2S}, thus keeping the function f(u) independent of skin'.

4.3 Efect of λ

The value of Xe^{-2S} determines whether or not pressure response for a two-porosity reservoir will be distinguishable from a homogeneous reservoir. Examining the function f(u), early time effects are present in the time range of interest only for $\lambda \leqslant C_D-1$. For λ and C_D based on effective wellbore radius, early time effects are observed when:

$$\lambda e^{-2S} \leqslant 1/C_De2s$$

This expression suggests that for $\lambda e^{-2S} > 1/C_D$, no early time effects are seen. Figure 4.1 shows that for $C_D e^{2S}=10^{10}$ and $\lambda e^{-2S}=0.1$ (too large), no early time effects($C_D e^{2S}=10^{12}$) are observed. Large values of λe^{-2S} result in homogeneous behavior following the late time response($C_{Df+m} e^{2S}=10^{10}$).

Figure. *4.2* indicates that for $C_D e^{2S}=10^{10}$ and $\lambda e^{-2S}=10^{-15}$, (too small) no double-porosity effects are observed. Pressure response is that of a homogeneous medium along the early time behavior ($C_D e^{2S}=10^{12}$), for $\omega=0.01$.

All three flow regimes are observed when:

$$1/(100\, C_D e^{2S}) \leqslant \lambda e^{-2S} \leqslant 1/10 C_D e^{2S}.$$

This is shown in Fig. *4.3* for $C_D e^{2S}=10^{10}$, and $\lambda e^{-2S}=10^{-11}$. Pressure follows the early time curve ($C_{Df} e^{2S}=10^{12}$) to t_D/C_D = 0.5. The matrix starts to contribute at this stage, and the pressure follows the transition curve. At about $t_D/C_D=10$, the late time effects are noticed with pressure following the late time curve ($C_{Df+m} e^{2S}=10^{10}$).

The time at which the matrix starts contributing for a fixed storativity ratio is dependent upon the value of λe^{-2s}. The beginning of transition shifts to later times as λe^{-2S} decreases. Figure. *4.4* shows the effect of a decrease in λe^{-2S} for a fixed storativity ratio. This result suggests that for $\lambda e^{-2s} \ll 1/C_D$, no late-time effects are observed. This is confirmed by the solution for $C_D e^{2S}=10^{10}$ and λe^{-2S}, previously shown in Fig. *4.2.*

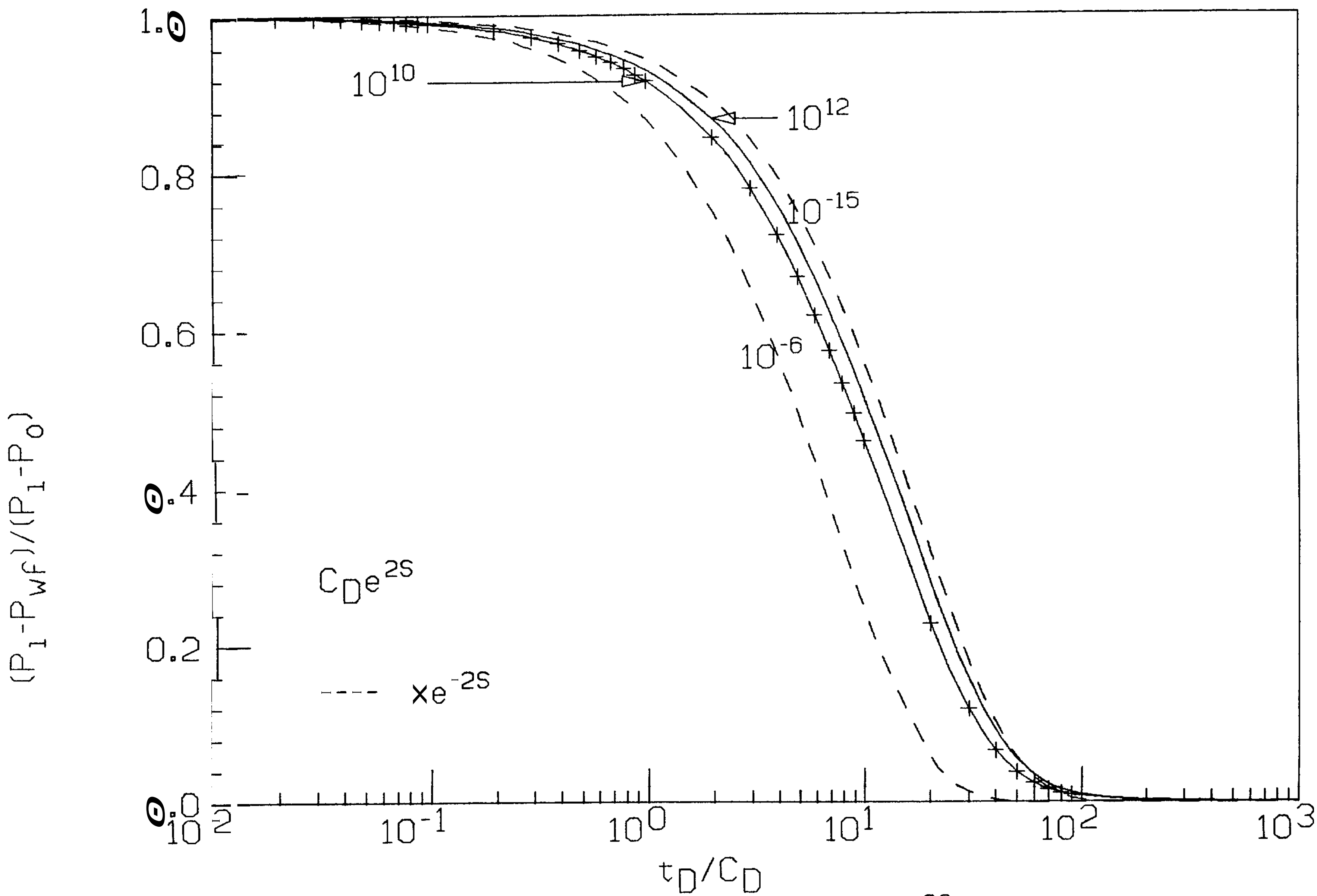

FIG. 4.1-Effect of very large λe^{-2S}
(ω=0.01, $\lambda^* e^{-2S}$=.1)

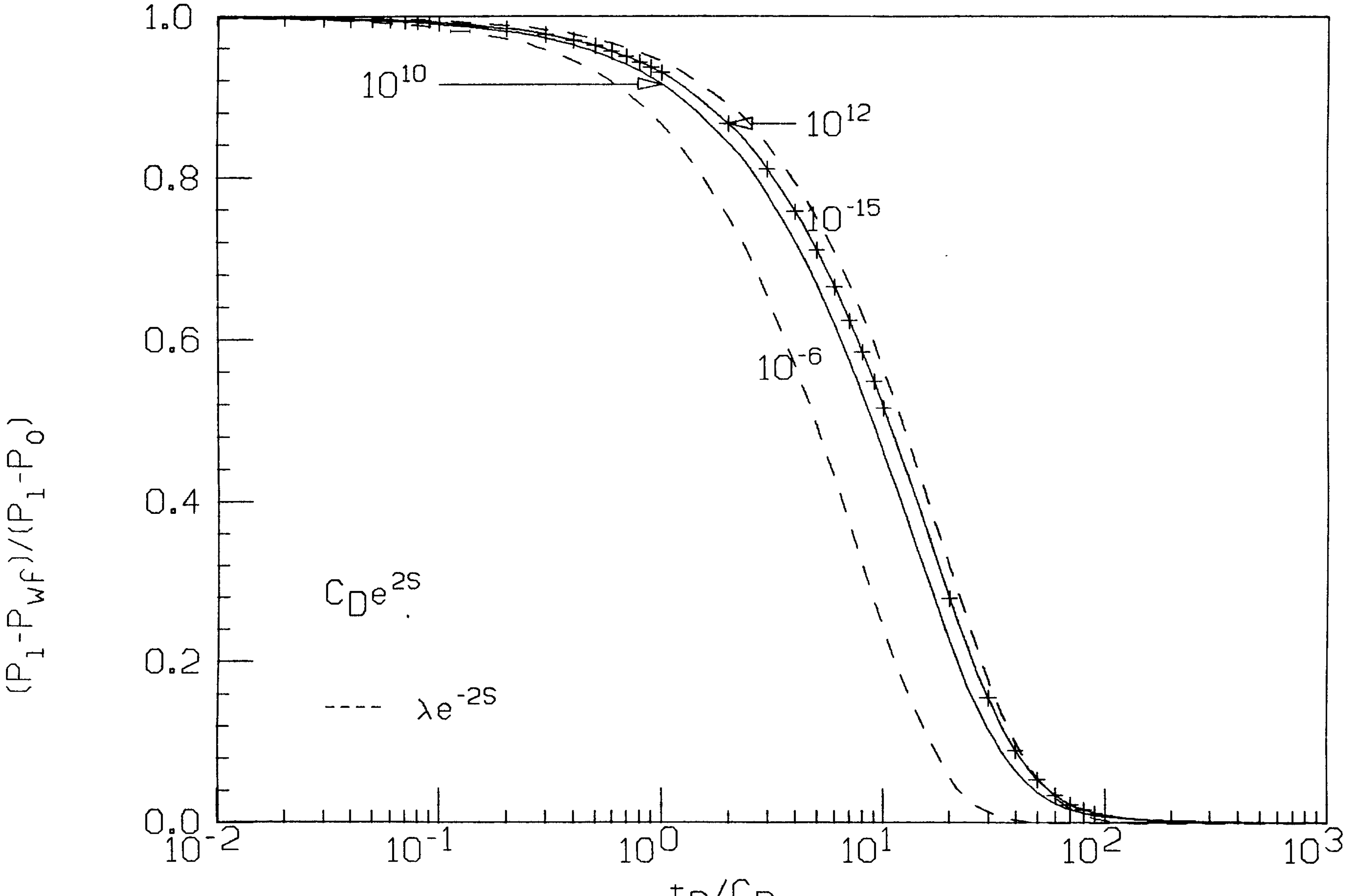

FIG. 4.2-Effect of very small λe^{-2S}

$(\omega=0.01, \lambda * e^{-2S}=1.0E-15)$

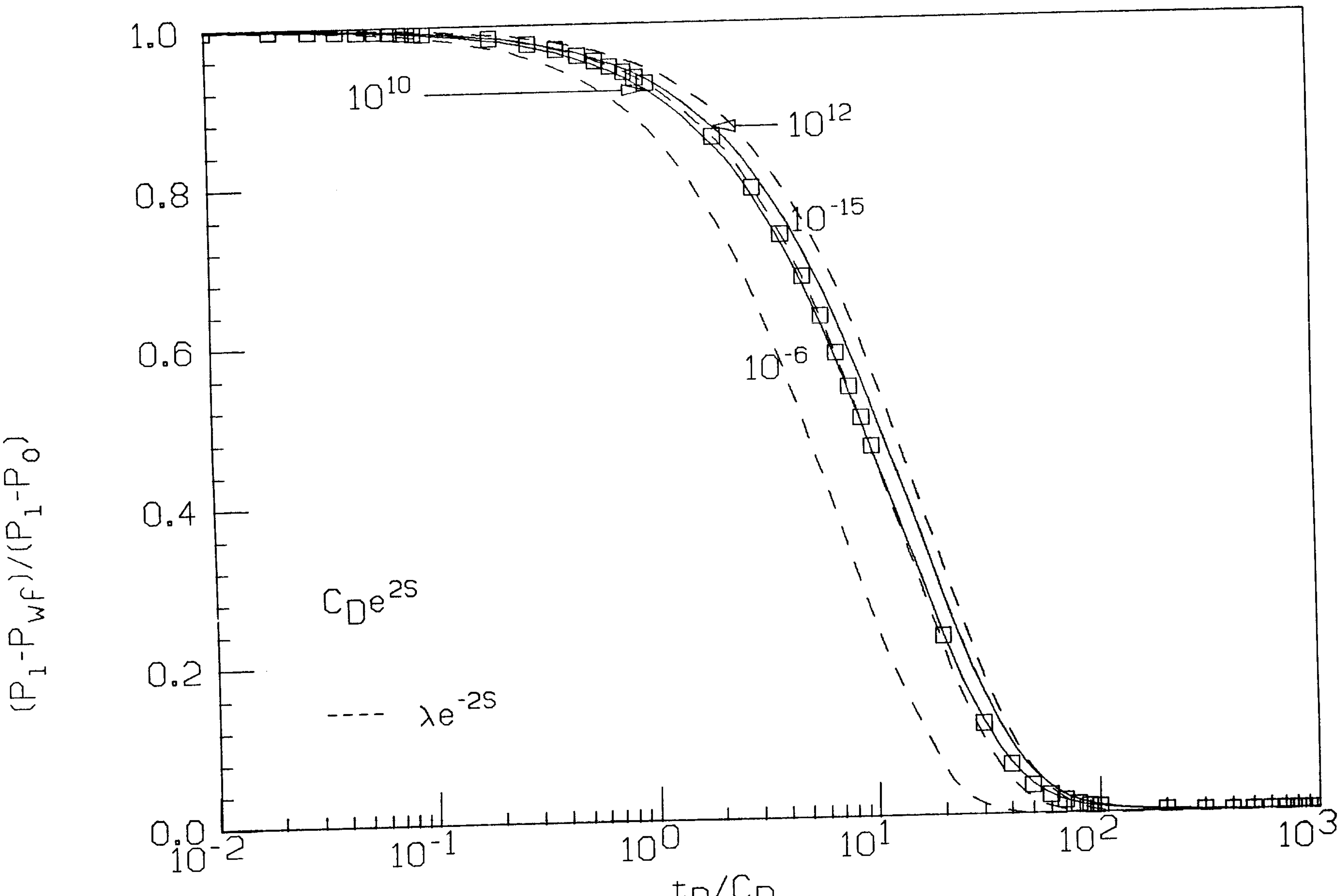

FIG. 4.3-Example of semilog double porosity behaviour(PSS)

$(\omega=0.01, \lambda * e^{-2S}=1.0D\text{-}11)$

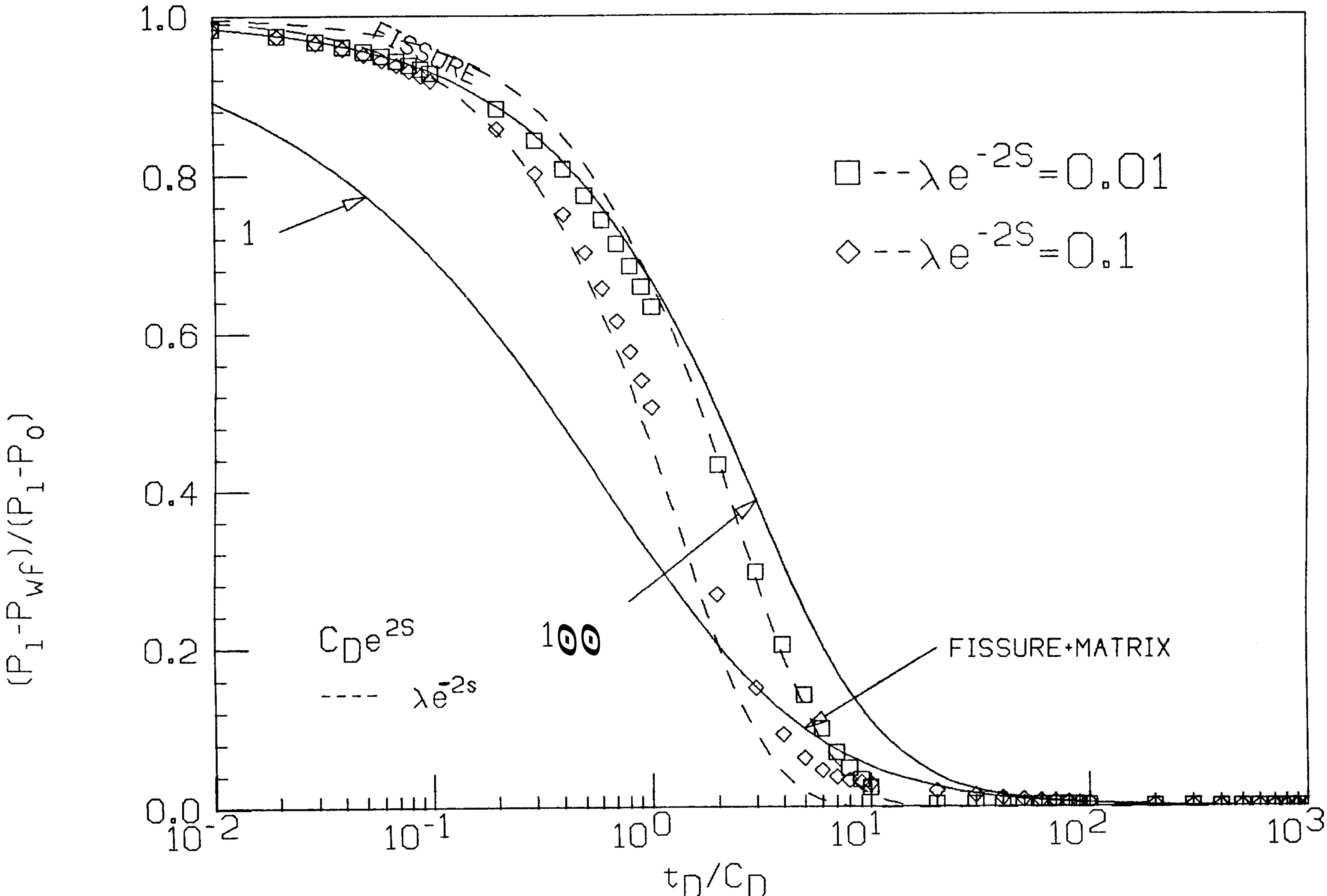

FIG. 4.4-Effect of decrease in λe^{-2S}
(ω=0.01)

4.3 Effect of ω

The effect of a change of storativity ratio for a fixed value of the group λe^{-2S} is depicted in Fig. 4.5. The matrix starts contributing much earlier for smaller values of ω. In other words, the beginning of the transition shifts to earlier times for smaller values of ω.

For very small values of storativity ratio $\omega \leq 10^{-3}$, that is when storativity of the fissure $(\phi V c_t)_f$ is very small as compared to the total storativity $[(\phi V c_t)_f + (\phi V c_t)_m]$, the pressure follows the transition curve from the very beginning. No early time effects are present for the case $C_D e^{2S} = 1$ and $\omega = 10^{-4}$, as shown in Fig. 4.6. It is not possible to determine the values of ω for such cases through the type curves presented. Only an upper limit of the value of ω can be estimated:

$$\omega = C_{Df+m}/C_{Df} \text{ (unknown but greater then } C_{Df+m})$$

The time for stabilization of pressure in the matrix and the fissures, that is the time at which pressure starts following the late time curve, is dependent upon the value of ω. The smaller the value of ω, the later is the beginning of the late time effects. This is shown in Fig. 4.7. This means that it takes more time for pressure in the matrix and fissures to stabilize when fissure storativity is much smaller than the matrix storativity.

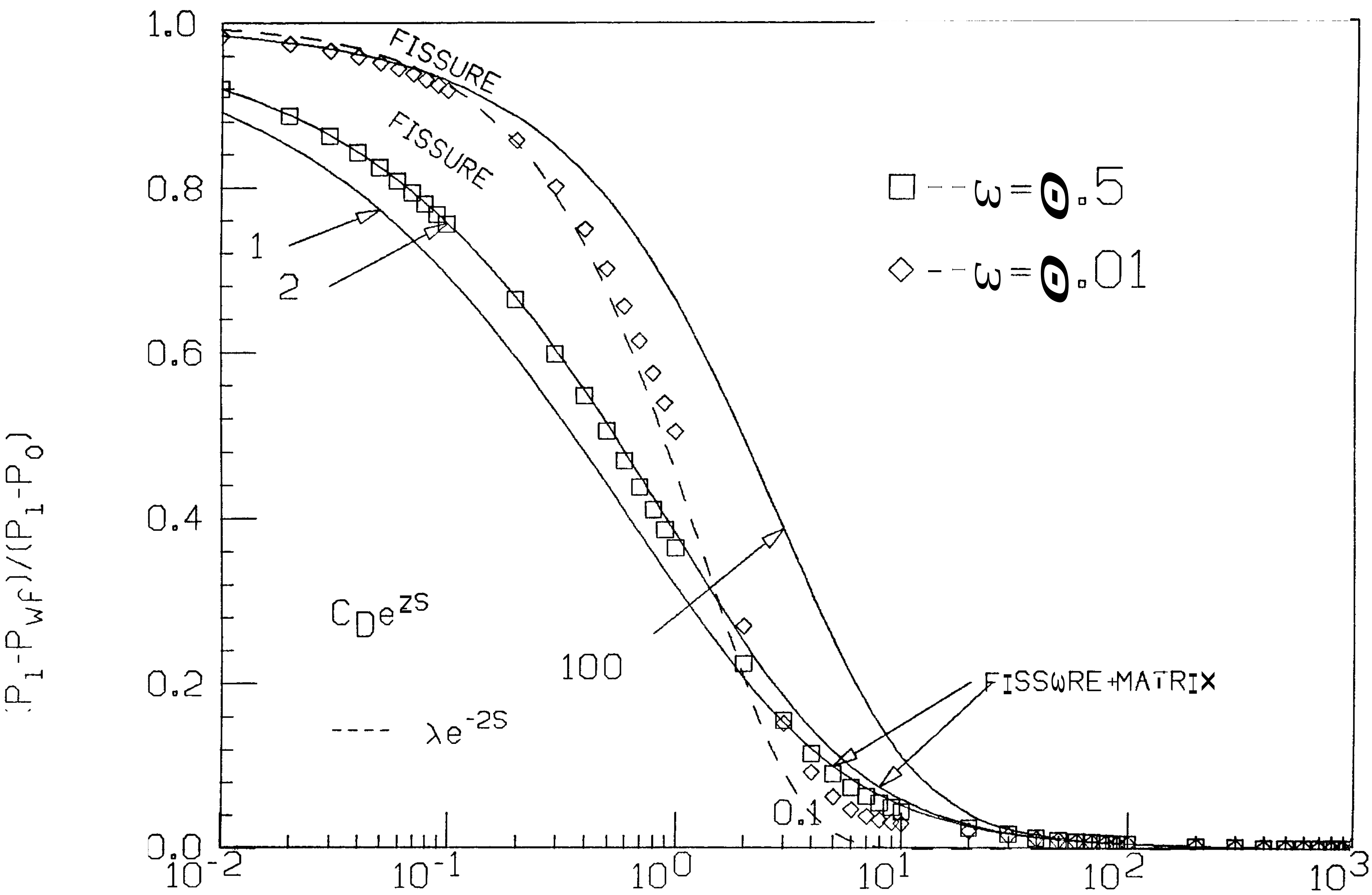

FIG. 4.5-Effect of decrease in ω

$\lambda^* e^{-2S}=0.1)$

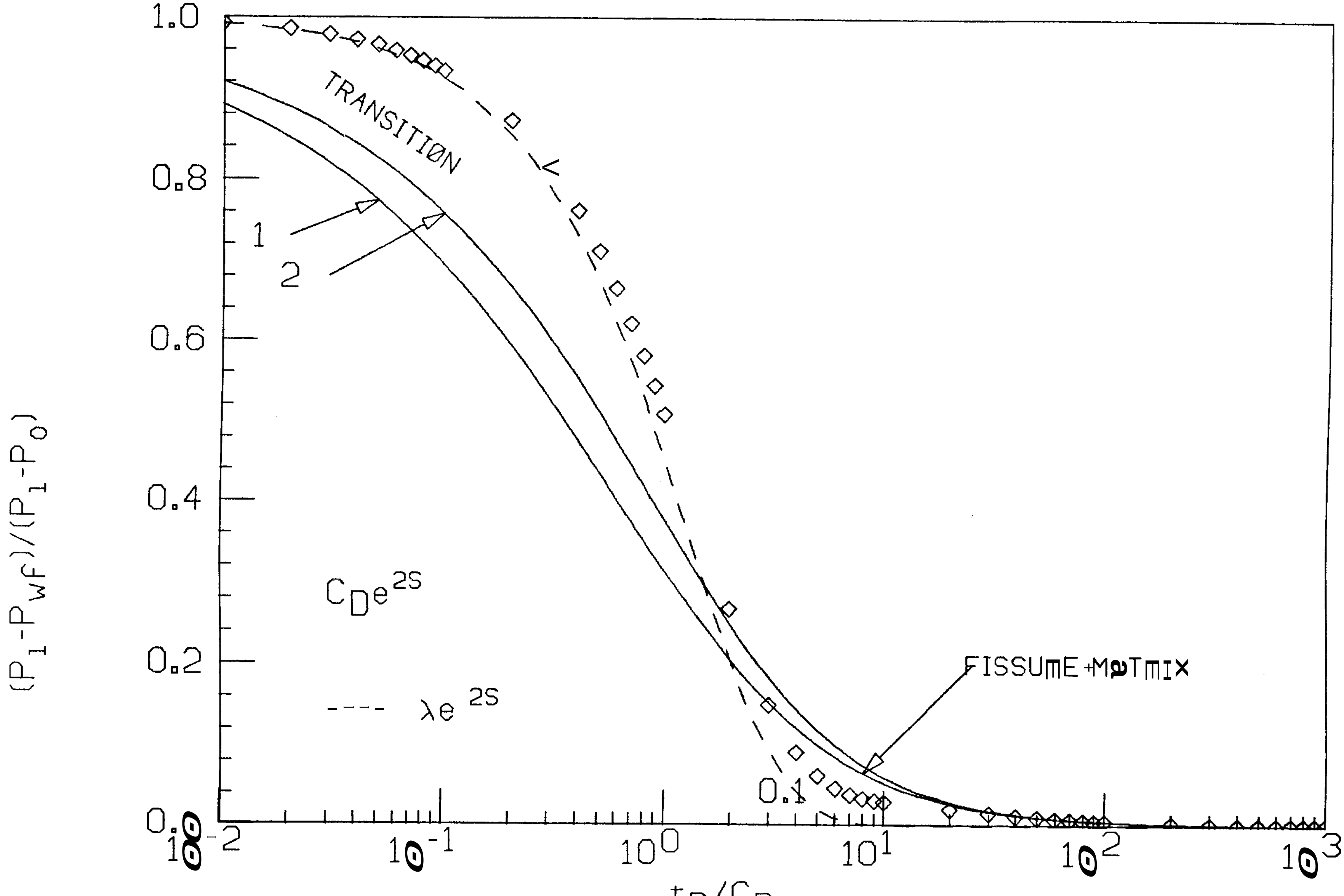

FIG. 4.6-Effect of very small ω
$\omega=0.0001, \lambda*e^{-2S}=0.1)$

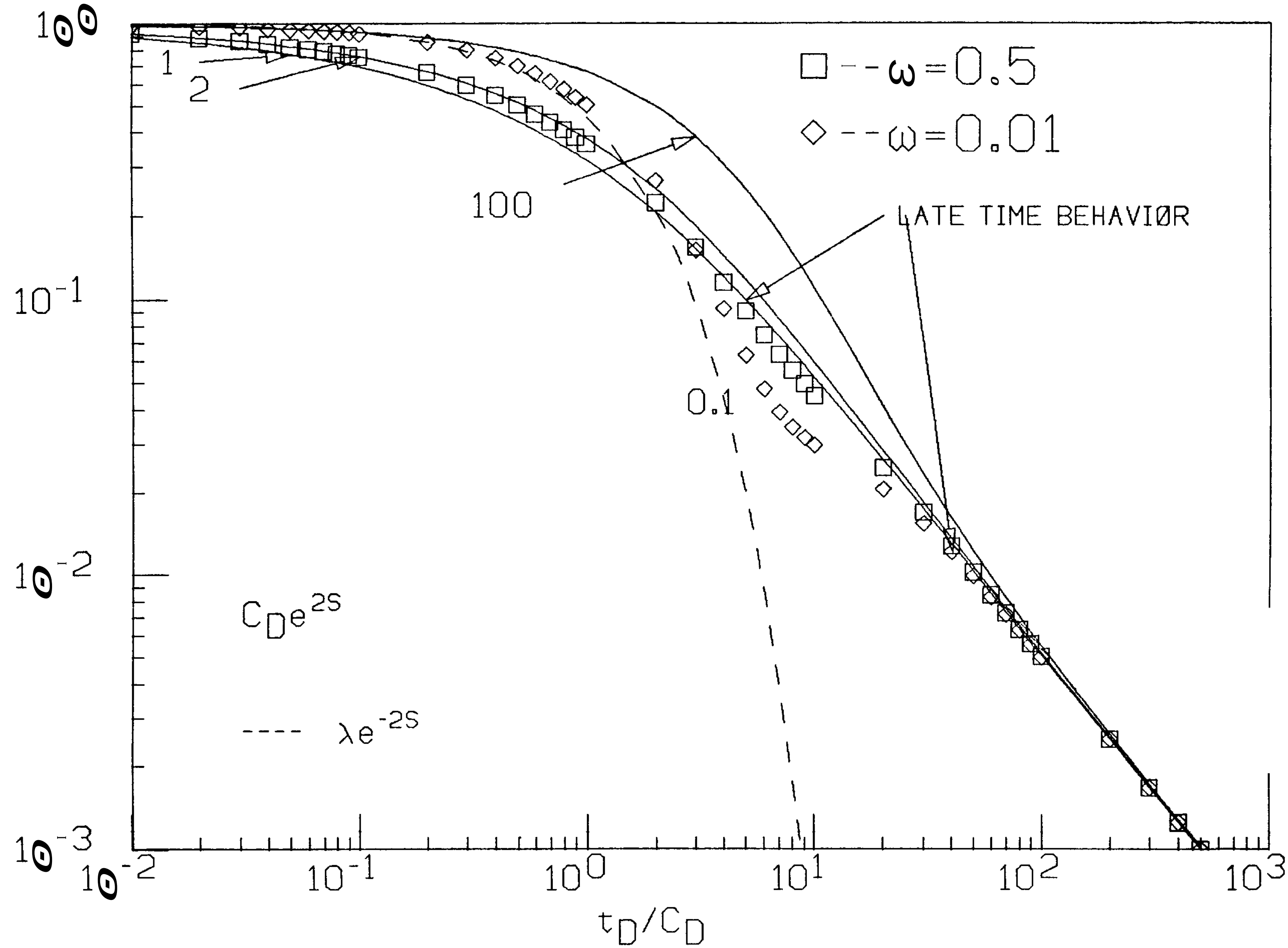

FIG. 4.7-Effect of ω on begining of late time effects
($\lambda^* e^{-2S}=0.1$)

Dimensionless pressure drops below the late time curve for certain time ranges shown in Fig. 4.8. This is explained by the fact that pressure follows the transition curve between the time range given by (Deruyck et al., 1982).

$$[\lambda << u << 1/\omega, \quad \text{for } \lambda << 1]$$

Smaller values of ω results in the continuation of the transition period to larger times. Pressure follows the transition curve and falls below the late time curve in the later stages of the transition period. This effect is diminished as the value of ω increases. In Fig. 4.9, no such effect appears because $\omega=0.5$ is large, which reduces the transition duration and pressure departs from the transition curve before the transition curve falls below the late time curve ($C_De^{2S}=1$). This effect is not observed for large values of C_De^{2S}, for instance in Fig. 4.1'0 with $C_De^{2S}=10^{10}$, as the transition curves cross the late time curves at relatively late times.

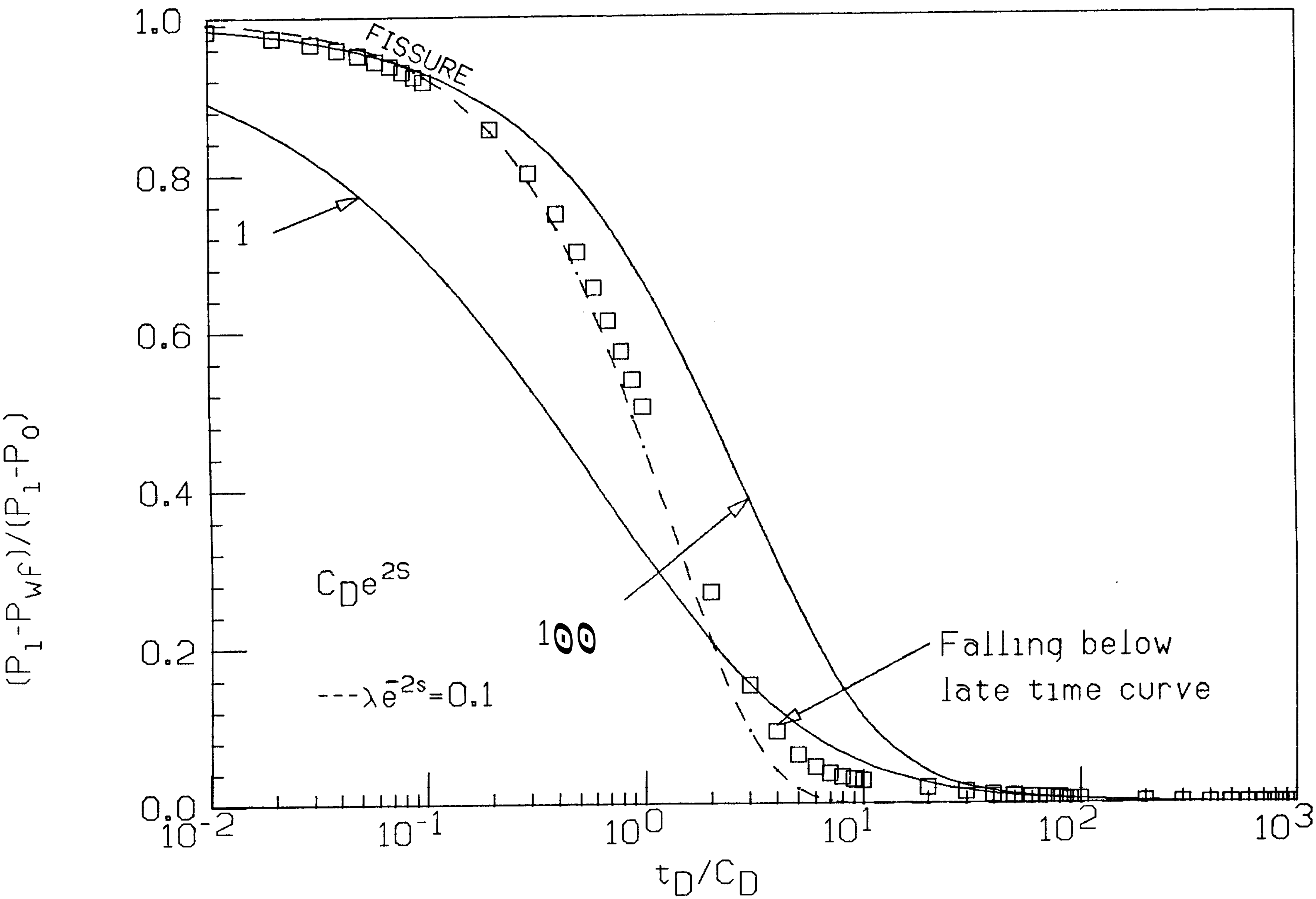

FIG. 4.8- Dimensionless pressure falling below the late time curv
$(\omega=0.01, \lambda^* e^{-2S}=.1)$

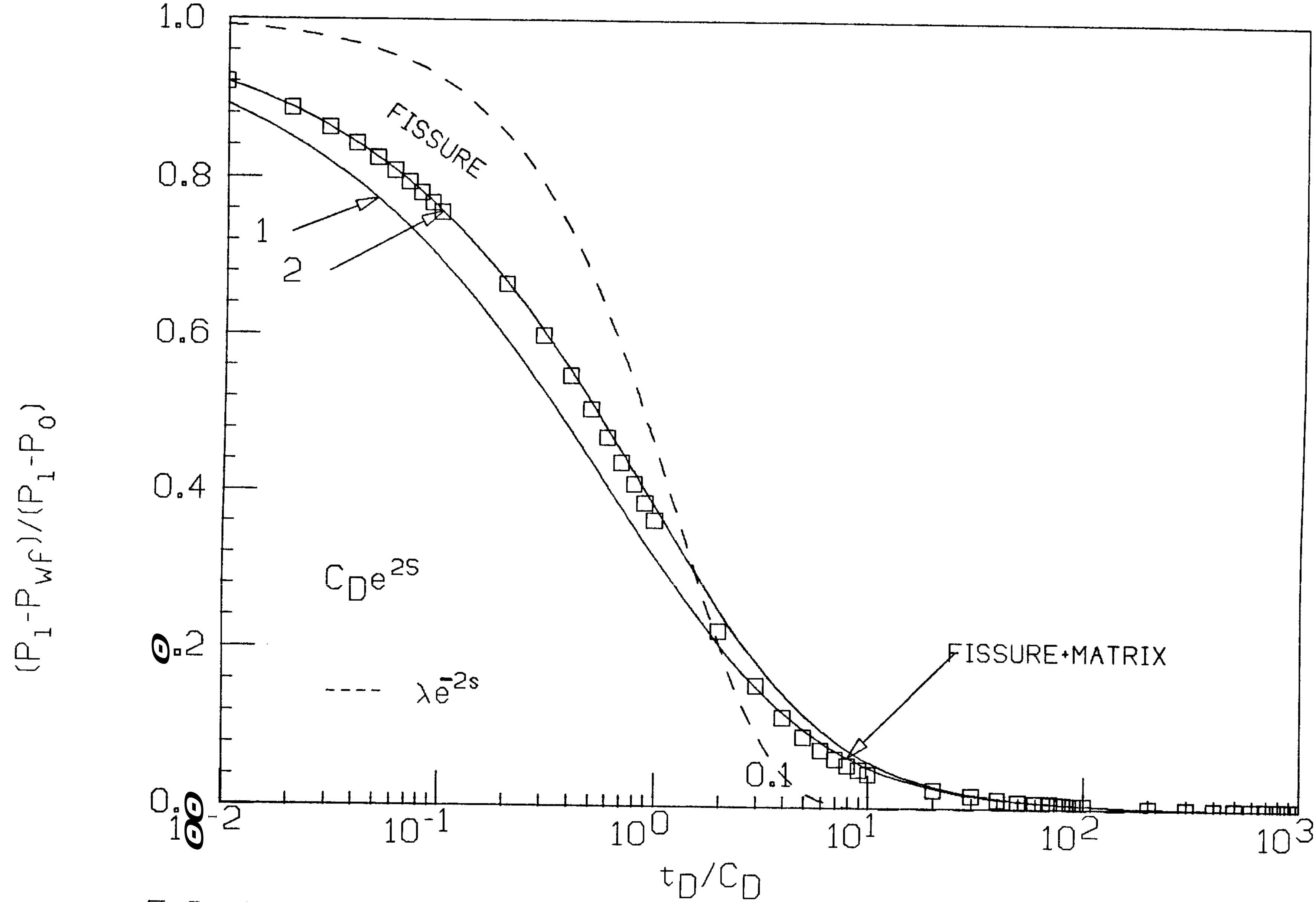

FIG. 4.9- Small transition duration because of large ω
(ω=0.5,λ*e^{-2S}=0.1)

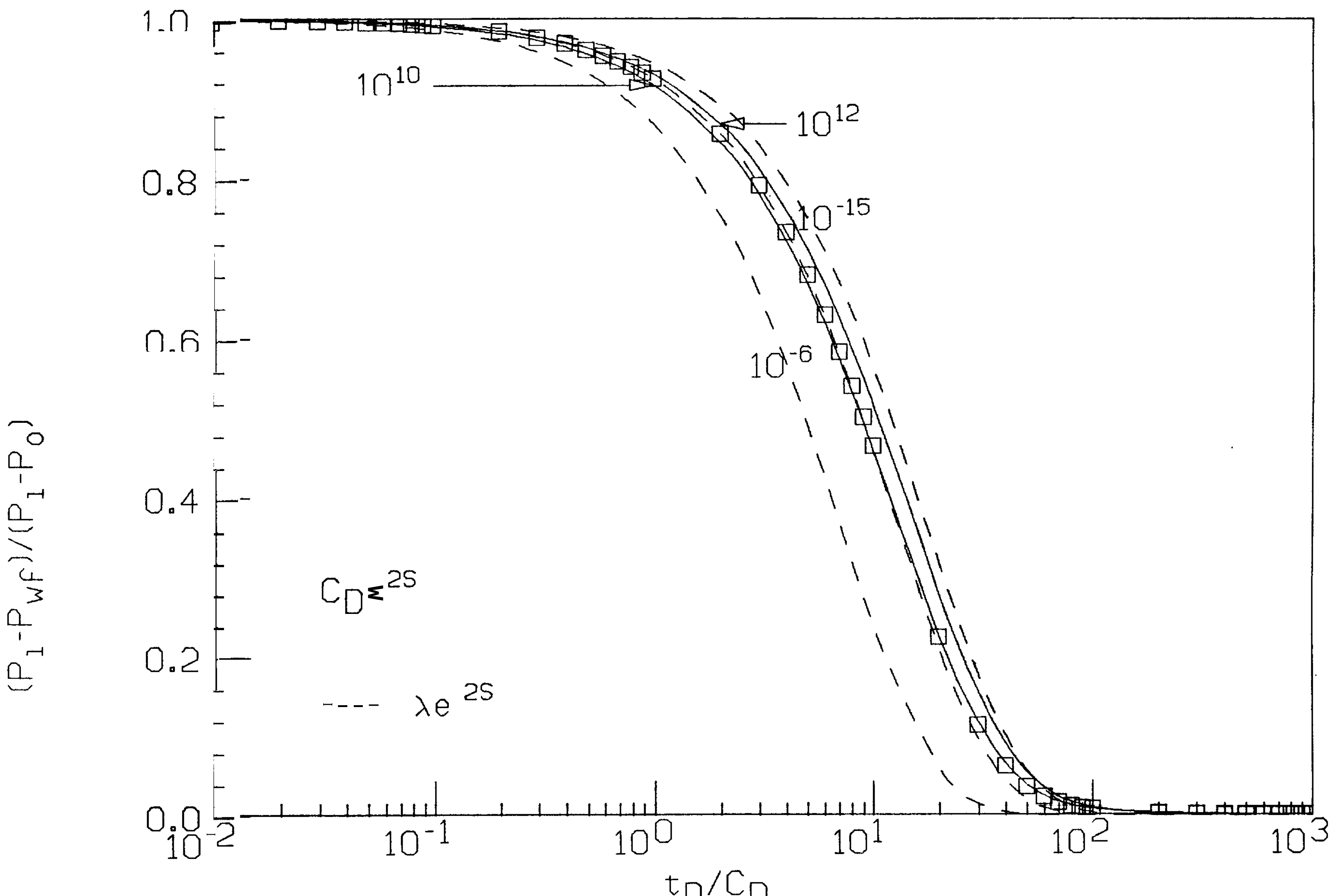

FIG. 4.10-Late merging of transition curve into late time curve (ω=0.01,λ*e^{-2S}=1.0D-11)

5. Conclusions

(1) The flowing period of a DST (Slug test) in a two-porosity reservoir has been mathematically posed. A dimensionless formulation for pressure reponse is presented.

(2) Results presented are valid both for fissured and layered reservoirs.

(3) Type curves based on pseudo-steady state and transient interporosity flow regimes are presented.

(4) For each flow regime three different type curves are presented: semi-log Figs. 3.7 and 3.16, log-log Fig$. 3.8 and 3.17, and different log-log Figs. 3.9 and 3.18. The semi-log type curve is best when both early and late time data is available and is to be given equal weight. Log-log curves may be more useful if either P_i or P_o are uncertain or unknown.

(5) The pressure response for two-porosity reservoirs is indistinguishable from that of a homogeneous reservoir for certain ranges of values λ and ω.

(6) The pseudo-steady state model and transient models yield the same flow pattern: fissure homogeneous at early times, transition at intermediate times, and total system homogeneous at late times.

(7) The transient interporosity flow model provides a smooth transition from early to late time curves for large values of λe^{-2S}, and does not show a relatively abrupt change in direction as does the steady state model. The pressure response for small values of λe^{-2s} is almost identical for the the two flow models.

(8) A semi-log or a log-log type curve matching based on the type curves presented can yield all the system pertinent parameters: transmissivity (kh/μ) from the time match, skin (S) from C_De^{2S} match, interporosity flow parameter (λ) from λe^{-2S} match, and storativity ratio (ω) from the ratio of the C_De^{2S} value of the last curve to the C_De^{2S} value of the first curve.

(9) A correlation quantifying the range of values of λe^{-2S} for which the two porosity effects are observed is presented.

(10) A formal solution showing C_De^{2S} as the sole governing group for the homogeneous reservoir is presented.

6. Nomenclature

c_t		total compressibility, atm
C		wellbore storage constant, cc/atm
C_D	:	dimensionless wellbore storage constant
h		formation thickness, cm
k_f	:	permeability of fissure, darcys
K_o	:	modified Bessel function of second kind, order zero
K_1	:	modified Bessel function of second kind, order one
p_f	:	pressure in the fissure medium on the formation side of the skin effect,atm
p_m	:	pressure in the matrix medium, atm
p_{wf}	:	pressure in the fissure medium within the wellbore, atm
p_{fD}	:	dimensionless pressure drop in the fissure medium on the formation side of the skin effect
p_{mD}	:	dimensionless pressure in the matrix medium
p_{wfD}	:	dimensionless pressure drop in the fissure medium within the wellbore
q	:	interporosity flow rate, cc/sec
q_{wb}	:	annulus loading rate,, cc/sec
q_{sf}	:	flow from sandface, cc/sec
r		radial distance from the well, cm
r_w		radius of the wellbore, cm
r_p	.	inside radius of the drill pipe, cm
S		dimensionless skin factor
t		time, sec

t_D : dimensionless time

u : Laplace variable

V : ratio of volume of one porous system to bulk volume

α : interporosity shape factor, cm2

β : interporosity flow group

η : diffusivity, cm^2/sec

γ : Euler's constant

λ : interporosity flow parameter

μ : viscosity, cp

ω : storativity ratio

ϕ : porosity of the medium, fraction

Subscripts and superscripts

D : dimensionless

f : fissure medium

i : initial

$\overline{}$: Laplace transform

7. References

Agarwal, R. G., Al-Husseiny, R., and Ramey, H. J., Jr.: "An Investigation of Wellbore Storage and Skin effect in Unsteady Liquid Flow" I. Analytical treatment, Soc. Pet. Eng. J. (Sept. 1970), 279-290.

Bourdet, D., and Gringarten, A. C.: "Determination of Fissure Volume and Block Size in Fractured Reservoirs by Type Curve Analysis," paper SPE 9293 presented at the SPE 55th Annual Fall Technical Conference and Exhibition, Dallas, Texas, Sept. 21-24, 1980.

Cooper, H. H., Jr., Bredehoeft, J. D., and Papadopulous, I. S.: "Response of a Finite-diameter Well to Instantaneous Charge of Water", Water Resources Research (1967), Vol. 3, No. , 263-269.

Deruyck, B. J., Bourdet, D. P., Da Prat, G., Ramey, H. J., Jr.: "Interpretation of Interference Tests in Reservoirs With Double Porosity Behavior-Theory and Examples," paper SPE 11025 presented at the SPE 57th Annual Fall Technical Conference, New Orleans, LA, Sept 26-29, 1982.

de Swan, A. O.: "Analytical Solutions for Determining Naturally Fractured Reservoir Properties by Well Testing," Soc. Pet. Eng. J., pp. 117-122, June 1976.

Earlougher, R. C., Jr., and Kersch, K. M.: "Analysis of Short-Time Transient Test Data by Type Curve Matching," J. Pet. Tech. (July 1974), 793.

Ferris, J. G., and Knowles, D. B.: "The Slug Test for Estimating Transmissibility," U.S. Geol. Survey Ground Water Note 26, 1-7, 1954.

Gringarten, A. C., Bourdet, D. P., Landel, P. A., and Kniazeff, V. J.: "A Comparison Between Different Skin and Wellbore Storage Type-Curves For Early-Time Transient Analysis," paper SPE 8205 presented at the 54th Annual Fall Technical Conference, Las Vegas, Nevada, Sept. 23-26, 1979.

Hurst, W.: Establishment of the Skin Effect and Its Impediment to Fluid-flow into a Well Bore, Pet. Eng. (Oct. 1953), Vol. 25, B-6.

Jaeger, J. C.: Conduction of Heat in an Infinite Region bounded Internally by a Circular Cylinder of a Perfect Conductor, Aust. J. Phys. (1956), Vol. 9, No. 2, 167-179.

Kohlhaas, C. A.: "A Method for Analyzing the Pressures Measured During Drill Stem-Test Flow Periods," Jour. Pet. Tech., Oct. 1972, p-1278.

Maier, L. F.: "DST Interpretation Calculations for Water Reservoirs," Haliberton Services Limited Report, Calgary, Alberta, Jan. 2, 1970.

Mavor, M. J., and Cinco-Ley, H.: "Transient Pressure Behaviour of Naturally-Fractured Reservoirs," paper SPE No. 7977 presented at the Soc. Pet. Engrs. California Regional Meeting, Ventura, California, April 18-20, 1979.

Ramey, H. J., Jr., and Agarwal, R. G.: "Annulus Unloading Rates as Influenced by Wellbore Storage and Skin Effect," Soc. Pet. Engrs. Jour., October, 1972, p-453.

Ramey, H. J., Jr., Agarwal, R. G., and Martin, I.: "Analysis of 'Slug Test' or DST Flow Period Data," Can. Pet. Jour., July, 1975, 1.

Saldana, M.: "Flow Phenomenon Of Drill Stem Test With Inertial and Frictional Wellbore Effects," Ph.D. Dissertation, Stanford University, October, 1983.

Stehfest, H.: "Algorithm 368: Numerical Inversion of Laplace Transforms," D-5 Communication of the ACM, 13 (1), January, 1970, 47-49.

Strelsova, T. D.: "Well Pressure Behavior of a Naturally-Fractured Reservoir," paper SPE 10782 presented at the SPE 1982 California Regional Meeting, San Francisco, California, March 24-26, 1982.

van Everdingen, A. F., and Hurst, W.: "The Application of the Laplace Transformation to Flow Problems in Reservoirs," Trans. AIME (1949), Vol. 186, 305-324.

van Everdingen, A. F.: "The Skin Effect and Its Influence on the Productive Capacity of a Well,," Trans. AIME (1953), Vol. 25, B-6.

van Poollen, H. K., and Weber,, J. D.: "Data Analysis for High Influx Wells," Paper SPE 3017 presented at 45th Annual Fall Meeting of the Soc. of Pet. Engrs. of AIME, Houston, Oct. 4-7, 1970.

Warren, J. E., and Root, P. J.: "The Behavior of Naturally-Fractured Reservoirs," Soc. Pet. Eng. J., (Sept., 1963), 245-255.

8. Appendix A

Derivation Of Characteristic Equation With Double-Porosity Behavior

This derivation is from the model by Warren and Root(1956), and is presented in detail in Deruyuck et al. (1982). It consists of two separate material balances over a differential element containing both fissures and matrix. Equations so obtained are combined later using pseudo-steady state or transient interporosity flow regimes.

(A-1) Material Balance

[Mass rate in]=[Mass rate out]+[Mass storage rate]+Source or Sink

....................(A-1)

Neglecting gravitational forces, and assuming Darcy's law holds for a radial system of constant thickness for fissures:

$$\left[-\frac{2\pi r h k_f \rho}{\mu}\frac{\partial p_f}{\partial r}\right] = \left[-\frac{2\pi r h k_f \rho}{\mu}\frac{\partial p_f}{\partial r}\right] + \left[\frac{2\pi r\, dr\, h\, \partial(\phi V)\rho}{\partial t}\right] - \left[2\pi r h \rho q\, dr\right]$$

....................(A-2)

for matrix:

$$\left[-\frac{2\pi rhk_f\rho}{\mu}\frac{\partial p_m}{\partial r}\right] = \left[-\frac{2\pi rhk_m\rho}{\mu}\frac{\partial p_m}{\partial r}\right] + \left[\frac{2\pi rdrh\partial(\phi V)_m\rho}{\partial t}\right] + 2\pi rdrh\rho q \qquad \text{................(A-3)}$$

Dividing and rearranging, we have for the fissures:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{rk_f\rho}{\mu}\frac{\partial p_f}{\partial r}\right) = \frac{\partial(\phi V)_f\rho}{\partial t} - \rho q \qquad \text{...............(A-4)}$$

For matrix:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{rk_m\rho}{\mu}\frac{\partial p_m}{\partial r}\right) = \frac{\partial(\phi V)_m\rho}{\partial t} + \rho q \qquad \text{............(A-5)}$$

For a fluid of constant compressibility and assuming small pressure gradients, we have for the fissure medium:

$$\frac{k_f}{} \nabla^2 p_f = (\phi V c_t)_f \frac{\partial p_f}{\partial t} - q \qquad \ldots\ldots\ldots\ldots(A-6)$$

For the matrix medium:

$$\frac{{}_m}{\mu} \nabla^2 p_m = (\phi V c_t)_m \frac{\partial p_m}{\partial t} + q \qquad \ldots\ldots\ldots\ldots(A-7)$$

(A-2) Derivation of characteristic equation with pseudo-steady state interporosity flow assumption

The pseudo-steady state interporosity flow assumption can be expressed as:

$$q = \frac{\alpha\, k_m (p_m - p_f)}{\mu} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(A-8)$$

Assuming the matrix permeability is low and the left side of Eq. (A-7) can be neglected, yields:

$$q = -(\phi V c_t)_m \frac{\partial p_m}{\partial t} \quad \text{..................(A-9)}$$

Substituting for q in Eq. (A-8):

$$\frac{\alpha k_m}{\mu} (p_m - p_f) = -(\phi V c_t)_m \frac{\partial p_m}{\partial t} \quad \text{...........(A-10)}$$

Equation (A-10) can be expressed in dimensionless form, using dimensionless parameters λ and ω defined by Warren and Root (1956):

$$\lambda \; (p \qquad {}_{fD}) \qquad \frac{\partial p}{} \qquad \text{(A-loa)}$$

Multiplying both sides by the dimensionless storage coefficient to change the independent variable from t_D to t_D/C_D, we have:

$$\lambda C_D (p \qquad _{fD}) \qquad \frac{\partial p}{\partial(\frac{t_D}{C_D})} \qquad \ldots\ldots\ldots\ldots\ldots\ldots$$

Dimensionless time and wellbore storage coefficient here are based upon the total fissure and matrix storativity.

The dimensionless pressure in the matrix can be defined in Laplace space by taking the Laplace transform of Eq. (A-11):

$$\bar{p}_{mD} = \frac{\lambda C_D}{(1-\omega) + \lambda C_D} \bar{p}_{fD} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(A\text{-}12)$$

The equation for fissures, Eq. (A-6), can be written in dimensionless form as:

$$\frac{\partial^2 p_{fD}}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_{fD}}{\partial r_D} = \frac{\omega C_D \partial p_{fD}}{\partial(\frac{t_D}{C_D})} + \frac{\mu r_w^2}{k_f} \frac{q}{(p_i - p_o)} \qquad \ldots\ldots\ldots\ldots\ldots(A\text{-}13)$$

Taking the Laplace transform we have:

$$\frac{d^2\bar{p}_{fD}}{dr_D^{\ 2}} + \frac{1}{r_D}\frac{d\bar{p}_{fD}}{dr_D} = \frac{\omega u}{C_D}\bar{p}_{fD} - \frac{\mu r_w^{\ 2}}{k_f}\frac{\bar{q}}{(p_i - p_f)} \quad \text{...........(A-14)}$$

where:

$$\bar{q} = \frac{\alpha k_m (p_i - p_o)}{\mu}(\bar{p}_{fD} - \bar{p}_{mD}) \quad \text{.................(A-15)}$$

Substituting for dimensionless pressures in Laplace space, interporosity flow is given by:

$$\bar{q} = \frac{\alpha\ k_m (p_i - p_o)}{\mu}\left[\frac{(1-\omega)\ u}{(1-\omega)\ u + \lambda C_D}\right]\bar{p}_{fD} \quad \text{........(A-16)}$$

Substituting in **Eq.(A-14)**, we have:

$$\frac{d^2\bar{p}_{fD}}{dr_D^{\ 2}} + \frac{1}{r_D dr_D}d\bar{p}_{fD} = \frac{\omega u}{C_D}\bar{p}_{fD} - \frac{\alpha k_m r_w^{\ 2}}{k_f}\left[\frac{(1-\omega)\ u}{(1-\omega)\ u + \lambda c_D}\right]\bar{p}_{fD} \quad \text{....(A-17)}$$

which can be simplified to:

$$\frac{d^2\bar{p}_{fD}}{dr_D^{\,2}} + \frac{1}{r_D}\frac{d\bar{p}_{fD}}{dr_D} - \frac{u\,f(u)}{C_D}\,\bar{p}_{fD} = 0 \quad \text{.....................(A-18)}$$

where

$$f(u) = \frac{\omega(1-\omega)u + \lambda C_D}{(1-\omega)u + \lambda C_D} \quad \text{.......................(A-19)}$$

Equation (A-18) is the characteristic equation for a double-porosity reservoir, with the assumption of pseudo-steady state interporosity flow.

(A-3) Derivation of characteristic equation for transient interporosity flow

The dimensionless equation for fissures in Laplace space **is** given by **Eq.** (A-14):

$$\frac{d^2\bar{p}_{fD}}{dr_D^{\,2}} + \frac{1}{r_D}\frac{d\bar{p}_{fD}}{dr_D} = \frac{\omega\, u}{C_D}\,\bar{p}_{fD} + \frac{\mu r_w^{\,2}}{k_f}\,\frac{\bar{q}}{(p_i - p_o)} \quad \text{............(A-20)}$$

The fluid flow from matrix to fissure is described by:

$$\nabla^2 p_m = \frac{1}{\eta_m} \frac{\partial p_m}{\partial t} \qquad \text{.............................(A-21)}$$

Equation (A-21) can be expressed considering horizontal symmetry for the matrix, for slab shaped matrix blocks:

$$\frac{\partial^2 p_m}{\partial z^2} = \frac{1}{\eta_m} \frac{\partial p_m}{\partial t} \qquad \text{........................(A-22)}$$

where "z" is the vertical coordinate.

The boundary and initial conditions to solve Eq. (A-22) are:

$$p_m = p_i \,, \qquad \text{at } t=0 \qquad \text{..............(A-23)}$$

$$\frac{\partial p_m}{\partial z} = 0 \qquad \text{at } z = \frac{h_m}{2} \,, \quad \text{for all } t \qquad \text{..........(A-24)}$$

$$p_m = p_f \quad \text{at } z=0, \text{ for all } t \qquad \text{.............(A-25)}$$

where h_m is the thickness of a matrix block. Equation (A-21) can be expressed as:

$$\nabla^2 p_m = \frac{k_f}{C_D \eta_m [(\phi V c_t)_f + (\phi V c_t)] \mu r_w^2} \frac{\partial p_m}{\partial(\frac{t_D}{C_D})} \quad \text{......(A-26)}$$

which can be expressed in terms of λ and ω as:

$$\nabla^2 p_m = \frac{(1-\omega)\, a}{C_D\, \lambda \quad \partial(\frac{t_D}{C_D})} \quad \text{..............(A-27)}$$

where α is given by:

$$a = \frac{4n\ (n+2)}{1}$$

For slab-shaped matrix blocks, $n=1$, and the characteristic dimension $1=h_m^2$ (Deryuck et al.,1982). Transforming Eq. (A-27) into Laplace space:

$$\nabla^2 \bar{p}_m - \frac{12\ (1-\omega)\ u}{h_m^2\ \lambda\ C_D}\ \bar{p}_m \quad \text{................(A-28)}$$

Equation **(A-29)** is a linear, homogeneous equation and solution is, Streltsova **(1982):**

$$\bar{p}_m = \bar{p}_f \frac{\cosh\left(\frac{h_m}{2} - z\right) \frac{12\,(1-\omega)\,u}{h_m^2\,\lambda C_D}}{\cosh\left(\frac{h_m}{2} - z\right) \frac{12\,(1-\omega)\,u}{h_m^2\,\lambda\, C_D}} \qquad \ldots\ldots\ldots\ldots\ldots(A-29)$$

The interporosity flow is defined in the Laplace space **as:**

$$\bar{q} = \frac{2\,k_m}{h_m\,\mu} \frac{\partial \bar{p}_m}{\partial z}\Big|_{z=0} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(A-30)$$

The pressure gradient is given by:

$$\left(\frac{\partial \bar{p}_m}{\partial z}\right)_{z=0} = \bar{p}_{fD} \frac{12\,(1-\omega)\,u}{h_m^2\,\lambda\,C_D} \tanh \frac{h_m}{2} \frac{12\,(1-\omega)\,u}{h_m^2\,\lambda C_D} \qquad \ldots\ldots(A-31)$$

$$\bar{q} = \frac{4\,k_m\,\bar{p}_{fD}\,(p_i - p_o)}{h_m^2\,\mu} \frac{3\,(1-\omega)\,u}{\lambda\,C_D} \tanh \frac{3\,(1-\omega)\,u}{\lambda\,C_D} \qquad \ldots\ldots(A-32)$$

Equation (A-20) then becomes:

$$\frac{d^2\bar{p}_{fD}}{dr_D^2} + \frac{1}{r_D}\frac{d\bar{p}_{fD}}{dr_D} = \frac{\omega u}{C_D} + \frac{k_m r_w^2}{k_f} \frac{3(1-\omega)u}{\lambda C_D} \tanh \frac{3(1-\omega)u}{\lambda C_D} \bar{p}_{fD}$$

....................(A-33)

which can be expressed in the same form as that for the pseudo-steady state model:

$$\frac{d^2\bar{p}_{fD}}{dr_D^2} + \frac{1}{r_D}\frac{d\bar{p}_{fD}}{dr_D} - \frac{u\, f(u)\, \bar{p}_{fD}}{C_D} = 0$$

..............(A-34)

where:

$$f(u) = \omega + \frac{C_D \lambda (1-\omega)}{3u} \tanh \frac{3(1-\omega)u}{\lambda C_D}$$

.......(A-39)

(A-4) Derivation of boundary conditions in Laplace space

For a Slug test there is no flow at the surface, flow coming from the sand face only changes the liquid level in the wellbore:

$$q_{wb} = \frac{C \, d(p_i - p_f)}{dt} \qquad \text{.................(A-36)}$$

This can be expressed in dimensionless form as:

$$q_{wb} = \frac{2\pi k_f h}{\mu} (p_i - p_o) \, C_D \frac{dp_{wfD}}{dt_D} \qquad \text{..............(A-37)}$$

The sand flow rate is given by:

$$q_{sf} = \left[- \frac{2\pi k_f r h}{\mu} \frac{\partial p_f}{\partial r}\right]_{r=r_w} \qquad \text{.....................(A-38)}$$

in dimensionless form:

$$q_{sf} = - \frac{2\pi k_f h}{\mu} (p_i - p_f) \left(\frac{\partial p_{fD}}{\partial r_D}\right)_{r_D = 1} \qquad \text{..............(A-39)}$$

as there is no flow at the surface:

$$q_{wb} + q_{sf} = 0 \qquad \text{..................(A-40)}$$

$$C_D \frac{\partial p_{wfD}}{\ } = \left(\frac{\partial p_{fD}}{\partial r_D}\right) \qquad \text{....................(A-41)}$$

Taking the Laplace transform, we have:

$$\left[u \bar{p}_{wfD} - \bar{p}_{wfD}(0) \right] = \left(\frac{\partial p}{}\right)_{r_-=1} \qquad \text{............(A-42)}$$

$$(u \bar{p}_{wfD} - 1) = \left(\frac{\partial \bar{p}_{fD}}{\partial r_D}\right)_{r_D=1} \qquad \text{...............(A-43)}$$

Pressure drop at the sand face is considered by using the Hurst and Everdingen skin factor:

$$\frac{2\pi k_f h}{q_{sf}\mu} \Delta p_s = S \qquad \text{.......................(A-44)}$$

Flow at the sand face is given by:

$$q_{sf} = \frac{2\pi k_f h \, (p_i - p_o)}{\mu} \left(\frac{\partial p_{fD}}{\partial r_D}\right)_{r_D=1} \qquad \text{........(A-45)}$$

Substituting in Eq. *(A-44):*

$$(p_{fD} - p_{wfD}) = S \left(\frac{\partial p_{fD}}{\partial r_-}\right)_{r_-=1} \qquad \text{..................(A-46)}$$

taking the Laplace transform:

$$\bar{p}_{wfD} = \left[\bar{p}_{fD} - S \frac{\partial \bar{p}_{fD}}{\partial r_D} \right]_{r_D=1} \qquad \text{...............(A-47)}$$

Outer boundary condition:

$$\ell imit_{r_D \to \infty} \bar{p}_{fD}(r_D, u) = 0 \qquad \text{...............(A-48)}$$

9. Appendix B

Solution of Slug test problem in two porosity medium

The problem of the slug test can now be presented in Laplace space as:

$$\frac{d^2\bar{p}_{fd}}{dr_D^2} + \frac{1}{r_D}\frac{d\bar{p}_{fD}}{dr_D} - \frac{u\ f(u)}{C_D}\bar{p}_{fD} = 0 \qquad \text{.................(B-1)}$$

$$(u\ \bar{p}_{wfD} - 1) = \left(\frac{\partial \bar{p}_{fD}}{\partial r_D}\right)_{r_D=1} \qquad \text{.................(B-2)}$$

$$\bar{p}_{wfD} = \left[\ \bar{p}_{fD} - S\frac{\partial \bar{p}}{\ }\right]_{r_-=1} \qquad \text{.................(B-3)}$$

$$\text{Rimit}_{r_D \to \infty}\ \bar{p}_{wfD}(r_D, u) = 0 \qquad \text{.................(B-4)}$$

Equation (B-1) is a modified Bessel equation of order zero, its solution is a linear combination of two Bessel functions:

$$\bar{p}_{fD} = A\ I_o\ r_D\ \frac{u\ f(u)}{C_D} + B\ K_o\ r_D\ \frac{u\ f(u)}{C_D} \qquad \text{.........(B-5)}$$

Here I_o and K_o are modified Bessel functions of order zero.

Arbitrary constant A must be zero to satisfy the outer boundary condition, *so:*

$$\bar{p}_{fD} = B\ K_o\ r_D\ \frac{u\ f(u)}{C_D} \qquad \text{...............(B-6)}$$

Differentiating with respect to r_D:

$$\frac{\partial p_{fD}}{\partial r_D} = - \frac{u\ f(u)}{C_D}\ K_1\ r_D\ \frac{u\ f(u)}{C_D} \qquad \text{..............(B-7)}$$

Substituting for the dimensionless pressure gradient in Eq. (B-2):

$$(u\,\bar{p}_{wfD} - 1) = -B \quad \frac{u\ f(u)}{C_D} \quad K_1 \quad \frac{u\ f(u)}{C_D} \qquad \text{.............(B-8)}$$

$$B = \frac{(1 - u\,\bar{p}_{wfD})}{\frac{u\ f(u)}{C_D} \; K_1 \; \frac{u\ f(u)}{C_D}} \qquad \text{..................(B-9)}$$

Relating the dimensionless pressure at the reservoir side to the wellbore pressure through Eq. (B-3):

$$\bar{p}_{wfD} = B \quad K_o \quad \frac{u\ f(u)}{C_D} + S \quad \frac{u\ f(u)}{C_D} \quad K_1 \quad \frac{u\ f(u)}{C_D} \qquad \text{.....(B-10)}$$

Substituting for B and rearranging, we have:

$$\bar{p}_{wfD} - \frac{K_o \; \frac{u\ f(u)}{C_D} \; - \; S \; \frac{u\ f(u)}{C_D} \; K_1 \; \frac{u\ f(u)}{C_D}}{C_D \quad K_1 \quad C_D \; + \; U \; K_o \; \frac{u\ f(u)}{C_D} \; - \; C_D \quad K_1 \quad \frac{u\ f(u)}{C_D}}$$

$$\text{.................(B-11)}$$

Equation (B-11) is the solution of the slug test problem in Laplace space. This can be approximated for all practical purposes by the limiting forms of the Bessel functions. For small values of arguments of Bessel functions:

$$K_o(z) \approx - \left[\ell n \left(\frac{z}{2}\right) + \gamma \right] \qquad \text{.................(B-12)}$$

$$K_1(z) \approx \frac{1}{z} \qquad \text{.................(B-13)}$$

Substituting Eqs. (B-12) and (B-13) in (B-11):

$$\bar{p}_{wfD} = \frac{\left[- \ell n\left(\frac{e^{\gamma}}{2} \frac{u\ f(u)}{C_D}\right) + S \right]}{1 + u \left[- \ell n\left(\frac{e^{\gamma}}{2} \frac{u\ f(u)}{C_D}\right) + S\right]} \qquad \text{.................(B-14)}$$

which can be simplified:

$$\bar{p}_{wfD} = \frac{1}{\mu + x \ \ell n \ \frac{2}{e} \ \frac{C_D e^{2S}}{u \, f(u)} \ -1} \qquad \text{................(B-15)}$$

where γ= 0.5778. ....., (Euler's constant).

10. APPENDIX C

COMPUTER PROGRAMS

```
C      THE FOLLOWING PROGRAM INVERTS THE SOLUTION OF
C      TWO POROSITY SLUG TESTING INTO REAL SPACE USING
C      THE STEHFEST ALGORITHIM. THE PROGRAM HAS THE
C      THE SOLUTION FOR BOTH PSEUDO STEADY STATE AND
C      TRANSIENT FLOW REGIMES.  IT CAN BE USED TO
C      DETERMINE THE EARLY AND LATE TIME RESPONSE OF THE
C      TWO REGIMES.  IT ALSO DETERMINES THE COMPLETE PRESSURE
C      REPONSE FOR THE TWO REGIMES.  INTERMEDIATE REPONSE FOR
C      THE TRANSIENT INTERPOROSITY FLOW REGIME CAN ALSO
C      BE OBTAINED. APPROPRATE EXPRESSION IS TO BE USED FOR
C      EACH CASE. EXPRESSION FOR EACH CASE ARE PROVIDED AND
C      ARE DOCUMENTED
C
C
C      NOMENCLATURE
C
C
C      BETA = TRANSIENT INTERPOROSITY FLOW GROUP
C      CD= WELLBPRE STORAGE CONSTANT
C      CD1 = CD*EXP(2S)
C      DLAMDA = INTERPOROSITY FLOW PARAMETER
       FUN= WARREN AND ROOTS FUNCTION
C         =1  FOR EARLY TIMES
C      OMEGA  = STORATIVITY RATIO
C      P= SOLUTION IN THE LAPLACE SPACE
C      S = SKIN FACTOR
C      TDCD= DIMENSIONLESS TIME
C
C
C
// JOB
// EXEC WATFIV
       IMPLICIT REAL*8(A-H,O-Z)
       INTEGER SN
       DIMENSION G(25),H(25),V(25)
       EXTERNAL BESK0,BESK1,DSQRT
       REAL*8 BESK0,BESK1
       TDCD=0.0D0
       N=16
       M=25
       DLOGTH=0.69314718056D0
       CD=1.0D0
       S=0.D0
       DLAMDA=0.10D0
       OMEGA=0.1D0
C       CD1=CD*DEXP(2.0D0*S)
C       CD1=1.0D+04
C      BETA = 10.0D+10
       WRITE(6,100) CD1,CD,S,DLAMDA,OMEGA
C
C
56     CONTINUE
C
       IF(TDCD.GE.0.09D0) GO TO 101
       TDCD=TDCD+0.01D0
       GO TO 50
101    CONTINUE
       IF(TDCD.GE.0.9D0) GO TO 102
       TDCD=TDCD+0.10D0
       GO TO 50
```

```
102   CONTINUE
      IF(TDCD.GE.09.0D0) GO TO 103
      TDCD=TDCD+1.0D0
      GO TO 50
103   CONTINUE
      IF (TDCD.GE.99.0D0) GO TO 104
      TDCD=TDCD+10.0D0
      GO TO 50
104   CONTINUE
      TDCD=TDCD+100.0D0
      IF(TDCD.GT.1000.0D0) GO TO 79
 50   CONTINUE
C
C
C     CALCULATION OF INVERSE FUNCTION
C
      IF(M.EQ.N) GO TO 80
C
C     CALCULATION OF V-ARRAY
C
      M=N
      G(1)=1.0D0
      NH=N/2
      DO 10 I=2,N
 10   G(I)=G(I-1)*DFLOAT(I)
      H(1)=2.0D0/G(NH-1)
      DO 30 I =2,NH
      FI=DFLOAT(I)
      IF(I.EQ.NH) GO TO 20
      H(I)=FI**NH*G(2*I)/(G(NH-I)*G(I)*G(I-1))
      GO TO 30
 20   H(I) =FI**NH*G(2*I)/(G(I)*G(I-1))
 30   CONTINUE
      SN=2*(NH-NH/2*2)-1
      DO 70 I=1,N
      V(I)=0.0D0
      K1=(I+1) /2
      K2=I
      IF(K2.GT.NH) K2=NH
      DO 60 K=K1,K2
      KK=2*K-I
      IF(KK.EQ.0.D0) GO TO 40
      IF (I.EQ.K) GO TO 75
      V(I)=V(I)+H(K)/(G(I-K)*G(2*K-I))
      GO TO 60
 40   V(I)= V(I)+ H(K)/ G(I-K)
      GO TO 60
 75   V(I)= V(I)+H(K)/G(KK)
 60   CONTINUE
      V(I)= SN* V(I)
      SN=-SN
 70   CONTINUE
 80   FA=0.D0
      A=DLOGTH/TDCD
      DO 90 I=1,N
      ARG= DFLOAT(I)*A
C
C
C     THIS IS THE  EXPRESSION OF THE WARREN AND ROOTS
C     FUNCTION FOR PSEUDO STEADY STATE INTERPOROSITY FLOW
```

```
C     REGIME AND IS TO BE USED TO GET THE FULL TIME PRESSURE
C     RESPONSE.
C
      FUN=(OMEGA*(1.0D0-OMEGA)*ARG+DLAMDA*CD)/((1-OMEGA)*ARG+DLAMDA
C
C
C
C     THIS IS THE EXPRESSION FOR FUNCTION
C     FOR TRANSIENT INTERPOOSITY FLOW REGIME. THIS
C     THIS IS USED TO GET THE FULL TIME PRESSURE
C     RESPONSE FOR THIS REGIME.
C
C
C      FUN=OMEGA+(DSQRT(CD*DLAMDA*(1-OMEGA)/ARG/3)
C     *    *DTANH(DSQRT(3*(1-OMEGA)*ARG/DLAMDA/CD)))
C
C     THIS VALUE OF FUNCTION IS USED FOR EARLY
C     TIME APPROXIMATION
C
C      FUN=1.0D0
C
C
C     THIS IS THE EXACT SOLUTION OF THE SLUG TEST IN TWO
C     POROSITY MEDIUM IN LAPLACE SPACE. VALUE OF
C     THE FUNCTION 'FUN' USED IS ACCORDING TO
C     THE REGIME FOR WHICH THE RESPONSE IS TO BE
C     DETERMINED.
C
      P=(BESK0(DSQRT(ARG*FUN/CD))+S*DSQRT(ARG*FUN/CD)
     *    *BESK1(DSQRT(ARG*FUN/CD)))/(DSQRT(ARG*FUN/CD)*
     *    BESK1(DSQRT(ARG*FUN/CD))+ARG*(BESK0(DSQRT(ARG*FUN/CD))+
     *    S*DSQRT(ARG*FUN/CD)*BESK1(DSQRT(ARG*FUN/CD))))
C      WRITE (6,149) FUN
C149   FORMAT(5X,D20.9)
C
C
C     THIS IS THE APPROXINATION TO THE EXACT
C     SOLUTION. THIS SOLUTION TREATS CD*EXP(2S)
C     AS ONE GROUP.
C
C      P=1.0D0/(ARG+1.0D0/DLOG(2.0D0/1.78*DSQRT(CD1/FUN*ARG)))
C
C
C     THIS THE EXPRESSION FOR THE INTERMEDIATE
C     RESPONSE FOR THE TRANSIENT MODEL. THIS IS
C     TO BE USED WHEN INTERMEDIATE RESPONSE
C     IS TO BE DETERMINED. THE VALUE OF TRNSIENT
C     INTERPOROSITY FLOW GROUP HAS TO BE SPECIFIED
C     IN THIS CASE.
C
C
C      P=1.0D0/(ARG+(1.0D0/(DLOG(2.0D0/1.78*((BETA/ARG)**0.25))
C     *  )))
      FA=FA+ V(I)*P
90    CONTINUE
      FA=FA*A
      WRITE (6,120) TDCD,FA
      GO TO 56
120   FORMAT(5X,F12.6,20X,F9.6)
100   FORMAT(1X,'          CD*exp(2*S)=',E12.6,//
```

```
     *        5X,' CD*******************=',E12.6,/
     *        5X,' S *******************=',F6.4,//
     *        5X,'DDLAMDA****************=',E12.6,/
     *        5X,'OMEGA****************=',E12.6,/
     *        5X,' TD/CD                                        PD     ',/)
79        STOP
          END
```