UNSTEADY–STATE PRESSURE RESPONSE
IN A SLOTTED LINER

A REPORT

SUBMITTED TO **THE** DEPARTMENT OF PETROLEUM ENGINEERING

OF STANFORD UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

'MASTER OF SCIENCE

by

David Spivak

October 1981

ACKNOWLEDGEMENTS

I wish to express my appreciation to Professor Roland Horne of the Department of Petroleum Engineering for his patience, encouragement and guidance as my research advisor. Also gratitude goes to the Department of Petroleum Engineering of Stanford University and to Tenneco Research Fund who have supported my studies. I am much indebted to my mother, Julia Spivak, for her continued encouragement, understanding and support during this work.

ABSTRACT

A semi-analytical solution has been obtained for the unsteady-state pressure response in a slotted liner.

The solution was obtained by means of a source function, assuming constant rate of production, penetration ratio of 0.5 (i.e. ratio of total open interval to total thickness) and various slot lengths (no storage or skin effects were considered)

For integration purposes, satisfactory results were obtained using Simpson's rule.

The resultu show that inside and outside the slotted liner, the pseudo-skin factor due to restricted flow to the well is negligible for 6 vertical rows of slots (3 pairs of vertical staggered rows,or 3 slots in one ring of slots), even at early time of production. On the other hand, on the surface of the liner , there is a pseudo-skin factor, unless the number of rows is increased.

There is not much difference in pressure drop in a liner with many and short slots, as compared to a liner with a fewer and longer slots, but with the same number of vertical rows.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF TABLES (Cont.)

LIST OF TABLES (Cont.)

LIST OF TABLES (Cont.)

LIST OF TABLES (Cont.)

Page

Table

LIST OF TABLES (Cont.)

Page

Table

LIST OF TABLES(Cont.)

1. INTRODUCTION

Sand problems have occurred in the oil and gas industry since shortly after the first oil well discovery. These problems rise due to formation movement under stresses resulting from fluid flow to the wellbore and pressure drop in the reservoir. Laboratory studies show formation failure by 3 mechanisms when sand control is inadequate. The first one is grain movement away from formation into the wellbore under low fluid velocities (or low pressure drop). At higher velocities, small masses of sand break away leading to rapid failure. The third mechanism occurs under some conditions; the formation becomes fluidized, resulting in gross flow of sand with produced fluid. Often sand control is achieved at the cost of reduced well production. Sometimes it is economically preferred to allow more sand to be produced in spite of high costs for pump maintenance, sand disposal, etc[3].

The main sand control methods are mechanical and consolidation ones. Mechanical sand'control provides a physical barrier to sand movement. It is either a screen (slotted liner) or a gravelpack between a screen and the formation[3]. These are wire wrapped and slotted pipe screens. The slotted liner is a pre-perforated tubing. Usually the slots are milled and machined in vertical rows, being parallel to the axis of the pipe. They are either straight-sided or have undercut sides (vee-shaped). The milling procedure produces a smooth finish for the slot sides, thus reducing the possibility of turbulance within the slot. On the other hand, undercut slots, which are narrower at the outside surface of the pipe than at the pipe bore, reduce the chance of clogging, since any grain of sand entering the slot will immediately find additional clearance to pass through it without wedging or bridging[4].

The most common patterns of arrangements of slots are the staggered vertical rows, multiple staggered vertical and horizontal slots (Fig. 1).

This work investigates the unsteady-state pressure response due to production through a liner with staggered vertical rows of slots.

Standard spacing of slots is 6" (center to center) between the slots in each vertical row. Slot lengths are usually 1½" to 2" depending on their widths (which are usually between 0.01" to 0.5"). To determine the number of rows of slots for any given size of pipe, the usual practice is to specify two rows for each nominal pipe diameter. But this oaries, however, for where sand conditions require slots of narrow width, a greater number of slots can be used[4].

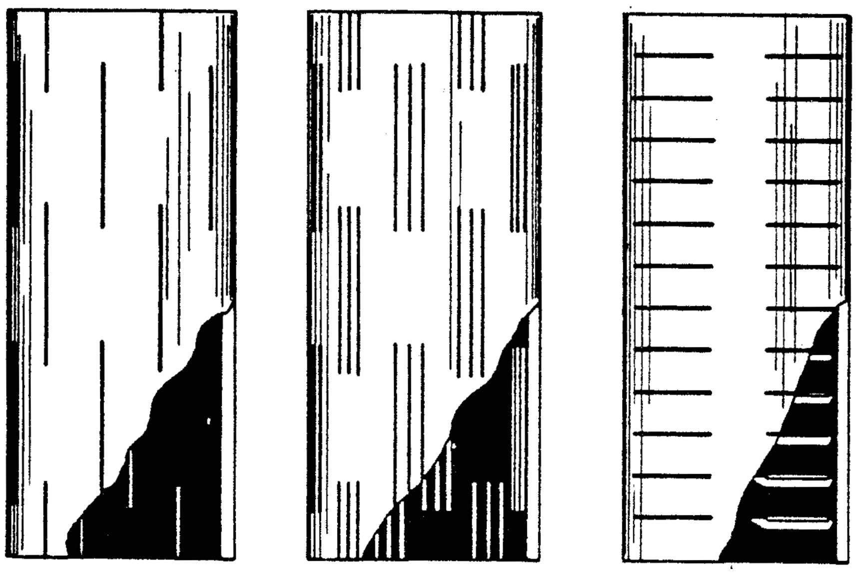


FIGURE I

SLOTTED LINERS

(After Buzardc, *et* al[3])

2. UNSTEADY-STATE PRESSURE RESPONSE IN A SLOTTED LINER

The following represents the scheme of the problem, the solution and the discussion of results.

2.1 Mathematical Derivation

The transient (unsteady-state) flow of a compressible fluid in a porous medium is described by the diffusivity equation derived from the continuity equation and Darcy's Law[9]. In isotropic and homogeneous porous medium, this equation can be expressed in cylindrical coordinates as*:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{\eta}\frac{\partial p}{\partial t} \qquad (1)$$

where $\eta = \frac{k}{\phi \mu c_t}$ is the diffusivity term, when the effect of gravity is neglected and the permeabilities, porosity and fluid viscosity are assumed constant.

In Cartesian coordinates, the differential equation is:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{\eta}\frac{\partial p}{\partial t} \qquad (1a)$$

The solution of a particular' problem is determined by the initial and boundary conditions.

Numerous analytical and numerical method have been developed to solve such flow problems. The first ones were used to solve heat conduction problems and later on were applied to petroleum engineering. Perhaps the most known method is Lord Kelvin's instantaneous point source solution[5]:

*Nomenclature defined at end of text

$$\Delta p(M,t) = \frac{q}{8\phi c_t(\pi\eta t)^{3/2}} \cdot \exp\left(-\frac{d^2}{4\eta t}\right) \quad (2)$$

Eq. 2 represents the pressure drop created at a certain point **M** in a reservoir of infinite extent by an instantaneous point **P, of** strength **q** at a distance d (=PM) from point M. The strength **q** is the withdrawal rate per volume **of** source; it is also defined as a withdrawal rate per unit length or area **of** source, depending on its nature, i.e., line source or surface cylinder source[7]. The distance d can be expressed in terms of Cartesian or cylindrical coordinates (see Fig. 2). From eq. 2 **all** other solutions are obtainable by integration. The instantaneous infinite line source case, for example, ***is*** obtained by expressing d in terms of x,y,z coordinates and integrating z from $-\infty$ to $+\infty$. The method was firstly applied in groundwater hydrology by **Theis**[15]. If there **is** a continuous withdrawal **of** fluid **of** strength q from time **0** to t, the pressure drop created by **a** continuous point source is given by:

$$\Delta p(M,t) = \frac{q}{8\phi c_t(\pi\eta)^{3/2}} \int_0^t \frac{1}{(t-t')^{3/2}} \exp\left[-\frac{d^2}{4\eta(t-t')}\right] \quad (3)$$

Let us now consider our actual problem. The reservoir of thickness **L** has an upper and lower impermeable boundaries. Therefore the boundary conditions are:

$$\frac{\partial p}{\partial z}(x,y,0,t) = \frac{\partial p}{\partial z}(x,y,L,t) = 0 \quad (4)$$

$$x\to\infty \text{ or } y\to\infty,\ P(x,y,z,t) = p_i \quad (5)$$

The initial condition is:

$$p(x,y,z,0) = p_i \quad (6)$$

In cylindrical coordinates the above conditions are:

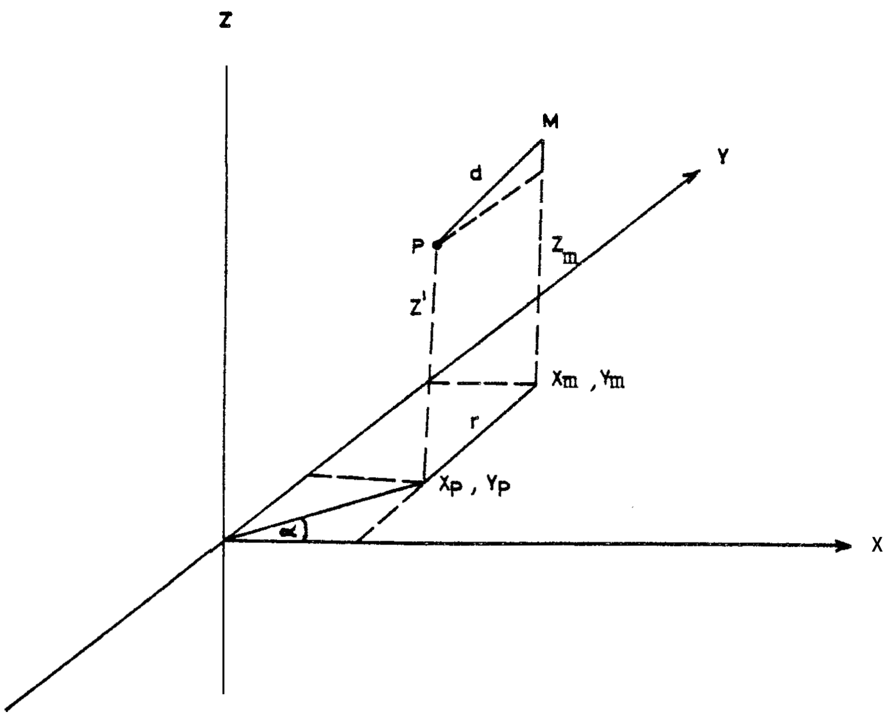


FIG. 2 - POINT SOURCE IN AN INFINITE MEDIUM

$$\frac{\partial p}{\partial z}(r,0,t) = \frac{\partial p}{\partial z}(r,L,t) = 0 \qquad (4a)$$

$$p(\infty,z,t) = P_i \qquad (5a)$$

$$p(r,z,0) = p_i \qquad (6a)$$

Also, since it is assumed that the rate over the open interval is constant, the boundary condition is:

$$\sum_{\substack{\text{slots}\\ \text{"i"}}} \int_h 2\pi \cdot w_i \frac{\partial p}{\partial w_i} dz \Big|_{w_i \to 0} + \sum_{\substack{\text{slots}\\ \text{"j"}}} \int_h 2\pi \cdot w_j \frac{\partial p}{\partial w_j} dz \Big|_{w_j \to 0} = \frac{q \cdot \mu}{k} \qquad (7)$$

where h is the slots length, and w is the radius for each slot*.

Note that in deriving and solving eq. 1, the following assumptions were made:

1. Constant Thickness and Permeability.
2. Darcy's Law applies, Laminar Flow.
3. Radial symmetry exists, Gravitational effects negligible.
4. Fluid Viscosity is constant.
5. The liquid is of small and constant compressibility, and the pressure gradients are small everywhere, i.e.,

   $c\left(\frac{\partial p}{\partial r}\right)^2$ is negligible.
6. Isothermal Flow.
7. Flow into the slots is radially symmetrical.

* However, Muskat[12] also showed that a simpler boundary condition can be introduced with a negligible error i.e.,:

$$\sum_{\substack{\text{slots}\\ \text{"i"}}} 2\pi \cdot w_i h \frac{\partial p}{\partial w_i} \Big|_{w_i \to 0} + \sum_{\substack{\text{slots}\\ \text{"j"}}} 2\pi \cdot w_j h \frac{\partial p}{\partial w_j} \Big|_{w_j \to 0} = \frac{q \cdot \mu}{k}$$

8. The porous Medium inside and outside the well is homogeneous.

A detailed diagram of all variables involved in computing the pressure drop at point M is shown on Fig. 3, and an analytic solution, based on the imape source method of Nisle[13] has been obtained as follows:

The pressure drop created at point M (Fig. 2) by a single slot is obtained by integrating a unit impulse point with respect to time and space. Consider eq. 4 for certain vertical slot "i" with a length from $z=0$ to $z=h$ oriented at an angle α with respect to the X axis (Fig. 3). The distance d_i from any point P on the slot to any point M in the porous medium will be:

$$d_i^2=(z'-z_m)^2+(x_m-r_w\cos\alpha)^2+(y_m-r_w\sin\alpha)^2 \tag{8}$$

where x_m, y_m, z_m are the coordinates of point M in space and r_w is the slotted liner radius.

Let $r_i^2=(x_m-r_w\cos\alpha)^2+(y_m-r_w\sin\alpha)^2$

so eq. 8 is given by:

$$d_i^2=(z'-z_m)+r_i^2 \tag{8a}$$

Integrating the position of the s'ource P along a staight line from 0 to h yields the instantaneous pressure drop:

$$\Delta p_i(M,t) = \frac{q_i}{8\phi c_t(\pi\eta t)^{3/2}} \int_0^h \exp[-d_i^2/4\eta t]\, dz' \tag{9}$$

and the continuous pressure drop is:

$$\Delta p_i(M,t) = \frac{q_i}{8\phi c_t(\pi\eta)^{3/2}} \int_0^t \frac{1}{(t-t')^{3/2}}\{\int_0^h \exp[-d_i^2/4\eta(t-t')]dz'\}dt' \tag{10}$$

or:

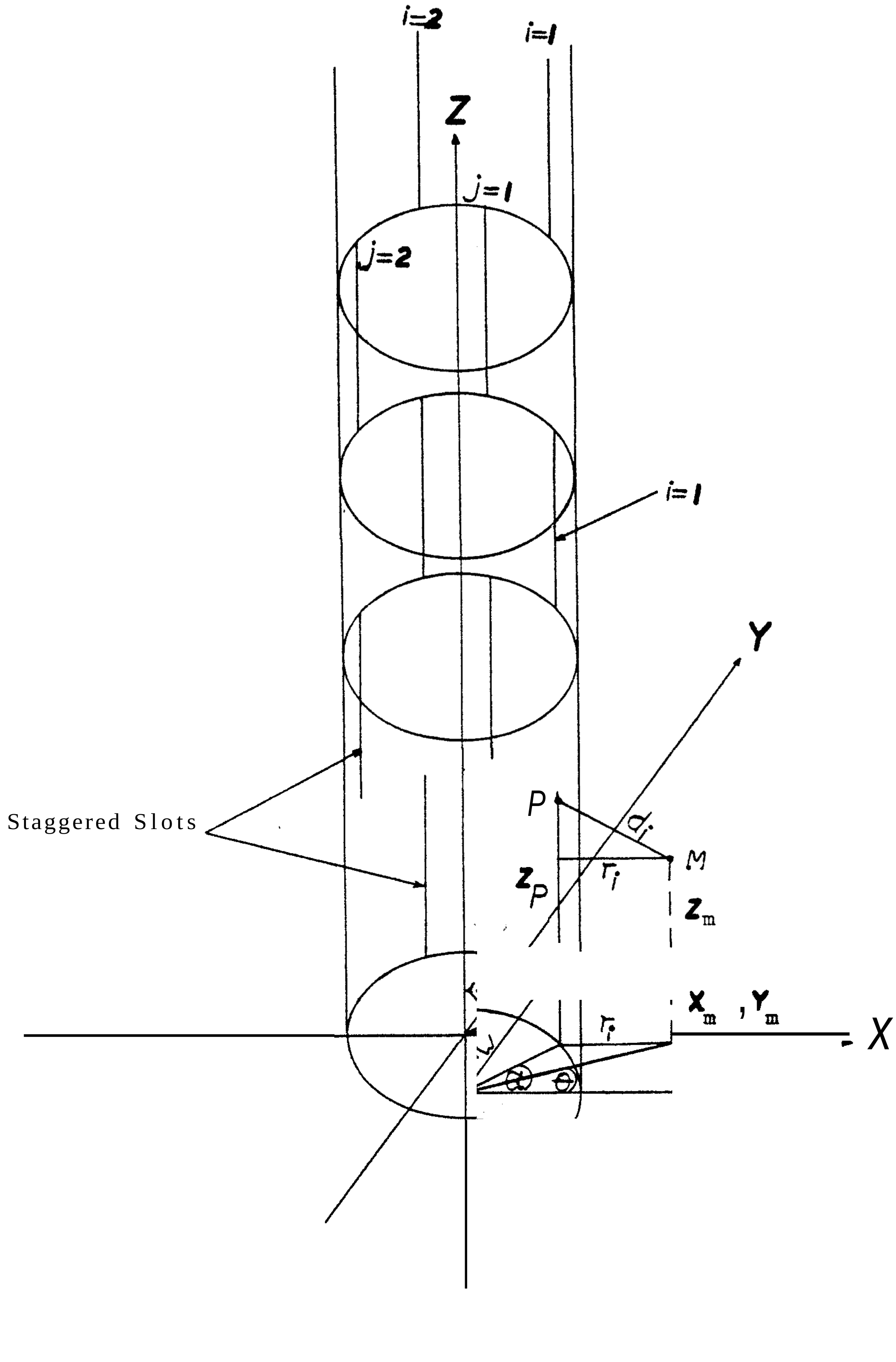


Fig. 3 – Section of a Liner with Staggered Vertical Rows of Slots

$$\Delta p_i(M,t) = \frac{q_i}{8\phi c_t(\pi\eta)^{3/2}} \int_0^t \frac{\exp[-r_i^2/4\eta(t-t')}{(t-t')^{3/2}} \cdot$$

$$\cdot\{\int_0^t \exp[-(z'-z_m)^2/4\eta(t-t')]\}dt' \qquad (10a)$$

Eq. 10a represents the continuous pressure drop created at point M due to production of strength q_i through one slot "i";

Let $\tau = t-t'$ and $dt = -d\tau$

also,

$$u = -\frac{z'-z_m}{2\sqrt{\eta\tau}} \quad a \qquad du = -\frac{dz'}{2\sqrt{\eta\tau}}$$

so, eq. 10a is written as:

$$\Delta p_i(M,t) = \frac{q_i}{8\phi c_t(\pi\eta)^{3/2}} \int_0^t \frac{\exp[-r_i^2/4\eta\tau]}{\tau^{3/2}} \cdot$$

$$\cdot\{-2\sqrt{\eta\tau} \int_{\frac{z_m}{2\sqrt{\eta t}}}^{\frac{z_m-h}{2\sqrt{\eta t}}} e^{-u^2} du\} d\tau \qquad (11)$$

The error function is defined as:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

Eq. 11 can therefore be written in terms of the error function as:

$$\Delta p_i(M,t) = \frac{q_i}{8\phi c_t \pi\eta} \int_0^t \frac{\exp[-r_i^2/4\eta\tau)}{\tau} \{\mathrm{erf}\frac{z_m}{2\sqrt{\eta\tau}} - \mathrm{erf}\frac{z_m-h}{2\sqrt{\eta\tau}}\} d\tau \qquad (12)$$

The total continuous pressure drop at point M is obtained by summing the Ap's created by all the slots in the liner(Figs. 3

& 4). It should be mentioned that the no-flow boundary conditions may be satisfied by adding an infinite series of image slots. Let us now introduce some dimensionless variables:

$$t_D = \frac{kt}{\phi \mu c_t r_w^2} = \frac{\eta \cdot t}{r_w^2}, \quad (t = \tau) \tag{13}$$

$$p_D = \frac{2\pi k L}{Q \cdot \mu} \Delta p \tag{14}$$

$$r_D = r/r_w \tag{15}$$

$$z_D = z/r_w \tag{16}$$

$$z_{mD} = z_m/r_w \tag{16a}$$

$$h_D = h/r_w \tag{17}$$

$$L_D = L/r_w \tag{18}$$

Note that in defining the dimensionless pressure drop p_D, the total production Q is considered. The relation between Q and the strength q_i (rate per unit length) of the one slot is given as follows: if NS is the number of pairs of staggered vertical rows (i.e., each pair contains one row of slots "i" and one row of slots "j", see Figs. 3 & 4). NS represents as well the number of slots in one ring of slots and n is the number of slots in one vertical row; also, the total cumulative length of n slots is L/2 (one half of the formation thickness), that is penetration ratio of ½, then the strength of 1 slot is:

$$q_i = \frac{Q}{(0.5L/n) 2 \cdot NS \cdot n} = \frac{Q}{L \cdot NS} \tag{19}$$

and the pressure drop created at point M by one slot is:

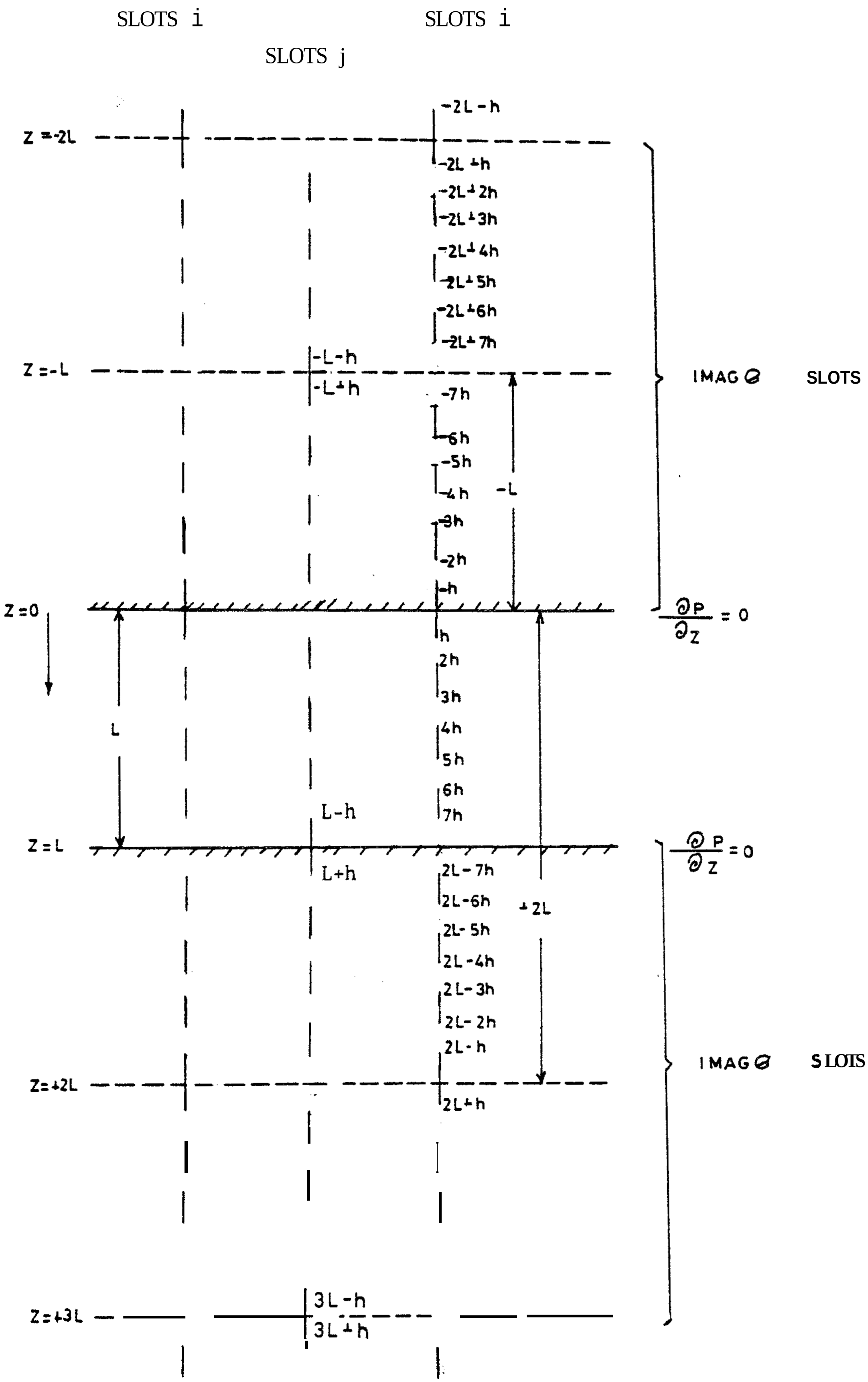


FIG. 4 — SCHEMATIC DIAGRAM OF THE MODEL

$$\Delta p_i = \frac{p_D Q\mu}{2\pi k L} = p_D \cdot \frac{q_i \mu L \cdot NS}{2\pi k L} = p_D \cdot \frac{q_i \mu \cdot NS}{2\pi k} \tag{20}$$

while the dimensionless pressure drop is given by:

$$p_{D_i}(M,tD) \cdot \frac{q_i \mu \cdot NS}{2\pi k} = \frac{q_i}{8\phi c_t \pi n} \int_0^{t_D} \frac{\exp[-r_{iD}^2/4t_D]}{t_D} \cdot$$

$$\cdot \left\{ \text{erf} \frac{z_{mD}}{\sqrt{4t_D}} - \text{erf} \frac{z_{mD}-h_D}{\sqrt{4t_D}} \right\} dt_D \tag{21}$$

or, after arrangement of terms:

$$p_{D_i}(M,t_D) = \frac{1}{4 \cdot NS} \int_0^{t_D} \frac{\exp[-r_{iD}^2/4t_D]}{t_D} \cdot$$

$$\cdot \left\{ \text{erf} \frac{z_{mD}}{\sqrt{4t_D}} - \text{erf} \frac{z_{mD}-h_D}{\sqrt{4t_D}} \right\} dt_D \tag{21a}$$

this integral can be evaluated numerically using Simpson's rule.

One of the last stages of the mathematical derivation is to apply eq. 21a over all the slots in one row (i.e., n slots) and then the summation is done 'over all the vertical rows in the liner, yielding the total dimensionless pressure drop created at point M by production through a slotted liner, This stage is carried out as follows: considering Fig. 4 which shows 3 vertical rows of slots; the right and the left ones are slots "i", and the staggered central one is a row of slots "j". As can be seen, the cumulative lengths of the slots in one row is L/2, which is one. half of the formation thickness, In addion to the real slots, there are also image ones in order to satisfy the boundary conditions. In the right row (and of course in all the other vertical rows of slots "i") there are two types of slots: one is adjacent to the upper boundary and the second type is slots which are away from the boundary. The slots "j" (staggered rows, Fig. 4) are also either adjacent to the lower boundary or away from it. So, for convenience purpose let us consider separately the contribution of each type of slots to the total pressure drop. We begin with the slot "i" adjacent to the upper boundary:

$$\Delta p^{i}_{D_1}(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{iD}/4t_D]}{t_D}\cdot$$

$$\cdot\{\operatorname{erf}\frac{z_{mD}+h_D}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}-h_D}{\sqrt{4t_D}}$$

$$+\sum_{n=1}^{\infty}[\operatorname{erf}\frac{z_{mD}+(2nL_D+h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}+(2nL_D-h_D)}{\sqrt{4t_D}}$$

$$+\operatorname{erf}\frac{z_{mD}-(2nL_D-h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}-(2nL_D+h_D)}{\sqrt{4t_D}}]\}dt_D \qquad (22)$$

Eq. 22 can be written also in the following way:

$$\Delta p^{i}_{D_1}(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{iD}/4t_D]}{t_D}\cdot$$

$$\cdot\{\sum_{n=-\infty}^{+\infty}[\operatorname{erf}\frac{z_{mD}+2nL+h}{\sqrt{4t_D}} + \operatorname{erf}\frac{z_{mD}+2nL_D-h_D)}{\sqrt{4t_D}}]\}dt_D \qquad (22a)$$

Then the pressure drop caused by slots "i" which are away from the upper boundary is given by:

$$\Delta p^{i}_{D_2}(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{iD}/4t_D]}{t_D}$$

$$\cdot\Big\{\mathrm{erf}\frac{z_{mD}+3h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2h_D}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}-2h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}-3h_D}{\sqrt{4t_D}}$$

$$+ \sum_{n=1}^{\infty}\Big[\mathrm{erf}\frac{z_{mD}+(2nL_D+3h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+(2nL_D+2h_D)}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}+(2nL_D-2h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+(2n1_D-3h_D)}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}-(2nL_D+2h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}-(2nL_D+3h_D)}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}-(2nL_D-3h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}-(2nL_D-2h_D)}{\sqrt{4t_D}}\Big]\Big\}dt_D \qquad (23)$$

Eq. 23 can be written in another form, which is:

$$\Delta p^{i}_{D_2}(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{iD}/4t_D)}{t_D}$$

$$\cdot\Big\{\sum_{n=-\infty}^{+\infty}\Big(\mathrm{erf}\frac{zmD+2nL_D+3h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{zmD+2nL_D+2h_D}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}+2nL_D-2h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D-3h_D}{\sqrt{4t_D}}\Big)\Big\}dt_D \qquad (23a)$$

and further going down to a third slot, results in:

$$\Delta p^i_{D_3}(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{iD}/4t_D]}{t_D}\cdot$$

$$\cdot\{\text{erf}\frac{z_{mD}+5h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+4h_D}{\sqrt{4t_D}}$$

$$+ \text{erf}\frac{z_{mD}-4h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}-5h_D}{\sqrt{4t_D}}$$

$$+ \sum_{n=1}^{\infty}[\text{erf}\frac{z_{mD}+(2nL_D+5h_D)}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+(2nL_D+4h_D)}{\sqrt{4t_D}}$$

$$+ \text{erf}\frac{z_{mD}+(2nL_D-4h_D)}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+(2nL_D-5h_D)}{\sqrt{4t_D}}$$

$$+ \text{erf}\frac{z_{mD}-(2nL_D+4h_D)}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}-(2nL_D+5h_D)}{\sqrt{4t_D}}$$

$$+ \text{erf}\frac{z_{mD}-(2nL_D-5h_D)}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}-(2nL_D-4h_D)}{\sqrt{4t_D}}]\}dt_D \qquad (24)$$

Again, eq. 24 can be expressed as follows:

$$\Delta p^i_{D_3}(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{iD}/4t_D]}{t_D}\cdot$$

$$\cdot\{\sum_{n=-\infty}^{+\infty}(\text{erf}\frac{z_{mD}+2nL_D+5h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+2nL_D+4h_D}{\sqrt{4t_D}}$$

$$+ \text{erf}\frac{z_{mD}+2nL_D-4h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+2nL_D-5h_D}{\sqrt{4t_D}})\}dt_D \qquad (24a)$$

These equations for Δp^i_D's should be written for all real slots which consist one row. Now, considering the staggered rows "j" , one gets the the pressure drop caused by a slot which is the nearest to the upper boundary (Pig. 4) :

$$\Delta p^j_{D_1}(M,t_D) = \frac{1}{4\cdot NS}\sum_{j=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{jD}/4t_D]}{t_D}$$

$$\cdot\{\mathrm{erf}\frac{z_{mD}+2h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+h_D}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}-h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}-2h_D}{\sqrt{4t_D}}$$

$$+ \sum_{n=1}^{\infty}[\mathrm{erf}\frac{z_{mD}+(2nL_D+2h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+(2nL_D+h_D)}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}+(2nL_D-h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+(2nL_D-2h_D)}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}-(2nL_D+h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}-(2nL_D+2h_D)}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}-(2nL_D-2h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}-(2nL_D-h_D)}{\sqrt{4t_D}}]\}dt_D \qquad (25)$$

Eq. 25 can be written in the following manner:

$$\Delta p^j_{D_1}(M,t_D) = \frac{1}{4\cdot NS}\sum_{j=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{jD}/4t_D]}{t_D}\cdot$$

$$\cdot\{\sum_{n=-\infty}^{+\infty}(\mathrm{erf}\frac{z_{mD}+2nL_D+2h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D+h_D}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}+2nL_D-h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D-2h_D}{\sqrt{4t_D}})\}dt_D \qquad (25a)$$

Moving to a lower slot, just beneath the first one, we obtain:

$$\Delta p^j_{D_2}(M,t_D) = \frac{1}{4\cdot NS}\sum_{j=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{jD}/4t_D]}{t_D}\cdot$$

$$\cdot\left\{\operatorname{erf}\frac{z_{mD}+4h_D}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}+3h_D}{\sqrt{4t_D}}\right.$$

$$+\operatorname{erf}\frac{z_{mD}-3h_D}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}-4h_D}{\sqrt{4t_D}}$$

$$+\sum_{n=1}^{\infty}\left[\operatorname{erf}\frac{z_{mD}+(2nL_D+4h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}+(2nL_D+3h_D)}{\sqrt{4t_D}}\right.$$

$$+\operatorname{erf}\frac{z_{mD}+(2nL_D-3h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}+(2nL_D-4h_D)}{\sqrt{4t_D}}$$

$$+\operatorname{erf}\frac{z_{mD}-(2nL_D+3h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}-(2nL_D+4h_D)}{\sqrt{4t_D}}$$

$$\left.\left.+\operatorname{erf}\frac{z_{mD}-(2nL_D-4h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}-(2nL_D-3h_D)}{\sqrt{4t_D}}\right]\right\}dt_D \qquad (26)$$

Eq. 26 can be expressed as:

$$\Delta p^{j}_{D_2}(M,t_D) = \frac{1}{4\cdot NS}\sum_{n=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{jD}/4t_D]}{t_D}\cdot$$

$$\cdot\left\{\sum_{n=-\infty}^{+\infty}\left(\operatorname{erf}\frac{z_{mD}+2nL_D+4h_D}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}+2nL_D+3h_D}{\sqrt{4t_D}}\right.\right.$$

$$\left.\left.+\operatorname{erf}\frac{z_{mD}+2nL_D-3h_D}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}+2nL_D-4h_D}{\sqrt{4t_D}}\right)\right\}dt_D \qquad (26a)$$

and so on, going down and writing all expressions for the various slots, until reaching the last one which is adjacent to the lower boundary (Fig. 4) :

$$\Delta p^j_{D_{Last}}(M,t_D) = \frac{1}{4\cdot NS} \sum_{j=1}^{NS} \int_0^{t_D} \frac{\exp[-r^2_{jD}/4t_D]}{t_D}$$

$$\cdot\{\operatorname{erf}\frac{z_{mD}-(L_D-h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}-(L_D+h_D)}{\sqrt{4t_D}}$$

$$+ \sum_{n=1}^{\infty} [\operatorname{erf}\frac{z_{mD}+((2n-1)L_D+h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}+((2n-1)L_D-h_D)}{\sqrt{4t_D}}$$

$$+ \operatorname{erf}\frac{z_{mD}-((2n+1)L_D-h_D)}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}-((2n+1)L_D+h_D)}{\sqrt{4t_D}}]\}dt_D \qquad (27)$$

The above equation can be written in a condensed form **as**:

$$\Delta p^j_{D_{Last}}(M,t_D) = \frac{1}{4\cdot NS} \sum_{j=1}^{NS} \int_0^{t_D} \frac{\exp[-r^2_{jD}/4t_D]}{t_D}\cdot$$

$$\cdot\{ \sum_{n=-\infty}^{+\infty} (\operatorname{erf}\frac{z_{mD}+(2n-1)L_D+h_D}{\sqrt{4t_D}} - \operatorname{erf}\frac{z_{mD}+(2n-1)L_D-h_D}{\sqrt{4t_D}})\}dt_D \qquad (27a)$$

Again, **this** summation **of** Δp_D**'s** should be carried **on** until **the** last real slot "j" is considered.

The total dimensionless pressure drop will be then:

$$p_D(M,t_D) = \Delta p^i_{D_1} + \Delta p^i_{D_2} + \Delta p^i_{D_3} + \cdots + \Delta p^j_{D_1} + \Delta p^j_{D_2} + \Delta p^j_{D_3} + \cdots \qquad (28)$$

Some important facts should be noticed in eqs. 22-27: the summations of all error functions inside the integrals are constant for all vertical rows "i" and rows "j". Therefore, this summation **is** done only once. However, these constants are different for the vertical rows "i" and "j". There **is** a second way to calculate the total pressure drop. **In** this way the contribution **of** all slots in one vertical row **is** calculated before considering another row, while in the present method the contribution to the pressure drop of all slots which have the same elevation in the vertical rows "i" and "j" is calculated first. As a matter of fact, the computer program (App. A) was compiled using the second method ; handling first all real and image slots in vertical row and then considering the other rows. The second method was chosen only because of convenience (see also section 2.3, eq. 33). Note also that with a alight modification, the system on Fig. 4 could be reduced to a more basic unit consisting of only one i and one j slots.

2.2 Short-Time Behaviour

In Studying the short-time behaviour of eq. 28, three cases were considered:

i) $0<z_{mD}<h_D$ or $2h_D<z_{mD}<3h_D$ or $4h_D<z_{mD}<5h_D$, etc. This means that the elevation of the considered point is between the lower and upper tips of one of slots "i".

ii) $h_D<z_{mD}<2h_D$ or $3h_D<z_{mD}<4h_D$ or $5h_D<z_{mD}<6h_D$, etc., i.e., the elevation of the considered point is between the. lower and upper tips of one of slots "j".

iii) $z_{mD}=h_D$ or $z_{mD}=2h_D$ or $z_{mD}=3h_D$, etc.

'Case (i): for $t_D \to 0$ and $0<z_{mD}<h_D$

$$\operatorname{erf} \frac{z_{mD}+h_D}{\sqrt{4t_D}} \to 1$$

$$\operatorname{erf} \frac{z_{mD}-h_D}{\sqrt{4t_D}} \to -1$$

Then considering eq. 22:

$$\Delta p^i_{D_1}(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{iD}/4t_D]}{t_D}\cdot\{1-(-1)+\underline{0}\}dt_D$$

$$=\frac{1}{2\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{iD}/4t_D]}{t_D}\cdot dt_D \qquad (29)$$

changing variable:

$$u=r_{iD}{}^2/4t_D \ , \quad du = -\frac{r^2_{iD}\cdot dt_D}{4t^2_D}$$

$$\Delta p^i_{D_1}(M,t_D) = \frac{1}{2\cdot NS}\sum_{i=1}^{NS}\int_{\infty}^{r^2_{iD}/4t_D}\frac{-\exp(-u)}{u}\,du$$

$$= -\frac{1}{2\cdot NS}\sum_{i=1}^{NS} Ei(-r^2_{iD}/4t_D) \qquad (29a)$$

where $-Ei(-x)$ the Exponential integral (see Nomenclature). It can easily **be** demonstrated that all other pressure drops(i.e. **eq.** 23-27) at early times are zero. In this case:

$$p_D(M,t_D) = \Delta p^i_{D_1}(M,t_D) = -\frac{1}{2\cdot NS}\sum_{i=1}^{NS} Ei(-r^2_{iD}/4t_D) \qquad (29b)$$

By the same course **of** manipulation , if

$2h_D<z_{mD}<3h_D$, one gets for $t_D\rightarrow 0$:

$$\mathrm{erf}\,\frac{z_{mD}-2h_D}{\sqrt{4t_D}}\rightarrow 1$$

$$\text{erf}\ \frac{z_{mD}-3h_D}{\sqrt{4t_D}} \rightarrow -1$$

Then, considering eq. 23:

$$\Delta p^{i}_{D_2}(M,t_D) = \frac{1}{4 \cdot NS} \sum_{i=1}^{NS} \int_0^{t_D} \frac{\exp\left[-r_{iD}^2/4t_D\right]}{t_D} \cdot$$

$$\cdot \left\{1-1+1-(-1)+ \underline{0} \right\} dt_D \qquad (30)$$

or:

$$\Delta p^{i}_{D_2}(M,t_D) = -\frac{1}{2 \cdot NS} \sum_{i=1}^{NS} Ei(-r_{iD}^2/4t_D) \qquad (30a)$$

All other summations **of** the error functions in eqs. 22,24-27 becom zero, and again eq. 28 is given by:

$$p_D(M,t_D) = \Delta p^{i}_{D_2}(M,t_D) = -\frac{1}{2 \cdot NS} \sum_{i=1}^{NS} Ei(-r_{iD}^2/4t_D) \quad (30b)$$

Case (ii)

By the same reasoning as in the previous case, it can be easily shown that eq. 28 becomes (for early times) :

$$p_D(M,t_D) = -\frac{1}{2 \cdot NS} \sum_{j=1}^{NS} Ei(-r_{jD}^2/4t_D) \qquad (31)$$

Case (iii)

In this case, **eq**. 28 simply becomes:

$$p_D(M,t_D) = -\frac{1}{4\cdot NS}\sum_{i=1}^{NS} Ei(-r_{iD}^2/4t_D) - \frac{1}{4\cdot NS}\sum_{j=1}^{NS} Ei(-r_{jD}^2/4t_D) \quad (32)$$

Since for $z_{mD}=h_D$, $\text{erf}\frac{z_{mD}-h_D}{\sqrt{4t_D}} = 0$ and $\text{erf}\frac{z_{mD}-h_D}{\sqrt{4t_D}} \to -1$

2.3 Long-Time Behaviour

As mentioned at the end of section 2.1 the total pressure drop can also be calculated by handling firstly the effects of all slots in one row and then considering other vertical rows. Therefore, eq. 28 can be written in the following way:

$$p_D(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r_{iD}^2/4t_D]}{t_D}\cdot$$

$$\cdot\{\sum_{n=-\infty}^{+\infty}(\text{erf}\frac{z_{mD}+2nL_D+h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+2nL_D-h_D}{\sqrt{4t_D}})$$

$$+\sum_{n=-\infty}^{+\infty}[\text{erf}\frac{z_{mD}+2nL_D+3h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+2nL_D+2h_D}{\sqrt{4t_D}}$$

$$+\text{erf}\frac{z_{mD}+2nL_D-2h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+2nL_D-3h_D}{\sqrt{4t_D}}]$$

$$+\sum_{n=-\infty}^{+\infty}[\text{erf}\frac{z_{mD}+2nL_D+5h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+2nL_D+4h_D}{\sqrt{4t_D}}$$

$$+\text{erf}\frac{z_{mD}+2nL_D-4h_D}{\sqrt{4t_D}} - \text{erf}\frac{z_{mD}+2nL_D-5h_D}{\sqrt{4t_D}}]$$

$$+\cdots\cdots+\cdots\cdots\}dt_D$$

$$+\frac{1}{4\cdot NS}\sum_{j=1}^{NS}\int_0^{t_D}\frac{\exp[-r_{jD}^2/4t_D]}{t_D}\cdot$$

$$\cdot\{\sum_{n=-\infty}^{+\infty}(\mathrm{erf}\frac{z_{mD}+2nL_D+2h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D+h_D}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}+2nL_D-h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D-2h_D}{\sqrt{4t_D}})$$

$$+ \sum_{n=-\infty}^{+\infty}(\mathrm{erf}\frac{z_{mD}+2nL_D+4h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D+3h_D}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}+2nL_D-3h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D-4h_D}{\sqrt{4t_D}})$$

$$+\cdots\cdot+\sum_{n=-\infty}^{+\infty}(\mathrm{erf}\frac{z_{mD}+((2n-1)L_D+h_D)}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+((2n-1)L_D-h_D)}{\sqrt{4t_D}})\}dt_D \qquad (33)$$

Eq. 33 is the second way of calculation of the total dimensionless pressure drop, and the computer program is based on this equation (see App. A).

Let us now examine the first summation of the error function in eq. 33:

$$\sum_{n=-\infty}^{+\infty}(\mathrm{erf}\frac{z_{mD}+2nL+h}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D-h_D}{\sqrt{4t_D}}) \qquad (34)$$

By changing variable, eq. 34 becomes:

$$\sum_{n=-\infty}^{+\infty}(\mathrm{erf}\frac{z_{mD}+2nL+h}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D-h_D}{\sqrt{4t_D}} =$$

$$= \frac{2}{\sqrt{\pi}} \sum_{n=-\infty}^{+\infty} \int_{\frac{z_{mD}+2nL_D-h_D}{\sqrt{4t_D}}}^{\frac{z_{mD}+2nL_D+h_D}{\sqrt{4t_D}}} e^{-u^2} du =$$

$$= \frac{1}{\sqrt{\pi t_D}} \sum_{n=-\infty}^{+\infty} \int_{-h_D}^{h_D} \exp[- \frac{(z_{mD}-z_D+2nL_D)^2}{4t_D}] dz_D \qquad (34a)$$

The above change of variable is done by letting

$$u = \frac{z_{mD}-z_D+2nL_D}{\sqrt{4t_D}}$$

Eq. 34a can be written in a different way as:

$$\frac{1}{\sqrt{\pi t_D}} \sum_{n=-\infty}^{+\infty} \int_{-h_D}^{h_D} \exp[- \frac{(z_{mD}-z_D+2nL_D)^2}{4t_D}] dz_D =$$

$$= \frac{1}{\sqrt{\pi t_D}} \int_{-h_D}^{h_D} \sum_{n=-\infty}^{+\infty} \exp[- \frac{(z_D-z_{mD}+2nL_D)^2}{4t_D}] dz_D \qquad (35)$$

From Poisson's summation formula[6] :

$$\sum_{n=-\infty}^{+\infty} \exp[- \frac{(z_D-z_{mD}+2nL_D)^2}{4t_D}] =$$

$$= \frac{\sqrt{\pi t_D}}{L_D} \{1+ 2 \sum_{n=1}^{\infty} \cos \frac{n\pi(z_D-z_{mD})}{L_D} \cdot \exp[- \frac{n^2\pi^2 t_D}{L_D^2}]\}$$

Therefore:

$$\sum_{n=-\infty}^{+\infty}\left(\operatorname{erf}\frac{z_{mD}+2nL_D+h_D}{\sqrt{4t_D}}-\operatorname{erf}\frac{z_{mD}+2nL_D-h_D}{\sqrt{4t_D}}\right)=$$

$$=\frac{1}{L_D}\left\{2h_D+2\sum_{n=1}^{\infty}\exp\left[-\frac{n^2\pi^2t_D}{L_D^2}\right]\cdot\int_{-h_D}^{h_D}\cos\frac{n\pi(z_D-z_{mD})}{L_D}\,dz_D\right\} \tag{36}$$

and:

$$\int_{-h_D}^{h_D}\cos\frac{n\pi(z_D-z_{mD})}{L_D}\,dz_D=\frac{L_D}{n\pi}\left[\sin\left(n\pi\frac{h_D-z_{mD}}{L_D}\right)+\sin\left(n\pi\frac{h_D+z_{mD}}{L_D}\right)\right]$$

$$=\frac{2L_D}{n\pi}\sin\frac{n\pi}{L_D}h_D\cdot\cos\frac{n\pi}{L_D}z_D$$

Hence, eq. 36 becomes:

$$\sum_{n=-\infty}^{+\infty}\left(\operatorname{erf}\frac{z_{mD}+2nL_D+h_D}{\sqrt{4t_D}}-\operatorname{erf}\frac{z_{mD}+2nL_D-h_D}{\sqrt{4t_D}}\right)=$$

$$=\frac{1}{L_D}\left\{2h_D+\frac{4L_D}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left[-\frac{n^2\pi^2t_D}{L_D^2}\right]\sin\frac{n\pi}{L_D}h_D\cdot\cos\frac{n\pi}{L_D}z_{mD}\right\} \tag{37}$$

In the same way, it is not difficult to demons'trate that the second summation of the error function in eq. 33 can be expressed as:

$$\sum_{n=-\infty}^{+\infty}\left(\operatorname{erf}\frac{z_{mD}+2nL_D+3h_D}{\sqrt{4t_D}}-\operatorname{erf}\frac{z_{mD}+2nL_D-3h_D}{\sqrt{4t_D}}\right)=$$

$$=\frac{1}{L_D}\left\{6h_D+\frac{4L_D}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left[-\frac{n^2\pi^2t_D}{L_D^2}\right]\sin\frac{n\pi}{L_D}3h_D\cdot\cos\frac{n\pi}{L_D}z_{mD}\right\} \tag{38}$$

and:

$$\sum_{n=-\infty}^{+''} (\mathrm{erf}\frac{z_{mD}+2nL_D-2h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D+2h_D}{\sqrt{4t_D}} =$$

$$= - \frac{1}{L_D}\{4h_D + \frac{4L_D}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp[- \frac{n^2\pi^2 t_D}{L_D^2}] \sin\frac{n\pi}{L_D}2h_D \cdot \cos\frac{n\pi}{L_D}z_{mD}\} \qquad (39)$$

So, combining eqs. 37+38 we obtain:

$$\sum_{n=-\infty}^{+\infty} (\mathrm{erf}\frac{z_{mD}+2nL_D+3h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D+2h_D}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}+2nL_D-2h_D}{f4t_D} - \mathrm{erf}\frac{z_{mD}+2nL_D-3h_D}{\sqrt{4t_D}}) =$$

$$= \frac{1}{L_D}\{2h_D + \frac{4L_D}{\pi} \sum_{n=1}^{w} \frac{1}{n} \exp(- \frac{n^2\pi^2 t_D}{L_D^2})[\sin \frac{n\pi}{L_D} 3h_D - \sin\frac{n\pi}{L_D}2h_D] \cdot$$

$$\cdot \cos\frac{n\pi}{L_D}z_{mD}\} \qquad (40)$$

In a similar way the next summation of error functions in eq. 33 is expressed as:

$$\sum_{n=-\infty}^{+\infty} (\mathrm{erf}\frac{z_{mD}+2nL_D+5h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{zmD+2nLD+4h_D}{\sqrt{4t_D}}$$

$$+ \mathrm{erf}\frac{z_{mD}+2nL_D-4h_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nL_D-5h_D}{\sqrt{4t_D}}) =$$

$$= \frac{1}{L_D}\{2h_D + \frac{4L_D}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp(- \frac{n^2\pi^2 t_D}{L_D^2})[\sin\frac{n\pi}{L_D}5h_D - \sin\frac{n\pi}{L_D}4h_D] \cdot$$

$$\cdot \cos\frac{n\pi}{L_D} z_{mD} \Big\} \qquad (41)$$

Therefore, the pressure drop created at point M by all **rows** "i" can be written as:

$$\Delta p^{i}_{D_{Total}}(M,t_D) = 1 \sum_{i=1}^{NS} \int_0^{t_D} \frac{\exp(-r^2_{iD}/4t_D)}{t_D} \cdot$$

$$\cdot \Big\{ \frac{1}{L_D}\Big(2k \cdot h_D + \frac{L_D}{n} \sum_{n=1}^{\infty} \frac{1}{n} \exp\Big(-\frac{n^2\pi^2 t}{L_D^2}\Big) \Big[\Big\{\Big(\sin\frac{n\pi}{L_D} h_D + \sin\frac{n\pi}{L_D} 3h_D$$

$$+ \sin\frac{n\pi}{L_D} 5h_D + \cdots + \sin\frac{n\pi}{L_D}(2k-1)h_D\Big) - \Big(\sin\frac{n\pi}{L_D} 2h_D + \sin\frac{n\pi}{L_D} 4h_D$$

$$+ \cdots + \sin\frac{n\pi}{L_D}(2k-1)h_D\Big)\Big\} \cdot \cos\frac{n\pi}{L_D} z_{mD}\Big]\Big)\Big\} dt_D \qquad (42)$$

Where **k** *is* the number **of** slots in one vertical row and $2k \cdot h_D$ equal to L_D.

It is well known that:

$$\sin(x)+\sin(3x)+\sin(5x)+\ldots\ldots+\sin((2\ell-1)x) = \frac{\sin^2(\ell x)}{\sin(x)}$$

Also: $$\sin(0)+\sin(2x)+\sin(4x)+\ldots\ldots+\sin((2\ell-2)x) = $$

$$= \frac{\sin(\ell x)\cdot\sin((\ell-1)x)}{\sin(x)}$$

and: $$\sin(3x)+\sin(5x)+\ldots\ldots+\sin((2\ell+1)x) = $$

$$= \frac{\sin(\ell x)\cdot\sin((\ell+2)x)}{\sin(x)}$$

So, **eq**. 42 is written in the following form:

$$\Delta p^{i}_{D_{Total}}(M,t_D) = \frac{1}{4\cdot NS} \sum_{i=1}^{NS} \int_0^{t_D} \frac{\exp[-r^2_{iD}/4t_D]}{t_D} \cdot$$

$$\cdot\left\{1+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2 t_D}{L_D^2}\right)\left[\frac{2\sin\left(\frac{n\pi}{L_D}h_D\cdot k\right)\cdot\sin\left(\frac{n\pi}{2L_D}h_D\right)\cdot\cos\left(\frac{n\pi}{L_D}h_D\frac{2k-1}{2}\right)}{\sin\frac{n\pi}{L_D}h_D}\right]\right.$$

$$\left.\cdot\cos\frac{n\pi}{L_D}z_{mD}\right\}dt_D \qquad (42a)$$

It can easily be shown that the pressure drop created at point M by all vertical rows "j" is given by:

$$\Delta p^j_{D_{Total}}(M,t_D) = \frac{1}{4\cdot NS}\sum_{j=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{jD}/4t_D]}{t_D}\cdot$$

$$\cdot\left\{1+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2 t_D}{L_D^2}\right)\left[\left(\sin\frac{n\pi}{L_D}0h_D+\sin\frac{n\pi}{L_D}2h_D+\sin\frac{n\pi}{L_D}4h_D+\cdots\right)\right.\right.$$

$$\left.\left.-\left(\sin\frac{n\pi}{L_D}3h_D+\sin\frac{n\pi}{L_D}5h_D+\cdots\right)\right]\cdot\cos\frac{n\pi}{L_D}z_{mD}\right\}dt_D \qquad (43)$$

or:

$$\Delta p^j_{D_{Total}}(M,t_D) = \frac{1}{4\cdot NS}\sum_{j=1}^{NS}\int_0^{t_D}\frac{\exp[-r^2_{jD}/4t_D]}{t_D}\cdot$$

$$\cdot\left\{1+\frac{4}{\pi}\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2 t_D}{L_D^2}\right)\left[\frac{\sin\left(\frac{n\pi}{L_D}h_D\cdot k\right)\cdot\sin\left(\frac{n\pi}{L_D}h_D(k-1)\right)}{\sin\frac{n\pi}{L_D}h_D}\right.\right.$$

$$\left.\left.-\frac{\sin\left(\frac{n\pi}{L_D}h_D\cdot k\right)\cdot\sin\left(\frac{n\pi}{L_D}h_D(k+2)\right)}{\sin\frac{n\pi}{L_D}h_D}\right]\cdot\cos\frac{n\pi}{L_D}z_{mD}\right\}dt_D \qquad (43a)$$

Again, **k** is the number of slots in one vertical row.

Eq. 43a can be further simplified to:

$$\Delta p_{D_{Total}}(M,t_D) = \frac{1}{4\cdot NS}\sum_{j=1}^{NS}\int_0^{t_D}\frac{\exp[-r_{jD}^2/4t_D]}{t_D}\cdot$$

$$\cdot\left\{1+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2 t_D}{L_D^2}\right)\left[\frac{2\sin(\frac{n\pi}{L_D}h_D\cdot k)\cdot\sin(-\frac{2n\pi}{2L_D}h_D)\cdot\cos(\frac{n\pi}{L_D}h_D\frac{2k+1}{2})}{\sin\frac{n\pi}{L_D}h_D}\right]\right.$$

$$\left.\cdot\cos\frac{n\pi}{L_D}z_{mD}\right\}dt_D \qquad (43b)$$

Adding eq. 43b to eq. 42a we obtain the total pressure drop created at point M by all rows of slots in the liner, **as was** previously written:

$$\Delta p_D(M,t) = \Delta p^i_{D_{Total}}(M,t_D) + \Delta p^j_{D_{Total}}(M,t_D) \qquad (28a)$$

It will be convenient to define the functions:

$$R^i(r_{iD},t_D) = \frac{\exp[-r_{iD}^2/4t_D]}{t_D} \qquad (44)$$

$$R^j(r_{jD},t_D) = \frac{\exp[-r_{jD}^2/4t_D]}{t_D} \qquad (45)$$

$$Z^i(t_D/L_D^2, z_{mD}, h_D, k) = 1+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2 t_D}{L_D^2}\right)\cdot$$

$$\cdot\left\{\frac{2\sin(\frac{n\pi}{L_D}h_D\cdot k)\cdot\sin(\frac{n\pi}{2L_D}h_D)\cdot\cos(\frac{n\pi}{L_D}h_D\frac{2k+1}{2})}{\sin\frac{n\pi}{L_D}h_D}\right\}\cdot\cos\frac{n\pi}{L_D}z_{mD} \qquad (46)$$

$$Z^j(t_D/L_D^2, z_{mD}, h_D, k) = 1 + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \exp\left(-\frac{n^2\pi^2 t_D}{L_D^2}\right) \cdot$$

$$\cdot \left\{ \frac{2\sin\left(\frac{n\pi}{L_D}h_D \cdot k\right) \cdot \sin\left(-\frac{3n\pi}{2L_D}h_D\right) * \cos\left(\frac{n\pi}{L_D}h_D \frac{2k+1}{2}\right)}{\sin\frac{n\pi}{L_D}h_D} \right\} \cdot \cos\frac{n\pi}{L_D} z_{mD} \qquad (47)$$

Combination of eqs. 28a, 42a, 43b, 44-47 yields:

$$p_D(M, t_D) = \frac{1}{4 \cdot NS}\left[\sum_{i=1}^{NS} \int_0^{t_D} R^i(r_{iD}, t_D) \cdot Z^i(t_D/L_D^2, z_{mD}, h_D, k)\, dt_D\right.$$

$$\left. + \sum_{j=1}^{NS} \int_0^{t_D} R^j(r_{jD}, t_D) \cdot Z^j(t_D/L_D^2, z_{mD}, h_D, k)\, dt_D \right] \qquad (48)$$

It is well known that the line source solution is expressed by the formula:

$$p_D(r_D, t_D) = -\frac{1}{2} Ei(-r_D^2/4t_D) \qquad (49)$$

Where $-Ei(-r_D^2/4t_D)$ is the Exponential integral.
For $t_D/r_D^2 > 70$, the line source solution can be approximated within one percent error by[14]:

$$p_D(r_D, t_D) \approx \frac{1}{2}\left(\ln\frac{t_D}{r_D^2} + 0.80907\right) \qquad (50)$$

By adding and substracting the line source solution on the right hand side of eq. *48*, we obtain:

$$p_D(M,t_D) = \frac{1}{4\cdot NS}\Big[\sum_{n=1}^{NS} \int_0^{t_D} R^i(r_{iD},t_D)\cdot Z^i(t_D/L_D^2,z_{mD},h_D,k)dt_D$$

$$+ \sum_{j=1}^{NS} \int_0^{t_D} R^j(r_{jD},t_D)\cdot Z^j(t_D/L_D^2,z_{mD},h_D,k)dt_D\Big]$$

$$+ p_D(r_D,t_D) - p_D(r_D,t_D) \qquad (51)$$

Let us define the function:

$$s(r_{iD},r_{jD},z_{mD},t_D,h_D,L_D,k,NS) = - p_D(r_D,t_D)$$

$$+ \frac{1}{4\cdot NS}\Big[\sum_{i=1}^{NS} \int_0^{t_D} R^i(r_{iD},t_D)\cdot Z^i(t_D/L_D^2,z_{mD},h_D,k)dt_D$$

$$+ \sum_{j=1}^{NS} \int_0^{t_D} R^j(r_{jD},t_D)\cdot Z^j(t_D/L_D^2,z_{mD},h_D,k)dt_D\Big] \qquad (52)$$

The function s^* is the so-called pseudo-skin factor due to analogy with the skin factor defined by Van Everdingen[16] and Hurst[10]. Note that R^i, R^j and hence s are implicitly functions of θ. Eq. 51 can be therefore written as:

$$p_D(M,t_D) = p_D(r_D,t_D) + s(r_{iD},r_{jD},z_{mD},t_D,h_D,k,NS) \qquad (53)$$

As shown by eq. 53, the dimensionless pressure drop created at point M due to production through a liner, can be expressed as the sum of two terms, the first one of which is the line source solution and the second is the pseudo-skin function, s. The pseudo-skin function is a function of dimensionless time, but it will be shown that this function can be apprroximated by

* The dimensionless thickness, L_D, can be expressed as: $L_D = 2kh_D$

a constant for large values of time.

By definition:

$$R^i(r_{iD},t_D) = \frac{\exp[- r_{iD}/4t_D]}{t_D}$$

$$R^j(r_{jD},t_D) = \frac{\exp[- r_{jD}^2/4t_D]}{t_D}$$

Expansion of the exponential function gives:

$$e^{-x} = 1 - x + x^2 + 0(x)$$

so:

$$R^i(r_{iD},t_D) = \frac{1}{t_D}[1 - (r_{iD}^2/4t_D) + (r_{iD}^4/16t_D^2) + 0(1/t_D^3)]$$

$$R^j(r_{jD},t_D) = \frac{1}{t_D}[1 - (r_{jD}^2/4t_D) + (r_{jD}^4/16t_D^2) + 0(1/t_D^3)]$$

We can approximate the R functions by the first term within one percent error if:

$$\frac{1}{t_D} > \text{Max} \begin{cases} 25r_{iD}^2/t_D^2 \\ 25r_{jD}^2/t_D^2 \end{cases}$$

or:

$$t_D > \text{Max} \begin{cases} 25r_{iD}^2 \\ 25r_{jD}^2 \end{cases}$$

In that case, the R functions can be approximated by:

$$R^i(r_{iD},t_D) \cong 1/t_D \tag{54}$$

$$R^j(r_{jD},t_D) \cong 1/t_D \tag{55}$$

Now, considering the Z functions defined by eqs. 46, 47, we notice that the value of each term of the infinite series decreases rapidly when t_D increases, and the Z functions will be different from unity by less than 1% as soon as:

$$t_D > \frac{5L_D^2}{\pi}, \quad \text{(since for } \exp\left[-\frac{{}^2 t_D}{L_D^2}\right] < 0.01 \rightarrow t_D > \frac{4.61 \cdot L_D^2}{\pi}$$

For large values of t_D, the pseudo-skin function can be approximated by using eqs. 50,54,55:

$$s(r_{iD},r_{jD},z_{mD},t_D,h_D,L_D,k,NS) = -\tfrac{1}{2}(\ln(t_D/r_D^2) + 0.80907)$$

$$+ \frac{1}{4\cdot NS}\Big[\sum_{i=1}^{NS} \int_0^{t_{D1}} R^i(r_{iD},t_D)\cdot Z^i(t_D/L_D^2,z_{mD},h_D,k)\,dt_D$$

$$+ \sum_{j=1}^{NS} \int_0^{t_{D1}} R^j(r_{jD},t_D)\cdot Z^j(t_D/L_D^2,z_{mD},h_D,k)\,dt_D\Big]$$

$$+ \frac{1}{4\cdot NS}\Big[\sum_{i=1}^{NS} \int_{t_{D1}}^{t_D} R^i(r_{iD},t_D)\cdot Z^i(t_D/L_D^2,z_{mD},h_D,k)\,dt_D$$

$$+ \sum_{j=1}^{NS} \int_{t_{D1}}^{t_D} R^j(r_{jD},t_D)\cdot Z^j(t_D/L_D^2,z_{mD},h_D,k)\,dt_D\Big] =$$

$$= \frac{1}{4\cdot NS}\Big[\sum_{i=1}^{NS} \int_0^{t_{D1}} R^i(r_{iD},t_D)\cdot Z^i(t_D/L_D^2,z_{mD},h_D,k)\,dt_D$$

$$+ \sum_{j=1}^{NS} \int_0^{t_{D1}} R^j(r_{jD},t_D)\cdot Z^j(t_D/L_D^2,z_{mD},h_D,k)\,dt_D\Big]$$

$$+ \frac{2\cdot NS}{4\cdot NS} \int_{t_{D1}}^{t_D} \frac{1}{t_D}\,dt_D - \frac{1}{2}(\ln t_D/r_D^2 + 0.80907) \tag{56}$$

Eq. 56 can further be simplified to:

$$s(r_{iD},r_{jD},z_{mD},t_D,h_D,L_D,k,NS) =$$

$$= \frac{1}{4\cdot NS}\Big[\sum_{i=1}^{NS} \int_0^{t_{D1}} R^i(r_{iD},t_D)\cdot Z^i(t_D/L_D^2)dt_D$$

$$+ \sum_{j=1}^{NS} \int_0^{t_{D1}} R^j(r_{jD},t_D)\cdot Z^j(t_D/L_D^2,z_{mD},h_D,k)dt_D\Big]$$

$$- \frac{1}{2}(\ln\frac{t_{D1}}{r_D^2} + 0.80907) \qquad (57)$$

where:

$$t_{D1} > \text{Max} \begin{cases} 70r_D^2 \\ 25r_{iD}^2 \\ 25r_{jD}^2 \\ 5L_D^2/\pi^2 \end{cases}$$

As can be seen from eq. 57, the pseudo-skin function becomes a constant within one percent error at large values of t_D ($t_D > t_{D_1}$). The values of the pseudo-skin factor are printed on Tables B-1÷ B-31.

3. COMPUTATIONAL PROCEDURE AND NUMERICAL RESULTS

The solution has been evaluated from eq. 33 using Simpson's rule for integration purposes.

The computer program was compiled using the following parameters as entry values (see also App. A) :
Reservoir thickness, slot length, number of pairs of staggered vertical rows of slots (this is also the number of slots in one ring), well radius, dimensionless distance of point for which the pressure drop is calculated, elevation of point M and its orientation (angle = θ) with respect to the origin of X-Y coordinates and a tolerance (for convergence purposes). The dimensionless time should also be specified.

The program considers penetration ratio of 0.5 , i.e., the total cumulative lengths of slots in one vertical row *is* half of the formation thickness.

For representation of results, the elevation of point M was located at the center of formation and its position is between two vertical rows of "i" slots (for $r_D=1$, the position is half of the circumferencial distance).

Burns[2] also uses "observation height" as halfway between top and bottom of the interval, while Muskat[11] concluded that the exact steady-state wellbore pressure for a partially penetrating well could be obtained within ½% accuracy (assuming a uniform flux distribution at the wellbore) by computing the pressure at a location 3/4 of the distance from the top of the layer to the bottom of the well. The calculations were also made by using 10 intervals of dimensionless time Δt_D per log cycle, starting from $\Delta t_D = 0.001$. Accuracy was checked (with satisfactory results) in the following manner:

1) For $r_D = 0$, the results were compared with the semi-analytical solution given in Table 1, which was derived by Gringarten and Ramey[a]. This solution is numerically equal to the line source solution. No difference is noticed between the results presented in Table 2 and Table 1. The line source is always obtained for $r_D = 0$, no matter what is the number of vertical rows of slots. The limit case is, of course, for infinite number of vertical rows ($NS \to \infty$) which is the surface cylinder solution (see also Fig. 5, on which the line source solution was plotted for $r_D = 0$).

2) For $r_D = 0.5$, 6 pairs of staggered vertical rows were sufficient to reproduce to within 5 digits accuracy the results of the semi-analytical solution for the surface cylinder presented in Table 1. The reproduced results are shown in Table 3 and on Fig. 6.

3) For $r_D = 2$, again for $NS = 6$, there is agreement within 5 digits accuracy of the results given in Table 4 and the solution presented in Table 1 (see also Fig. 7)

4) For $r_D = 1$, this case was the last one used to check the validity of the computer program. No complete agreement for this case could be obtained with the surface cylinder solution no matter what was the number of vertical rows of slots, probably since it is a limit case. However, for 30 pairs of vertical rows

(or 30 slots in each ring of slots), for $t_D=3$ and **on**, the difference between the surface cylinder solution given in Table 1 and the obtained results which are shown in Table **5**, is only −0.0022. This difference is constant and therefore becomes negligible later **on** (see also Fig. **5**, and App. A for more details).

3.1 Computer Runs

After the validity of the computer program was checked, the following cases were run:

i) Firstly, the number of rows had to be specified. All cases were run with 4 different pairs of staggered vertical rows of slots (=NS), namely 1 pair, **2**, **3**, 6 and 12 pairs. The 1 pair case was run for the sake of representation; since the usual practice is to specify **two** rows for each nominal pipe diameter[4] and the smallest one is **of** 2 3/8" which requires more than 1 pair of rows. However, for $r_D=1$, additional cases of 4,8 and 10 pairs **also** considered.

ii) Secondly, the influence of slot length was checked. Four different cases of dimensionless slots lengths were run, i.e. 0.2, 0.4167, 0.5714 and 1.0. Because the cumulative lengths **of** slots in one row should be always (as imposed) one half of the formation thickness, the shorter the slots become, their number **in** one row is increased (see Tables B-1 to B-23).

iii) Thirdly, dimensionless pressure drops were obtained for various dimensionless distances which were: $r_D=0$ (representing the line source case), $r_D=0.5$ and $r_D=1$ **(on** the surface) and $r_D=2$.

iv) Variation of formation thickness was also checked (but not shown;see section 3.2).

Three facts should be noticed: almost all cases were run by considering an elevation of a point at the center of formation and location was half-way between two vertical rows of slots. Only Tables B-32 and B-33 show different cases where elevation of the point was at the center of formation + $\frac{1}{2}$ of dimensionless slot length and the location of point was $\frac{1}{4}$ of the circumferencial distance between two vertical rows of "i" slots; these cases were run for $r_D=1$.

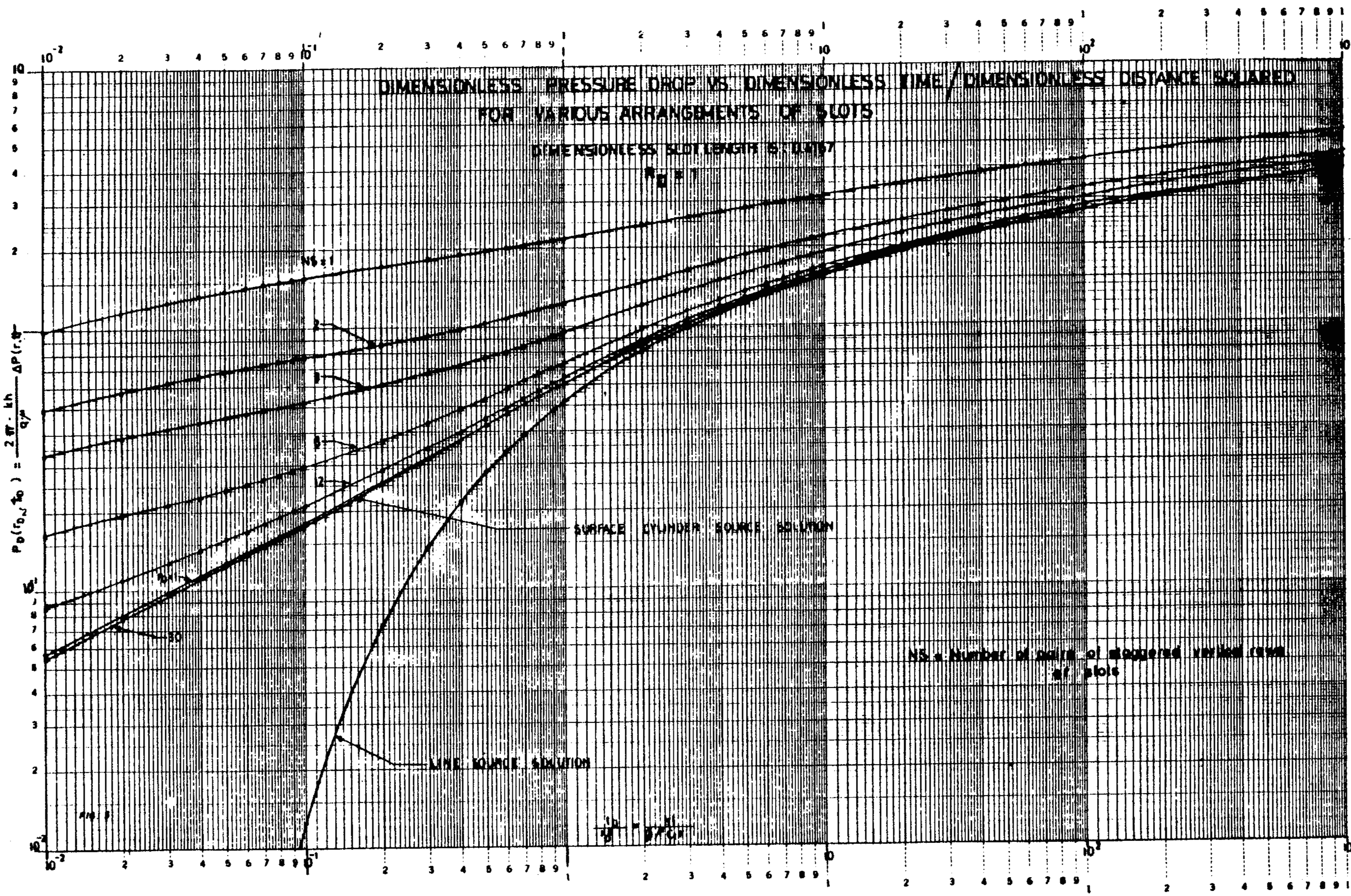
3.2 Results and Calculation of Pseudo-Skin Factors

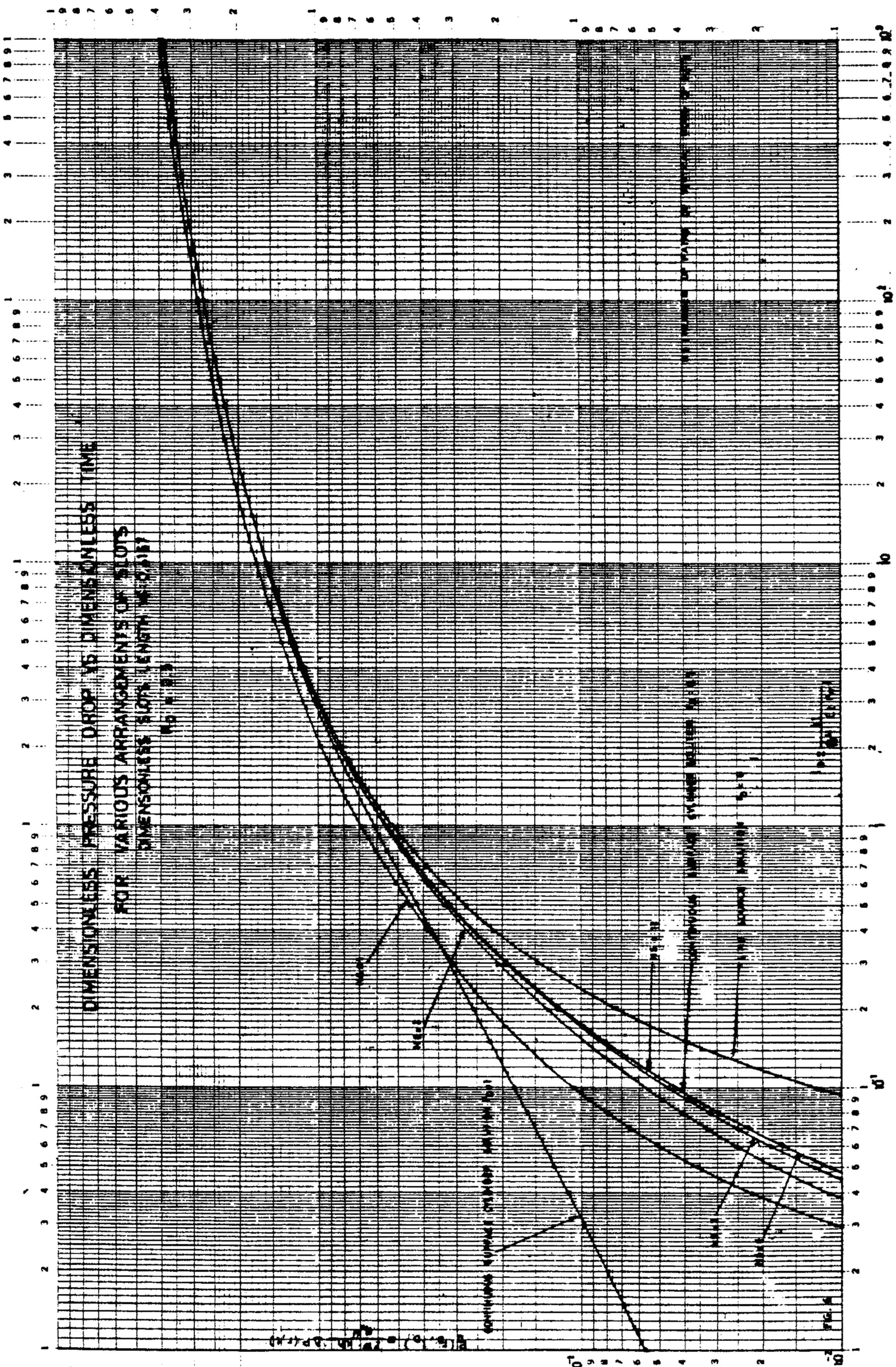
The results of all runs with relevant data are shown in Tables 2-5 and B-1 to B-33.

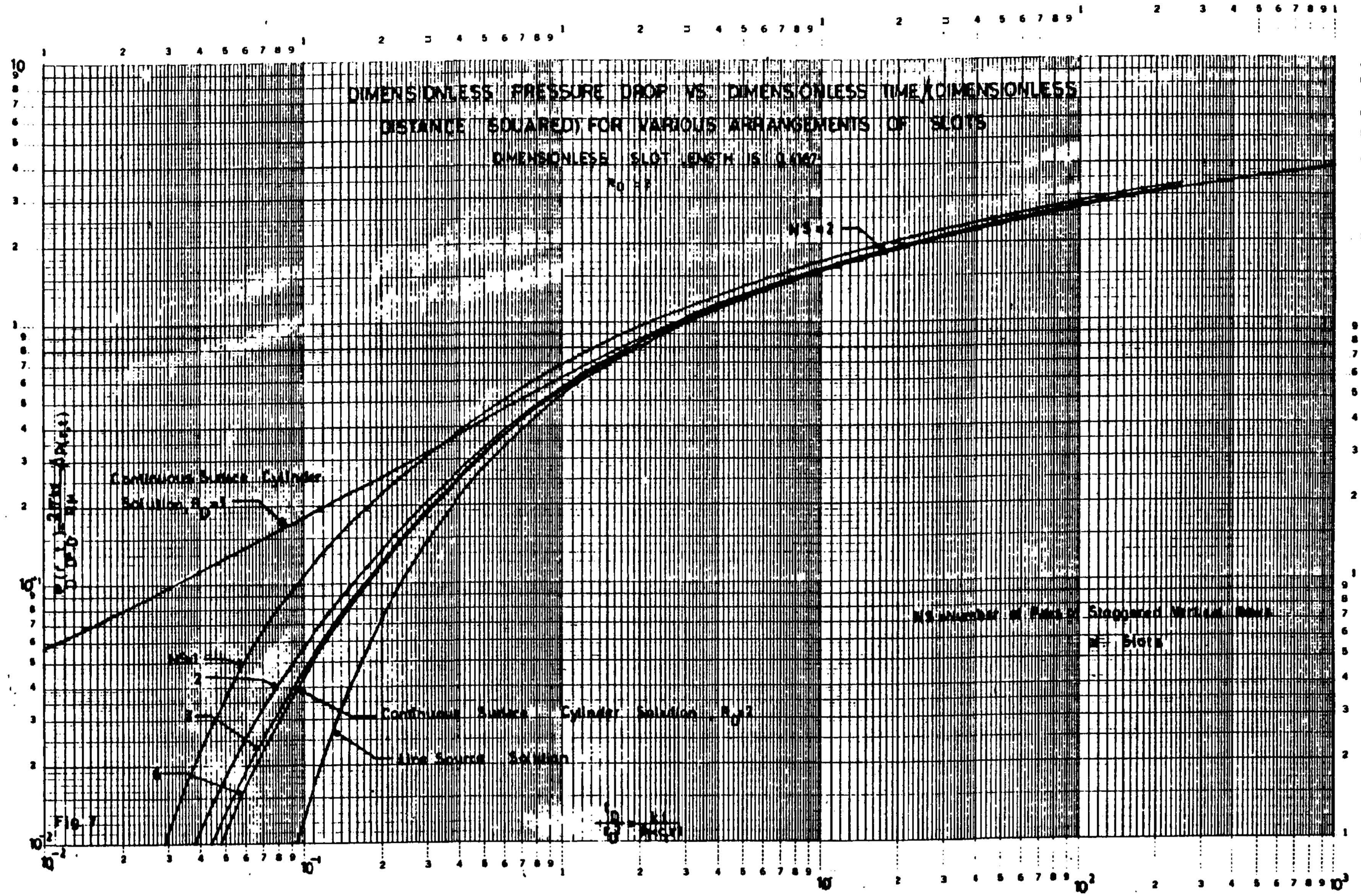
It was found that the variation of dimensionless formation has no actual influence on the dimensionless pressure drop.

The variation of dimensionless pressure drop with dimensionless slot length is negligible (see Tables B-1 to B-23)

There is a pseudo-skin factor which becomes a constant after a certain time and decreaseswhen the number of vertical rows is increased. The pseudo-skin factor becomes negligible when the number of pairs of staggered vertical rows is more than 3, in all cases. All values of the pseudo-skin factor are printe in Tables B-1 to B- 31.


DIMENSIONLESS PRESSURE DROP VS. DIMENSIONLESS TIME / DIMENSIONLESS DISTANCE SQUARED
FOR VARIOUS ARRANGEMENTS OF SLOTS
DIMENSIONLESS SLOT LENGTH IS 0.4167
NS = 1
2
8
12
30
SURFACE CYLINDER SOURCE SOLUTION
LINE SOURCE SOLUTION
NS = Number of pairs of staggered vertical rows of slots


DIMENSIONLESS PRESSURE DROP VS DIMENSIONLESS TIME
FOR VARIOUS ARRANGEMENTS OF SLOTS



DIMENSIONLESS PRESSURE DROP VS. DIMENSIONLESS TIME/(DIMENSIONLESS DISTANCE SQUARED) FOR VARIOUS ARRANGEMENTS OF SLOTS
DIMENSIONLESS SLOT LENGTH IS 0.1067
Continuous Surface Cylinder Solution
Continuous Surface Cylinder Solution
Line Source Solution
N = 2
2
Slots

Fig. 7

t_D \ r_D	0	0.25	0.50	0.75	1	2	3	5	10
1				.0025	.0565				
1.5			.0001	.0067	.0692				
2			.0006	.0117	.0799				
3		.0002	.0027	.0228	.0980				
4	.0001	.0007	.0063	.0341	.1132				
5	.0006	.0020	.0111	.0451	.1267				
6	.0015	.0040	.0167	.0558	.1389	.0001			
8	.0056	.0103	.0296	.0760	.1607	.0006			
$1 \cdot 10^{-1}$	.0125	.0192	.0436	.0949	.1800	.0014			
1.5	.0392	.0492	.0809	.1379	.2216	.0053			
2	.0732	.0844	.1187	.1766	.2574	.0111	.0001		
3	.1463	.1576	.1911	.2458	.3194	.0262	.0007		
4	.2161	.2265	.2573	.3071	.3735	.0436	.0023		
5	.2799	.2893	.3172	.3622	.4223	.0621	.0051		
6	.3376	.3462	.3714	.4123	.4670	.0809	.0089		
8	.4378	.4449	.4660	.5002	.5462	.1187	.0190	.0001	
1	.5221	.5282	.5462	.5755	.6151	.1556	.0315	.0005	
1.5	.6873	.6917	.7047	.7262	.7554	.2414	.0690	.0030	
2	.8117	.8152	.8254	.8423	.8654	.3172	.1097	.0085	
3	.9947	.9971	1.0042	1.0160	1.0323	.4439	.1897	.0271	
4	1.1285	1.1303	1.1358	1.1448	1.1574	.5462	.2629	.0522	.0002
5	1.2340	1.2354	1.2399	1.2472	1.2575	.6316	.3285	.0804	.0008
6	1.3210	1.3223	1.3260	1.3322	1.3408	.7047	.3873	.1097	.0019
8	1.4598	1.4607	1.4635	1.4682	1.4748	.8254	.4887	.1678	.0063
$1 \cdot 10^{1}$	1.5683	1.5690	1.5713	1.5751	1.5806	.9227	.5736	.2228	.0135
1.5	1.7669	1.7674	1.7689	1.7715	1.7750	1.1060	.7394	.3431	.0407
2	1.9086	1.9090	1.9102	1.9121	1.9148	1.2399	.8641	.4424	.0750
3	2.1093	2.1096	2.1103	2.1116	2.1134	1.4325	1.0472	.5981	.1481
4	2.2521	2.2523	2.2529	2.2539	2.2552	1.5713	1.1811	.7174	.2178
5	2.3631	2.3632	2.2637	2.3645	2.3655	1.6798	1.2866	.8139	.2814
6	2.4538	2.4539	2.4543	2.4550	2.4559	1.7689	1.3737	.8949	.3309
8	2.5971	2.5972	2.5975	2.5980	2.5987	1.9102	1.5124	1.0259	.4390
$1 \cdot 10^{2}$	2.7084	2.7085	2.7087	2.7091	2.7096	2.0202	1.6209	1.1296	.5231
1.5	2.9107	2.9108	2.9109	2.9112	2.9115	2.2209	1.8195	1.3218	.6880
2	3.0543	3.0544	3.0545	3.0547	3.0550	2.3637	1.9613	1.4604	.8123
3	3.2569	3.2569	3.2570	3.2571	3.2573	2.5654	2.1620	1.6578	.9950
4	3.4006	3.4006	3.4007	3.4008	3.4009	2.7087	2.3048	1.7989	1.1288
5	3.5121	3.5121	3.5122	3.5122	3.5124	2.8200	2.4157	1.9089	1.2342
6	3.6032	3.6032	3.6033	3.6033	3.6034	2.9109	2.5065	1.9990	1.3212
8	3.7470	3.7470	3.7471	3.7471	3.7472	3.0545	2.6498	2.1415	1.4599

Table 1 -- **Continuous Surface** Cylinder **Source**
(After Gringarten, A.C., et al.[a])

TABLE 2

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 6 STAGGERED VERTICAL ROWS(3 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 0.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	Pit	TD	PO	TD	PO
.100E-02	.57863E-43	.200E+00	.73207E-01	.200E+02	.19086E+01
.200E-02	.70756E-43	.300E+00	.14628E+00	.300E+02	.21093E+01
.300E-02	.44348E-38	.400E+00	.21613E+00	.400E+02	.22521E+01
.400E 02	.57942E-29	.500E+00	.27989E+00	.500E+02	.23630E+01
.500E-02	.18998E-23	.600E+00	.33762E+00	.600E+02	.24538E+01
.600E-02	.94177E-20	.700E+00	.39001E+00	.700E+02	.25306E+01
.700E-02	.42077E-17	.800E+00	.43781E+00	.800E+02	.25971E+01
.800E-02	.41608E-15	.900E+00	.46167E+00	.900E+02	.26558E+01
.900E-02	.15021E-13	.100E+01	.52214E+00	.100E+03	.27064E+01
.100E-01	.26745E-12	.200E+01	.81171E+00	.200E+03	.30543E+01
.200E-01	.13875E-06	.300E+01	.99466E+00	.300E+03	.32569E+01
.300E-01	.13003E-04	.400E+01	.11285E+01	.400E+03	.34306E+01
.400E-01	.13524E-03	.500E+01	.12339E+01	.500E+03	.35121E+01
.500E-01	.57415E-03	.600E+01	.13210E+01	.600E+03	.36032E+01
.600E-01	.15451E-02	.700E+01	.13952E+01	.700E+03	.36803E+01
.700E-01	.31906E-02	.800E+01	.14598E+01	.800E+03	.37470E+01
.800E-01	.55685E-02	.900E+01	.15169E+01	.900E+03	.38059E+01
.900E-01	.86726E-02	.100E+02	.15683E+01	.100E+04	.38585E+01
.100E+00	.12457E-01				

TABLE 3

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 12 STAGGERED VERTICAL ROWS(6 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

ZD= .50

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.82515E-30	.200E+00	.11866E+00	.200E+02	.19102E+01
.200E-02	.30901E-16	.300E+00	.19110E+00	.300E+02	.21103E+01
.300E-02	.15342E-11	.400E+00	.25731E+00	.400E+02	.22529E+01
.400E-02	.37415E-09	.500E+00	.31719E+00	.500E+02	.23637E+01
.500E-02	.10625E-07	.600E+00	.37142E+00	.600E+02	.24543E+01
.600E-02	.10223E-06	.700E+00	.42079E+00	.700E+02	.25310E+01
.700E-02	.52831E-06	.800E+00	.46599E+00	.800E+02	.25975E+01
.800E-02	.18485E-05	.900E+00	.50762E+00	.900E+02	.26562E+01
.900E-02	.43813E-05	.100E+01	.54618E+00	.100E+03	.27087E+01
.100E-01	.11171E-04	.200E+01	.82539E+00	.200E+03	.30545E+01
.200E-01	.57245E-03	.300E+01	.10042E+01	.300E+03	.32569E+01
.300E-01	.26554E-02	.400E+01	.11357E+01	.400E+03	.34006E+01
.400E-01	.62760E-02	.500E+01	.12399E+01	.500E+03	.35121E+01
.500E-01	.11068E-01	.600E+01	.13260E+01	.600E+03	.36032E+01
.600E-01	.16093E-01	.700E+01	.13995E+01	.700E+03	.36803E+01
.700E-01	.22965E-01	.800E+01	.14635E+01	.800E+03	.37478E+01
.800E-01	.29534E-01	.900E+01	.15253E+01	.900E+03	.38559E+01
.900E-01	.36460E-01	.100E+02	.15713E+01	.100E+04	.38586E+01
.100E+00	.43602E-01				

TABLE 4

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 12 STAGGERED VERTICAL ROWS(6 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 2.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.57863E-43	.200E+00	.11106E-01	.200E+02	.12399E+01
.200E-02	.70756E-43	.300E+00	.26215E-01	.300E+02	.14326E+01
.300E-02	.33155E-39	.400E+00	.43641E-01	.400E+02	.15713E+01
.400E-02	.43956E-30	.500E+00	.62078E-01	.500E+02	.16798E+01
.500E-02	.14559E-24	.600E+00	.88927E-01	.600E+02	.17669E+01
.600E-02	.72714E-21	.700E+00	.99365E-01	.700E+02	.18445E+01
.700E-02	.32680E-18	.800E+00	.11870E+00	.800E+02	.19102E+01
.800E-02	.32472E-16	.900E+00	.13731E+00	.900E+02	.19682E+01
.900E-02	.11771E-14	.100E+01	.15562E+00	.100E+03	.20202E+01
.100E-01	.21036E-13	.200E+01	.31723E+00	.200E+03	.23637E+01
.200E-01	.11198E-07	.300E+01	.44391E+00	.300E+03	.25654E+01
.300E-01	.10785E-05	.400E+01	.54622E+00	.400E+03	.27087E+01
.400E-01	.11666E-04	.500E+01	.63161E+00	.500E+03	.28200E+01
.500E-01	.51959E-04	.600E+01	.70476E+00	.600E+03	.29109E+01
.600E-01	.14714E-03	.700E+01	.76868E+00	.700E+03	.29878E+01
.700E-01	.31948E-03	.800E+01	.82543E+00	.800E+03	.30545E+01
.800E-01	.58479E-03	.900E+01	.87643E+00	.900E+03	.31133E+01
.900E-01	.95214E-03	.100E+02	.92273E+00	.100E+04	.31659E+01
.100E+00	.14250E-02				

TABLE 5

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 60 STAGGERED VERTICAL ROWS(30 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PG
.100E-02	.16371E-01	.200E+00	.25524E+00	.200E+02	.13126E+01
.200E-02	.23265E-01	.300E+00	.31719E+00	.300E+02	.21112E+01
.300E-02	.28798E-01	.400E+00	.37134E+00	.400E+02	.22530E+01
.400E-02	.33541E-01	.500E+00	.42016E+00	.500E+02	.23634E+01
.500E-02	.37745E-01	.600E+00	.46478E+00	.600E+02	.24537E+01
.600E-02	.41553E-01	.700E+00	.50591E+00	.700E+02	.25302E+01
.700E-02	.45059E-01	.800E+00	.54407E+00	.800E+02	.25965E+01
.800E-02	.48324E-01	.900E+00	.57963E+00	.900E+02	.26550E+01
.900E-02	.51391E-01	.100E+01	.61293E+00	.100E+03	.27074E+01
.100E-01	.54293E-01	.200E+01	.86322E+00	.200E+03	.30528E+01
.200E-01	.77750E-01	.300E+01	.10301E+01	.300E+03	.32551E+01
.300E-01	.95797E-01	.400E+01	.11552E+01	.400E+03	.33967E+01
.400E-01	.11105E+00	.500E+01	.12553E+01	.500E+03	.35102E+01
.500E-01	.12453E+00	.600E+01	.13387E+01	.600E+03	.36013E+ [illegible]
.600E-01	.13675E+00	.700E+01	.14101E+01	.700E+03	.36783E+01
.700E-01	.14801E+00	.800E+01	.14726E+01	.800E+03	.37450E+01
.800E-01	.15853E+00	.900E+01	.15292E+01	.900E+03	.38038E+01
.900E-01	.16844E+00	.100E+02	.15782E+01	.100E+04	.38565E+01
.100E+00	.17785E+00				

4. NOMENCLATURE*

c_t = total compressibility

d_i = distance in space

h = slot length

h_D = dimensionless slot length

k = formation permeability

k = total number of slots in one vertical row

NS = number of pairs of vertical rows of slots (also number of slots in one ring of slots)

Δp = pressure drop

p_i = initial pressure in the reservoir

p_D = dimensionless pressure drop

q = fluid withdrawal per unit length or unit area (or volume) of a source = strength

Q = total withdrawal from a well (rate of production)

r_i, r_j = distances in X-Y plane (to slots)

r_{iD} = dimensionless distance in X-Y plane (to slots)

r_{jD} = dimensionless distance in X-Y plane (to slots)

r_D = dimensionless radial distance

r_w = wellbore radius

s = pseudo-skin factor

t = producing time

t_D = dimensionless producing time

x = abscissa of a point

y = ordinate of a point

* c-g-s units

z = elevation of a point

z_m = elevation of point M in a reservoir

z_{mD} = dimensionless elevation of a point a in a reservoir

η = diffusivity

μ = viscosity

ϕ = porosity

θ = angle which the projection of point M on the X-Y plane makes with the origin of coordinates

Subscripts and Superscripts

D = dimensionless

i = pertaining to vertical row of slots "i"

j = pertaining to vertical row of slots "j"

m = any point in space

t = total

w = well

SPECIAL FUNCTIONS

Error Function: $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$

Exponential Integral: $-Ei(-x) = \int_x^\infty \frac{e^{-u}}{u} du$

5. REFERENCES

1. Brons, F., Marting, V. E.: "The Effect of Restricted Fluid Entry on Well Productivity", JPT, Feb. 1961, 172-174.
2. Burns, W. A., Jr.: "New Single-Well Test for Determining Vertical Permeability", Trans. AIME (1969), 246, 743-752.
3. Buzarde, L. E., Kastor, R. L., Bell. W. T., and DePriester, C. L.: "Production Operation Course I-Well completions", SPE of AIME, 1972, Chap. IV.
4. Composite Catalog, World Oil ed. 1974-5, 2894.
5. Carslaw, H. S., and Jaeger, J. C.: Conduction of heat in Solids, Oxford at the Clarendon Press (1959), 255-256.
6. Carslaw, H. S., and Jaeger, J. C.: Op. cit., 275-276.
7. Gringarten, A. C., and Ramey, H. J., Jr.: "The Use of Source Functions in Solving Unsteady-Flow Problems in Reservoirs", SPEJ, Oct. 1973; Trans. AIME (1975), 255, 285-296.
8. Gringarten, A. C., and Ramey, H. J., Jr.: "A Comparison of Different Solutions to the Radial Flow Problems", paper SPE 3817, submitted for publication.
9. Hubbert, M. K.: "Darcy's Law and the Field equations the flow of underground Fluids", Trans. AIME (1956), 207, 222-239.
10. Hurst, W.: "Establishment of the Skin Effect and its Impediment to Fluid Flow into a Wellbore", Pet. Eng., Oct. 1953, 25, B-6.

11. Muskat, M.: The Flow of Homogeneous Fluids Through Porous Media, T. W. Edwards, Inc., Ann Arbor, Mich. (194.6) 273.

12. Muskat, M.: Physical Principles of Oil Production, McGraw-Hill Book Co., Inc., New York (1949), 210-220.

13. Nisle, G. R.: "The Effect of Partial Penetration on pressure Build-Up in Oil Wells", Trans. AIME (1958), 213, 85-90.

14. Ramey, H. J.,Jr.: "Application of the Line Source Solution to Porous Media-A Review", Producers Monthly, May 1967, 4.

15. Theis, C. V.: "The Relationship between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge Using Ground-Water Storage", Trans. AGU (1935) (1935), 519-524.

16. Van Everdingen, A. F.: "The Skin Effect and its Influence on the Productive Capacity of a Well", Trans. AIME (1953), 198, 171-176.

APPENDIX A

Computer Program

1....General

The computer program was compiled in order to calculate the dimensionless pressure drop given by eq. 33 in the text. It is divided into two parts: The main program and the subroutine. The subroutine computes the infinite summation of all error functions in the integrands in eq. 33, while the main program uses Simpson's rule for integration purposes. It should be mentioned again that the program firstly computes the pressure drops created by all real and image slots in one vertical row and then considers another row; while in the text the explanations refer to another method where all pressure drops created by slots which have the same elevations are firstly computed and then slots in other elevations are considered (eqs. 22-27)

2....The Main Program

The following parameters are needed for the calculations:

1) Reservoir thickness = L (ft.)

2) Slot length = SL (inch)

3) Number of vertical staggered rows of slots (or number of Slots in one ring) = NS

4) Well radius = r_w (ft.)

5) Dimensionless distance of point M = r_D

6) Dimensionless time = t_D

7) The elevation of point M = z_{mD}

8) The angle of point M with respect to the origin of X-Y coordinates (Fig. 3) = Theta

9) The tolerance, which is used for testing the convergence of the infinite summation of the error functions = 10^{-10}. This tolerance can be changed and is used in the eubroutine.

3. Subroutine Sum

As said above, this subroutine calculates all error function summations, using a certain tolerance which decreases when t_D increase. Three hundred terms are considered at most, but more terms can be added.

```
      PROGRAM LINER(INPUT,OUTPUT,TAPE6=OUTPUT)
      REAL  L,LD
      DIMENSION POT(800),TDT(800),FIL(360),FJL(360),FIC(360),FJC(360),
     AFIR(360),FJR(360),RI(360),RJ(360),ALFA(360),BETA(360)
C
C     FIL=VALUE OF FUNCTION TO BE INTEGRATED AT THE LEFT SIDE OF
C     TIME INTERVAL FOR ≠I≠ SLOTS.
C     FJL=DITTO,FOR ≠J≠ SLOTS.
C     FIC=VALUE OF FUNCTION TO BE INTEGRATED AT THE CENTEP OF TIME
C     INTERVAL FOR #I# SLOTS.
C     FJC=DITTO,FOR ≠J≠ SLOTS.
C     FIR=VALUE OF FUNCTION TO BE INTEGRATED AT THE RIGHT SIDE OF
C     TIME INTERVAL FOR ≠I≠ SLOTS.
C     FJR=DITOO, FOR ≠J≠ SLOTS.
C     NS=NUMBER OF STACGZQED (PAIRS) ROUS OF SLOTS. THERE ARE TWO
C     KINDS OF SLOTS: ONE IS SLOTS ≠I≠,AND THE SECOND ONE IS SLOTS ≠J≠
C     WHICH ARE ROTATED IN RELATION TO SLOTS ≠I≠.
C     RI=DISTANCE OF THE POINT FOR WHICH THE PRESSURE DROP IS TO BE
C     CALCULATE3 DUE TO PROOUCTION THRU ≠I≠ SLOTS.
C     RJ=DITTO,FOR ≠J≠ SLOTS.
C     SILC=SUM OF ERROR FUNCTIONS AT THE CENTER OF TIME INTERVAL FOR
C     ≠I≠ SLOTS.
C     SJLC=DITTO,FOR ≠J≠ SLOTS.
C     SILR=SUM OF ERROR FUNCTIONS AT THE LEFT SIDE OF TIME INTERVAL
C     FOR ≠I≠ SLOTS.
C     SJLR=DITTO,FOR ≠J≠ SLOTS.
C-----INPUT DATA
C     RESERVOIR THICKNESS L(FT.)
      L=4.
C     SLOT LENGTH (INCH)
      SL=2.
C     NO. OF SLOTS IN ONE ROW IS CALCULATED
      NSL=L*6/SL
C     WELL RADIUS
      RW=1./6.
C     RD,LD,HD=DIMENSIONLESS DISTANCE,THICKNESS AND SLOT LENGTH,RESPECTIVELY
      RD=1.0
      LD=L/RW
      HD=SL/12./RW
      ZMD=LD/2.+0.01
      TOL=1.E-10
      TD=0.
      NS=12
      THETA=15.
      DTI=0.001
      DT=DTI/5.
      NCT=0
      XM=RW*RD*COSD(THETA)
      YM=RW*RD*SIND(THETA)
      WRITE(6,100)
      DO 12 I=1,NS
      ALFA(I)=360./NS*I
      WRITE(6,125) I,ALFA(I)
      RI(I)=SQRT((XM-RW*COSD(ALFA(I)))**2+(YM-RW*SIND(ALFA(I)))**2)/RW
   12 CONTINUE
      DO 13 J=1,NS
      BETA(J)=360./NS*J-180./NS
```

```
      WRITE(6,126) J,BETA(J)
      RJ(J)=SQRT((XM-RW*COSD(BETA(J)))**2+(YM-RW*SIND(BETA(J)))**2)/RW
   13 CONTINUE
      PD=0.
      PDI=0.
      PDJ=0.
      DO 8 I=1,NS
      FIL(I)=0.
      FJL(I)=0.
      WRITE(6,127) I,RI(I),I,RJ(I)
    8 CONTINUE
      NC=6
C
      DO 1 K=1,NC
      IF(K-1) 2,2,3
    2 NINT=10
      GO TO 4
    3 NINT=9
    4 DO 5 J=1,NINT
      DO 6 L1=1,5
      T1=TD+DT/2.
      IF(TD.GE.0.1) TOL=1.E-08
C
      CALL SUM(T1,NSL,LD,HD,ZMD,TOL,NS,SILC,SJLC)
      TD=TD+DT
      CALL SUM(TD,NSL,LD,HD,ZMD,TOL,NS,SILR,SJLR)
      DO 7 I=1,NS
      E=RI(I)*RI(I)
      F=RJ(I)*RJ(I)
      E1=E/(4.*T1)
      E11=E/(4.*TD)
      F1=F/(4.*TD)
      F11=F/(4.*T1)
      IF(E1.GE.100.) E1=100.
      IF(F1.GE.100.) F1=100.
      IF(E11.GE.100.)  E11=100.
      IF(F11.GE.100.)  F11=100.
      FIC(I)=1./T1*EXP(-E1)*SILC
      FJC(I)=1./T1*EXP(-F11)*SJLC
      FIR(I)=1./TD*EXP(-E11)*SILR
      FJR(I)=1./TD*EXP(-F1)*SJLR
      PDI=PDI+.25*DT/6.*(FIL(I)+4.*FIC(I)+FIR(I))
      PDJ=PDJ+.25*DT/6.*(FJL(I)+4.*FJC(I)+FJR(I))
      FIL(I)=FIR(I)
      FJL(I)=FJR(I)
    7 CONTINUE
      PD=PD+PDI+PDJ
      PDI=0.
      PDJ=0.
      PPD=PD/NS
    6 CONTINUE
      NCT=NCT+1
      TDT(NCT)=TD
      PDT(NCT)=PPD
    5 CONTINUE
      DT=10.*DT
    1 CONTINUE
```

```
      PRINT 99,NCT
      WRITE(6,134)
      N=2*NS
      WRITE(6,135) N,NS
      WRITE(6,136) RD,HD
      WRITE(6,131)
      NCP1=3*NC+1

      NCP=3+NC
      DO 15 I=1,NCP
      K=NCP1+I
      N=NCP2+I
      WRITE(6,132) TOT(I),PDT(I),TOT(K),PDT(K),TOT(N),PDT(N)
   15 CONTINUE
      WRITE(6,133) TOT(NCP1),PDT(NCP1)
   99 FORMAT(4X,*NCT=*,I4)
  100 FORMAT(1H1)
  125 FORMAT(4X,*ALFA(*,I3,*)=*,F8.2)
  126 FORMAT(4X,*BETA(*,I3,*)=*,F8.2)
  127 FORMAT(4X,*RI(*,I3,*)=*,F10.5,5X,*RJ(*,I3,*)=*,F10.5)
  131 FORMAT(12X,*TD*,12X,*PD*,10X,*TD*,11X,*PD*,11X,*TD*,11X,*PD*,//)
  132 FORMAT(7X,E10.3,2X,E11.5,2(03X,E10.3,2X,E11.5))
  133 FORMAT(07X,E10.3,2X,E11.5)
  134 FORMAT(1H1,///,22X,*UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER*
     A,//)
  135 FORMAT(12X,*ARRANGEMENT OF SLOTS:*,I3, 2X,*STAGGERED VERTICAL ROWS
     A(*,I3,2X,*PAIRS)*,/)
  136 FORMAT(12X,*DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF
     A FORMATION*,//, 22X,*RD=*,F5.2,//,12X,*DIMENSIONLESS SLOT LENGTH I
     BS:*,F6.4,//)
      STOP
      END
```

```
      SUBROUTINE  SUM(T,NSL,LD,HD,ZMD,TOL,NS,SIL,SJL)
      REAL LD
      SQT=SQRT(T)
      FAC=2.*SQT
      C1=(ZMD+HD)/FAC
      C2=(ZMD-HD)/FAC
      C3=(ZMD+HD-LD)/FAC
      C4=(ZMD-HD-LD)/FAC
      SEI=ERF(C1)-ERF(C2)
      SEJ=ERF(C3)-ERF(C4)
      K=300
      I2=NSL-1
      DO 1 I1=1,I2
      SEI=SEI+ERF((ZMD+(2*I1+1)*HD)/FAC)-ERF((ZMD+2*I1*HD)/FAC)
     A+ERF((ZMD-2*I1*HD)/FAC)-ERF((ZMD-(2*I1+1)*HD)/FAC)
      SEJ=SEJ+ERF((ZMD+2*I1*HD)/FAC)-ERF((ZMD+(2*I1-1)*HD)/FAC)
     A+ERF((ZMD-(2*I1-1)*HD)/FAC)-ERF((ZMD-2*I1*HD)/FAC)
    1 CONTINUE
      SI=SEI
      SJ=SEJ
      DO 2 N=1,K
      F1=(ZMD+HD+2*N*LD)/FAC
      F2=(ZMD-HD+2*N*LD)/FAC
      F3=(ZMD+HD-2*N*LD)/FAC
      F4=(ZMD-HD-2*N*LD)/FAC
      F5=(ZMD+HD+(2*N-1)*LD)/FAC
      F6=(ZMD-HD+(2*N-1)*LD)/FAC
      F7=(ZMD+HD-(2*N+1)*LD)/FAC
      F8=(ZMD-HD-(2*N+1)*LD)/FAC
      SPI=ERF(F1)-ERF(F2)+ERF(F3)-ERF(F4)
      SPJ=ERF(F5)-ERF(F6)+ERF(F7)-ERF(F8)
      DO 4 I1=1,I2
      A=ERF((ZMD+(2*I1+1)*HD+2*N*LD)/FAC)
      B=ERF((ZMD+2*I1*HD+2*N*LD)/FAC)
      C=ERF((ZMD-2*I1*HD+2*N*LD)/FAC)
      D=ERF((ZMD-(2*I1+1)*HD+2*N*LD)/FAC)
      E=ERF((ZMD-2*I1*HD-2*N*LD)/FAC)
      F=ERF((ZMD-(2*I1+1)*HD-2*N*LD)/FAC)
      G=ERF((ZMD+(2*I1+1)*HD-2*N*LD)/FAC)
      H=ERF((ZMD+2*I1*HD-2*N*LD)/FAC)
      SPI=SPI+A-B+C-D+E-F+G-H
      A1=ERF((ZMD+2*I1*HD+2*N*LD)/FAC)
      B1=ERF((ZMD+(2*I1-1)*HD+2*N*LD)/FAC)
      C1=ERF((ZMD-(2*I1-1)*HD+2*N*LD)/FAC)
      D1=ERF((ZMD-2*I1*HD+2*N*LD)/FAC)
      E1=ERF((ZMD-(2*I1-1)*HD-2*N*LD)/FAC)
      F1=ERF((ZMD-2*I1*HD-2*N*LD)/FAC)
      G1=ERF((ZMD+2*I1*HD-2*N*LD)/FAC)
      H1=ERF((ZMD+(2*I1-1)*HD-2*N*LD)/FAC)
      SPJ=SPJ+A1-B1+C1-D1+E1-F1+G1-H1
    4 CONTINUE
      SI=SI+SPI
      SJ=SJ+SPJ
C     TEST OF CONVERGENCE
      IF(SJ.LE.1.E-50) SJ=1.E-50
      AI=SPI/SI
      AJ=SPJ/SJ
```

```
      D=AMAX1(AI,AJ)
      IF(D-TOL) 3,3,2
    2 CONTINUE
      WRITE(6,100)
  100 FORMAT(//,1X,*NO CONVERGENCE FOR 300 TERMS*,)
    3 SIL=SI
      IF(SJ.EQ.1.E-50) SJ=0.
      SJL=SJ
      RETURN
      END
```

APPENDIX B

Tabulated Results

The numerical results of all computer runs with calculated pseudo-skin factors are printed in the following pages.

TABLE B-1

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 2 STAGGERED VERTICAL ROWS(1 PAIR)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .2000

TD	PD	TD	PD	TD	PD
.100E-02	.48020E+00	.200E+00	.17508E+01	.200E+02	.35188E+01
.200E-02	.62760E+00	.300E+00	.18540E+01	.300E+02	.37175E+01
.300E-02	.71785E+00	.400E+00	.19300E+01	.400E+02	.38593E+01
.400E-02	.78367E+00	.500E+00	.19918E+01	.500E+02	.39696E+01
.500E-02	.83577E+00	.600E+00	.20448E+01	.600E+02	.40599E+01
.600E-02	.87900E+00	.700E+00	.20915E+01	.700E+02	.41364E+01
.700E-02	.91599E+00	.800E+00	.21337E+01	.800E+02	.42027E+01
.800E-02	.94832E+00	.900E+00	.21722E+01	.900E+02	.42613E+01
.900E-02	.97704E+00	.100E+01	.22077E+01	.100E+03	.43137E+01
.100E-01	.10029E+01	.200E+01	.24661E+01	.200E+03	.46590E+01
.200E-01	.11749E+01	.300E+01	.26348E+01	.300E+03	.48613E+01
.300E-01	.12762E+01	.400E+01	.27606E+01	.400E+03	.50350E+01
.400E-01	.13482E+01	.500E+01	.28609E+01	.500E+03	.51164E+01
.500E-01	.14039E+01	.600E+01	.29445E+01	.600E+03	.52075E+01
.600E-01	.14495E+01	.700E+01	.30160E+01	.700E+03	.52845E+01
.700E-01	.14881E+01	.800E+01	.30786E+01	.800E+03	.53512E+01
.800E-01	.15214E+01	.900E+01	.31342E+01	.900E+03	.54101E+01
.900E-01	.15509E+01	.100E+02	.31843E+01	.100E+04	.54627E+01
.100E+00	.15772E+01				

PSEUDO-SKIN FACTOR=1.6042

TABLE B-2

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 2 STAGGERED VERTICAL ROWS(1 PAIR)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.46020E+00	.200E+00	.17327E+01	.200E+02	.35307E+01
.200E-02	.62752E+00	.300E+00	.18360E+01	.300E+02	.36994E+01
.300E-02	.71735E+00	.400E+00	.19120E+01	.400E+02	.38412E+01
.400E-02	.78238E+00	.500E+00	.19738E+01	.500E+02	.39515E+01
.500E-02	.83347E+00	.600E+00	.20267E+01	.600E+02	.40419E+01
.600E-02	.87558E+00	.700E+00	.20735E+01	.700E+02	.41183E+01
.700E-02	.91143E+00	.800E+00	.21156E+01	.800E+02	.41847E+01
.800E-02	.94265E+00	.900E+00	.21541E+01	.900E+02	.42432E+01
.900E-02	.97032E+00	.100E+01	.21897E+01	.100E+03	.42956E+01
.100E-01	.99516E+00	.200E+01	.24481E+01	.200E+03	.46409E+01
.200E-01	.11610E+01	.300E+01	.26167E+01	.300E+03	.48433E+01
.300E-01	.12600E+01	.400E+01	.27425E+01	.400E+03	.49869E+01
.400E-01	.13309E+01	.500E+01	.28429E+01	.500E+03	.50983E+01
.500E-01	.13863E+01	.600E+01	.29264E+01	.600E+03	.51894E+01
.600E-01	.14317E+01	.700E+01	.29980E+01	.700E+03	.52664E+01
.700E-01	.14701E+01	.800E+01	.30606E+01	.800E+03	.53332E+01
.800E-01	.15034E+01	.900E+01	.31162E+01	.900E+03	.53920E+01
.900E-01	.15329E+01	.100E+02	.31662E+01	.100E+04	.54447E+01
.100E+00	.15592E+01				

PSEUDO-SKIN FACTOR=1.5862

TABLE B–3

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 2 STAGGERED VERTICAL ROWS(1 PAIR)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .5714

TD	PD	TD	PD	TD	PD
.100E-02	.48020E+00	.200E+00	.17282E+01	.200E+02	.34962E+01
.200E-02	.62752E+00	.300E+00	.18315E+01	.300E+02	.36349E+01
.300E-02	.71735E+00	.400E+00	.19075E+01	.400E+02	.3e367E+01
.400E-02	.78238E+00	.500E+00	.19692E+01	.500E+02	.39470E+01
.500E-02	.83347E+00	.600E+00	.20222E+01	.600E+02	.40373E+01
.600E-02	.87558E+00	.700E+00	.20690E+01	.700E+02	.41138E+01
.700E-02	.91142E+00	.800E+00	.21111E+01	.800E+02	.41801E+01
.800E-02	.94263E+00	.900E+00	.21496E+01	.900E+02	.42387E+01
.900E-02	.97027E+00	.100E+01	.21852E+01	.100E+03	.42311E+01
.100E-01	.99509E+00	.200E+01	.24435E+01	.200E+03	.46364E+01
.200E-01	.11602E+01	.300E+01	.26122E+01	.300E+03	.48397E+01
.300E-01	.12582E+01	.400E+01	.27380E+01	.400E+03	.49824E+01
.400E-01	.13283E+01	.500E+01	.28383E+01	.500E+03	.50938E+01
.500E-01	.13831E+01	.600E+01	.29219E+01	.600E+03	.51849E+01
.600E-01	.14281E+01	.700E+01	.29934E+01	.700E+03	.52619E+01
.700E-01	.14662E+01	.800E+01	.30560E+01	.800E+03	.53266E+31
.800E-01	.14994E+01	.900E+01	.31116E+01	.900E+03	.53875E+01
.900E-01	.15287E+01	.100E+02	.31617E+01	.100E+04	.54401E+01
.100E+00	.15549E+01				

PSEUDO-SKIN FACTOR=1.5816

TABLE B-4

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 2 STAGGERED VERTICAL ROWS(1 PAIR)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS:1.0000

TD	PD	TD	PD	TD	PD'
[illegible]00E-02	.48020E+00	.200E+00	.17235E+01	.200E+02	.34310E+01
.200E-02	.62752E+00	.300E+00	.18264E+01	.300E+02	.36897E+01
.300E-02	.71735E+00	.400E+00	.19023E+01	.400E+02	.38315E+01
.400E-02	.78238E+00	.500E+00	.19641E+01	.500E+02	.39418E+01
.500E-02	.83347E+00	.600E+00	.20170E+01	.600E+02	.40321E+01
.600E-02	.87558E+00	.700E+00	.20638E+01	.700E+02	.41186E+01
.700E-02	.91142E+00	.800E+00	.21059E+01	.800E+02	.41750E+01
.800E-02	.94263E+00	.900E+00	.21445E+01	.900E+02	.42335E+01
.9003-02	.97027E+00	.100E+01	.21800E+01	.100E+03	.42859E+01
.100E-01	.99500E+00	.200E+01	.24384E+01	.200E+03	.46312E+01
.200E-01	.11501E+01	.300E+01	.26070E+01	.300E+03	.48336E+01
.300E-01	.12578E+01	.400E+01	.27328E+01	.400E+03	.49772E+01
.400E-01	.13276E+01	.500E+01	.28332E+01	.500E+03	.50866E+01
.500E-01	.13810E+01	.600E+01	.29167E+01	.600E+03	.51797E+01
.600E-01	.14264E+01	.700E+01	.29883E+01	.700E+03	.52567E+01
.700E-01	.14641E+01	.800E+01	.30508E+01	.800E+03	.53234E+01
.800E-01	.14968E+01	.900E+01	.31065E+01	.900E+03	.53823E+01
.900E-01	.15258E+01	.100E+02	.31565E+01	.100E+04	.54350E+01
.100E+00	.15517E+01				

PSEUDO-SKIN FACTOR=1.5765

TABLE B-5

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 4 STAGGERED VERTICAL ROWS(2 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .2000

TD	PD	TD	PD	TD	PD
.100E-02	.24010E+00	.200E+00	.88163E+00	.200E+02	.25435E+01
.200E-02	.31380E+00	.300E+00	.94660E+00	.300E+02	.27422E+01
.300E-02	.35893E+00	.400E+00	.10016E+01	.400E+02	.28839E+01
.400E-02	.39184E+00	.500E+00	.10508E+01	.500E+02	.29943E+01
.500E-02	.41789E+00	.600E+00	.10955E+01	.600E+02	.30846E+01
.600E-02	.43950E+00	.700E+00	.11367E+01	.700E+02	.31611E+01
.700E-02	.45799E+00	.800E+00	.11749E+01	.800E+02	.32274E+01
.800E-02	.47416E+00	.900E+00	.12105E+01	.900E+02	.32859E+01
.900E-02	.48852E+00	.100E+01	.12438E+01	.100E+03	.33384E+01
.100E-01	.50144E+00	.200E+01	.14941E+01	.200E+03	.36837E+01
.200E-01	.58746E+00	.300E+01	.16610E+01	.300E+03	.38360E+01
.300E-01	.63812E+00	.400E+01	.17861E+01	.400E+03	.40296E+01
.400E-01	.67408E+00	.500E+01	.18862E+01	.500E+03	.41411E+01
.500E-01	.70197E+00	.600E+01	.19696E+01	.600E+03	.42322E+01
.600E-01	.72477E+00	.700E+01	.20410E+01	.700E+03	.43092E+01
.700E-01	.74405E+00	.800E+01	.21035E+01	.800E+03	.43759E+01
.800E-01	.76079E+00	.900E+01	.21591E+01	.900E+03	.44347E+01
.900E-01	.77559E+00	.100E+02	.22091E+01	.100E+04	.44874E+01
.100E+00	.78890E+00				

PSEUDO-SKIN FACTOR=0.6289

TABLE B-6

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 4 STAGGERED VERTICAL ROWS(2 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.24010E+00	.200E+00	.87263E+00	.200E+02	.25345E+01
.200E-02	.31376E+00	.300E+00	.93757E+00	.300E+02	.27331E+01
.300E-02	.35867E+00	.400E+00	.99259E+00	.400E+02	.28749E+01
.400E-02	.39119E+00	.500E+00	.10417E+01	.500E+02	.29852E+01
.500E-02	.41674E+00	.600E+00	.10865E+01	.600E+02	.30756E+01
.600E-02	.43779E+00	.700E+00	.11277E+01	.700E+02	.31521E+01
.700E-02	.45571E+00	.800E+00	.11659E+01	.800E+02	.32184E+01
.800E-02	.47133E+00	.900E+00	.12015E+01	.900E+02	.32769E+01
.900E-02	.48516E+00	.100E+01	.12348E+01	.100E+03	.33293E+01
.100E-01	.49756E+00	.200E+01	.14851E+01	.200E+03	.36747E+01
.200E-01	.58051E+00	.300E+01	.16520E+01	.300E+03	.38770E+01
.300E-01	.62996E+00	.400E+01	.17771E+01	.400E+03	.40206E+01
.400E-01	.66545E+00	.500E+01	.18772E+01	.500E+03	.41320E+01
.500E-01	.69313E+00	.600E+01	.19605E+01	.600E+03	.42231E+01
.600E-01	.71583E+00	.700E+01	.20320E+01	.700E+03	.43001E+01
.700E-01	.73507E+00	.800E+01	.20945E+01	.800E+03	.43669E+01
.800E-01	.75178E+00	.900E+01	.21501E+01	.900E+03	.44257E+01
.900E-01	.76656E+00	.100E+02	.22001E+01	.100E+04	.44784E+01
.100E+00	.77988E+00				

PSEUDO-SKIN FACTOR=0.6199

TABLE B-7

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 4 STAGGERED VERTICAL ROWS(2 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .5714

TD	PD	TD	PD	TD	PD
.100E-02	.24010E+00	.200E+00	.87036E+00	.200E+02	.25322E+01
.200E-02	.31376E+00	.300E+00	.93532E+00	.300E+02	.27309E+01
.300E-02	.35867E+00	.400E+00	.99034E+00	.400E+02	.28727E+01
.400E-02	.39119E+00	.500E+00	.10395E+01	.500E+02	.29830E+01
.500E-02	.41673E+00	.600E+00	.10842E+01	.600E+02	.30733E+01
.600E-02	.43779E+00	.700E+00	.11254E+01	.700E+02	.31498E+01
.700E-02	.45571E+00	.800E+00	.11636E+01	.800E+02	.32161E+01
.800E-02	.47131E+00	.900E+00	.11992E+01	.900E+02	.32747E+01
.900E-02	.48514E+00	.100E+01	.12326E+01	.100E+03	.33271E+01
.100E-01	.49754E+00	.200E+01	.14828E+01	.200E+03	.36724E+01
.200E-01	.58010E+00	.300E+01	.16497E+01	.300E+03	.38747E+01
.300E-01	.62998E+00	.400E+01	.17748E+01	.400E+03	.40184E+01
.400E-01	.66416E+00	.500E+01	.18749E+01	.500E+03	.41298E+01
.500E-01	.69155E+00	.600E+01	.19583E+01	.600E+03	.42209E+01
.600E-01	.71404E+00	.700E+01	.20297E+01	.700E+03	.42979E+01
.700E-01	.73314E+00	.800E+01	.20922E+01	.800E+03	.43646E+01
.800E-01	.74975E+00	.900E+01	.21478E+01	.900E+03	.44235E+01
.900E-01	.76448E+00	.100E+02	.21978E+01	.100E+04	.44761E+01
.100E+00	.77773E+00				

PSEUDO-SKIN FACTOR=0.6176

TABLE B-8

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 4 STAGGERED VERTICAL ROWS(2 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS:1.0000

TD	PD	TD	PD	TD	PD
.100E-02	.24010E+00	.200E+00	.86805E+00	.200E+02	.25298E+01
.200E-02	.31376E+00	.300E+00	.93288E+00	.300E+02	.27284E+01
.300E-02	.35867E+00	.400E+00	.98787E+00	.400E+02	.28702E+01
.400E-02	.39119E+00	.500E+00	.10370E+01	.500E+02	.29805E+01
.500E-02	.41673E+00	.600E+00	.10818E+01	.600E+02	.30708E+01
.600E-02	.43779E+00	.700E+00	.11230E+01	.700E+02	.31473E+01
.700E-02	.45571E+00	.800E+00	.11612E+01	.800E+02	.32136E+01
.800E-02	.47131E+00	.900E+00	.11967E+01	.900E+02	.32722E+01
.900E-02	.48514E+00	.100E+01	.12300E+01	.100E+03	.33246E+01
.100E-01	.49754E+00	.200E+01	.14804E+01	.200E+03	.36699E+01
.200E-01	.53006E+00	.300E+01	.16472E+01	.300E+03	.38722E+01
.300E-01	.62891E+00	.400E+01	.17724E+01	.400E+03	.40159E+01
.400E-01	.66376E+00	.500E+01	.18724E+01	.500E+03	.41273E+01
.500E-01	.69094E+00	.600E+01	.19558E+01	.600E+03	.42164E+01
.600E-01	.71320E+00	.700E+01	.20273E+01	.700E+03	.42954E+01
.700E-01	.73208E+00	.800E+01	.20898E+01	.800E+03	.43621E+01
.800E-01	.74849E+00	.900E+01	.21453E+01	.900E+03	.44210E+01
.900E-01	.76304E+00	.100E+02	.21953E+01	.100E+04	.44736E+01
.100E+00	.77615E+00				

PSEUDO-SKIN FACTOR=0.6151

TABLE B-9

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 6 STAGGERED VERTICAL ROWS(3 PAIRS?

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .2000

TD	PD	TD	PD	TD	PD
.100E-02	.16007E+00	.200E+00	.60886E+00	.200E+02	.22664E+01
.200E-02	.20920E+00	.300E+00	.67092E+00	.300E+02	.24650E+01
.300E-02	.23928E+00	.400E+00	.72509E+00	.400E+02	.26068E+01
.400E-02	.26122E+00	.500E+00	.77390E+00	.500E+02	.27171E+01
.500E-02	.27859E+00	.600E+00	.81853E+00	.600E+02	.28074E+01
.600E-02	.29300E+00	.700E+00	.85966E+00	.700E+02	.28839E+01
.700E-02	.30533E+00	.800E+00	.89782E+00	.800E+02	.29502E+01
.800E-02	.31611E+00	.900E+00	.93338E+00	.900E+02	.30088E+01
.900E-02	.32568E+00	.100E+01	.96668E+00	.100E+03	.30612E+01
.100E-01	.33429E+00	.200E+01	.12170E+01	.200E+03	.34065E+01
.200E-01	.39164E+00	.300E+01	.13839E+01	.300E+03	.36088E+01
.300E-01	.42542E+00	.400E+01	.15090E+01	.400E+03	.37525E+01
.400E-01	.44943E+00	.500E+01	.16090E+01	.500E+03	.36639E+01
.500E-01	.46817E+00	.600E+01	.16924E+01	.600E+03	.33550E+01
.600E-01	.48369E+00	.700E+01	.17639E+01	.700E+03	.40320E+01
.700E-01	.49708E+00	.800E+01	.18264E+01	.800E+03	.40987E+01
.800E-01	.50900E+00	.900E+01	.18819E+01	.900E+03	.41576E+01
.900E-01	.51986E+00	.100E+02	.19319E+01	.100E+04	.42102E+01
.100E+00	.52991E+00				

PSEUDO-SKIN FACTOR=0.3517

TABLE B-10

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 6 STAGGERED VERTICAL ROWS(3 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.16007E+00	.200E+00	.50285E+00	.200E+02	.22604E+01
.200E-02	.20917E+00	.300E+00	.66491E+00	.300E+02	.24590E+01
.300E-02	.23912E+00	.400E+00	.71908E+00	.400E+02	.26008E+01
.400E-02	.26079E+00	.500E+00	.76789E+00	.500E+02	.27111E+01
.500E-02	.27782E+00	.600E+00	.81252E+00	.600E+02	.28014E+01
.600E-02	.29186E+00	.700E+00	.85365E+00	.700E+02	.28779E+01
.700E-02	.30381E+00	.800E+00	.89181E+00	.800E+02	.29442E+01
.800E-02	.31422E+00	.900E+00	.92737E+00	.900E+02	.30028E+01
.900E-02	.32344E+00	.100E+01	.96067E+00	.100E+03	.30552E+01
.100E-01	.33172E+00	.200E+01	.12110E+01	.200E+03	.34005E+01
.200E-01	.38700E+00	.300E+01	.13778E+01	.300E+03	.36028E+01
.300E-01	.41999E+00	.400E+01	.15830E+01	.400E+03	.37465E+01
.400E-01	.44368E+00	.500E+01	.16030E+01	.500E+03	.38579E+01
.500E-01	.46226E+00	.600E+01	.16664E+01	.600E+03	.39490E+01
.600E-01	.47774E+00	.700E+01	.17578E+01	.700E+03	.40260E+01
.700E-01	.49110E+00	.800E+01	.18204E+01	.800E+03	.40927E+01
.800E-01	.50301E+00	.900E+01	.18759E+01	.900E+03	.41516E+01
.900E-01	.51385E+00	.100E+02	.19259E+01	.100E+04	.42042E+01
.100E+00	.52393E+00				

PSEUDO-SKIN FACTOR=0.*3457*

TABLE B-11

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 6 STAGGERED VERTICAL ROWS(3 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .5714

TD	PD	TD	PD	TD	PC
.100E-02	.16007E+00	.200E+00	.60139E+00	.200E+02	.22569E+01
.200E-02	.20917E+00	.300E+00	.66345E+00	.300E+02	.24575E+01
.300E-02	.23912E+00	.400E+00	.71762E+00	.400E+02	.25993E+[illegible]
.400E-02	.26079E+00	.500E+00	.76643E+00	.500E+02	.27096E+01
.500E-02	.27782E+00	.600E+00	.81105E+00	.600E+02	.28000E+01
.600E-02	.29186E+00	.700E+00	.85219E+00	.700E+02	.28765E+01
.700E-02	.30381E+00	.800E+00	.89034E+00	.800E+02	.29428E+01
.800E-02	.31421E+00	.900E+00	.92591E+00	.900E+02	.30013E+01
.900E-02	.32342E+00	.100E+01	.95921E+00	.100E+03	.30537E+01
.100E-01	.33170E+00	.200E+01	.12095E+01	.200E+03	.33991E+[illegible]
.200E-01	.38673E+00	.300E+01	.13764E+01	.300E+03	.36014E+01
.300E-01	.41939E+00	.400E+01	.15015E+01	.400E+03	.37450E+01
.400E-01	.44282E+00	.500E+01	.16016E+01	.500E+03	.38564E+01
.500E-01	.46123E+00	.600E+01	.16849E+01	.600E+03	.39475E+31
.600E-01	.47655E+00	.700E+01	.17564E+01	.700E+03	.4[illegible]245E+01
.700E-01	.48982E+00	.800E+01	.18189E+01	.800E+03	.40913E+01
.800E-01	.50166E+00	.900E+01	.18745E+01	.900E+03	.41501E+01
.900E-01	.51247E+00	.100E+02	.19245E+01	.100E+04	.42028E+01
.100E+00	.52249E+00				

PSEUDO-SKIN FACTOR=0.3443

TABLE B-12

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 6 STAGGERED VERTICAL ROWS(3 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS:1.0000

TD	PD	TD	PD	TD	PD
.100E-02	.16007E+00	.200E+00	.60003E+00	.200E+02	.22575E+01
.200E-02	.20917E+00	.300E+00	.66204E+00	.300E+02	.24561E+01
.300E-02	.23912E+00	.400E+00	.71620E+00	.400E+02	.25979E+01
.400E-02	.26079E+00	.500E+00	.76502E+00	.500E+02	.27082E+01
.500E-02	.27782E+00	.600E+00	.80964E+00	.600E+02	.27986E+01
.600E-02	.29186E+00	.700E+00	.85078E+00	.700E+02	.28750E+01
.700E-02	.30381E+00	.800E+00	.88893E+00	.800E+02	.29414E+01
.800E-02	.31421E+00	.900E+00	.92450E+00	.900E+02	.29999E+01
.900E-02	.32342E+00	.100E+01	.95779E+00	.100E+03	.30523E+01
.100E-01	.33170E+00	.200E+01	.12081E+01	.200E+03	.33976E+01
.200E-01	.38671E+00	.300E+01	.13750E+01	.300E+03	.36000E+01
.300E-01	.41928E+00	.400E+01	.15001E+01	.400E+03	.37436E+01
.400E-01	.44257E+00	.500E+01	.16001E+01	.500E+03	.38550E+01
.500E-01	.46082E+00	.600E+01	.16835E+01	.600E+03	.39461E+01
.600E-01	.47599E+00	.700E+01	.17550E+01	.700E+03	.40231E+01
.700E-01	.48912E+00	.800E+01	.18175E+01	.800E+03	.40899E+01
.800E-01	.50084E+00	.900E+01	.18731E+01	.900E+03	.41487E+01
.900E-01	.51155E+00	.100E+02	.19231E+01	.100E+04	.42014E+01
.100E+00	.52148E+00				

PSEUDO-SKIN FACTOR=0.3429

TABLE B-13

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 8 STAGGERED VERTICAL ROWS(4 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.12005E+00	.200E+00	.48065E+00	.200E+02	.21380E+01
.200E-02	.15686E+00	.300E+00	.54261E+00	.300E+02	.23367E+01
.300E-02	.17934E+00	.400E+00	.59676E+00	.400E+02	.24784E+01
.400E-02	.19566E+00	.500E+00	.64557E+00	.500E+02	.25888E+01
.500E-02	.20837E+00	.600E+00	.69026E+00	.600E+02	.26791E+01
.600E-02	.21890E+00	.700E+00	.73133E+00	.700E+02	.27556E+01
.700E-02	.22786E+00	.800E+00	.76949E+00	.800E+02	.28219E+01
.800E-02	.23566E+00	.900E+00	.80505E+00	.900E+02	.28805E+01
.900E-02	.24258E+00	.100E+01	.83835E+00	.100E+03	.29329E+01
.100E-01	.24879E+00	.200E+01	.10886E+01	.200E+03	.32782E+01
.200E-01	.29026E+00	.300E+01	.12555E+01	.300E+03	.34805E+01
.300E-01	.31516E+00	.400E+01	.13806E+01	.400E+03	.36241E+01
.400E-01	.33345E+00	.500E+01	.14807E+01	.500E+03	.37356E+01
.500E-01	.34837E+00	.600E+01	.15641E+01	.600E+03	.38267E+01
.600E-01	.36131E+00	.700E+01	.16355E+01	.700E+03	.39037E+01
.700E-01	.37295E+00	.800E+01	.16980E+01	.800E+03	.39784E+01
.800E-01	.38367E+00	.900E+01	.17536E+01	.900E+03	.40293E+01
.900E-01	.39369E+00	.100E+02	.18036E+01	.100E+04	.40819E+01
.100E+00	.40316E+00				

PSEUDO-SKIN FACTOR=0.2234

TABLE *B-14*

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 12 STAGGERED VERTICAL ROWS(6 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .2000

TD	PD	TD	PD	TD	PD
.100E-02	.80033E-01	.200E+00	.37544E+00	.200E+02	.20328E+01
.200E-02	.10460E+00	.300E+00	.43739E+00	.300E+02	.22315E+01
.300E-02	.11964E+00	.400E+00	.49155E+00	.400E+02	.23732E+01
.400E-02	.13061E+00	.500E+00	.54036E+00	.500E+02	.24836E+01
.500E-02	.13930E+00	.600E+00	.58498E+00	.600E+02	.25739E+01
.600E-02	.14650E+00	.700E+00	.62612E+00	.700E+02	.26504E+01
.700E-02	.15266E+00	.800E+00	.66427E+00	.800E+02	.27167E+01
.800E-02	.15806E+00	.900E+00	.69984E+00	.900E+02	.27752E+01
.900E-02	.16285E+00	.100E+01	.73313E+00	.100E+03	.28277E+01
.100E-01	.16716E+00	.200E+01	.96343E+00	.200E+03	.31730E+01
.200E-01	.19652E+00	.300E+01	.11503E+01	.300E+03	.33753E+01
.300E-01	.21568E+00	.400E+01	.12754E+01	.400E+03	.35189E+01
.400E-01	.23117E+00	.500E+01	.13755E+01	.500E+03	.36304E+01
.500E-01	.24471E+00	.600E+01	.14589E+01	.600E+03	.37215E+01
.600E-01	.25694E+00	.700E+01	.15303E+01	.700E+03	.37985E+01
.700E-01	.26821E+00	.800E+01	.15928E+01	.800E+03	.38652E+01
.800E-01	.27874E+00	.900E+01	.16484E+01	.900E+03	.39240E+01
.900E-01	.28865E+00	.100E+02	.16984E+01	.100E+04	.39767E+01
.100E+00	.29805E+00				

PSEUDO-SKIN FACTOR=0.1182

TABLE B-15

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 12 STAGGERED VERTICAL ROWS(6 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.80033E-01	.200E+00	.37262E+00	.200E+02	.20300E+01
.200E-02	.10459E+00	.300E+00	.43458E+00	.300E+02	.22286E+01
.300E-02	.11956E+00	.400E+00	.48673E+00	.400E+02	.23704E+01
.400E-02	.13040E+00	.500E+00	.53754E+00	.500E+02	.24807E+01
.500E-02	.13391E+00	.600E+00	.58216E+00	.600E+02	.25711E+01
.600E-02	.14593E+00	.700E+00	.62330E+00	.700E+02	.26476E+01
.700E-02	.15191E+00	.800E+00	.66145E+00	.800E+02	.27139E+01
.800E-02	.15711E+00	.900E+00	.69702E+00	.900E+02	.27724E+01
.900E-02	.16173E+00	.100E+01	.73031E+00	.100E+03	.28248E+01
.100E-01	.16538E+00	.200E+01	.93061E+00	.200E+03	.31702E+01
.200E-01	.19423E+00	.300E+01	.11475E+01	.300E+03	.33725E+01
.300E-01	.22304E+00	.400E+01	.12726E+01	.400E+03	.35161E+01
.400E-01	.22642E+00	.500E+01	.13727E+01	.500E+03	.36276E+01
.500E-01	.24191E+00	.600E+01	.14560E+01	.600E+03	.37186E+01
.600E-01	.25413E+00	.700E+01	.15275E+01	.700E+03	.37957E+01
.700E-01	.26540E+00	.800E+01	.15900E+01	.800E+03	.38624E+01
.800E-01	.27592E+00	.900E+01	.16456E+01	.900E+03	.39212E+01
.900E-01	.28583E+00	.100E+02	.16956E+01	.100E+04	.39739E+01
.100E+00	.29523E+00				

PSEUDO-SKIN FACTOR=0.1154

TABLE B-16

[redacted] PRESSURE RESPONSE IN A SLOTTED [redacted]

ARRANGEMENT OF SLOTS: 12 STAGGERED VERTICAL ROWS(6 PAIRS)

DIMENSIONLESS PRESSURE [redacted] IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .5714

TD	PD	TD	PD	TD	PD
.100E-02	.80033E-01	.200E+00	.37214E+00	.200E+02	.20295E+01
.200E-02	.10450E+00	.300E+00	.43409E+00	.300E+02	.22281E+01
.300E-02	.11956E+00	.400E+00	.48824E+00	.400E+02	.23639E+01
.400E-02	.13040E+00	.500E+00	.53706E+00	.500E+02	.24303E+01
.500E-02	.13891E+00	.600E+00	.58168E+00	.600E+02	.25706E+01
.600E-02	.14593E+00	.700E+00	.62281E+00	.700E+02	.26471E+01
.700E-02	.15190E+00	.800E+00	.66097E+00	.800E+02	.27134E+01
.800E-02	.15711E+00	.900E+00	.69653E+00	.900E+02	.27719E+01
.900E-02	.16172E+00	.100E+01	.72983E+00	.100E+03	.28243E+01
.100E-01	.16586E+00	.200E+01	.98012E+00	.200E+03	.31697E+01
.200E-01	.19410E+00	.300E+01	.11470E+01	.300E+03	.33720E+01
.300E-01	.21277E+00	.400E+01	.12721E+01	.400E+03	.35156E+01
.400E-01	.22805E+00	.500E+01	.13722E+01	.500E+03	.36271E+01
.500E-01	.24149E+00	.600E+01	.14555E+01	.600E+03	.37132E+01
.600E-01	.25368E+00	.700E+01	.15270E+01	.700E+03	.37952E+01
.700E-01	.26493E+00	.800E+01	.15895E+01	.800E+03	.38619E+01
.800E-01	.27544E+00	.900E+01	.16451E+01	.900E+03	.39207E+01
.900E-01	.28535E+00	.100E+02	.16951E+01	.100E+04	.39734E+01
.100E+00	.29475E+00				

PSEUDO-SKIN FACTOR=0.1149

TABLE B-17

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 12 STAGGERED VERTICAL ROWS(6 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS:1.0000

TD	PO	TD	PO	TD	PO
.100E-02	.80033E-01	.200E+00	.37189E+00	.200E+02	.20293E+01
.200E-02	.10459E+00	.300E+00	.43384E+00	.300E+02	.22279E+01
.300E-02	.11956E+00	.400E+00	.48799E+00	.400E+02	.23697E+01
.400E-02	.13040E+00	.500E+00	.53681E+00	.500E+02	.24800E+01
.500E-02	.13891E+00	.600E+00	.58143E+00	.600E+02	.25703E+01
.600E-02	.14593E+00	.700E+00	.62256E+00	.700E+02	.26468E+01
.700E-02	.15190E+00	.800E+00	.66072E+00	.800E+02	.27131E+01
.800E-02	.15711E+00	.900E+00	.69628E+00	.900E+02	.27717E+01
.900E-02	.16172E+00	.100E+01	.72958E+00	.100E+03	.28241E+01
.100E-01	.16586E+00	.200E+01	.97987E+00	.200E+03	.31694E+01
.200E-01	.19408E+00	.300E+01	.11468E+01	.300E+03	.33717E+01
.300E-01	.21272E+00	.400E+01	.12719E+01	.400E+03	.35154E+01
.400E-01	.22795E+00	.500E+01	.13719E+01	.500E+03	.36268E+01
.500E-01	.24135E+00	.600E+01	.14553E+01	.600E+03	.37179E+01
.600E-01	.25351E+00	.700E+01	.15268E+01	.700E+03	.37949E+01
.700E-01	.26473E+00	.800E+01	.15893E+01	.800E+03	.38616E+01
.800E-01	.27523E+00	.900E+01	.16448E+01	.900E+03	.39205E+01
.900E-01	.28512E+00	.100E+02	.16948E+01	.100E+04	.39731E+01
.100E+00	.29452E+00				

PSEUDO-SKIN FACTOR=0.1146

TABLE B-18

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 16 STAGGERED VERTICAL ROWS(8 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .4167

.100E-02	.60025E-01	.200E+00	.32612E+00	.200E+02	.19835E+01
.200E-02	.75439E-01	.300E+00	.38808E+00	.300E+02	.21821E+01
.300E-02	.89668E-01	.400E+00	.44223E+00	.400E+02	.23239E+01
.400E-02	.97738E-01	.500E+00	.49104E+00	.500E+02	.24342E+01
.500E-02	.10419E+00	.600E+00	.53566E+00	.600E+02	.25246E+01
.600E-02	.10946E+00	.700E+00	.57680E+00	.700E+02	.26011E+01
.700E-02	.11397E+00	.800E+00	.61495E+00	.800E+02	.26674E+01
.800E-02	.11793E+00	.900E+00	.65052E+00	.900E+02	.27259E+01
.900E-02	.12148E+00	.100E+01	.68381E+00	.100E+03	.27783E+01
.100E-01	.12471E+00	.200E+01	.93411E+00	.200E+03	.31237E+01
.200E-01	.14877E+00	.300E+01	.11010E+01	.300E+03	.33260E+01
.300E-01	.16673E+00	.400E+01	.12261E+01	.400E+03	.34696E+01
.400E-01	.18195E+00	.500E+01	.13262E+01	.500E+03	.35811E+01
.500E-01	.19542E+00	.600E+01	.14095E+01	.600E+03	.36721E+01
.600E-01	.20763E+00	.700E+01	.14810E+01	.700E+03	.37491E+01
.700E-01	.21890E+00	.800E+01	.15435E+01	.800E+03	.38159E+01
.800E-01	.22942E+00	.900E+01	.15991E+01	.900E+03	.38747E+01
.900E-01	.23933E+00	.100E+02	.16491E+01	.100E+04	.39274E+01
.100E+00	.24873E+00				

PSEUDO-SKIN FACTOR=0.0689

TABLE B-19

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 20 STAGGERED VERTICAL ROWS(10 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.05

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.48020E-01	.200E+00	.30154E+00	.200E+02	.19589E+01
.200E-02	.62752E-01	.300E+00	.36350E+00	.300E+02	.21576E+01
.300E-02	.71736E-01	.400E+00	.41765E+00	.400E+02	.22993E+01
.400E-02	.78256E-01	.500E+00	.46646E+00	.500E+02	.24097E+01
.500E-02	.83418E-01	.600E+00	.51108E+00	.600E+02	.25008E+01
.600E-02	.87744E-01	.700E+00	.55222E+00	.700E+02	.25765E+01
.700E-02	.91519E-01	.800E+00	.59037E+00	.800E+02	.26428E+01
.800E-02	.94913E-01	.900E+00	.62594E+00	.900E+02	.27013E+01
.900E-02	.96030E-01	.100E+01	.65923E+00	.100E+03	.27538E+01
.100E-01	.10094E+00	.200E+01	.90953E+00	.200E+03	.30991E+01
.200E-01	.12414E+00	.300E+01	.10764E+01	.300E+03	.33014E+01
.300E-01	.14212E+00	.400E+01	.12015E+01	.400E+03	.34450E+01
.400E-01	.15736E+00	.500E+01	.13016E+01	.500E+03	.35565E+01
.500E-01	.17083E+00	.600E+01	.13850E+01	.600E+03	.36476E+01
.600E-01	.18305E+00	.700E+01	.14564E+01	.700E+03	.37246E+01
.700E-01	.19432E+00	.800E+01	.15189E+01	.800E+03	.37913E+01
.800E-01	.20484E+00	.900E+01	.15745E+01	.900E+03	.38521E+01
.900E-01	.21475E+00	.100E+02	.16245E+01	.100E+04	.39028E+01
.100E+00	.22415E+00				

PSEUDO-SKIN FACTOR=0.0443

TABLE B-20

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 24 STAGGERED VERTICAL ROWS(12 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .2000

TD	PD	TD	PD	TD	PD
.100E-02	.40016E-01	.200E+00	.28770E+00	.200E+02	.19451E+01
.200E-02	.52301E-01	.300E+00	.34966E+00	.300E+02	.21437E+01
.300E-02	.59845E-01	.400E+00	.40381E+00	.400E+02	.22855E+01
.400E-02	.65431E-01	.500E+00	.45262E+00	.500E+02	.23958E+01
.500E-02	.69997E-01	.600E+00	.49725E+00	.600E+02	.24862E+01
.600E-02	.73959E-01	.700E+00	.53838E+00	.700E+02	.25626E+01
.700E-02	.77525E-01	.800E+00	.57654E+00	.800E+02	.26290E+01
.800E-02	.80810E-01	.900E+00	.61210E+00	.900E+02	.26875E+01
.900E-02	.83881E-01	.100E+01	.64540E+00	.100E+03	.27399E+01
.100E-01	.86781E-01	.200E+01	.89569E+00	.200E+03	.30852E+01
.200E-01	.11022E+00	.300E+01	.10626E+01	.300E+03	.32876E+01
.300E-01	.12826E+00	.400E+01	.11877E+01	.400E+03	.34312E+01
.400E-01	.14352E+00	.500E+01	.12877E+01	.500E+03	.35426E+01
.500E-01	.15700E+00	.600E+01	.13711E+01	.600E+03	.36337E+01
.600E-01	.16921E+00	.700E+01	.14426E+01	.700E+03	.37107E+01
.700E-01	.18048E+00	.800E+01	.15051E+01	.800E+03	.37775E+01
.800E-01	.19100E+00	.900E+01	.15607E+01	.900E+03	.38363E+01
.900E-01	.20091E+00	.100E+02	.16107E+01	.100E+04	.38890E+01
.100E+00	.21032E+00				

PSEUDO-SKIN FACTOR=0.0305

TABLE B-21

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 24 STAGGERED VERTICAL ROWS(12 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.40016E-01	.200E+00	.28691E+00	.200E+02	.19443E+01
.200E-02	.52234E-01	.300E+00	.34886E+00	.300E+02	.21429E+01
.300E-02	.59803E-01	.400E+00	.40301E+00	.400E+02	.22847E+01
.400E-02	.65325E-01	.500E+00	.45183E+00	.500E+02	.23950E+01
.500E-02	.69810E-01	.600E+00	.49645E+00	.600E+02	.24854E+01
.600E-02	.73687E-01	.700E+00	.53758E+00	.700E+02	.25618E+01
.700E-02	.77173E-01	.800E+00	.57574E+00	.800E+02	.26282E+01
.800E-02	.80384E-01	.900E+00	.61130E+00	.900E+02	.26867E+01
.900E-02	.83391E-01	.100E+01	.64460E+00	.100E+03	.27391E+01
.100E-01	.86236E-01	.200E+01	.89489E+00	.200E+03	.30644E+01
.200E-01	.10945E+00	.300E+01	.10618E+01	.300E+03	.32868E+01
.300E-01	.12747E+00	.400E+01	.11869E+01	.400E+03	.34304E+01
.400E-01	.14272E+00	.500E+01	.12869E+01	.500E+03	.35418E+01
.500E-01	.15620E+00	.600E+01	.13703E+01	.600E+03	.36329E+01
.600E-01	.16842E+00	.700E+01	.14418E+01	.700E+03	.37099E+01
.700E-01	.17968E+00	.800E+01	.15043E+01	.800E+03	.37767E+01
.800E-01	.19020E+00	.900E+01	.15599E+01	.900E+03	.38355E+01
.900E-01	.20011E+00	.100E+02	.16099E+01	.100E+04	.38882E+01
.100E+00	.20952E+00				

PSEUDO-SKIN FACTOR= 0.0297

TABLE B-22

UNSTEADY PRESSURE RESPONSE IN 4 SLOTTED LINER

ARRANGEMENT OF SLOTS: 24 STAGGERED VERTICAL ROWS(12 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS: .5714

TD	PD	TD	PD	TD	PD
.100E-02	.40016E-01	.200E+00	.28686E+00	.200E+02	.19442E+01
.200E-02	.52294E-01	.300E+00	.34882E+00	.300E+02	.21429E+01
.300E-02	.59803E-01	.400E+00	.41297E+00	.400E+02	.22847E+01
.400E-02	.65325E-01	.500E+00	.45178E+00	.500E+02	.23958E+01
.500E-02	.69810E-01	.600E+00	.49641E+00	.600E+02	.24853E+01
.600E-02	.73687E-01	.700E+00	.53754E+00	.700E+02	.25618E+01
.700E-02	.77172E-01	.800E+00	.57570E+00	.800E+02	.26281E+01
.800E-02	.80383E-01	.900E+00	.61126E+00	.900E+02	.26867E+01
.900E-02	.83388E-01	.100E+01	.64456E+00	.100E+03	.27391E+01
.100E-01	.86231E-01	.200E+01	.89485E+00	.200E+03	.30844E+01
.200E-01	.10942E+00	.300E+01	.10617E+01	.300E+03	.32857E+01
.300E-01	.12743E+00	.400E+01	.11858E+01	.400E+03	.34304E+01
.400E-01	.14268E+00	.500E+01	.12869E+01	.500E+03	.35418E+01
.500E-01	.15616E+00	.600E+01	.13703E+01	.600E+03	.36329E+01
.600E-01	.16837E+00	.700E+01	.14417E+01	.700E+03	.37099E+01
.700E-01	.17964E+00	.800E+01	.15042E+01	.800E+03	.37766E+01
.800E-01	.19016E+00	.900E+01	.15598E+01	.900E+03	.38355E+01
.900E-01	.20037E+00	.100E+02	.16098E+01	.100E+04	.38881E+01
.100E+00	.20948E+00				

PSEUDO-SKIN FACTOR=0.0296

TABLE B-23

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 24 STAGGERED VERTICAL ROWS(12 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 1.00

DIMENSIONLESS SLOT LENGTH IS:1.0000

TD	PD	TD	PD	TD	PD
.100E-02	.40016E-01	.200E+00	.28686E+00	.200E+02	.19442E+01
.200E-02	.52294E-01	.300E+00	.34881E+00	.300E+02	.21429E+01
.300E-02	.59803E-01	.400E+00	.40297E+00	.400E+02	.22846E+01
.400E-02	.65325E-01	.500E+00	.45178E+00	.500E+02	.23950E+01
.500E-02	.69810E-01	.600E+00	.49640E+00	.600E+02	.24853E+01
.600E-02	.73687E-01	.700E+00	.53754E+00	.700E+02	.25618E+01
.700E-02	.77172E-01	.800E+00	.57569E+00	.800E+02	.26281E+01
.800E-02	.80383E-01	.900E+00	.61126E+00	.900E+02	.26867E+01
.900E-02	.83388E-01	.100E+01	.64455E+00	.100E+03	.27391E+01
.100E-01	.86231E-01	.200E+01	.89484E+00	.200E+03	.30844E+01
.200E-01	.10942E+00	.300E+01	.10617E+01	.300E+03	.32867E+01
.300E-01	.12743E+00	.400E+01	.11868E+01	.400E+03	.34303E+01
.400E-01	.14268E+00	.500E+01	.12869E+01	.500E+03	.35418E+01
.500E-01	.15615E+00	.600E+01	.13703E+01	.600E+03	.36329E+01
.600E-01	.16837E+00	.700E+01	.14417E+01	.700E+03	.37099E+01
.700E-01	.17964E+00	.800E+01	.15042E+01	.800E+03	.37766E+01
.800E-01	.19015E+00	.900E+01	.15598E+01	.900E+03	.38355E+01
.900E-01	.20007E+00	.100E+02	.16098E+01	.100E+04	.38881E+01
.100E+00	.20947E+00				

PSEUDO-SKIN FACTOR=0.0296

TABLE B-24

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 2 STAGGERED VERTICAL ROWS(1 PAIR)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= .50

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.49509E-29	.200E+00	.22236E+00	.200E+02	.20533E+01
.200E-02	.16541E-15	.300E+00	.31120E+00	.300E+02	.22535E+01
.300E-02	.92051E-11	.400E+00	.38545E+00	.400E+02	.23960E+01
.400E-02	.22441E-08	.500E+00	.44981E+00	.500E+02	.25068E+01
.500E-02	.63628E-07	.600E+00	.50680E+00	.600E+02	.25975E+01
.600E-02	.60980E-06	.700E+00	.55796E+00	.700E+02	.26742E+01
.700E-02	.31295E-05	.800E+00	.60441E+00	.800E+02	.27407E+01
.800E-02	.10838E-04	.900E+00	.64694E+00	.900E+02	.27993E+01
.900E-02	.28820E-04	.100E+01	.68615E+00	.100E+03	.28518E+01
.100E-01	.63618E-04	.200E+01	.96768E+00	.200E+03	.31976E+01
.200E-01	.26776E-02	.300E+01	.11469E+01	.300E+03	.34061E+01
.300E-01	.10608E-01	.400E+01	.12787E+01	.400E+03	.35438E+01
.400E-01	.22349E-01	.500E+01	.13829E+01	.500E+03	.36553E+01
.500E-01	.36052E-01	.600E+01	.14691E+01	.600E+03	.37464E+01
.600E-01	.50557E-01	.700E+01	.15426E+01	.700E+03	.38235E+01
.700E-01	.65219E-01	.800E+01	.16066E+01	.800E+03	.38902E+01
.800E-01	.79698E-01	.900E+01	.16634E+01	.900E+03	.39491E+01
.900E-01	.93822E-01	.100E+02	.17144E+01	.100E+04	.40017E+01
.100E+00	.10752E+00				

PSEUDO-SKIN FACTOR=0.1432

TABLE B-25

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 4 STAGGERED VERTICAL ROWS(2 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= .50

DIMENSIONLESS SLOT LENGTH IS: a4167

TD	PD	TD	PD	TD	PD
.100E-02	.24754E-29	.200E+00	.13398E+00	.200E+02	.19260E+01
.200E-02	.92704E-16	.300E+00	.20677E+00	.300E+02	.21261E+01
.300E-02	.46026E-11	.400E+00	.27307E+00	.400E+02	.22687E+01
.400E-02	.11220E-08	.500E+00	.33297E+00	.500E+02	.23795E+01
.500E-02	.31814E-07	.600E+00	.38722E+00	.600E+02	.24701E+01
.600E-02	.30490E-06	.700E+00	.43659E+00	.700E+02	.25468E+01
.700E-02	.15647E-05	.800E+00	.48180E+00	.800E+02	.26133E+01
.800E-02	.54188E-05	.900E+00	.52343E+00	.900E+02	.26720E+01
.900E-02	.14413E-04	.100E+01	.56199E+00	.100E+03	.27245E+01
.100E-01	.31889E-04	.200E+01	.84120E+00	.200E+03	.30703E+01
.200E-01	.13388E-02	.300E+01	.10208E+01	.300E+03	.32727E+01
.300E-01	.53046E-02	.400E+01	.11616E+01	.400E+03	.34165E+01
.400E-01	.11186E-01	.500E+01	.12557E+01	.500E+03	.35280E+01
.500E-01	.18094E-01	.600E+01	.13418E+01	.600E+03	.36191E+01
.600E-01	.25504E-01	.700E+01	.14153E+01	.700E+03	.36961E+01
.700E-01	.33152E-01	.800E+01	.14793E+01	.800E+03	.37628E+01
.800E-01	.40910E-01	.900E+01	.15361E+01	.900E+03	.38217E+01
.900E-01	.48718E-01	.100E+02	.15871E+01	.100E+04	.38744E+01
.100E+00	.56546E-01				

PSEUDO-SKIN FACTOR=0.0159

TABLE B-26

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 6 STAGGERED VERTICAL ROWS(3 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= .50

DIMENSIONLESS SLOT LEEJGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.16503E-29	.200E+00	.12109E+00	.200E+02	.19126E+01
.200E-02	.61603E-16	.300E+00	.19354E+00	.300E+02	.21127E+01
.300E-02	.30684E-11	.400E+00	.25974E+00	.400E+02	.22553E+01
.400E-02	.74803E-09	.500E+00	.31952E+00	.500E+02	.23661E+01
.500E-02	.21209E-07	.600E+00	.37386E+00	.600E+02	.24567E+01
.600E-02	.20327E-06	.700E+00	.42322E+00	.700E+02	.25334E+01
.700E-02	.10432E-05	.800E+00	.46843E+00	.800E+02	.25999E+01
.800E-02	.36126E-05	.900E+00	.51006E+00	.900E+02	.26586E+01
.900E-02	.96067E-05	.100E+01	.54861E+00	.100E+03	.27111E+01
.100E-01	.21206E-04	.200E+01	.82782E+00	.200E+03	.30569E+01
.200E-01	.83396E-03	.300E+01	.10066E+01	.300E+03	.32594E+01
.300E-01	.35820E-02	.400E+01	.11382E+01	.400E+03	.34031E+01
.400E-01	.77304E-02	.500E+01	.12423E+01	.500E+03	.35146E+01
.500E-01	.12882E-01	.600E+01	.13284E+01	.600E+03	.36057E+01
.600E-01	.19734E-01	.700E+01	.14019E+01	.700E+03	.36827E+01
.700E-01	.25087E-01	.800E+01	.14660E+01	.800E+03	.37495E+01
.800E-01	.31803E-01	.900E+01	.15227E+01	.900E+03	.38043E+01
.900E-01	.38785E-01	.100E+02	.15737E+01	.100E+04	.38610E+01
.100E+00	.45962E-01				

PSEUDO-SKIN FACTOR=0.0025

TABLE B-27

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 24 STAGGERED VERTICAL ROWS(12 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= .56

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.41281E-30	.200E+00	.11868E+00	.200E+02	.19102E+01
.200E-02	.15955E-16	.300E+00	.19112E+00	.300E+02	.21103E+01
.300E-02	.86508E-12	.400E+00	.25733E+00	.400E+02	.22529E+01
.400E-02	.23439E-09	.500E+00	.31721E+00	.500E+02	.23637E+01
.500E-02	.73305E-08	.600E+00	.37144E+00	.600E+02	.24543E+01
.600E-02	.76469E-07	.700E+00	.42081E+00	.700E+02	.25310E+01
.700E-02	.42186E-06	.800E+00	.46601E+00	.800E+02	.25975E+01
.800E-02	.15545E-05	.900E+00	.50764E+00	.900E+02	.26562E+01
.900E-02	.43632E-05	.100E+01	.54620E+00	.100E+03	.27087E+01
.100E-01	.10100E-04	.200E+01	.82541E+00	.200E+03	.30545E+01
.200E-01	.57115E-03	.300E+01	.10042E+01	.300E+03	.32570E+01
.300E-01	.26641E-02	.400E+01	.11358E+01	.400E+03	.34007E+01
.400E-01	.62915E-02	.500E+01	.12399E+01	.500E+03	.35122E+01
.500E-01	.11086E-01	.600E+01	.13260E+01	.600E+03	.36033E+01
.600E-01	.16712E-01	.700E+01	.13995E+01	.700E+03	.36803E+01
.700E-01	.22925E-01	.800E+01	.14635E+01	.800E+03	.37470E+01
.800E-01	.29553E-01	.900E+01	.15253E+01	.900E+03	.38059E+01
.900E-01	.36480E-01	.100E+02	.15713E+01	.100E+04	.38586E+01
.100E+00	.43621E-01				

PSEUDO-SKIN FACTOR=0.000

TABLE B-28

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 2 STAGGERED VERTICAL ROWS(1 PAIR)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 2.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.57863E-43	.200E+00	.36592E-01	.200E+02	.13836E+01
.200E-02	.70756E-43	.300E+00	.73144E-01	.300E+02	.15763E+01
.300E-02	.19889E-38	.400E+00	.10819E+00	.400E+02	.17151E+01
.400E-02	.26374E-29	.500E+00	.14045E+00	.500E+02	.18236E+01
.500E-02	.87354E-24	.600E+00	.17008E+00	.600E+02	.19127E+01
.600E-02	.43629E-20	.700E+00	.19748E+00	.700E+02	.19884E+01
.700E-02	.19608E-17	.800E+00	.22304E+00	.800E+02	.20540E+01
.800E-02	.19483E-15	.900E+00	.24705E+00	.900E+02	.21121E+01
.900E-02	.70627E-14	.100E+01	.26975E+00	.100E+03	.21641E+01
.100E-01	.12622E-12	.200E+01	.45050E+00	.200E+03	.25075E+01
.200E-01	.67069E-07	.300E+01	.58241E+00	.300E+03	.27092E+01
.300E-01	.63682E-05	.400E+01	.68685E+00	.400E+03	.28525E+01
.400E-01	.66735E-04	.500E+01	.77331E+00	.500E+03	.29638E+01
.500E-01	.28461E-03	.600E+01	.84708E+00	.600E+03	.30548E+01
.600E-01	.76810E-03	.700E+01	.91138E+00	.700E+03	.31317E+01
.700E-01	.15889E-02	.800E+01	.96838E+00	.800E+03	.31983E+01
.800E-01	.27762E-02	.900E+01	.10196E+01	.900E+03	.32571E+01
.900E-01	.43271E-02	.100E+02	.10660E+01	.100E+04	.33098E+01
.100E+00	.62187E-02				

PSEUDO-SKIN FACTOR=0.1440

TABLE B-29

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 4 STAGGERED VERTICAL ROWS(2 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 2.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.57863E-43	.200E+00	.18364E-01	.200E+02	.12560E+01
.200E-02	.70756E-43	.300E+00	.37344E-01	.300E+02	.14487E+01
.300E-02	.99449E-39	.400E+00	.56879E-01	.400E+02	.15874E+01
.400E-02	.13187E-29	.500E+00	.76454E-01	.500E+02	.16959E+01
.500E-02	.43677E-24	.600E+00	.95348E-01	.600E+02	.17850E+01
.600E-02	.21814E-20	.700E+00	.11525E+00	.700E+02	.18606E+01
.700E-02	.98039E-18	.800E+00	.13432E+00	.800E+02	.19263E+01
.800E-02	.97415E-16	.900E+00	.15307E+00	.900E+02	.19843E+01
.900E-02	.35314E-14	.100E+01	.17148E+00	.100E+03	.20363E+01
.100E-01	.63108E-13	.200E+01	.33331E+00	.200E+03	.23798E+01
.200E-01	.33534E-07	.300E+01	.46001E+00	.300E+03	.25815E+01
.300E-01	.31841E-05	.400E+01	.56233E+00	.400E+03	.27248E+01
.400E-01	.33368E-04	.500E+01	.64772E+00	.500E+03	.28361E+01
.500E-01	.14231E-03	.600E+01	.72087E+00	.600E+03	.29270E+01
.600E-01	.38405E-03	.700E+01	.78480E+00	.700E+03	.30040E+01
.700E-01	.79445E-03	.800E+01	.84154E+00	.800E+03	.30736E+01
.800E-01	.13881E-02	.900E+01	.89254E+00	.900E+03	.31294E+01
.900E-01	.21636E-02	.100E+02	.93884E+00	.100E+04	.31820E+01
.100E+00	.31094E-02				

PSEUDO-SKIN FACTOR=0.0163

TABLE B-30

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 6 STAGGERED VERTICAL ROWS(3 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 2.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.57863E-43	.200E+00	.13054E-01	.200E+02	.12425E+01
.200E-02	.70756E-43	.300E+00	.28606E-01	.300E+02	.14352E+01
.300E-02	.66302E-39	.400E+00	.46166E-01	.400E+02	.15739E+01
.400E-02	.87912E-30	.500E+00	.64648E-01	.500E+02	.16824E+01
.500E-02	.29118E-24	.600E+00	.83554E-01	.600E+02	.17715E+01
.600E-02	.14543E-20	.700E+00	.10246E+00	.700E+02	.18471E+01
.700E-02	.65359E-18	.800E+00	.12130E+00	.800E+02	.19128E+01
.800E-02	.64944E-16	.900E+00	.13991E+00	.900E+02	.19708E+01
.900E-02	.23542E-14	.100E+01	.15822E+00	.100E+03	.20228E+01
.100E-01	.42072E-13	.200E+01	.31983E+00	.200E+03	.23663E+01
.200E-01	.22356E-07	.300E+01	.44651E+00	.300E+03	.25680E+01
.300E-01	.21227E-05	.400E+01	.54882E+00	.400E+03	.27113E+01
.400E-01	.22245E-04	.500E+01	.63421E+00	.500E+03	.28226E+01
.500E-01	-.94875E-04	.600E+01	.70736E+00	.600E+03	.29135E+01
.600E-01	.25608E-03	.700E+01	.77129E+00	.700E+03	.29904E+01
.700E-01	.52995E-03	.800E+01	.82803E+00	.800E+03	.30571E+01
.800E-01	.92679E-03	.900E+01	.87903E+00	.900E+03	.31159E+01
.900E-01	.14467E-02	.100E+02	.92533E+00	.100E+04	.31685E+01
.100E+00	.20839E-02				

PSEUDO-SKIN FACTOR=0.0028

TABLE B-31

UNSTEADY PRESSURE RESPONSE IN A SLOTTED LINER

ARRANGEMENT OF SLOTS: 24 STAGGERED VERTICAL ROWS(12 PAIRS)

DIMENSIONLESS PRESSURE DROP IS CALCULATED AT CENTER OF FORMATION

RD= 2.00

DIMENSIONLESS SLOT LENGTH IS: .4167

TD	PD	TD	PD	TD	PD
.100E-02	.57863E-43	.200E+00	.11087E-01	.200E+02	.12309E+31
.200E 02	.70756E-43	.300E+00	.26196E-01	.300E+02	.14325E+01
.300E-02	.16582E-39	.400E+00	.43622E-01	.400E+02	.15713E+01
.400E-02	.21987E-30	.500E+00	.62059E-01	.500E+02	.16798E+01
.500E-02	.72961E-25	.600E+00	.80908E-01	.600E+02	.17669E+01
.600E-02	.36611E-21	.700E+00	.99846E-01	.700E+02	.18445E+01
.700E-02	.16594E-18	.800E+00	.11868E+00	.800E+02	.19102E+01
.800E-02	.16696E-16	[illegible]00E+00	.13729E+00	.900E+02	.19682E+01
.900E-02	.61514E-15	.100E+01	.15560E+00	.100E+03	.20202E+01
.100E-01	.11207E-13	.200E+01	.31721E+00	.200E+03	.23637E+01
.200E 01	.75272E-08	.300E+01	.44389E+00	.300E+03	.25654E+01
.300E-01	.85487E-06	.400E+01	.54620E+00	.400E+03	.27087E+01
.400E-01	.10175E-04	.500E+01	.63159E+00	.500E+03	.28199E+01
.500E-01	.47840E-04	.600E+01	.7t474E+00	.600E+03	.29109E+01
.600E 01	.13973E-03	.700E+01	.76867E+00	.700E+03	.29878E+31
.700E-01	.30894E-03	.800E+01	.82541E+00	.800E+03	.30545E+01
.800E-01	.57170E-03	.900E+01	.87641E+00	.900E+03	.31133E+01
.900E-01	.93715E-03	.100E+02	.92271E+00	.100E+04	.31659E+01
.100E+00	.14087E-02				

PSEUDO-SKIN FACTOR=0.0000

TABLE B-32

unsteady pressure response in a slotted liner-

arrangement of slots: & staggered vertical rows(3 pairs)

location of calculated pressure drop is 1/4 of circumferencial distance between two "i" vertical rows of slots

dimensionless pressure drop is calculated at center of formation

rd= 1.00

dimensionless slot length(hd) is: .4167

td	pd	td	pd	td	pd
0.100E-02	.45045E-31	0.500E+00	.30681E+00	0.800E+02	.24831E+01
0.200E-02	.14136E-16	0.600E+00	.35143E+00	0.900E+02	.25417E+01
0.300E-02	.14380E-11	0.700E+00	.39256E+00	0.100E+03	.25941E+01
0.400E-02	.50231E-09	0.800E+00	.43072E+00	0.200E+03	.29394E+01
0.500E-02	.17660E-07	0.900E+00	.46628E+00	0.300E+03	.31417E+01
0.600E-02	.19527E-06	0.100E+01	.49958E+00	0.400E+03	.32854E+01
0.700E-02	.11095E-05	0.200E+01	.74987E+00	0.500E+03	.33968E+01
0.800E-02	.41460E-05	0.300E+01	.91675E+00	0.600E+03	.34879E+01
0.900E-02	.11695E-04	0.400E+01	.10419E+01	0.700E+03	.35649E+01
0.100E-01	.27056E-04	0.500E+01	.11419E+01	0.800E+03	.36316E+01
0.200E-01	.14008E-02	0.600E+01	.12253E+01	0.900E+03	.36905E+01
0.300E-01	.59288E-02	0.700E+01	.12968E+01	0.100E+04	.37431E+01
0.400E-01	.12905E-01	0.800E+01	.13593E+01	0.200E+04	.40896E+01
0.500E-01	.21235E-01	0.900E+01	.14148E+01	0.300E+04	.42923E+01
0.600E-01	.30186E-01	0.100E+02	.14648E+01	0.400E+04	.44361E+01
0.700E-01	.39337E-01	0.200E+02	.17993E+01	0.500E+04	.45477E+01
0.800E-01	.48457E-01	0.300E+02	.19979E+01	0.600E+04	.46388E+01
0.900E-01	.57426E-01	0.400E+02	.21397E+01	0.700E+04	.47159E+01
0.100E+00	.66184E-01	0.500E+02	.22500E+01	0.800E+04	.47827E+01
0.200E+00	.14201E+00	0.600E+02	.23403E+01	0.900E+04	.48415E+01
0.300E+00	.20386E+00	0.700E+02	.24168E+01	0.100E+05	.48942E+01
0.400E+00	.25800E+00				

TABLE B-33

unsteady pressure response in a slotted liner

arrangement of slots: 6 staggered vertical rows(3 pairs)

location of calculated pressure drop is 1/2 of circumferencial distance between two "i" vertical rows of slots

dimensionless pressure drop is calculated at center+hd/2. of formation

rd= 1.00

dimensionless slot length(hd) is: .4167

td	pd	td	pd	td	pd
0.100E-02	.45192E-07	0.500E+00	.40025E+00	0.800E+02	.25766E+01
0.200E-02	.24614E-04	0.600E+00	.44487E+00	0.900E+02	.26351E+01
0.300E-02	.24756E-03	0.700E+00	.48600E+00	0.100E+03	.26875E+01
0.400E-02	.85706E-03	0.800E+00	.52416E+00	0.200E+03	.30329E+01
0.500E-02	.18952E-02	0.900E+00	.55972E+00	0.300E+03	.32352E+01
0.600E-02	.33151E-02	0.100E+01	.59302E+00	0.400E+03	.33788E+01
0.700E-02	.50483E-02	0.200E+01	.84331E+00	0.500E+03	.34903E+01
0.800E-02	.70222E-02	0.300E+01	.10102E+01	0.600E+03	.35813E+01
0.900E-02	.91785E-02	0.400E+01	.11353E+01	0.700E+03	.36584E+01
0.100E-01	.11470E-01	0.500E+01	.12354E+01	0.800E+03	.37251E+01
0.200E-01	.35452E-01	0.600E+01	.13187E+01	0.900E+03	.37839E+01
0.300E-01	.59517E-01	0.700E+01	.13902E+01	0.100E+04	.38366E+01
0.400E-01	.79251E-01	0.800E+01	.14527E+01	0.200E+04	.41830E+01
0.500E-01	.96128E-01	0.900E+01	.15083E+01	0.300E+04	.43857E+01
0. OWE-01	.11080E+00	0.100E+02	.15583E+01	0.400E+04	.45296E+01
0.700E-01	.12379E+00	0.200E+02	.18927E+01	0.500E+04	.46411E+01
0.800E-01	.13553E+00	0.300E+02	.20913E+01	0.600E+04	.47323E+01
0.900E-01	.14629E+00	0.400E+02	.22331E+01	0.700E+04	.48093E+01
0.100E+00	.15629E+00	0.500E+02	.23435E+01	0.800E+04	.48761E+01
0.200E+00	.23520E+00	0.600E+02	.24338E+01	0.900E+04	.49350E+01
0.300E+00	.29726E+00	0.700E+02	.25103E+01	0.100E+05	.49877E+01
0.400E+00	.35143E+00				

SPE

Society of Petroleum Engineers of AIME

APPENDIX C

Paper SPE 10785-by D. 'Spivak and R. N. Horne

UNSTEADY-STATE PRESSURE RESPONSE DUE TO PRODUCTION WITH A SLOTTED LINER COMPLETION

ABSTRACT

The transient behavoiur of production from a well with a slotted liner has been solved analytically by the source function method. The unsteady-state pressure response was examined as a function of slot frequency, slot length and penetration ratio of *0.5*. The results indicate that due to limited entry into the well, skin effects appear. However, the effects become negligible when the number of slots distributed around the circumference of the liner is six or more. For more than six slots, the well behaves effectively like a continuous surface cylindrical source (or a line source well, depending on time and location).

This work concludes that there is likely to be little reduction in flow efficiency in an ideal slotted liner completion, unless some slots are clogged.

INTRODUCTION

A well is completed with a slotted liner when sand problems occur, mainly in unconsolidated formations[1]. The slotted liner is a pre-perforated tubing. Usually the slots are milled and machined in vertical rows, parallel to the axis of the pipe. The most common patterns of arrangements of slots are the staggered vertical rows, multiple staggered vertical and horizontal slots, as shown on **Fig. 1.** Standard spacing of slots is 6" between the slots in each vertical row, while their length is usually 1½" to 2" depending on their width. The common practice is to specify two rows for each inch of nominal pipe diameter[2].

This work arrived at an analytical solution to the problem of transient pressure response due to production through a liner with staggered vertical rows of Slots. The technique used was as suggested by Gringarten and Ramey[4]. It applies the use of a line source of finite length which in return represents the system of slots. The solution technique was verified by comparison with the continuous surface cylinder solution in the case of a large number of vertical rows of slots. No storage or damaged zone effects were considered.

References and illustrations at end of paper

PROBLEM DESCRIPTION

Consider an infinite. homogeneous and isotropic porous medium. The transient (unsteady-state) flow of a slightly compressible fluid is governed by the diffusivity equation:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{\eta}\frac{\partial p}{\partial t} \qquad (1)$$

where $\eta = \frac{k}{\phi \mu c_t}$ is the diffusivity term. The solution to a particular problem is determined by the initial and boundary conditions. One of the most well-known solutions is Lord Kelvin's instantaneous point source function, which is:

$$\Delta p(M,t) = \frac{q}{8\phi c_t (\pi \eta t)^{3/2}} \exp\left(-\frac{d^2}{4\eta t}\right) \qquad (2)$$

Eq. 2 represents the pressure drop created at point M in a reservoir of infinite extent by an instantaneous point source of strength q at point P, distanced from point M (**Fig. 2**). The strength q is the withdrawal rate per unit volume of source; it is also defined as e withdrawal rate per unit length or area of source, depending on its nature, i.e., line source or surface cylinder source[3]. From Eq. 2 solutions to many other problems are obtainable by integration . The instantaneous infinite line source case, for example, is obtained by expressing d in Eq. 2 in terms of x, y, z coordinates and integrating z from $-\infty$ to $+\infty$.
The method was first applied in pressure transient analysis by Theis[4].

If there is a continuous withdrawal of fluid, of strength q from time 0 to t, the pressure drop created by this continuous point source is given by integration over time :

$$\Delta p(M,t) = \frac{q}{8\phi c_t (\pi \eta)^{3/2}} \int_0^t \frac{1}{(t-t')^{3/2}} \cdot \exp\left(-\frac{d^2}{4\eta (t-t')}\right) dt', \qquad (3)$$

Now consider the actual problem at hand. Fig. 3 shows the coordinate sketch of one slot. The pressure drop created at point M by a single slot is obtained by integrating an instantaneous point source with respect to time and the z coordinate, in a manner similar to that used by Nisle[5]. Appendix A gives details of the derivation of the solution. Briefly stated, the continuous pressure drop created at point M by a single slot extending from $z = 0$ to $z = \ell$ is expressed by the following equation:

$$\Delta p_i(M,t) = \frac{q_i}{8\phi c_t(\pi\eta)^{3/2}} \int_0^t \frac{\exp[-r_i^2/4\eta(t-t')]}{(t-t')^{3/2}}$$

$$\cdot\{\int_0^\ell \exp[-(z'-z_m)^2/4\eta(t-t')dz'\}dt' \qquad (4)$$

Note that there are two sets of vertical rows of slots; one set being staggered from the other. The base set of slots is referred to as the "i" slots and the staggered set as the "j" slots. Eq. 4 therefore refers to one of the i slots.

Eq. 4 can be expressed in terms of the error function (after making a change of variables) as[6]:

$$\Delta p_i(M,t) = \frac{q_i}{8\phi c_t \pi\eta} \int_0^t \frac{\exp(-r_i^2/4\eta\tau)}{\tau}$$

$$\cdot\{\mathrm{erf}\frac{z_m}{\sqrt{4\eta\tau}} - \mathrm{erf}\frac{z_m-\ell}{\sqrt{4\eta\tau}}\}d\tau \qquad (5)$$

Introducing some dimensionless variables (described in Appendix A) and considering a penetration ratio of ½ result in the following dimensionless pressure drop caused by one slot[6]:

$$p_{D_i}(M,t_D) = \frac{1}{4\cdot NS} \int_0^{t_D} \frac{\exp[-r_{iD}^2/4t_D]}{t_D}$$

$$\cdot\{\mathrm{erf}\frac{z_{mD}}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}-\ell_D}{\sqrt{4t_D}}\}dt_D \qquad (6)$$

where NS is the number of slots in each ring of slots. The integral in Eq. 6 can be evaluated numerically using Simpson's rule.

One of the last stages of the mathematical derivation is to sum Eq. 6 over all the slots in the liner.

In this work a reservoir of thickness h with upper and lower impermeable boundaries was considered as shown on Fig. 4. Therefore, the boundary conditions are:

$$\frac{\partial p_D}{\partial z_D}(r_D,0,t_D) = \frac{\partial p_D}{\partial z_D}(r_D,h_D,t_D) = 0 \qquad (7)$$

and

$$\lim_{r_D\to\infty} p_D(r_D,z_D,t_D) = 0 \qquad (8)$$

The upper and lower impermeable reservoir boundaries can be represented by considering image slots and adding their effects to those caused by the real ones. There will therefore be an infinite summation over all real and image slots. Assuming that the formation is penetrated by an integral number of staggered pairs, with half length slots at top and bottom, the general model shown on Fig. 4 can be reduced to a basic unit as illustrated on Fig. 5. It should be noted that both on Figs. 4 and 5 the total cumulative length of the slots in one vertical row (either an i row or a staggered j row) is one half of the formation thickness, that is the penetration ratio is ½.

Finally, the total dimensionless pressure drop created at point M by all real and image slots is given by[6]:

$$p_D^*(M,t_D) = \frac{1}{4\cdot NS}\sum_{i=1}^{NS}\int_0^{t_D}\frac{\exp[-r_{iD}^2/4t_D]}{t_D}$$

$$\cdot\{\sum_{n=-\infty}^{+\infty}(\mathrm{erf}\frac{z_{mD}+2nh_D+\ell_D}{\sqrt{4t_D}} - \mathrm{erf}\frac{z_{mD}+2nh_D-\ell_D}{\sqrt{4t_D}})\}dt_D$$

$$+\frac{1}{4\cdot NS}\sum_{j=1}^{NS}\int_0^{t_D}\frac{\exp[-r_{jD}^2/4t_D]}{t_D}$$

$$\cdot\{\sum_{n=-\infty}^{+\infty}(\mathrm{erf}\frac{z_{mD}+(2n-1)h_D+\ell_D}{\sqrt{4t_D}}$$

$$-\mathrm{erf}\frac{z_{mD}+(2n-1)h_D-\ell_D}{\sqrt{4t_D}})\}dt_D \qquad (9)$$

Although this configuration of the line source segments represents the arrangement of slots, the resulting pressure distribution will not correctly represent the impermeable liner between the slots. Since the system modelled has "reservoir" both inside and outside the rings of slots, flow into the inner face of the ring of line source segments must pass between the slots from the outside. In the steady-state case, this difficulty is easily avoided by placing line source at the axis $r = 0$ of ½ the total strength of the NS line source segments around the circumference, and opposite in sign (i.e., a source as opposed to a sink). This configuration results in a circular no-flow boundary passing through each of the slots (provided they are regularly arranged around the ring), with the "inside" production into the slots being provided by the line source at the centerline. For unsteady state the situation is a little more complex, since the pressure and stream function are no longer harmonic; however, for $t_D/r_D^2 > 70$, the pressure function is approximately logarithmic and is therefore harmonic in a region close to the well and becomes harmonic over a wider radius as time progresses. For small radii (close to the well), the pressure and stream function are harmonic for very small times, and the circular line joining the slots is a very good approximation to a no-flow boundary. This is demonstrated on Fig. 6 which shows the isopotential lines and the directions of their normals for an arrangement of four slots (only one quadrant is shown).

Thus to obtain the correct representation of the slotted liner, it is necessary to multiply the pressure drop given by Eq. 9 by two (to double the flow rate) and subtract the line source (exponential integral) pressure function:

$$p_D(M,t_D) = 2\cdot p_D^*(M,t_D) - \left[-\tfrac{1}{2}Ei(-r_D^2/4t_D)\right] \quad (10)$$

In this way the sum of the flow rates into a ring of slots will be 2q, made up of a flow rate q from the reservoir to the exterior of the slots, and a flow rate q from the centerline line source to the interior of the slots.

SHORT-TIME BERAVIOUR

In studying the ahort time behaviour of Eq. 10, three cases were considered (see Fig.4):

1) $0<z_{mD}<\ell_D$

2) $h_D-\ell_D<z_{mD}<h_D$

3) $z_{mD} = \ell_D$

Case (1) : for $t_D \to 0$ and

$$0<z_{mD}<\ell_D,$$

$$erf\frac{z_{mD}+e_D}{\sqrt{4t_D}} \to 1$$

$$erf\frac{z_{mD}-\ell_D}{\sqrt{4t_D}} \to -1$$

and Eq. 10 reduces to the following expression[6]:

$$p_D(M,t_D) = -\frac{1}{NS}\sum_{i=1}^{NS} Ei(-r_{iD}^2/4t_D) + \tfrac{1}{2}Ei(-r_D^2/4t_D) \quad (11)$$

Case (2) : by the same reasoning as in case (1), it can easily be shown that Eq. 10 becomes[6]:

$$p_D(M,t_D) = -\frac{1}{NS}\sum_{i=1}^{NS} Ei(-r_{jD}^2/4t_D) + \tfrac{1}{2}Ei(-r_D^2/Gt_D) \quad (12)$$

Case (3) : in this case, Eq. 10 simply reduces to[6]:

$$p_D(M,t_D) = -\frac{1}{2\cdot NS}\sum_{i=1}^{NS} Ei(-r_{1D}^2/4t_D) - \frac{1}{2\cdot NS}\sum_{i=1}^{NS} Ei(-r_{jD}^2/4t_D) + \tfrac{1}{2}Ei(-r_D^2/4t_D) \quad (13)$$

LONG-TIME BEHAVIOUR

For sufficent long time, Eq. 10 becomes:

$$p_D(M,t_D) = -\tfrac{1}{2}Ei(-r_D^2/4t_D) + s(r_{iD},r_{jD},z_{mD},\theta,t_D,h_D,h_D,n_s,NS) \quad (14)$$

where the first term is the line source solution and s is the pseudo-skin factor which is dependent on time, but can be approximated by a constant (to any degree of accuracy) for large values of time. For accuracy of 1 percent s becomes a constant for a cutoff time given by:

$$t_D > \text{Max}\begin{cases} 70r_D^2 \\ 25r_{iD}^2 \\ 25r_{jD}^2 \\ 5h_D^2/\pi^2 \end{cases}$$

Full details of this derivation are given by Spivak[6].

RESULTS

The integral in Eq. 9 was calculated using Simpson's rule with 100 time divisions per log cycle. Figs. 7,8 show the semi-log behaviour of p_D vs. t_D and t_D/r_D^2 considering different number of vertical rows of slots, $z_D = h_D/2$ (i.e., center of formation), different radii (r_D) and a penetration ratio of ½ .

The slot length was found to have an overall negligible effect.

The position of the pressure measurement (z_{mD}) does influence the pressure drop significantly, especially when p_D is calculated at $r_D = 1$. (on the surface of the liner). Away from the liner ($r_D>1$), the importance of z_{mD} decreases as r_D increases. It was found that the maximum value of the pseudo-skin factor s for a liner which has only two staggered vertical rows (one slot per ring, or NS=1) is $s \cong 3$ (on the surface). This parameter depends on the location of the pressure measurement, and takes smaller values further from the slot tip. When the number of vertical rows is increased to six (NS = 3), the pseudo-skin factor rapidly decreases to $s \cong 0.7$. For more than six vertical rows, the well behaves effectively like a continuous surface cylindrical source (or a line source well, depending on time and location) for which values of p_D vs. t_D for different r_D were calculated by Gringarten and Ramey[8]. Also, in interference tests the well would behave essentially as a line source even for less than six vertical rows of slots, depending on the observation radius unless some slots are clogged.

CONCLUSIONS

In this study, the effect of a slotted liner completion on pressure translent analysis was determined using a simple analytical model. It was concluded that:

(1) For penetration ratio of 0.5, the slotted liner adds negligible skin effect provided there are 3 (or more) alots around a circumference of the liner. This means that actually in most cases there is no flow restriction due to the slotted liner provided all the slots remain open (or at least 3 remain unclogged). This is in direct agreement with the rule of thumb that recommends 2 slots for each inch of nominal pipe diameter.

(2) The length of the slots is not significant from a flow point of view.

NOMENCLATURE

c_t = total compressibility

d = distance in space

ℓ = slot length

ℓ_D = dimensionless slot length

h = formation thickness

h_D = dimensionless formation thickness

k = formation permeability

n_s = number of slots in one vertical row

NS = number of pairs of staggered vertical rows of slots (also number of slots in one ring)

Δp = pressure drop

p_D = dimensionless pressure drop

q = strength = fluid withdrawal per unit length, area or volume of a source

Q = total withdrawal from a well (rate of production)

r = distance in x-y plane

r_{iD} = dimensionless distance (to slots) in x-y plane

r_{jD} = dimensionless distance (to slots) in x-y plane

r_D = dimensionless distance

r_w = wellbore radius

s = pseudo-skin factor

t = producing time

t_D = dimensionless producing time

x = abscissa of a point

y = ordinate of a point

z = elevation of a point

z_m = elevation of point M in a reservoir

η = diffusivity

μ = viscosity

ϕ = porosity

θ = angle which the projection of point M on the x-y plane makes with the origin of x-y-z coordinates.

Subscripts

D = dimensionless

i = pertaining to slot i

j = pertaining to slot j

m = any point in space

w = well

Special Functions

Error function: $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$

Exponential integral: $-\mathrm{Ei}(-x) = \int_x^\infty \frac{e^{-u}}{u} du$

ACKNOWLEDGEMENT

Portions of this study were done in partial fulfillment of graduate degree requirements at Stanford University. Funding was provided through a contract with the U.S. Department of Energy, under contract DE-AT03-80SF11459.

REFERENCES

1. Buzarde, L. E., Jr., Kastor, R. L., Bell, W. T., and DePriester, C. L.: Production Operation Course I-Well Completions, SPE of AIME, 1972, Chap. IV.

2. Composite Catalog, World Oil ed., 1974-5, 2843.

3. Gringarten, A. C., and Ramey, H. J., Jr.: "The use of source and Greens Functions in solving Unsteady-Flow Problems in Reservoirs", SPEJ (Oct. 1973), Trans. AIME(1973), 255, 285-296.

4. Theis, C. V.: "The relationship Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge Using Ground-Water Storage", Trans. AGU (1935), 519.

5. Nisle, G. R :The Effect of Partial Penetration on Pressure Build-Up in Oil Wells", Trans. AIME (1958) 213, 85-90.

6. Spivak, D.: "Unsteady-State Pressure Response in a Slotted Liner", MS thesis, Stanford U. (1981)

7. Carslaw, H. S., and Jaeger, J. C: Conduction of Heat In Solids, Oxford at the Clarendon Press (1959), 275-276.

8. Grinparten, A C., and Ramey, H. J., Jr.: "A Comparison of Different Solutions to the Radial Flow Problem", Paper SPE 3817, submitted for publication.

APPENDIX A: Derivation of Solution

The solution to Eq. 1 in an infinite medium may be derived using the source function method described by Gringarten and Ramey[3]. The pressure drop at (r,z,t) due to an instantaneous point source at z = z', t = 0 is given by:

$$\Delta p(r,z,z',t) = \frac{q}{8\phi c_t(\pi\eta t)^{3/2}}\cdot\exp\left[-\frac{r^2+(z-z')^2}{4\eta t}\right] \quad (A-1)$$

here q is the strength of the source. The pressure rop due to an instantaneous line source of finite ength ℓ (e.g., one slot) may be obtained by superposing oint sources along a line of z' from z' = 0 to z' = ℓ, hus:

$$\Delta p(r,z,t) = \frac{q}{8\phi c_t(\pi\eta t)^{3/2}}\int_0^{\ell}\exp\left[-\{r^2+(z-z')^2\}/4\eta t\right]dz' \quad (A-2)$$

y changing variables, i.e., $u = (z-z')/\sqrt{4\eta t}$, the bove integral may be represented in terms of the error unction as:

$$\Delta p(r,z,t) = \frac{q}{8\phi c_t\pi\eta t}\left[\text{erf}\frac{z}{\sqrt{4\eta t}} - \text{erf}\frac{z-\ell}{\sqrt{4\eta t}}\right] \quad (A-3)$$

o obtain the continuous pressure drop created by a inite line source of length ℓ, we use again superposition, now in time:

$$\Delta p(r,z,t) = \frac{q}{8\phi c_t\pi\eta}\int_0^t \frac{\exp(-r^2/4\eta\tau)}{\tau}\left[\text{erf}\frac{z}{\sqrt{4\eta\tau}} - \text{erf}\frac{z-\ell}{\sqrt{4\eta\tau}}\right]d\tau \quad (A-4)$$

ith $\tau = t-t'$ and $d\tau = -dt'$

q. A-4 represents the continuous pressure drop due to single slot (finite line source).

The strength of one slot, for a penetration ratio f ½, is given by:

$$q = \frac{Q}{(0.5h/n_s)2\cdot NS\cdot n_s} = \frac{Q}{h\cdot NS} \quad (A-5)$$

here n_s is the number of slots in one vertical row and Q is the total withdrawal rate. By introducing the ollowing dimensionless variables:

$$h_D = h/r_w \quad (A-6)$$

$$\ell_D = \ell/r_w \quad (A-7)$$

$$r_D = r/r_w \quad (A-8)$$

$$t_D = \frac{kt}{\phi\mu c_t r_w^2} \quad (A-9)$$

$$z_D = z/r_w \quad (A-10)$$

$$p_D = \frac{2\pi kh\Delta p}{Q\mu} \quad (A-11)$$

Eq. A-4 can be written in the nondimensional form as:

$$p_D(r_D,z_D,t_D) = \frac{1}{4\cdot NS}\int_0^{t_D}\frac{\exp[-r_D^2/4t_D]}{t_D}\cdot\left\{\text{erf}\frac{z_D}{\sqrt{4t_D}} - \text{erf}\frac{z_D-\ell_D}{\sqrt{4t_D}}\right\}dt_D \quad (A-12)$$

In our actual problem it is necessary to superpose the pressure drops due to the real and (infinite) image slots. When this is done, Eq. A-12 becomes Eq. 9 in the text.

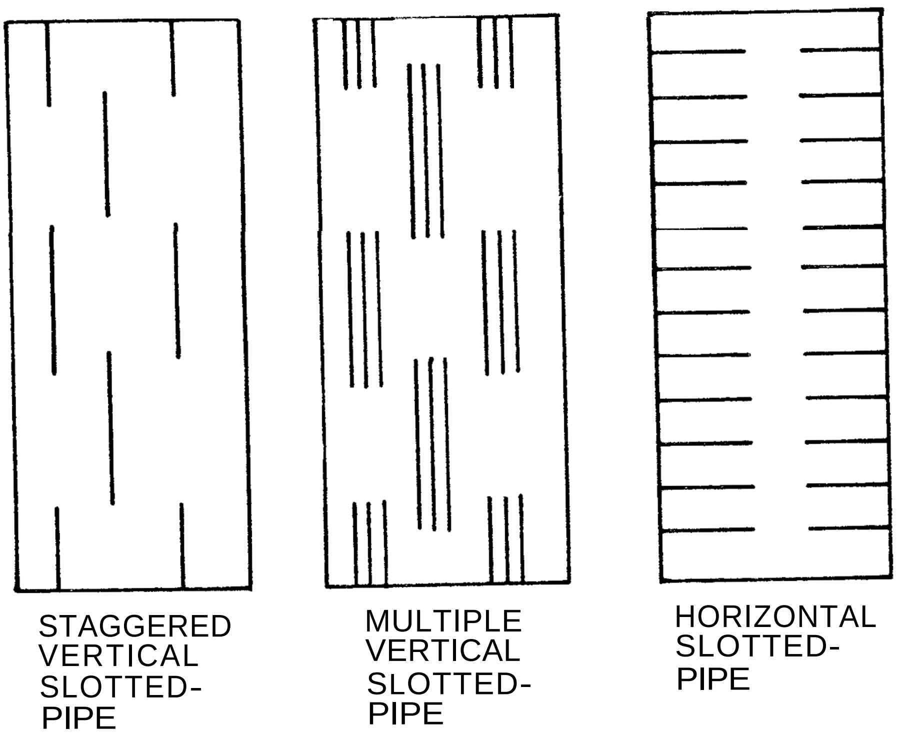


Fig. 1– Slotted Liners (After Buzarde et al.[1])

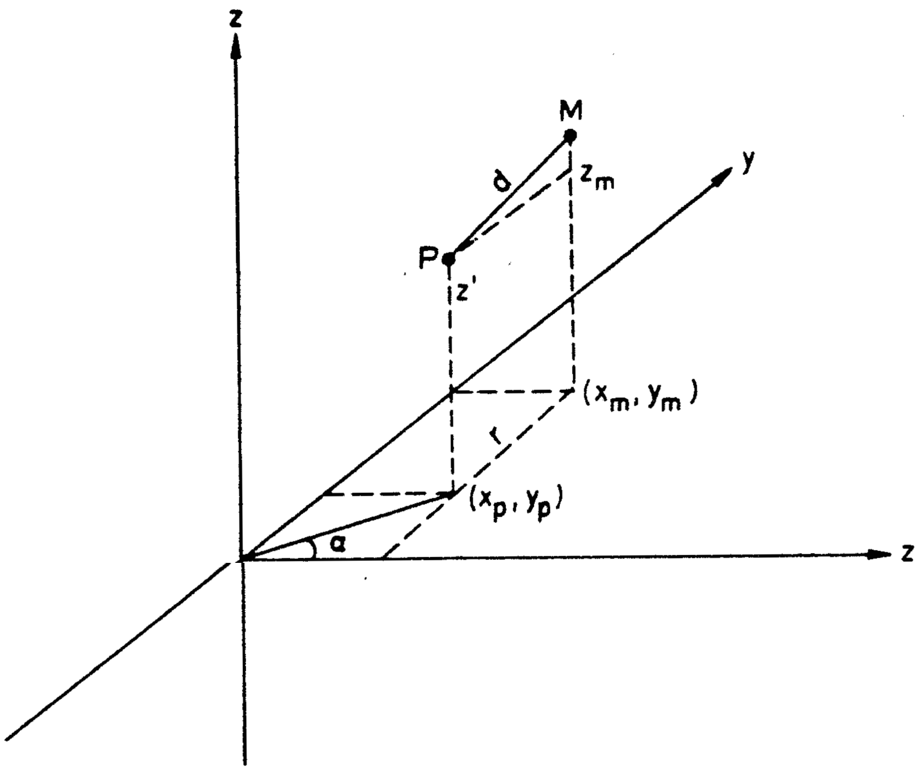


Fig. 2- Point Source in an Infinite Medium

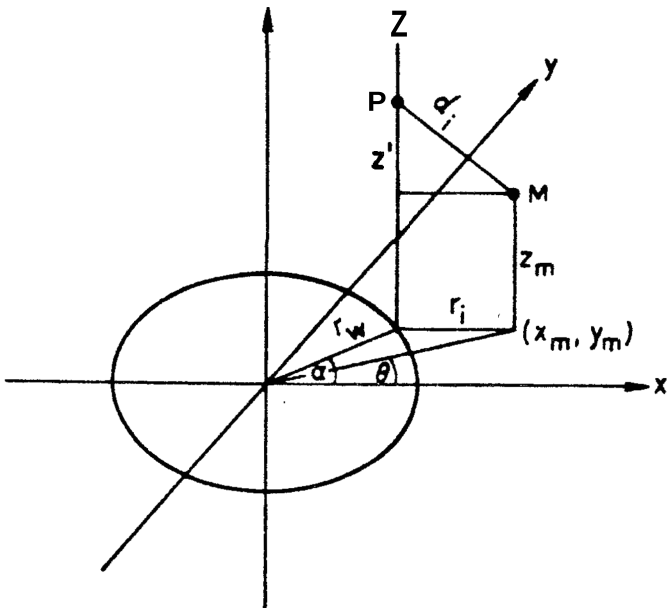


Fig. 3- Coordinate Sketch of one Slot

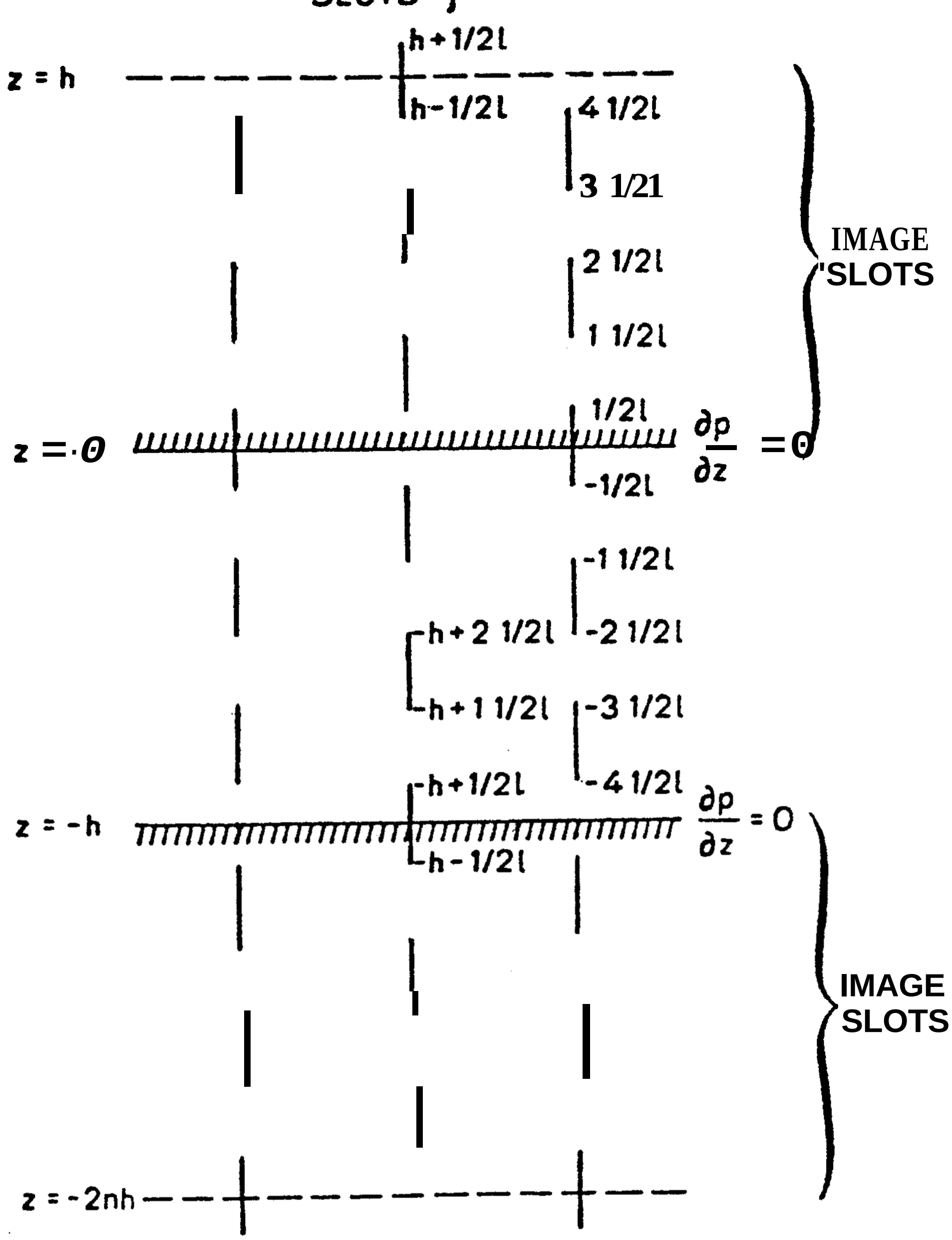


Fig. 4- Schematic "unwrapped" Diagram of the Model

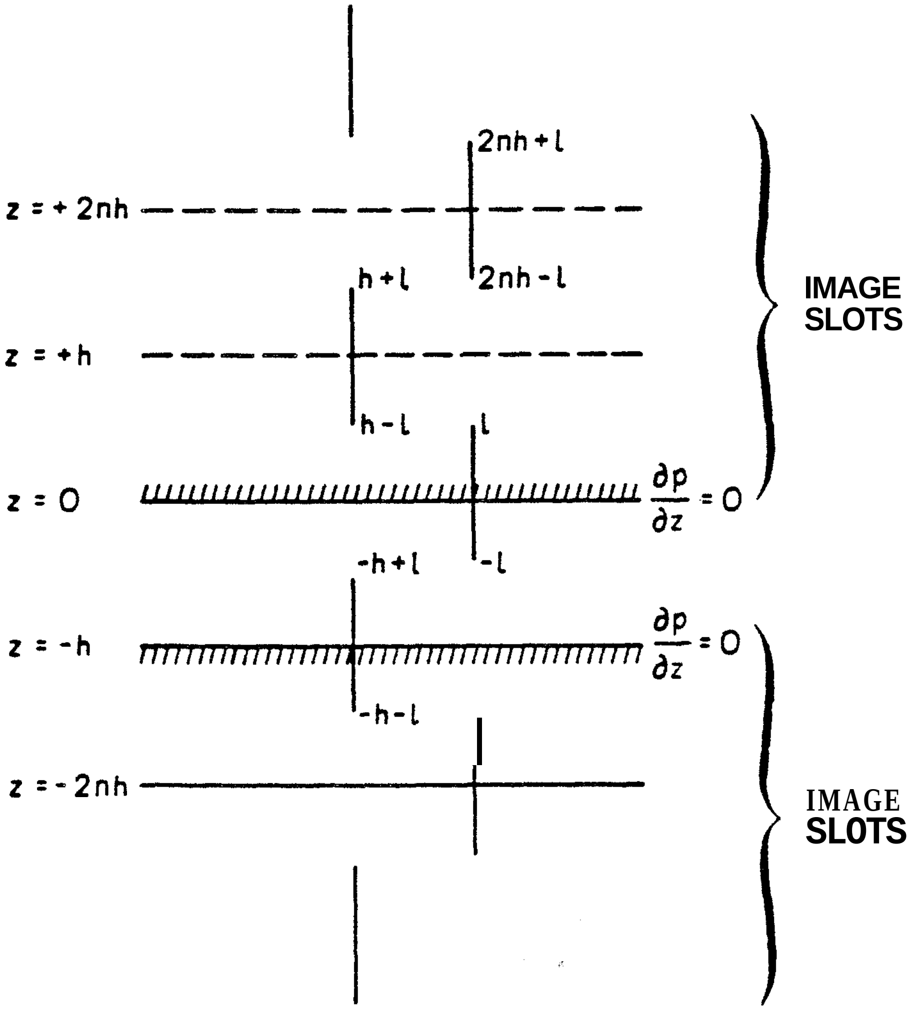


Fig. 5- Basic Unit of the Model

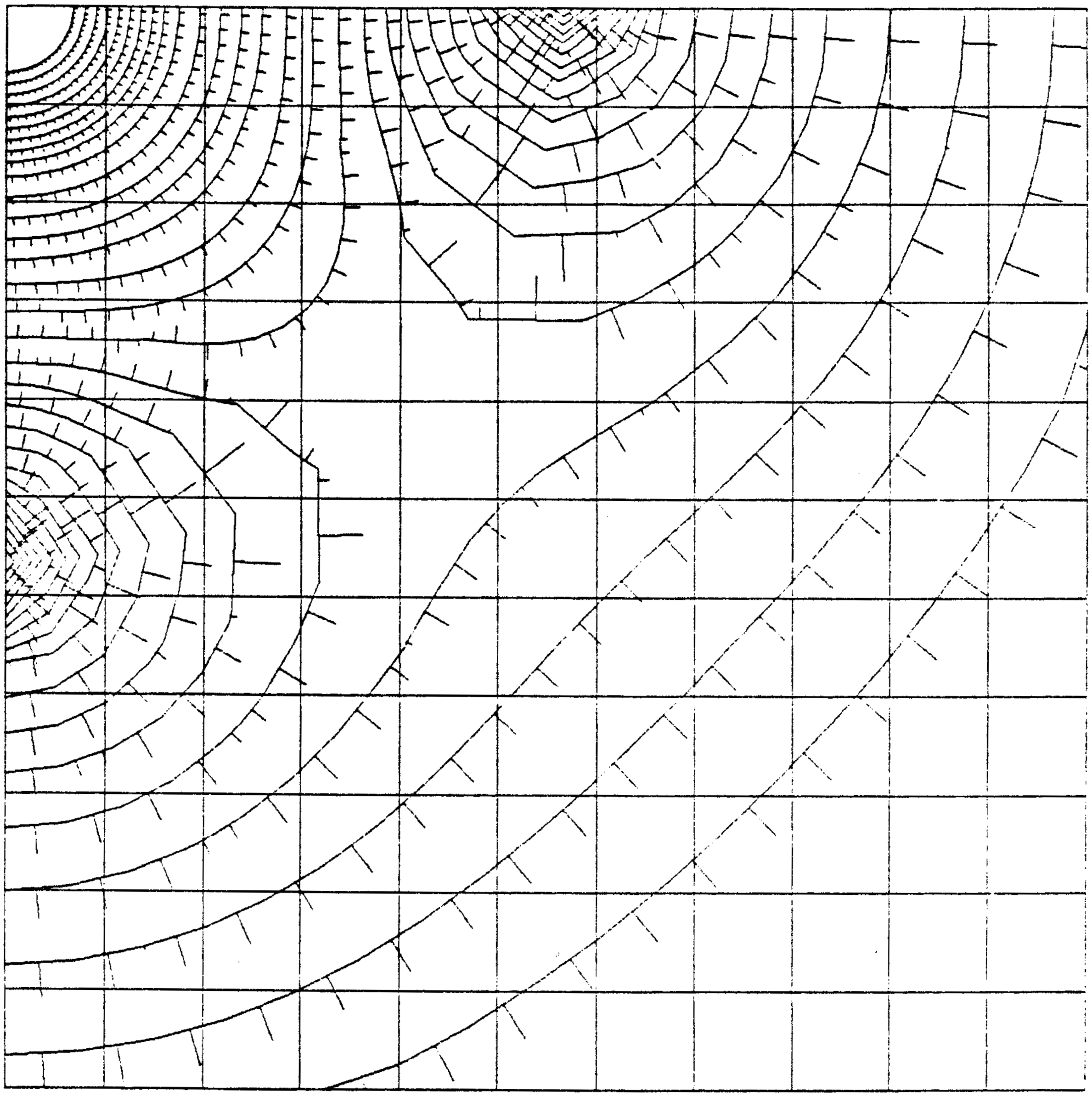


Fig. 6- Isopotential Contours for an Arrangement of Four Slots (One quadrant **only is** shown). Circular **No-Flow Boundary** is achieved by the Subtraction of the **Line** Source at the centerline (The origin in this diagram)

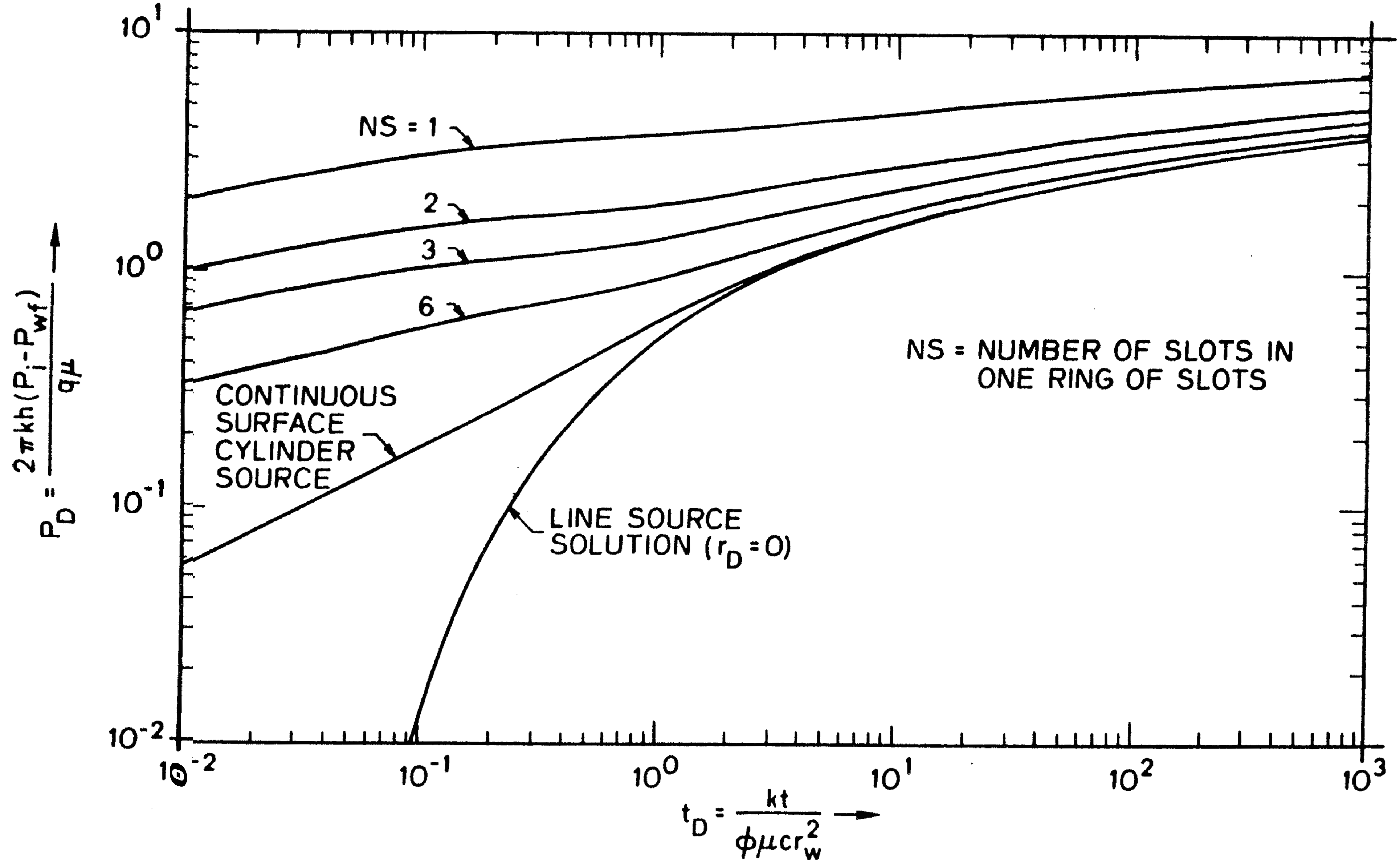


Fig. 7- Dimensionless Pressure Drop Vs. Dimensionless Time For a Slotted Liner With Various Numbers of Staggered Vertical Rows, R_D=1

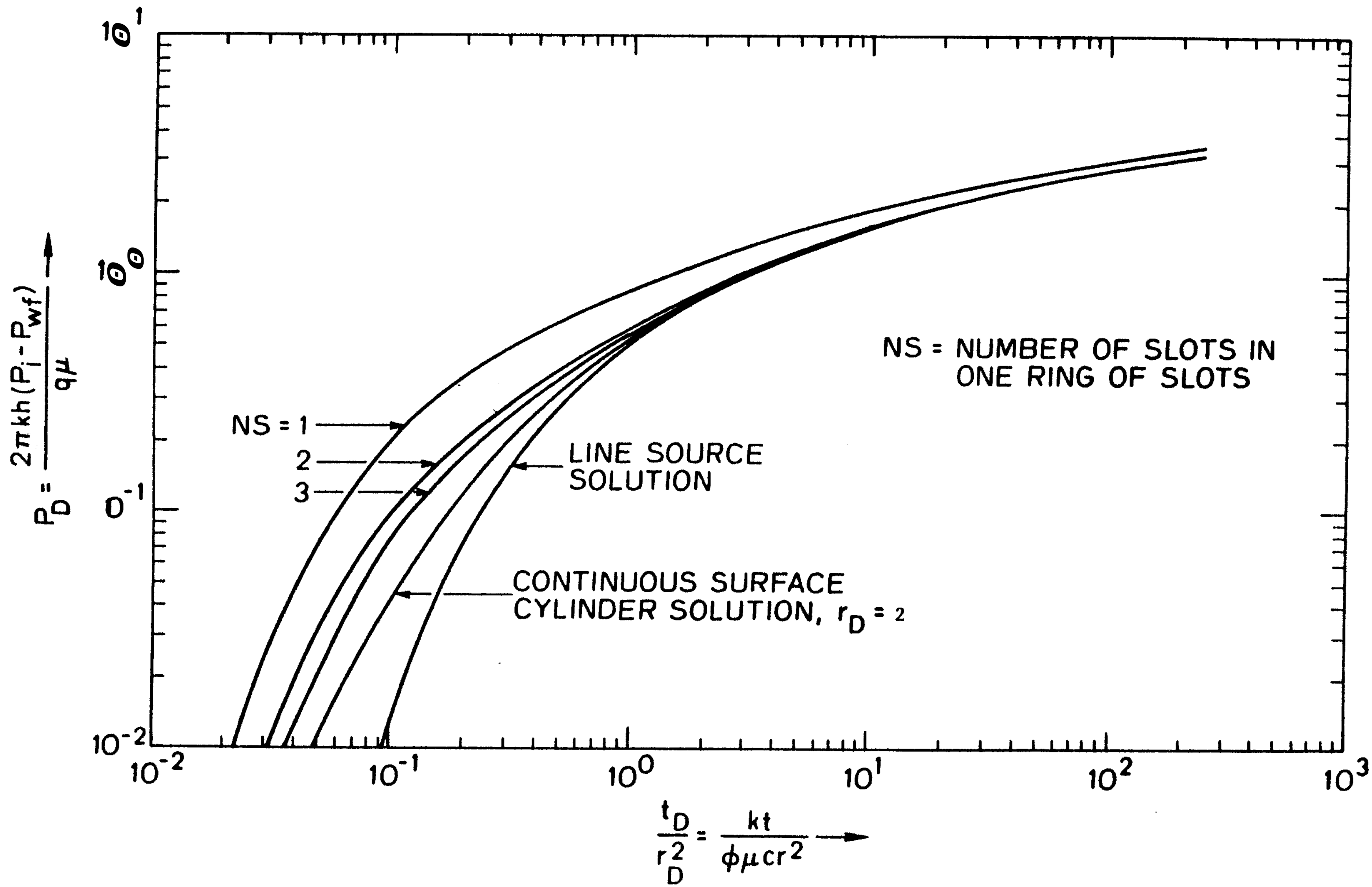


Fig 8- Dimensionless Pressure Drop vs. Dimensionless Time/Distance Sqared For a Slotted Liner With Various Numbers of Staggered Vertical Rows, $R_D=2$