LIMITATIONS OF THE $\Delta p/q$ APPROXIMATION IN THE ANALYSIS OF PRESSURE DRAWDOWN INTERFERENCE WITH VARIABLE FLOW RATE

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By
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Abstract

Conventional drawdown interference analysis assumes that the producing well flows at a constant flow rate. Several methods have been developed to handle variation of flow rate during the test.

In this work, the method of superposition of continuously changing rates was used to generate dimemsionless pressure solutions based on the instantaneous rate for a variety of functional forms of rate variation.

Curves relating dimensionless pressure, dimensionless time, and a correlation parameter defined by reservoir properties and rate variation were constructed. Error curves were constructed that can be used as diagnostic tools to determine the validity of the Aplq approximation.

When the method of stepwise change in rate is to be used in analyzing the data, it was shown that the pressure data at the end of the step period should be used. An equation has been developed to determine the duration of the flow rate in each step change to provide desired accuracy.

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1. Introduction

Pressure drawdown interference analysis generally assumes that the producing well is flowing at a constant rate. In some cases it is-difficult to maintain constant flow rates. Several techniques have been published in the literature to handle the changes in flow rate during a drawdown test. Due to the difficulty of obtaining an analytical expression to describe pressure behavior during production at variable flow rates, several approximations have been developed. The rate of variation is either handled by treating a continuous change in flow rate as discrete step changes[10,13] and then applying the principle of superposition, or using the $\Delta p/q$ approximation.[5,14,20] The $\Delta p/q$ approximation is commonly used in interference testing.[2,3,5,8,16,17]

The objective of this work is to study the limitations of the $\Delta p/q$ approximation.

The approach taken in this study was to assume a functional form of varying flow rate, obtain a solution, and then compare the solution obtained with the line source solution because the analysis of the drawdown interference data is based on the line source solution.

2. Literature

The basis for transient flow of fluids through porous media was presented by the fundamental study done by van Everdingen and Hurst[19] in 1949, and the pressure buildup studies by Miller, Dyes, and Hutchinson,[12] and by Homer.[9]

The analysis of variable flow rate data has been studied by several authors. When flow rate variation is a result of wellbore storage, the Gladfelter et al.[7] method of correcting the pressure difference can be used. Ramey[14] and Winestock and Colpitts[20] introduced the concept of the normalized drawdown method. This method requires graphing the $\Delta p/q$ vs the log of producing time. This theoretical development was based on analyzing the pressure data of the producing well. This method leads to useful results if the pressure data of the producing well are to be analyzed. This work did not test the method for interference tests. In 1976, Ramey15 established the Gladfelter et al.[7] method of graphing the pressure difference divided by the instantaneous flow rate vs the log of time.

Aron and Scott[3] examined the effect of rate variation on water well test analysis. Cooper and Jacob[5] developed a method for analyzing pressure data of aquifers pumped at a variable discharge rate. Odeh and Jones13 in 1965, and Sternberg[17] in 1968 also presented a method for analyzing variable flow rate data. The method was based on the superpositionprinciple, and approximating the variable flow rate data by step changes. They assumed that the logarithmic approximation to the line source solution was applicable. For the case of interference tests, the logarithmic approximation to the line source solution might not apply, and therefore this method cannot be used.

Jargon and van Poollen[10] introduced a method by which varying flow rate data can be converted to a constant rate pressure response called the unit response function. Economides et al.[6] studied the use of influence or response functions in analyzing variable flow rate and demonstrated application to geothermal well testing.

Tsang et al.[18] demonstrated the use of digital computers in analyzing well test data with variable flow rate. The concept was based on reproducing the pressure response by assuming several reservoir parameters, and selecting the set that best matches the actual field data.

3. Mathematical Model

Figure 1 is a schematic diagram of the reservoir in this study. The following assumptions were made:

1. The reservoir is infinite, homogeneous, isotropic, and of uniform thickness.
2. The fluid is slightly compressible and has constant viscosity.
3. The flow of fluid is isothermal radial flow.
4. The pressure gradients are small.
5. Gravity forces are negligible.

The mathematical equations describing the flow system are:

(1) The pressure response due to a line source with varying flow rate, $q(t)$, results in the expression[4]:

$$p_i - p_{r,t} = \frac{70.6\ B\mu}{k\ h} \int_0^t q(\tau) \frac{e^{-\frac{r^2}{4\eta(t-\tau)}}}{t - \tau} dt \qquad (3.1)$$

(2) The initial and boundary conditions are:

$p = p_i$ at $t = 0$ for all r

$p \to p_i$ as $r \to \infty$, for all t

$q(t) = f(t)$ at $r = r_w$ for all $t > 0$

There is no unique analytical solution to the above integral equation subject to the conditions specified. In this study, several functional forms of varying flow rate will be assumed.

3.1 Linear Variation of Flow Rate

The flow rate at any time is given by:

$$q(t) = q_i + bt \qquad (3.2)$$

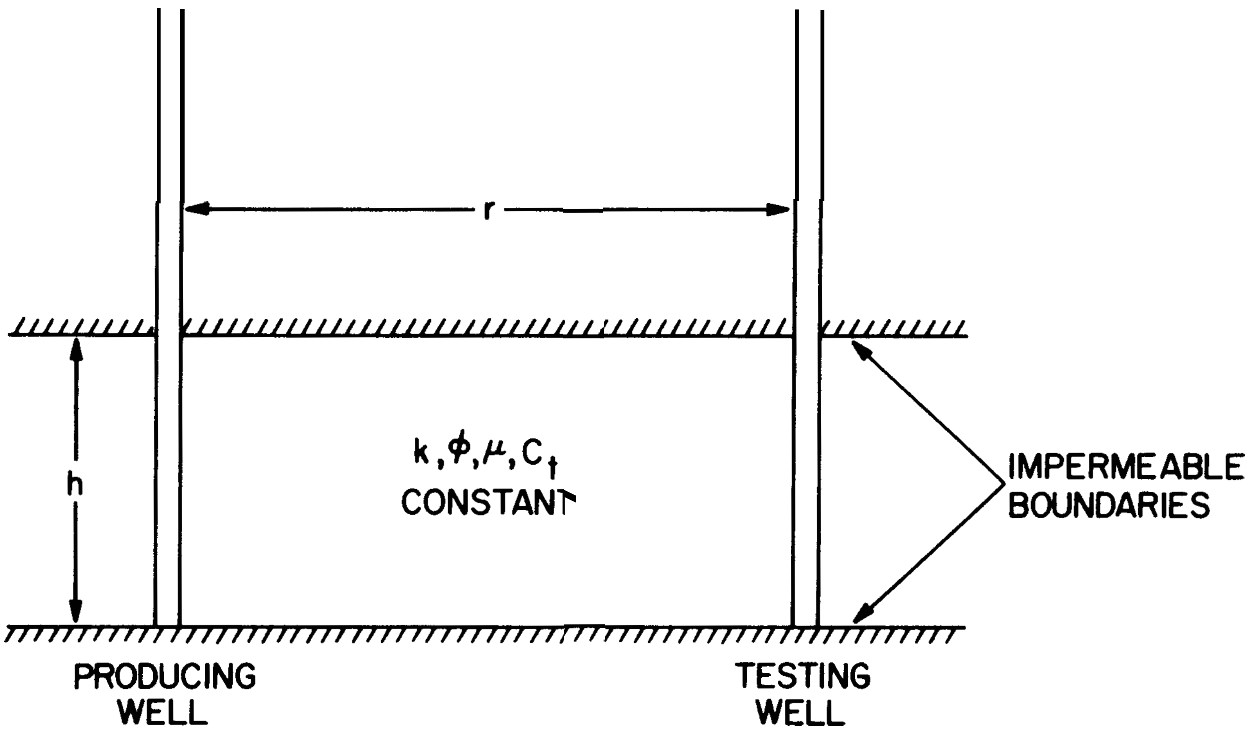


FIGURE 1 - SCHEMATIC DIAGRAM OF THE RESERVOIR

Substituting eq. 3.2 into eq. 3.1:

$$\frac{kh\ \Delta p(r,t)}{70.6\ B\mu} = \int_{0}^{t} (q_i+b\tau)\ \frac{e^{-\frac{r^2}{4\eta(t-\tau)}}}{t-\tau}\ dt$$

$$\int \frac{\overline{4\eta(t-\tau)}}{}\ dt + b\int \frac{\overline{4\eta(t-\tau)}}{t-\tau}\ dt \qquad (3.3)$$

The solution to the first integral is well documented in the literature[11].

$$2p_D(t_D,r_D) = -E_i\left(-\frac{r_D^2}{4t_D}\right) \qquad (3.4)$$

The solution to the second integral is given by[18]:

$$2bt\ p_D(t_D,r_D) + b\ \frac{r^2}{4\eta}\left[2\,p_D(t_D,r_D) - 4t_D/r_D^2\ e^{-\frac{1}{4t_D/r_D^2}}\right] \qquad (3.5)$$

$$\frac{kh\Delta p\ (r,t)}{70.6\ B\mu} = 2\,q_i\,p_D + 2\,b\,t\,p_D + 2\,\frac{br^2}{4\eta}\left[p_D - 2\,t_D/r_D^2\,e^{-\frac{1}{4\,t_D/r_D^2}}\right]$$

$$= 2\,q(t)\,p_D + 2\,\frac{br^2}{4\eta}\left[p_D - 2\,t_D/r_D^2\ e^{-\frac{1}{4\,t_D/r_D^2}}\right]$$

Divide by 2q(t):

$$\frac{kh\ \Delta p\ (r,t)}{141.2\ B\mu\ q(t)} = p_D + \frac{b\,r^2/4\eta}{q_i + b\,t}\left[p_D - 2\,t_D/r_D^2\ e^{-\frac{1}{4\,t_D/r_D^2}}\right] \qquad (3.6)$$

Let's define:

$$p_{DV} = \frac{k\,h\,\Delta p(r,t)}{141.2\ B\mu\ q(t)} \qquad (3.7)$$

$$\beta = \frac{b\,r^2}{q_i\ \eta} \qquad (3.8)$$

Then:

$$p_{DV}(t_D,r_D) = p_D(t_D,r_D) \left[1 + \frac{1}{4(\frac{1}{\beta} + t_D/r_D^2)}\right] - \frac{t_D/r_D^2 \; e^{-\frac{1}{4\,t_D/r_D^2}}}{2(\frac{1}{\beta} + t_D/r_D^2)} \quad (3.9)$$

3.2 Exponential Variation of Flow Rate

The flow rate is given by:

$$q(t) = q_i\, e^{-at} \quad (3.10)$$

Substituting eq. 3.10 into 3.1:

$$\frac{k\,h\,\Delta p(r,t)}{141.2\ B\ \mu} = \tfrac{1}{2}\int_0^t q_i\, e^{a\tau}\, \frac{e^{-\frac{r^2}{4\eta(t-\tau)}}}{t-\tau}\, dt \quad (3.11)$$

Divide by $q(t) = q_i\, e^{-at}$:

$$p_{DV}(t_D,r_D) = \tfrac{1}{2}\int_0^t e^{a(t-\tau)}\, \frac{e^{-\frac{r^2}{4\eta(t-\tau)}}}{t-\tau}\, dt \quad (3.12)$$

Let $u = \dfrac{r^2}{4\eta(t-\tau)}$ (3.13)

$$p_{DV} = \tfrac{1}{2} \Bigg|_{4t_D/r_D^2} \frac{-\frac{a\,r^2}{4\eta u}}{}\ du \quad (3.14)$$

Let $x = \dfrac{1}{u}$ (3.15)

$\beta = \dfrac{ar^2}{4\eta}$ (3.16)

$$p_{DV}(t_D,r_D) = \tfrac{1}{2}\int_0^{4t_D/r_D^2} \frac{e^{(\beta x - 1/x)}}{x}\ dx \quad (3.17)$$

3.3 Stepwise Change of Flowrate

The pressure drop due to the flow rates q_1 and q_2 is given by:

$$\frac{kh\Delta p(r,t)}{141.2\ B\mu} = q_1 p_D(t_D) - q_1 p_D(t_D - t_{D_1}) + q_2 p_D(t_D - t_{D_1})$$

$$= q_1 p_D(t_D) + (q_2 - q_1) p_D(t_D - t_{D_1}) \qquad (3.18)$$

Divide by q_2:

$$\frac{kh\Delta p(r,t)}{141.2\ B\mu\ q_2} = \frac{q_1}{q_2} p_D(t_D) + \left(1 - \frac{q_1}{q_2}\right) p_D(t_D - t_{D_1}) \qquad (3.19)$$

$$p_{DV}(t_D, r_D) = \frac{q_1}{q_2} p_D(t_D, r_D) + \left(1 - \frac{q_1}{q_2}\right) p_D\left[(t_D - t_{D_1}),\ r_D\right] \qquad (3.20)$$

4. Computer Programs

Computer programs were prepared to compute dimensionless pressure as a function of dimensionless time and the correlation parameter, β, for a variety of flow rate variations. To evaluate the dimensionless pressure for exponential variation of flow rate, given by Eq. 3.17, a numerical technique was used to carry out the integration.

The flow charts showing the main steps in the computer programs are presented in Appendix A.

5. Results and Discussion

The dimensionless pressure as defined by Eq. 3.7 was computed for a wide range of values of reservoir and flow rate parameters. The parameters are grouped in the dimensionless parameter β as defined by Eq. 3.8. These definitions are convenient in this study because they allow a comparison of the solutions obtained under certain flow rate variations with the line source solution.

5.1 Linear Change in Flow Rate

Figure 2 shows the dimensionless pressure as a function of dimensionless time for a range of the parameter β. In the case of decreasing flow rate, the curves start with the line source and then bend upwards. For increasing flow rate, the curves start with the line source and then bend downwards. The separation reaches a constant value at about 0.5. After this, all the curves provide a semi-log straight line of slope 1.151, low by 0.5 compared to the line source solution as shown on Fig. 3. In the case of well bore storage, Ramey[15] found a separation of about *0.4* and a slope of 1.151, then the curves bend upwards to join the line source solution.

Error curves have been prepared. See Figures *4* and 5. These results may be used to estimate the error that results in the $\Delta p/q$ approximation given the rate variation and the estimated reservoir properties.

5.2 Exponential Flow Rate

Figure 6 shows the dimensionless pressure as a function of dimensionless time for a range of the parameter B. The curves start with the line source then bend upwards in the case of decreasing flow rate; and bend downwards in the case of increasing flow rate.

The increasing flow rate case is interesting. Figure 7 shows a semi-log graph of the dimensionless pressure vs dimensionless time.

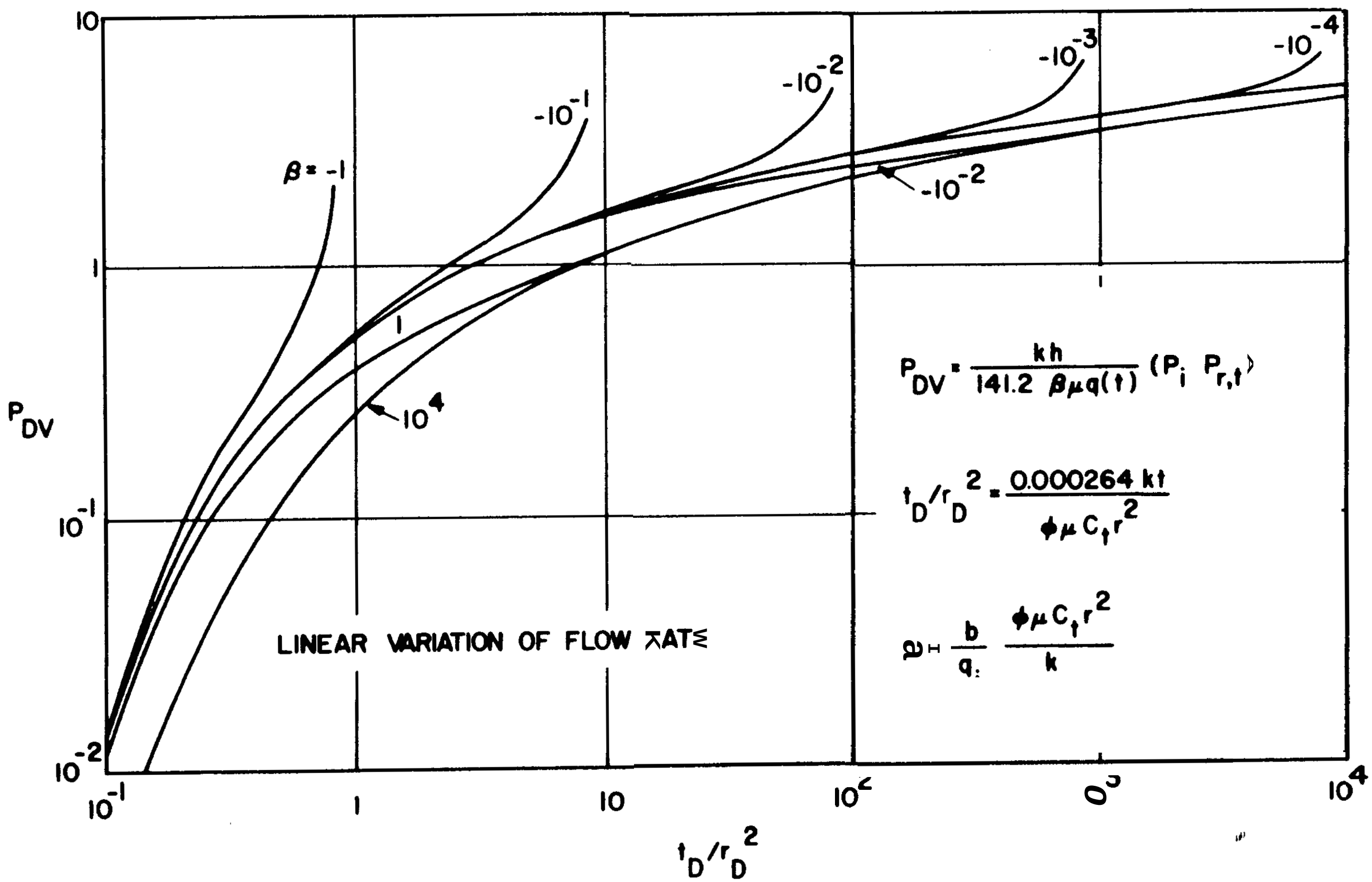


FIGURE 2 - LINEAR VARIATION OF FLOW RATE

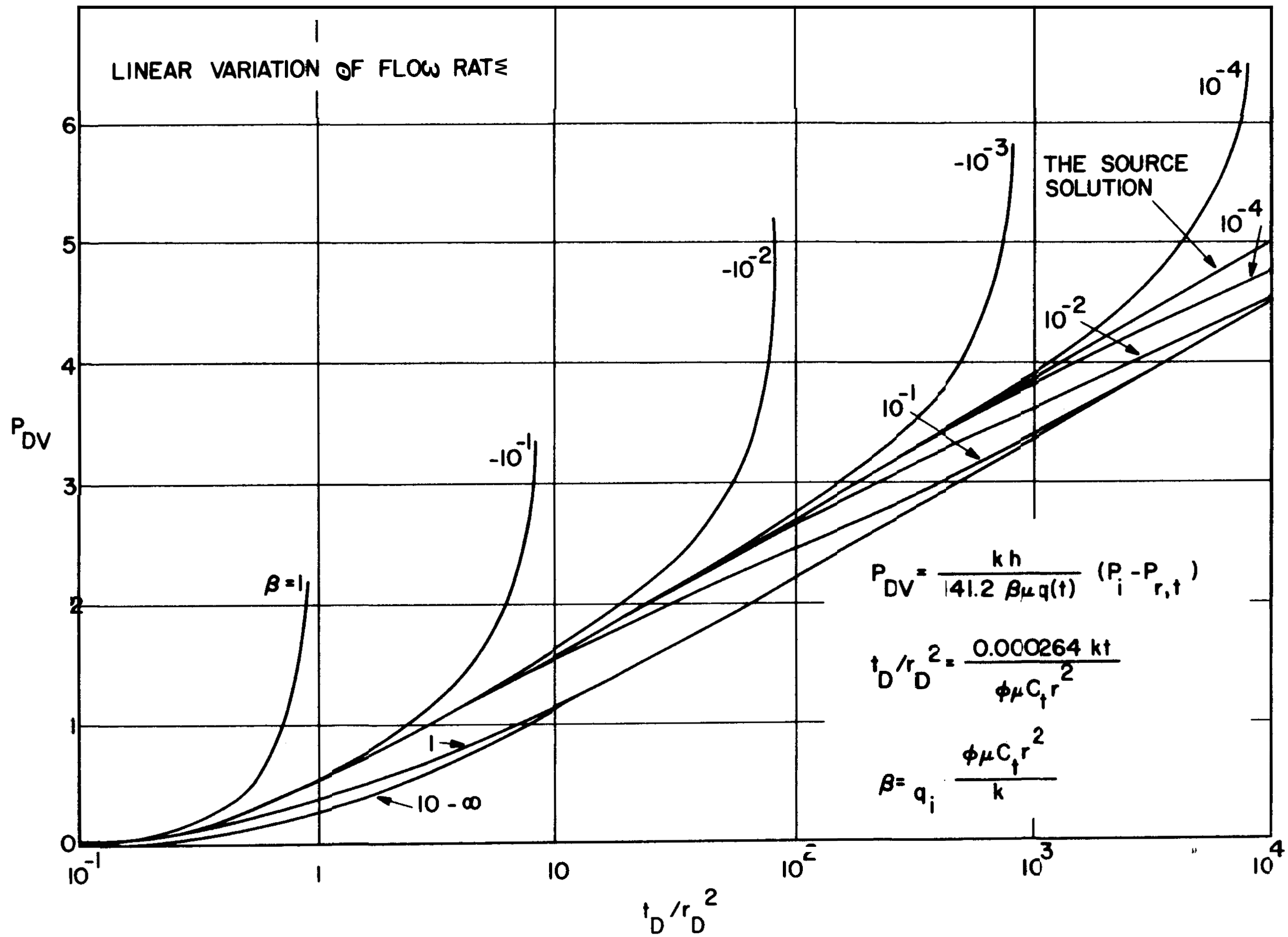


FIGURE 3 - LINEAR ω IATION OF FLOW RATE

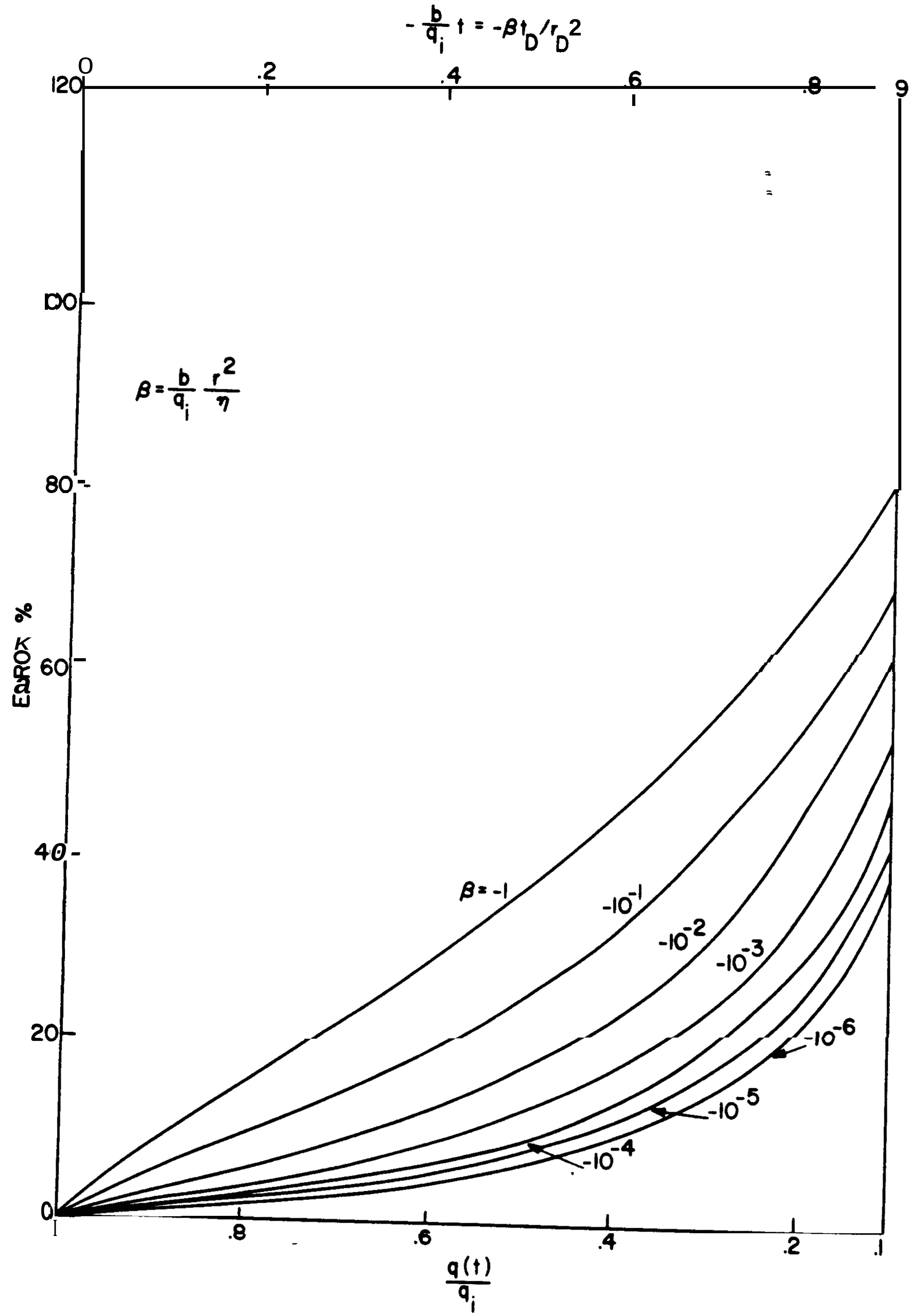


FIGURE 4 - ERROR CURVE FOR LINEARLY DECREASING FLOW RATE

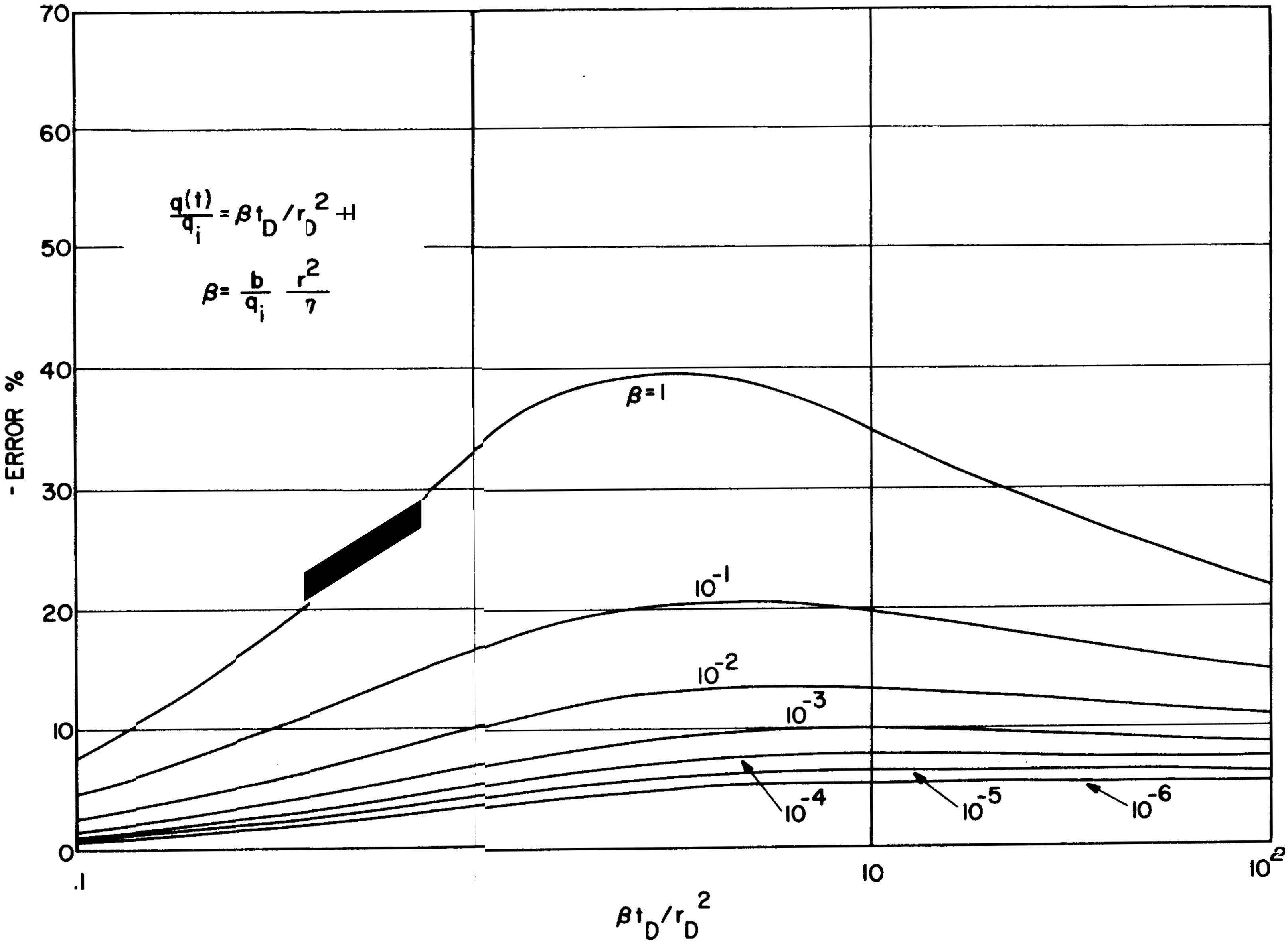


FIGURE 5 - ERROR CURVE FOR LINEARLY INCREASING FLOW RATE

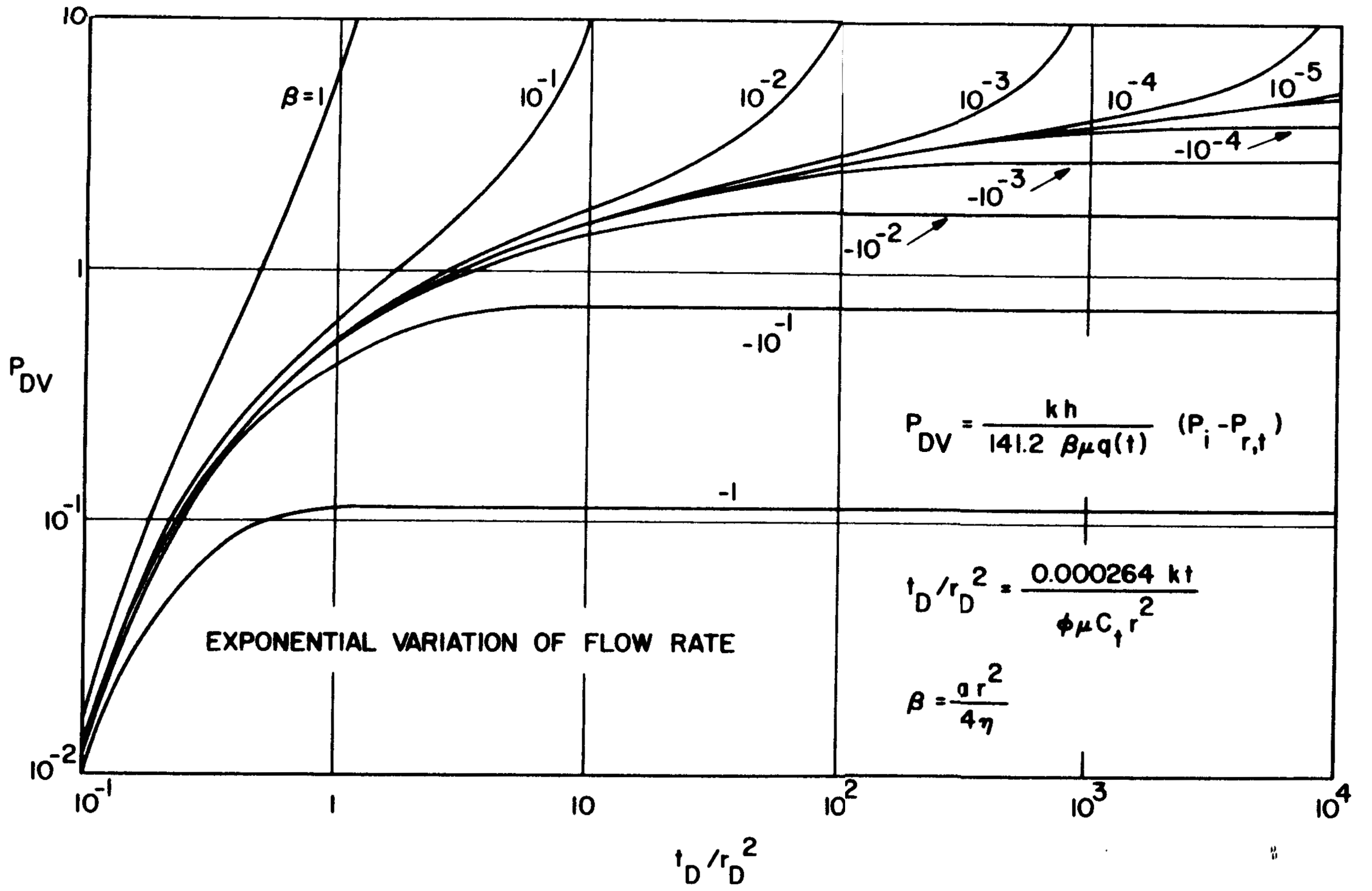


FIGURE 6 - EXPONENTIAL VARIATION OF FLOW RATE

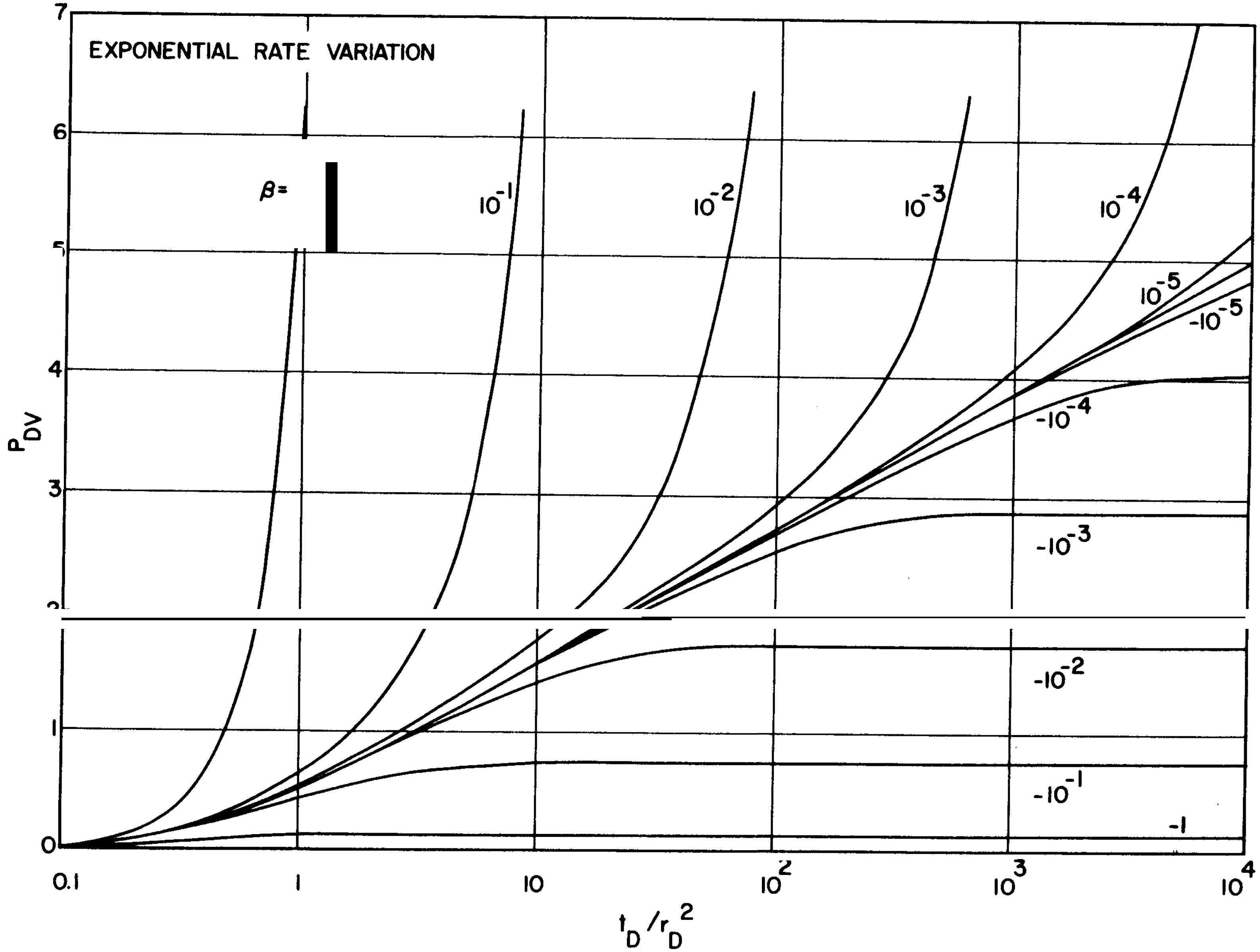


FIGURE 7 - EXPONENTIAL RATE VARIATION

After the curves depart from the line source, they become nearly a straight line of a unit slope, and then reach a constant value at about $t_D/r_D^2 = \left|\frac{1}{\beta}\right|$. This case is similar to the wellbore storage case, ,except in the wellbore storage case curves bend towards the line source solution.

Error curves have been prepared and are presented in Figures 8 and 9 to estimate the error that results in the $\Delta p/q$ approximation, given a rate variation and the estimated reservoir properties.

5.3 Stepwise Change of Flow Rate

For the case of step changes in flow rate there is a jump at the time of initiation of a new flow rate, then the curve bends towards the line source solution. An example is shown in Figure 10 for different flow rate ratios occuring at different dimensionless times. Figure 10 suggests that the points at the end of each step period should be used for analysis. The important question is how long should the rate in each step last in order to obtain sufficient accuracy? Using Eq. 3.20:

$$p_{DV}(t_D) = \frac{q_1}{q_2}\, p_D(t_D) + \left(1 - \frac{q_1}{q_2}\right) p_D(t_D - t_{D_1}) \qquad (3.20)$$

Subtracting $p_D(t_D)$ from both sides:

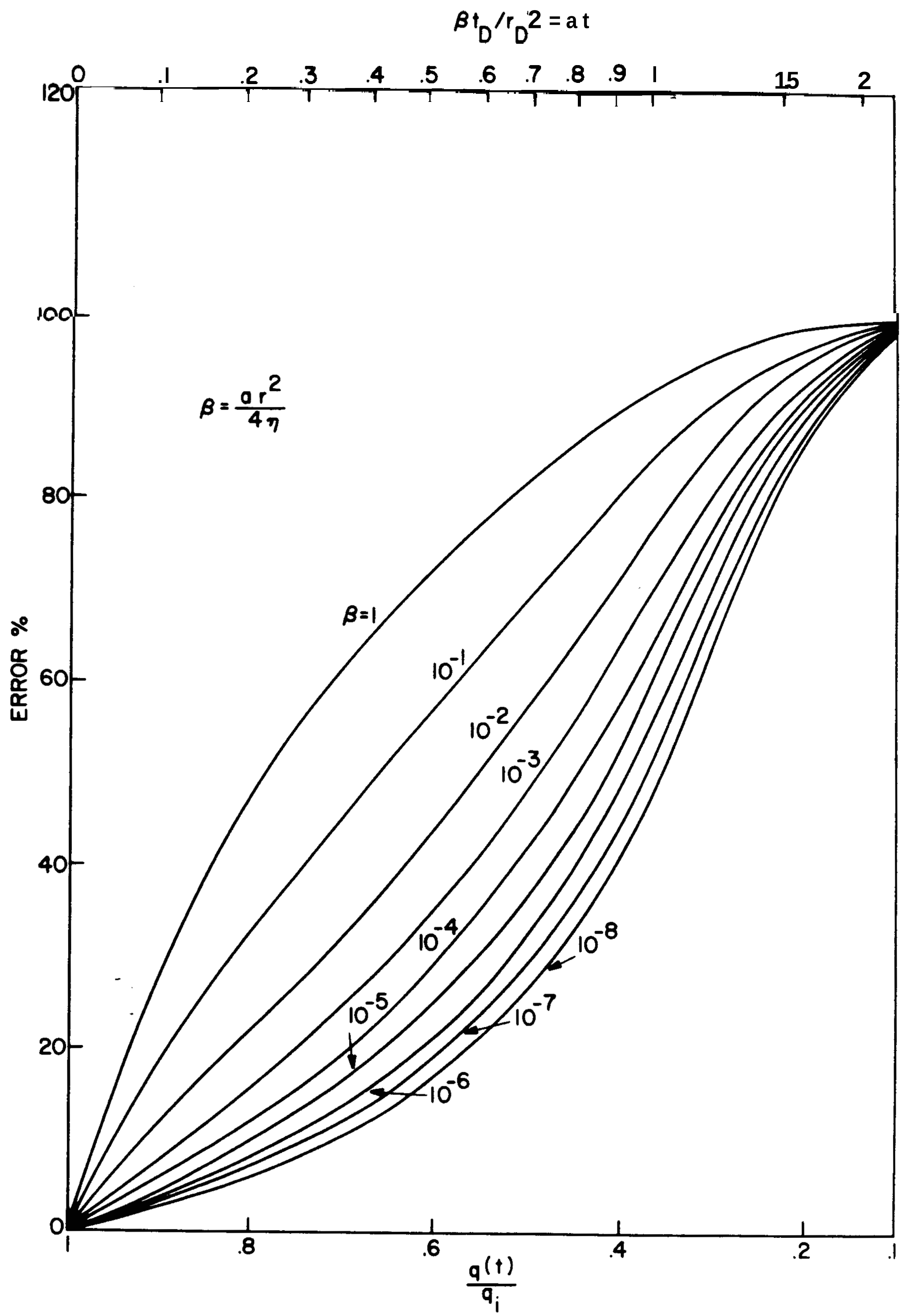
$$p_{DV}(t_D) - p_D(t_D) = \frac{q_1}{q_2}\, p_D(t_D) + \left(1 - \frac{q_1}{q_2}\right) p_D(t_D - t_{D_1}) - p_D(t_D) \qquad (5.1)$$

$$p_{DV}(t_D) - p_D(t_D) = \left(\frac{q_1}{q_2} - 1\right)\left[p_D(t_D) - p_D(t_D - t_{D_1})\right] \qquad (5.2)$$

If we require the difference to be zero, Eq. 5.2 suggests that $t_D >> t_{D_1}$. Let's require the difference to be δ. Then Eq. 5.2 becomes:

$$p_D(t_D) - p_D(t_D - t_{D_1}) = \delta\, \frac{q_2}{(q_1 - q_2)} \qquad (5.3)$$

FIGURE 8 - ERROR CURVE FOR EXPONENTIALLY DECREASING RATE



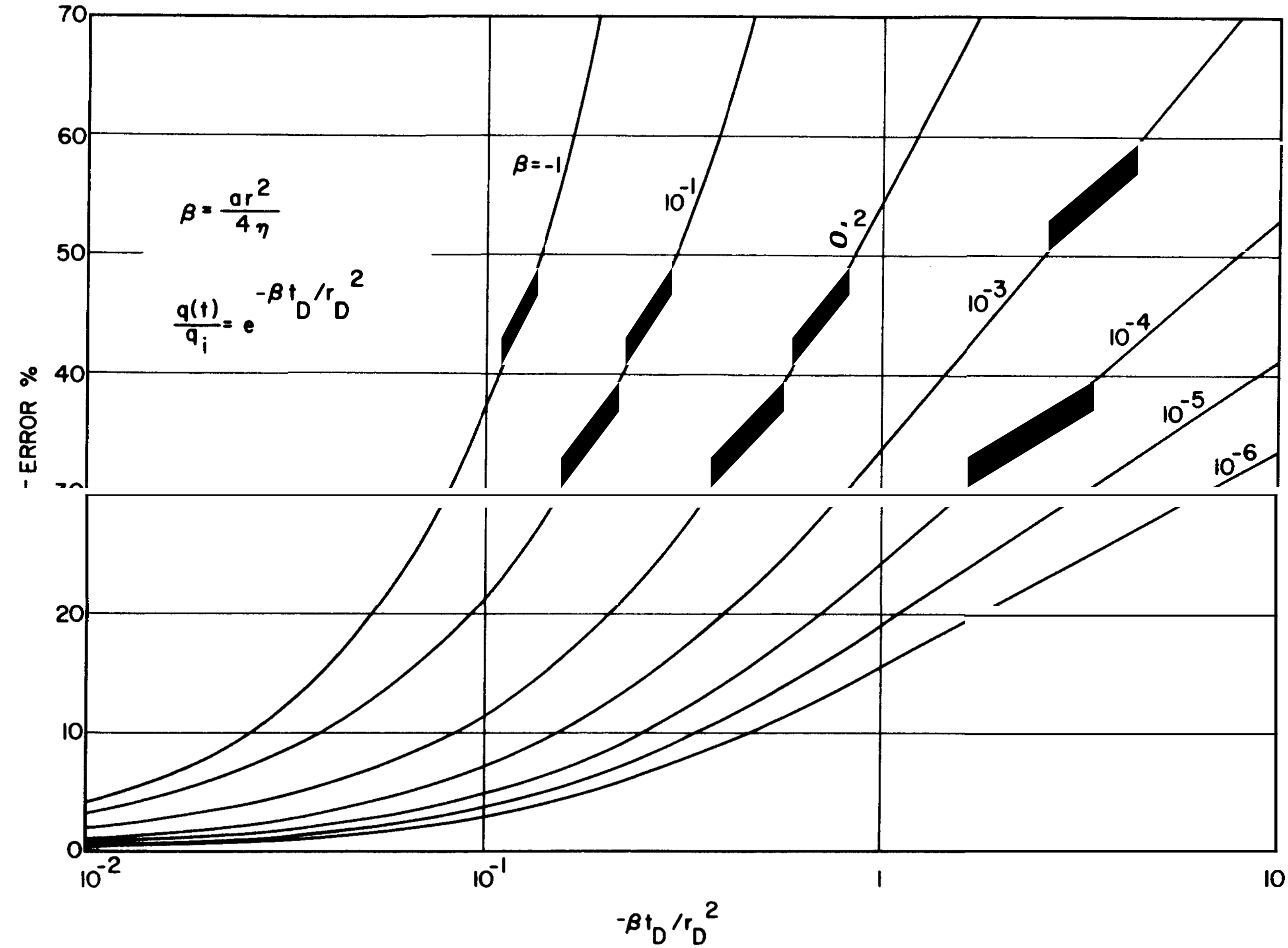


FIGURE 9 - ERROR CURVE FOR EXPONENTIALLY INCREASING FLOW RATE

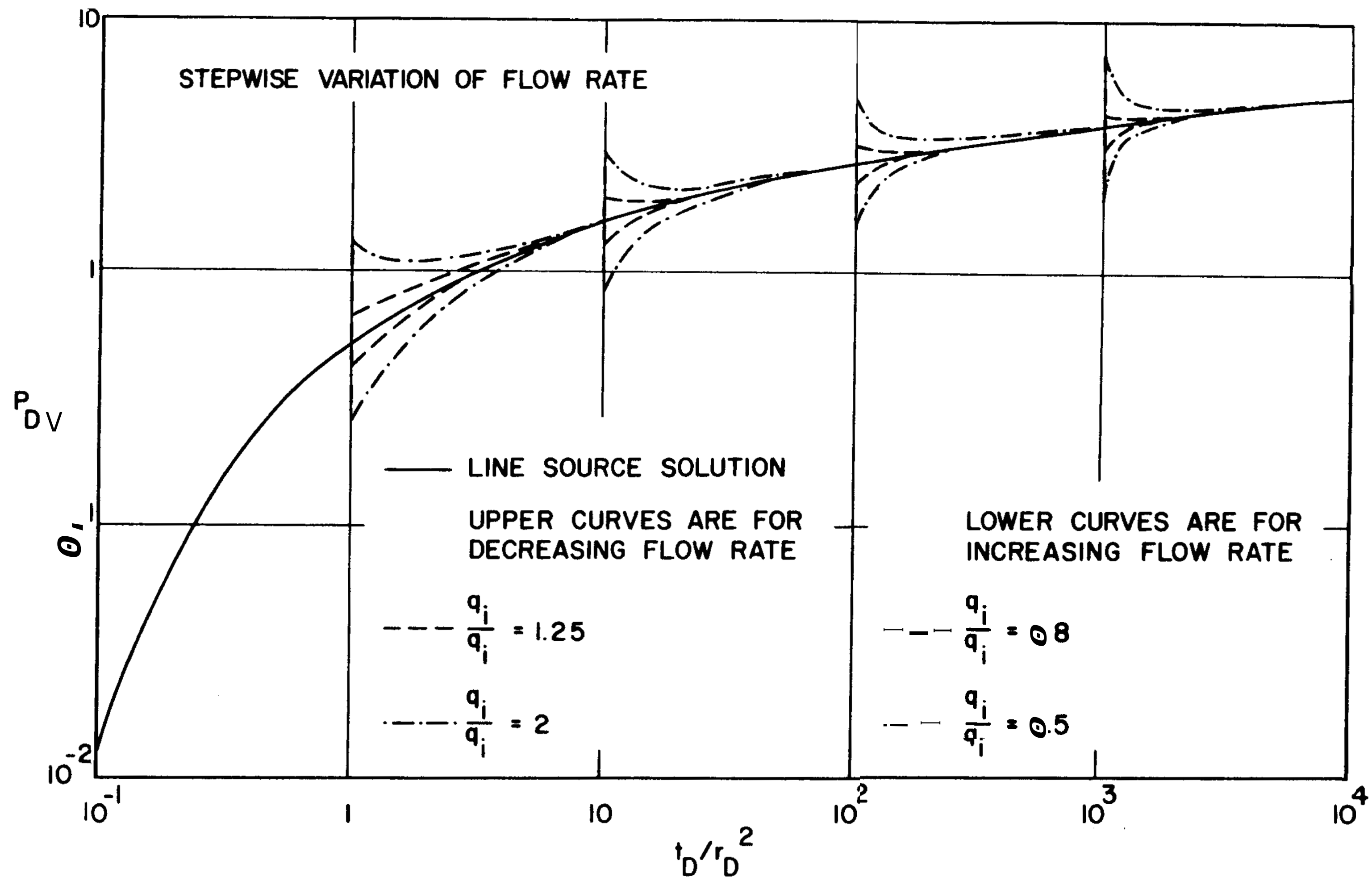


FIGURE 10 - STEPWISE VARIATION OF FLOW RATE

Define; $\Delta q = q_2 - q_1$ (5.4)

$$p_D(t_D) - p_D(t_D - t_{D_1}) = -\delta \frac{q_2}{\Delta q} \tag{5.5}$$

Equation 5.5 provides the relationship between Δt and Δq.

If we now assume that a logarithmic flow rate approximation applies, then:

$$p_D(t_D) = \tfrac{1}{2}\left[\ln t_D + 0.8091\right] = 1.151\left[\log t + \log \frac{k}{\phi \mu C_t r^2} + 0.351\right] \tag{5.6}$$

Substitute equation 5.6 into equation 5.5:

$$1.151 \log \frac{t}{t - t_1} = -\delta \frac{q_2}{\Delta q} \tag{5.7}$$

$$t - t_1 = t\, 10^{\delta q_2/1.151\Delta q} \tag{5.8}$$

$$t\left(1 - 10^{\delta q_2/1.151\Delta q}\right) = t_1 \tag{5.9}$$

$$t = \frac{t_1}{1 - 10^{\delta q_2/1.151\Delta q}} \tag{5.10}$$

$$\Delta t = t - \;=\; \frac{t_1}{1 - 10^{\delta q_2/1.151\Delta q}} \;- \tag{5.11}$$

δ is positive for decreasing flow rates, and negative for increasing flow rates.

Solving for Δq we obtain:

$$\Delta q = -\frac{\delta q_2}{1.151 \log \frac{t}{\Delta t}} \tag{5.12}$$

Equations 5.11 and 5.12 are useful in estimating Δt given Δq or vice versa, if the step changes in rate method is to be used.

6. Conclusions

The purpose of this work was to study the limitations of the $\Delta p/q$ approximation in the analysis of pressure drawdown interference testing with varying flow rates. A variety of functional forms of varying flow rates have been studied and the dimensionless pressure based on the instantaneous flow rate has been used to provide a comparison between the $\Delta p/q$ approximation, and the line source solution.

As a result of this study the following conclusions have been reached:

(1) The $\Delta p/q$ approximation leads to successful results only if the pressure data at the producing well is analyzed, or the change in flow rate is smooth.

(2) The error resulting from the $\Delta p/q$ approximation is a function of the distance between the producing well and the test well, and the nature of the change of the flow rate.

(3) Error curves for linear and exponential variations of flow rate were constructed. These curves can be used as diagnostic tools in analyzing pressure drawdown data obtained from varying flow rate tests.

(4) If a continuous change in flow rate can be approximated by discrete step changes, the pressure data at the end of each step period is recommended for analysis.

(5) Equations were developed to estimate the At or Aq necessary to obtain a desired accuracy for step changes in flow rate.

7. Nomenclature

a	The exponent of the exponential varying flow rate expression, hr^{-1}.
b	Slope of linear varying flow rate expression, STB/D/hour
B	Oil formation volume factor, Res bbl/STB
c_t	The total system compressibility, psi^{-1}
h	Formation thickness, ft
p	Pressure, psi
Δp	Pressure drop, psi
p_D	Dimensionless pressure, $\frac{kh(p_i - p_{r,t})}{141.2\, B\mu q}$
p_{DV}	Dimensionless pressure, $\frac{kh(p_i - p_{r,t})}{141.2\, B\mu q(t)}$
q	Flow rate, STB/D
q_i	Initial flow rate, STB/D
r	Radial distance, ft
r_w	Wellbore radius, ft
t	Time, hours
Δt	Time increment, hours
t_D/r_D^2	Dimensionless time, $\frac{2.637 \times 10^{-4} kt}{\phi \mu c_t r^2}$
β	Dimensionless parameter, $\frac{b}{q_i} \frac{r^2}{\eta}$ or $a \frac{r^2}{4\eta}$
η	Hydraulic diffusivity, $\frac{2.637 \times 10^{-4} k}{\phi \mu c_t}$
μ	Viscosity, cp
ϕ	Porosity, Fraction

8. References

1. Abramowitz, M. and Stegum, A. I.: Handbook of Mathematical Functions, Dover Publications, Inc., New York, 1972, 231.

2. Abu-Zied, M. A. and Scott, V. H.: "Non-Steady Flow for Wells with Decreasing Discharges," Proc. Am Soc. Civ. Eng., vol 89, HY 3, (1963) 119-132.

3. Aron, G. and Scott V. H.: "Simplified Solutions for Decreasing Flow in Wells," Proc. Am Soc. Civ. Eng., vol. 91, HY 5 (1965) 1-12.

4. Carslaw, H. S. and Jaeger, J. C.: Conduction of Heat in Solids, 2nd Edition, Oxford at Clardon Press, 1959, 261.

5. Cooper, H. H. and Jacob, C. E.: "A Generalized Graphical Method for Evaluating Formation Constants and Summarizing Well Field History," Am Geophys. Union Trans., vol. 27 (1946) 526-534.

6. Economides, M., Brigham, W. E., Cinco-Ley, H., Miller, F. G., Ramey, H. J., Jr., Barelli, A., and Mannetti, G.: "Influence Functions and their Application to Geothermal Well Testing," Geothermal Resources Council, Trans., vol 3, (Sept. 1979) 177.

7. Gladfelter, R. E., Tracy, G. W., and Wilsey, L. E.: "Selecting Wells Which Will Respond to Production-Stimulation Treatment," Drill. and Prod. Prac., API (1955) 117.

8. Hantush, M. S.: "Drawdown Around Wells of Variable Discharge," Journ. Geophys. Res., vol 69, (1964) 4221-4235.

9. Horner, D. R.: "Pressure Buildup in Wells," Proc. Third World Petroleum Congress, E. J. Brill, Leiden (1951) II, 503.

10. Jargon, J. R. and van Poollen, H. K.: "Unit Response Function From Varying Rate Data," J. Pet. Tech., (Aug. 1965) 965-969.

11. Matthews, C. S. and Russell, D. G.: Pressure Buildup and Flow Tests in Wells, Soc. Pet. Eng. Monograph Series, vol 1, SPE, Dallas, 1967, 130.

12. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr.,: "The Estimation of Permeability and Reservoir Pressure From Bottom-Hole Pressure Buildup Characteristics," Trans., AIME (1950) 189, 91-104.

13. Odeh, A. S. and Jones, L. G.: "Pressure Drawdown Analysis, Variable-Rate Case," J. Pet. Tech., (Aug. 1965) 960-964.

14. Ramey, H. J., Jr.: "Non-Darcy Flow and Wellbore Storage Effects in Pressure Buildup and Drawdown of Gas Wells," J. Pet. Tech., (1965) 223-233.

15. Ramey, H. J., Jr.: "Verification of the Gladfelter-Tracy-Wilsey Concept for Wellbore Storage-Dominated Transient Pressure During Production," J. Can. Pet. Tech., (April-June 1976) 1-2.

16. Sternberg, Y. M.: "Transmissibility Determination From Variable Discharge Pumping Tests," Groundwater, vol. 5, nr. (1967) 27-29.

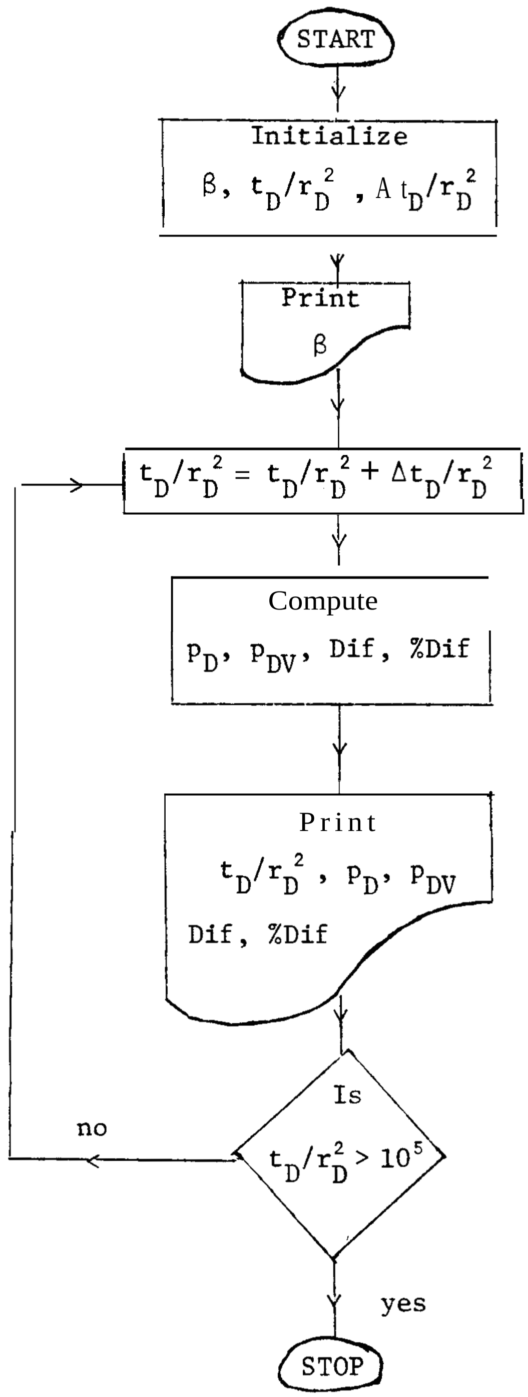
17. Sternberg, Y. M.: "Simplified Solution for Variable Rate Pumping Test," Proc. Am Soc. Civ. Eng., vol. 94, HY 1 (1968) 177-180.

18. Tsang, C. F., McEdwards, D. G., Narasimhan, T. N., and Witherspoon, P. A.: "Variable Flow Test Analysis by a Computer Assisted Matching Procedure," Paper no. SPE-6547, 47th Annual California Regional Meeting of SPE of AIME, Bakersfield, California, April 13-15, 1977.

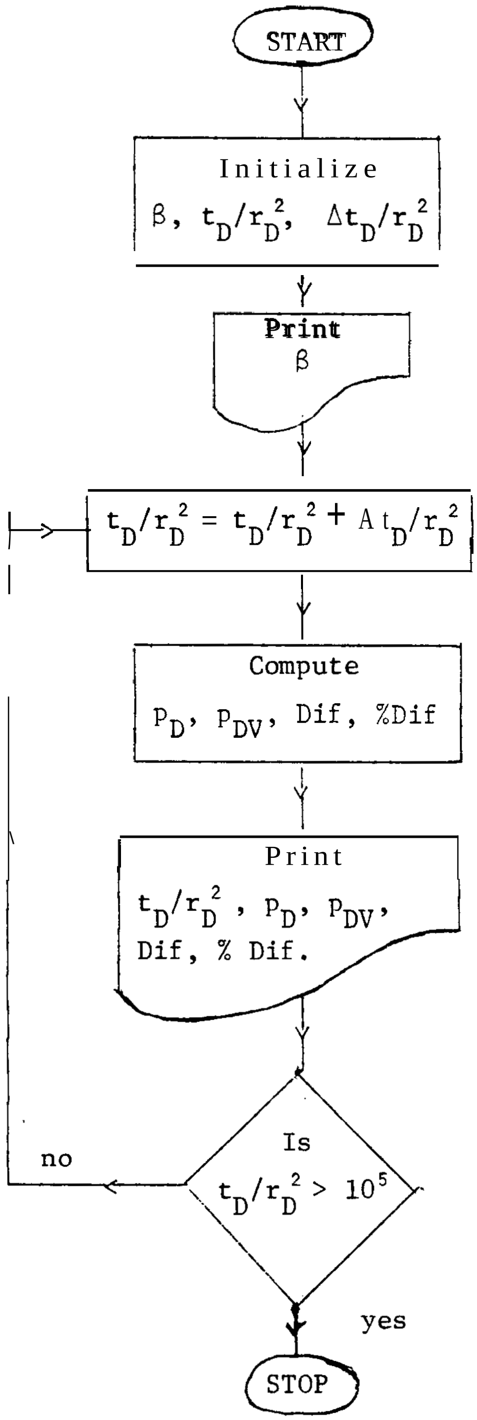
19. van Everdingen, A. F. and Hurst, W.: "The Application of the Laplace Transformation to Flow Problems in Reservoirs," Trans., AIME (1949) 186-305.

20. Winestoch, A. G. and Colpitts, G. P.: "Advances in Estimating Gas Well Deliverability," J. Can. Pet. Tech., (July-Sept. 1965) 111-119.

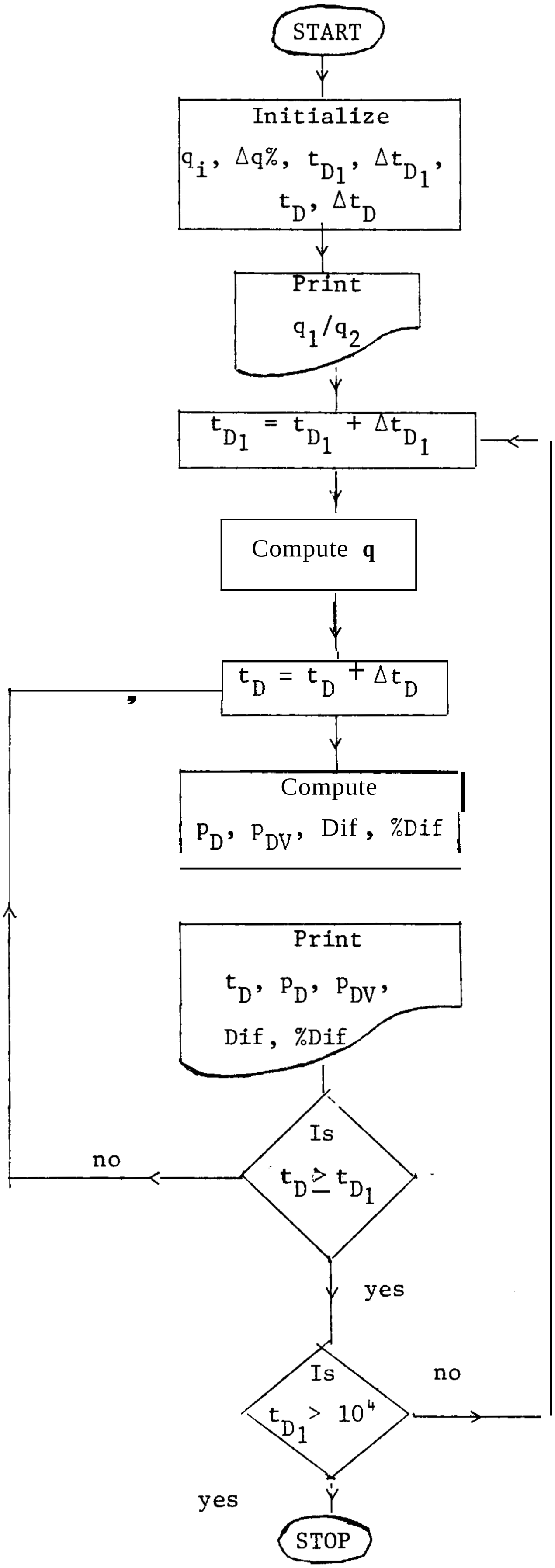
9. Appendix: Flow Charts and Listings of the Computer Programs



A-1: Flow Chart for Linear Variation of Flow Rate



A-2: Flow Chart for Exponential Variation of Flow Rate



A-3: Flow Chart for Step Change of **Flow** Rate

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C
C        THIS PROGRAM CALCULATES THE DIMENSIONLESS PRESSURE, PDV
C        AT DIMENSIONLESS TIME, TD FOR A WELL PRODUSING AT LINEAR
C        CHANGE OF FLOW RATE
C        DEFINITION OF VARIABLES
C        PD: DIMENSIONLESS PRESSURE FOR CONSTANT FLOW RATE
C        PDV: DIMENSIDNLESS PRESSURE FOR VARIABLE FLOW RATE
C        TDR: DIMENSIONLESS TIME
C        BETA: DIMENSIONLESS PARAMETER TO SPECIFY CHANGE OF FLOW RATE
C        C: -1 FOR DECREASING FLOW RATE , +1 FOR INCREASING FLOW RATE
C
         IMPLICIT REAL*8 (A-H,O-Z)
         EXTERNAL E1
         DATA C/1.0/
         WRITE (21,100)
  100    FORMAT (///,10X,'LINEAR CHANGE OF FLOW RATE',//)
         DO 10 I=1,9
         IP=I-5
         BETA=C*10.**IP
         COUNTR=0.2*DABS(1./BETA)
         WRITE (21,200) BETA
  200    FORMAT (10X,'BETA = ',E8.2,//)
         WRITE (21,300)
  300    FORMAT (11X,'TDR',16X,'PD',18X,'PDV',17X,'ERROR',11X,
      1  'ERROR PERCENT',/,5(5X,'---------------'),/)
         DO 20 J=1,7
         JP=J-3
         TDR1=10.**JP
         DO 20 K=1,5
         TDR=2.*K*TDR1
         DIF=1./BETA+TDR
         IF (DIF .LT. COUNTR) GO TO 10
         X=1./(4.*TDR)
         TEXP=TDR*DEXP(-X)
         ALFA=1./(2.*(1./BETA+TDR))
         PD=E1(X)/2.
         PDV=PD*(1.+ALFA/2.)-TEXP*ALFA
         ERROR=PDV-PD
         PERER=(ERROR/PDV)*100.
         WRITE (21,400) TDR,PD,PDV,ERROR,PERER
  400    FORMAT (5 (5X,E15.7))
   20    CONTINUE
   10    CONTINUE
         STOP
         END
C
```

```
C
      FUNCTION E1(X)
C
C     E1(X) = -EI(-X)
      IMPLICIT REAL*8 (A-H,O-Z)
      IF (X.GT.1.) GO TO 100
      Y = -ALOG(X)-.57721566+X*(.99999193+X*(-.24991055
     1 +X*(.05519968+X*(-.00976004+X*.00107857))))
      E1 = Y
      RETURN
  100 Y=(1.0/(X* EXP(X)))*(.2677737343+X*(8.6347608925+
     1 X*(18.059016973+X*(8.5733287401+X))))/(3.9549692288+
     2 X*(21.0996530827+X*(25.6329561486+X*(9.5733223454+X))))
      E1 = Y
      RETURN
      END
```

```
C
C        THIS PROGRAM CALCULATES THE DIMENSIONLESS PRESSURE, PDV
C        AT DIMENSIONLESS TIME, TD FOR A WELL PRODUSING AT EXPONENTIAL
C        CHANGE OF FLOW RATE
C        DEFINITION OF VARIABLES
C        PD: DIMENSIONLESS PRESSURE FOR CONSTANT FLOW RATE
C        PDV: DIMENSIONLESS PRESSURE FOR VARIABLE FLOW RATE
C        TDR: DIMENSIONLESS TIME
C        BETA: DIMENSIONLESS PARAMETER TO SPECIFY CHANGE OF FLOW RATE
C        C: +1 FOR DECREASING FLOW RATE , -1 FOR INCREASING FLOW RATE
C
         IMPLICIT REAL*8 (A-H,O-Z)
         COMMON BETA
         EXTERNAL E1,FUN
         DATA PER /1.0D-10/,AER/1.0D-10/,C/1.0/
         DO 10 I=1,6
         IP =I-6
         BETA = C*(10.**IP)
         A =6.0D-03
         PDV=0.0D00
         WRITE (21,100) BETA
 100     FORMAT (//,5X,'BETA= ',E8.1,//)
         WRITE (21,200)
 200     FORMAT (8X,'TDR',13X,'PD',13X,'PDV',13X,'DIF',9X,
     1   'ERROR PERCENT',4X,'ESTIM.ERROR',7X,'NUM.FUN',10X,'FLAG',
     2   /,8(3X,'-------------'),/)
         DO 20 J=1,10
         JP=J-3
         TDR1=10.**JP
         DO 20 K=1,5
         TDR =2.*K*TDR1
         T=4.*TDR
         IF (DABS(T*BETA).GT.174.6)GO TO 10
         CALL QUANC8 (FUN,A,T,AER,PER,Y,ERR,NOF,FLAG)
         X=1./(4.*TDR)
         PD=E1(X)/2.
         PDV=PDV+Y/2.
         DIF=PDV-PD
         PERDIF=DIF/PDV*100.
         A=T
         WRITE (21,300)TDR,PD,PDV,DIF,PERDIF,ERR,NOF,FLAG
 300     FORMAT (6(3X,E13.6),8X,I3,11X,F7.3)
 20      CONTINUE
 10      CONTINUE
         STOP
         END
C
```

```
C
      FUNCTION FUN (Z)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON BETA
      FUN=DEXP((BETA*Z-1./Z))/Z
      RETURN
      END
C
C
      FUNCTION E1(X)
C
C     .E1(X) = -EI(-X)
      IMPLICIT REAL*8 (A-H,O-Z)
      IF (X.GT.1.) GO TO 100
      Y = -ALOG(X)-.57721566+X*(.99999193+X*(-.24991055
     1 +X*(.05519968+X*(-.00976004+X*.00107857))))
      E1 = Y
      RETURN
100   Y=(1.0/(X* DEXP(X)))*(.2677737343+X*(8.6347608925+
     1 X*(18.059016973+X*(8.5733287401+X))))/(3.954969228+
     2 X*(21.0996530827+X*(25.6329561486+X*(9.5733223454+X))))
      E1 = Y
      RETURN
      END
```

```
C
C         THIS PROGRAM CALCULATES THE DIKENSIONLESS PRESSURE, PDV
C         AT DIMENSIONLESS TIME, TD FOR A WELL PRODUSING AT STEP
C         CHANGE OF FLOW RATE
C         DEFINITION OF VARIABLES
C         PD: DIMENSIONLESS PRESSURE FOR CONSTANT FLOW RATE
C         PDV: DIMENSIONLESS PRESSURE FOR VARIABLE FLOW RATE
C         TDR: DIMENSIONLESS TIME
C         BETA: DIMENSIONLESS PARAMETER TO SPECIFY CHANGE OF FLOW RATE
C         QI: INITIAL FLOW RATE
C
          IMPLICIT REAL*8 (A-H,O-Z)
          DIMENSION Q(60),TD(60)
          EXTERNAL E3
          DATA QI/2000./,DTI/.25/,I/50/
          WRITE (21,100)
  100     FORMAT (///,10X,'STEP  CHANGE OF FLOW RATE',//)
          DO 1000 NQ=1,2
          QI= QI/2.
          DO 900 ND=1,2
          DTD=ND*DTI
          DO 800 IS=1,3
C***      FLOW RATE SCHEDULE***
          IF (IS .EQ. 1) GO TO 1
          IF (IS .EQ. 2) GO TO 2
          QD=1.-0.05*(IS-2)
          GO TO 10
    1     QD= .99
          GO TO 10
    2     QD = .98
   10     Q(1) = QI
          TD(1) = DTD
          PQD= (1.-QD)*100
          WRITE (21,550) PQD
  550     FORMAT (/,30X,'PERCENT RATE CHANGE PER STEP = ',F5.2,//)
          WRITE (21,300)
  300     FORMAT (10X,'STEP', 3X,'FLOW RATE (STB/D)',3X,'DIMENS. TIME',
     1    2X,'DIMENS. PRESS.',6X,'PD',10X,'DIF',6X,'PERCENT DIF',/,
     2    10X,'----',3X,'-----------------',5(2X,'------------'),/)
          DO 250 L =2,I
          Q(L) = Q(L-1) *QD
          TD(L) = L*DTD
  250     CONTINUE
```

```
C***   COMPUTE DIMENSIONLESS PRESSURE    ***
       DO 200 N=1,I
       IF ( Q(N) .LT. 10.) GO TO 800
       SUM =0.0
       IF (N .EQ. 1) GO TO 450
       DO 400 J =2,N
       T =TD(N) -TD(J-1)
       X =1./(4.*T)
       SUM = SUM+(Q(J) -Q(J-1))*(E1(X)/2.+S)
 400   CONTINUE
 450   T =TD(N)
       X = 1./(4.*T)
       PD = E1(X)/2.
       PDV = (SUM + Q(1)*PD) /Q(N)
       DIF = PDV -PD
       PDIF = (DIF /PDV)*100.
       WRITE (21,600) N,Q(N),T,PDV,PD,DIF,PDIF
 600   FORMAT (11X,I2,9X,F6.1,1X,3(8X,F6.3),6X,F7.4,4X,F10.5)
 200   CONTINUE
 800   CONTINUE
 700   CONTINUE
1000   CONTINUE
       STOP
       END
C
C
       FUNCTION E1(X)
C
C      E1(X) = -EI(-X)
       IMPLICIT REAL*8 (A-H,O-Z)
       IF (X.GT.1.) GO TO 100
       Y = -ALOG(X)-.57721566+X*(.99999193+X*(-.24991055
     1 +X*(.05519968+X*(-.00976004+X*.001078571))))
       E1 = Y
       RETURN
 100   Y=(1.0/(X* EXP(X)))*(.2677737343+X*(8.6347608925+
     1 X*(18.059016973+X*(8.5733287401+X))))/(3.9584969228+
     2 X*(21.0996530827+X*(25.6329561486+X*(9.5733223454+X))))
       E1 = Y
       RETURN
       END
```