

WELL TEST ANALYSIS FOR WELLS PRODUCED AT A
CONSTANT PRESSURE

By

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ΘΑΡΣΕΙΝ ΧΡΗ ΤΑΧ'ΑΥΠΙΟΝ ΕΚCΕΤΑΙ ΑΜΕΙΝΟΝ

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ABSTRACT

Conventional well test analysis has been developed primarily for constant flow rate production. Constant pressure production results in a transient rate response. Pressure buildup after constant pressure flow is complicated by the transient rate prior to shut-in. Thus, the methods of draw-down and buildup analysis designed for constant rate production are not valid for constant pressure production.

Some transient rate analysis methods are outlined in the literature but a thorough study is lacking. The necessary analytical solutions for determination of reservoir permeability and porosity and wellbore skin factor are provided in this study. Reservoir limit testing and interference analysis are also discussed. In addition, analysis of flow at constant wellhead pressure is shown to be a simple extension of the existing theory for constant wellbore pressure production.

Most of the existing methods for pressure buildup analysis for wells with a constant pressure flow history are empirical. In this work, the method of superposition in time of continuously changing rates is used to generate an exact solution for pressure buildup following constant pressure

flow. The method is general. Wellbore storage and skin effects are incorporated into the theory, and both bounded and unbounded reservoirs are considered. Buildup solutions are graphed using conventional techniques for analysis. Horner's¹⁰ method for plotting buildup after variable rate flow is found to be accurate in a majority of cases. Curves for determination of static reservoir pressure similar to those developed by Matthews, Brons, and Hazebroek¹⁸ are provided for closed bounded reservoirs. Additional applications of the method of superposition in time of continuously changing rates are also included.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	iv
ABSTRACT	v
SECTION	page
1. INTRODUCTION	1
2. ANALYTICAL SOLUTIONS FOR TRANSIENT RATE DECLINE	6
Fundamental Partial Differential Equations	7
Method of Solution	11
Basic Transient Rate Solutions	16
Unbounded Reservoir	17
Closed Eounded Reservoir	22
Constant Pressure Bounded Circular Reservoir	27
Production at Constant Wellhead Pressure	30
Effect of Wellbore Storage	34
Interference Analysis	37
3. PRESSURE BUILDUP AFTER CONSTANT PRESSURE PRODUCTION	43
Theoretical Expression for Pressure Buildup	44
Analysis of Pressure Buildup	46
Early Shut-in Time	47
Horner Buildup Analysis	47
Outer Boundary Effects	50
Practical Limitations of the Theory	54
Short Flow Time Before Shut-in	54
Wellbore Effects	56
Outer Boundary Effects	57
Comparison with Previous Studies	57
Further Applications of the Solution Technique	60
The Critical Flow Phenomenon	60
Exponential Decline After Constant Rate Production	63
Interference among Flowing Wells	67
4. CONCLUSIONS	73
REFERENCES	76
NOMENCLATURE	79

Appendix	page
A . UNITS CONVERSIONS	82
B . TABULATED SOLUTIONS	83
C . COMPUTER PROGRAMS	111

SECTION 1

INTRODUCTION

Although constant-rate production is usually assumed in the development of well test analysis methods, several common reservoir production conditions result in flow at a constant pressure instead. Reservoir fluids are often produced into a constant pressure separator or pipeline; and constant pressure **flow** is also maintained during the rate decline period of reservoir depletion. Wells in low permeability reservoirs are often by necessity produced at constant pressure. In geothermal reservoirs, produced fluids may drive a back-pressured turbine. Finally, open wells, including artesian water wells, flow at constant atmospheric pressure.

Fundamental considerations instruct that conventional pressure drawdown and buildup analysis methods should not be appropriate **for** wells produced at constant pressure. However, analogous well test methods have been proposed. The purpose of this study is to review the existing methods for transient rate decline and pressure buildup analysis and to contribute new solutions where needed in order to produce a comprehensive well test analysis package **for** wells produced at constant pressure. The remainder of this section is a discussion of the methods available in the literature and the objectives of this work.

Many of the basic analytical solutions for transient rate decline have been available for some time. The first solutions were published by Moore, et al. (1933) and Hurst (1934). Results were presented in graphical form for bounded and unbounded reservoirs in which the flow was radial and the single phase fluid was slightly compressible. These solutions were not tabulated, however. Tables of dimensionless flow rate vs dimensionless time were provided later by Ferris, et al. (1962) for the unbounded system and by Tsarevich and Kuranov (1956) for the closed bounded circular reservoir. Tsarevich and Kuranov also provided tabulated solutions for the cumulative production from a closed bounded reservoir. Fetkovich (1973) developed the type curves for transient rate vs time in the closed bounded circular reservoir. Fetkovich was the first to determine the exponential form of the final rate decline for constant pressure production. Type curves for rate decline in closed bounded reservoirs with pressure sensitive rock and fluid properties were developed by Samaniego and Cinco (1978). A method for determining the skin effect was given by Earlougher (1977). Type curves for analysis of the transient rate response when the well penetrates a fracture were developed by Prats, et al. (1962) and by Locke and Sawyer (1975). Kucuk (1978) developed type curves for the transient rate and cumulative production for constant pressure production with elliptical flow.

Although the rate decline solutions present in the literature provide a fairly comprehensive list, certain problems have not been discussed. One such problem is the effect of production with constant pressure at the wellhead rather than the wellbore. Constant wellhead pressure production causes a variable wellbore pressure because the pressure drop due to friction in the wellbore is dependent on the transient rate. A second subject not found in the literature is interference analysis. Finally, a solution for the early transient rate response which allows for a more realistic finite initial rate has not been determined. These problems are discussed in Section 2 of this work.

Another subject which has not received a thorough treatment in the literature is the analysis of pressure buildup after constant pressure production. Horner (1951) suggested two methods for dealing with variable rate production prior to shut-in. The first method was exact, but required long calculations. The second method was to assume approximate constant rate production by using the last established rate in conjunction with a corrected flow time determined by dividing the cumulative production by the last established flow rate. The latter method was not theoretically justified at the time and has been questioned in other studies. Investigators who have found fault with the Horner approximate pressure buildup analysis method for variable rate production prior to shut-in include Odeh and Selig (1963), San-

drea (1971), and Clegg (1967). Their objections will be discussed in Section 3. Jacob and Lohman (1952) analyzed pressure buildup after constant pressure production for a number of wells for which transmissivity had already been determined by type curve analysis of the rate response. Their graph of residual drawdown versus the log of the total time divided by the shut-in time produced a semi-log straight line. Transmissivities calculated from the slope of the line and the average flow rate during the flow period agreed with the values determined from type curve matching.

In Section 3 of this study a solution for pressure buildup after constant pressure production is derived based on superposition in time of continuously varying rates. The resulting solution is general and can be used to justify the modified Horner method theoretically. The Jacob and Lohman method is shown to be of somewhat limited accuracy. In addition, methods for determination of wellbore storage and skin effect and the static reservoir pressure from the pressure buildup data are shown to be analogous to the constant rate case. Limitations of the methods for analysis of pressure buildup are also considered.

The method of superposition in time of continuously varying rates has many applications. In the last part of Section 3, three applications of the theory are presented: a constant initial rate followed by constant pressure production 1) during the early period of production, 2) after the

onset of pseudo-steady state, and 3) interference among flowing wells produced at constant rate or constant pressure.

SECTION 2

ANALYTICAL SOLUTIONS FOR TRANSIENT RATE DECLINE

Although many of the basic solutions for transient rate decline for wells produced at constant pressure have been published, no comprehensive analysis has been offered. In this section the problem of constant pressure production from the center of a circular reservoir is examined. In Section 2.1, equations which define the basic problem and the assumptions required for their derivation are given. In Section 2.2, the method used in this work for obtaining solutions to the equations is outlined. In Section 2.3 the analytical solutions in real space for the unbounded circular reservoir are presented. Included in this section are discussions of the application of the solutions to well test analysis.

Three important extensions of the basic solutions are derived in the final three sections. The solutions given in the first three sections apply for production at a constant wellbore pressure. Because the pressure is normally controlled at the wellhead, the effect of changing the inner boundary condition to include frictional pressure drop in the wellbore is examined in Section 2.1. An apparent advantage of constant pressure testing is the absence of wellbore

storage effects. This is discussed in Section 2.5. Finally, Section 2.6 contains a discussion of interference analysis for wells produced at constant pressure.

2.1 FUNDAMENTAL PARTIAL DIFFERENTIAL EQUATIONS

The fundamental partial differential equation representing idealized flow through porous media is the diffusivity equation. The diffusivity equation in radial geometry is given by:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{\phi \mu c_t}{k} \frac{\partial p}{\partial t} \quad (2.1)$$

The porous medium is contained in the region between the finite wellbore radius, r_w , and the reservoir radius, r_e , which may be infinite or finite. Implicit in the use of this equation are the following assumptions:

1. Flow through the porous medium is strictly radial with negligible gravity effects.
2. The porous medium is homogeneous and isotropic, with constant thickness, h , porosity, ϕ , and permeability, k .
3. The fluid viscosity, μ , is constant, and the total compressibility, c_t , of the fluid and the porous medium is small in magnitude and constant.

4. Pressure gradients are small everywhere such that gradient squared terms may be neglected.

The last two assumptions are essentially satisfied for a liquid saturated, one phase, isothermal reservoir.

A complete mathematical definition of the problem of constant pressure production from a circular reservoir requires additional equations which represent the appropriate initial and boundary conditions. For a reservoir initially at a constant pressure, p_i , the initial condition is given by:

$$p(r,0) = p_i \quad (2.2)$$

The inner boundary condition is:

$$p(r_w,t) = p_{wf} + s \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} + \quad (2.3)$$

where s is the wellbore skin factor, and p_{wf} is the flowing bottomhole pressure. Three different outer boundary conditions are often considered: an infinitely large reservoir, a closed outer boundary, and a constant-pressure outer boundary. The condition for an infinitely large reservoir is:

$$\lim_{r \rightarrow \infty} p(r,t) = p_i \quad (2.4)$$

For the closed outer boundary the condition is:

$$\frac{\partial p}{\partial r}(r_e,t) = 0 \quad (2.5)$$

and for the constant pressure outer boundary, the condition is:

$$p(r_-, t) = p_e \quad (2.6)$$

Fig. 2.1 is a schematic diagram of the system described by Eqs. 2.1-2.6. The flow into the wellbore is given by:

$$q(t) = \frac{-2\pi kh}{\mu} \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} + \quad (2.7)$$

In order to provide general solutions, dimensionless variables may be defined as follows:

$$r_D = r/r_w \quad (2.8)$$

$$t_D = kt/\phi\mu c_t r_w^2 \quad (2.9)$$

$$p_D(r_D, t_D) = \frac{p_i - p(r, t)}{p_i - p_{wf}} \quad (2.10)$$

$$q_D(t_D) = q(t) \mu / [2\pi kh(p_i - p_{wf})] \quad (2.11)$$

The resulting equations in dimensionless variables are

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} \quad (2.12)$$

$$p_D(r_D, 0) = 0 \quad (2.13)$$

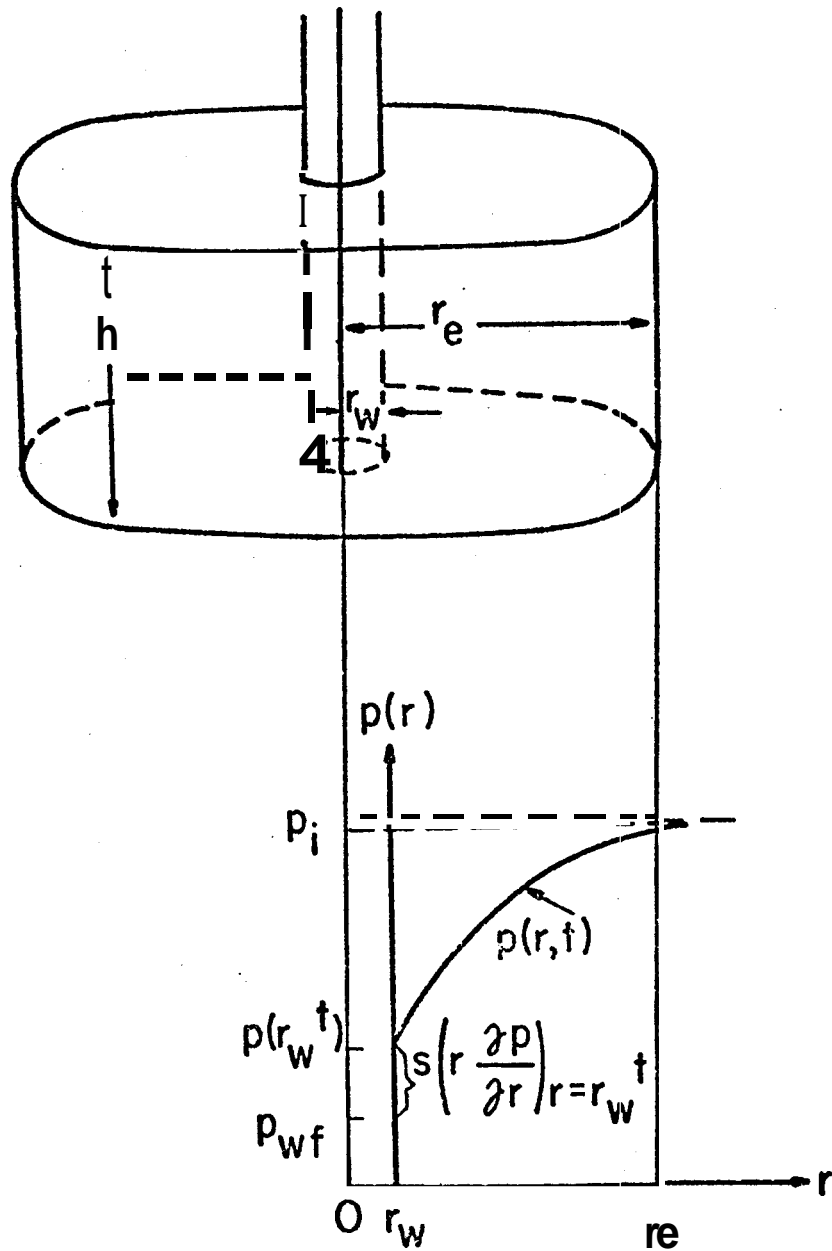


Figure 2.1: Schematic Diagram of a Well Producing at a Constant Wellbore Pressure from a Circular Reservoir

$$p_D(1, t_D) = 1 + s \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1^+} \quad (2.14)$$

with outer boundary condition one of the following:

$$\lim_{r_D \rightarrow \infty} p_D(r_D, t_D) = 0 \quad (2.15)$$

$$\frac{\partial p_D}{\partial r_D}(r_{eD}, t_D) = 0 \quad (2.16)$$

$$p_D(r_{eD}, t_D) = 0 \quad (2.17)$$

The flow rate is determined from:

$$q_D(t_D) = \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1^+} \quad (2.18)$$

Eqs. 2.12-2.14 and one of Eqs. 2.15, 2.16, or 2.17 completely describe the problem of a well producing at a constant wellbore pressure from the center of a circular reservoir under the assumptions listed in this section. In the next section, the method of solution used in this work is described.

2.2 METHOD OF SOLUTION

A straight-forward method for solving Eqs. 2.12-2.17 involves use of the Laplace transformation. Carslaw and Jaeger (1947) used the Laplace transformation to solve the diffusivity equation. By this method, the equations are transformed into a system of ordinary differential equations

which can be solved analytically. The resulting solution for the Laplace transform of the pressure, \bar{p}_D , is a function of the Laplace variable, ℓ , and the spacial variable, r_D . To determine the pressure, p_D , as a function of r_D and t_D , the Laplace space solutions must be inverted using the inverse Laplace transformation.

Application of the Laplace transformation to Eqs. 2.12-2.18 results in:

$$\frac{d^2 \bar{p}_D}{dr_D^2} + \frac{1}{r_D} \frac{d\bar{p}_D}{dr_D} = \ell \bar{p}_D \quad (2.19)$$

$$p_D(1, \ell) = \frac{1}{\ell} + s \left(\frac{dp_D}{dr_D} \right)_{r_D=1^+} \quad (2.20)$$

$$\text{Lim}_{r_D \rightarrow \infty} \bar{p}_D(r_D, \ell) = 0 \quad (2.21)$$

$$\frac{d\bar{p}_D}{dr_D}(r_{eD}, \ell) = 0 \quad (2.22)$$

$$\bar{p}_D(r_{eD}, \ell) = 0 \quad (2.23)$$

$$q_D(\ell) = \left(\frac{d\bar{p}_D}{dr_D} \right)_{r_D=1^+} \quad (2.24)$$

The solutions in Laplace space for all three boundary cases are given in Table 2.1.

A relationship exists between the Laplace transformed solutions for the constant pressure and constant rate problems which was indicated by van Everdingen and Hurst (1949). De-

Table 2.1: Laplace Space Solutions for a Well Producing at a Constant Pressure from the Center of a Circular Reservoir

INFINITE OUTER BOUNDARY

$$\bar{p}_D(r_D, \lambda) = K_0(r_D \sqrt{\lambda}) / \{\lambda [K_0(\sqrt{\lambda}) + s\sqrt{\lambda} K_1(\sqrt{\lambda})]\}$$

$$\bar{q}_D(\lambda) = K_1(\sqrt{\lambda}) / \{\sqrt{\lambda} [K_0(\sqrt{\lambda}) + s\sqrt{\lambda} K_1(\sqrt{\lambda})]\}$$

CLOSED OUTER BOUNDARY

$$\bar{p}_D(r_D, \lambda) = [K_1(r_{eD} \sqrt{\lambda}) I_0(r_D \sqrt{\lambda}) + K_0(r_D \sqrt{\lambda}) I_1(r_{eD} \sqrt{\lambda})] / D_1$$

$$\bar{q}_D(\lambda) = [K_1(\sqrt{\lambda}) I_1(r_{eD} \sqrt{\lambda}) - K_1(r_{eD} \sqrt{\lambda}) I_1(\sqrt{\lambda})] / D_1$$

$$D_1 = \lambda \cdot \{ [K_1(r_{eD} \sqrt{\lambda}) I_0(\sqrt{\lambda}) + K_0(\sqrt{\lambda}) I_1(r_{eD} \sqrt{\lambda})] \\ - [s\sqrt{\lambda} K_1(r_{eD} \sqrt{\lambda}) I_1(\sqrt{\lambda}) - K_1(\sqrt{\lambda}) I_1(r_{eD} \sqrt{\lambda})] \}$$

CONSTANT PRESSURE OUTER BOUNDARY

$$\bar{p}_D(r_D, \lambda) = [K_0(r_D \sqrt{\lambda}) I_0(r_{eD} \sqrt{\lambda}) - K_0(r_{eD} \sqrt{\lambda}) I_0(r_D \sqrt{\lambda})] / D_2$$

$$\bar{q}_D(\lambda) = [K_1(\sqrt{\lambda}) I_0(r_{eD} \sqrt{\lambda}) + K_0(r_{eD} \sqrt{\lambda}) I_1(\sqrt{\lambda})] / D_2$$

$$D_2 = \sqrt{\lambda} \cdot \{ [K_0(\sqrt{\lambda}) I_0(r_{eD} \sqrt{\lambda}) - K_0(r_{eD} \sqrt{\lambda}) I_0(\sqrt{\lambda})] \\ + s\sqrt{\lambda} [K_0(r_{eD} \sqrt{\lambda}) I_1(\sqrt{\lambda}) - K_1(\sqrt{\lambda}) I_0(r_{eD} \sqrt{\lambda})] \}$$

noting the dimensionless wellbore pressure under constant rate production by p_{wD} , and the dimensionless cumulative production under constant pressure production by Q_D , this relation is given by:

$$\ell \bar{p}_{wD}(\ell) \bar{Q}_D(\ell) = \frac{1}{\ell^2} \quad (2.25)$$

where Q_D is defined by:

$$Q_D(t_D) = Q(t) / [2\pi\phi c_t h r_w^2 (p_i - p_{wf})] \quad (2.26)$$

This result can be derived from the principle of superposition. The cumulative production is related to the transient rate by:

$$\bar{Q}_D(\ell) = \bar{q}_D(\ell) / \ell \quad (2.27)$$

This is easily verified from basic properties of the Laplace transformation. Finally, by combining Eqs. 2.25 and 2.27,

$$q_D(\ell) = \frac{1}{\ell^2 \bar{p}_{wD}(\ell)} \quad (2.28)$$

Thus, any solution for $\bar{p}_{wD}(\ell)$ for constant rate production has an analog solution, $\bar{q}_D(\ell)$ for constant pressure production.

Unfortunately, the inverse Laplace transformation of the solutions in Table 2.1 can only be obtained through use of the Mellin inversion integral, and the resulting integrals cannot be reduced to simple functions. The solutions tabu-

lated in the literature were obtained from numerical integrations of the inversion integrals. In this work, the solutions are determined using an algorithm for approximate numerical inversion of the Laplace space solutions. The tabulated solutions in the literature serve as a check of the "approximate" solutions determined herein. In general almost exact agreement was found with solutions obtained by numerical integration.

The algorithm for numerical inversion of the transformed solutions was presented by Stehfest (1970). This algorithm provides tabular solutions for a wide variety of problems of interest in well test analysis. The algorithm is based on the following formula given by Stehfest:

$$F(t) \approx \frac{\ln 2}{t} \sum_{i=1}^N v_i f\left(\frac{\ln 2}{t} i\right) \quad (2.29)$$

where $f(s)$ is the Laplace transformation of $F(t)$, and the v_i are:

$$v_i = (-1)^{[(N/2)+i]} \sum_{k=\frac{i+1}{2}}^{\min\{i, N/2\}} \frac{k^{N/2} (2k)!}{[(N/2)-k]! k! (k-1)! (i-k)! (2k-i)!} \quad (2.30)$$

N , the number of terms in the sum, may be determined by comparison with known analytical solutions. Stehfest observed that theoretically, the greater N is, the more accurate is the value computed for $F(t)$; but in practice roundoff errors increase with increasing N . Thus, there is an optimum value

for N which can only be determined by comparing values for $F(t)$ with known values.

The Stehfest algorithm provides a convenient method for obtaining real space solutions from the Laplace space solutions given in Table 2.1. Solutions calculated from the Stehfest algorithm are tabulated in Appendix B. The solutions tabulated in this work have been checked against existing solutions whenever possible. Generally, the solutions agree for at least three or, in most cases, four significant figures.

An alternative method for obtaining solutions for constant pressure flow was used by Juan (1977). He developed an algorithm for deriving the constant pressure solutions from the constant rate solutions using superposition. This derivation did not require Laplace transformations.

In the next section graphs of the solutions are presented along with a discussion of their use in well test analysis.

2.3 BASIC TRANSIENT RATE SOLUTIONS

Portions of the analytical solutions for transient rate decline discussed in this section have appeared elsewhere in the literature. A complete study of how they may be applied in well test analysis has been lacking. Three types of reservoirs are considered: the unbounded reservoir; the

closed, bounded reservoir; and the constant pressure bounded reservoir. For each type of reservoir, analogies with the analysis of pressure drawdown for the corresponding constant rate case are indicated.

2.3.1 Unbounded Reservoir

As in the case of constant rate production, the transient rate solutions for an unbounded reservoir represent the transient behavior before boundary effects become evident. The transient rate solution by Jacob and Lohman (1952), ignores the skin effect and assigns unrealistically high values to the flow rates during the early flow period. A log-log graph of this solution is shown in Fig. 2.2. Also shown in the figure is a graph of $1/p_{wD}$ where p_{wD} is the wellbore pressure drop determined from the finite wellbore radius solution for constant rate production.

The close similarity between the two solutions which are related exactly in Laplace space by Eq. 2.28 may be seen in Fig. 2.2. Earlougher (1977) determined that for $t_D > 8 \times 10^4$, p_{wD} and $1/q_D$ agree within 1%. Because the period when $1/q_D$ and p_{wD} coincide is in the semi-log straight portion of the p_{wD} function, a graph of $1/q_D$ vs $\log t_D$ produces a straight line if the flow period is long enough. Earlougher described the method for determining reservoir permeability from the slope, m_q , of the semi-log straight line:

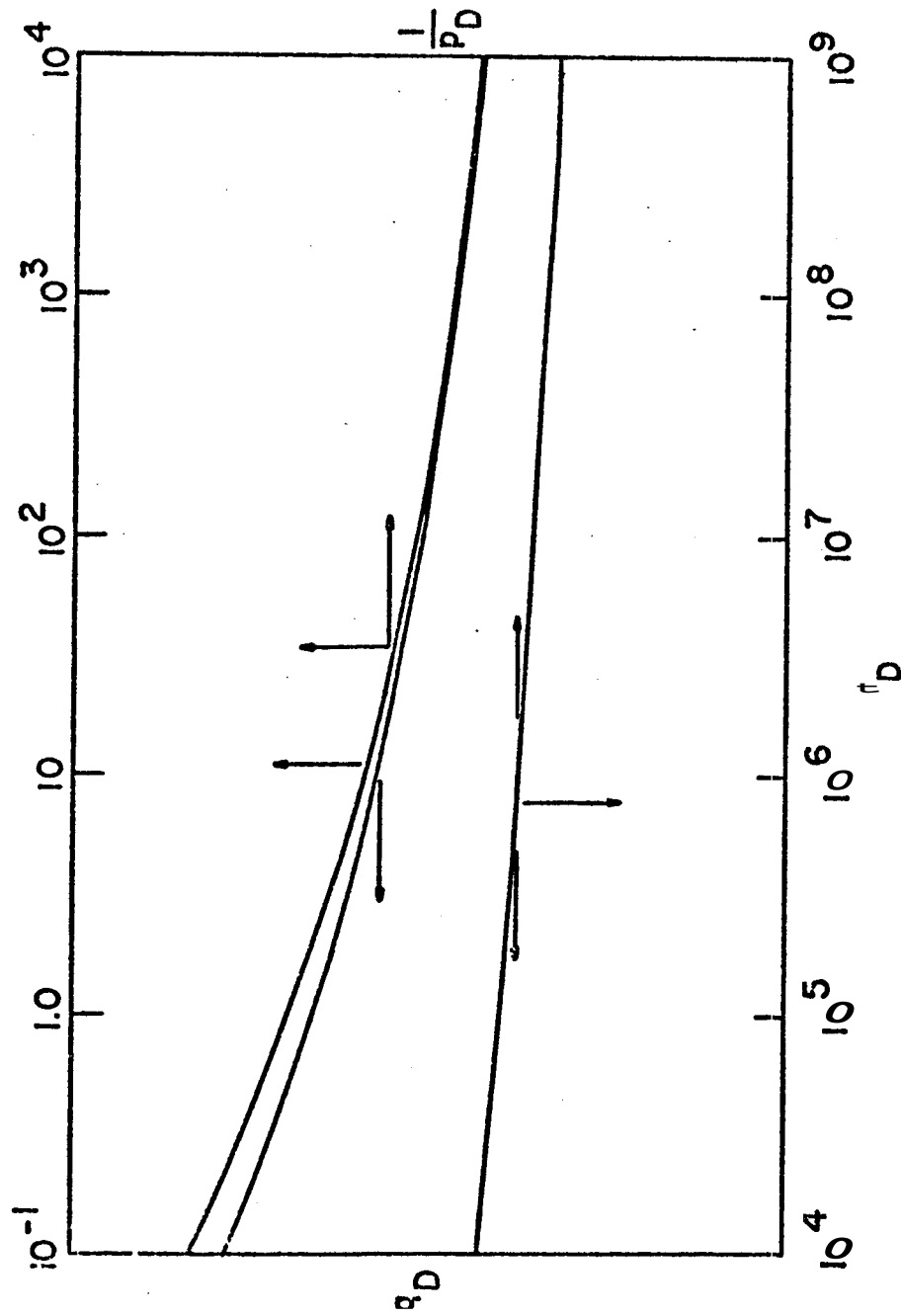


Figure 2.2: Dimensionless Flow Rate for Constant-Pressure Product q_D and Dimensionless Pressure Drop for Constant-Rate Product $1/p_D$ from an Unbounded Reservoir with a Finite Wellbore Radius and Zero Skin

$$k = \frac{\mu \cdot \ln 10}{4\pi m_q h (p_i - p_{wf})} \quad (2.31)$$

In addition, Earlougher indicated that the wellbore skin factor could be estimated from:

$$s = \frac{R_0 - 10}{2} \left[\frac{(\frac{1}{q})_{1 \text{ hr}}}{m_q} - \text{Rog} \frac{k}{\phi \mu c_t r_w^2} - 0.80907 \right] \quad (2.32)$$

where $(1/q)_{1 \text{ hr}}$ is the extrapolated value of the semi-log straight line at a flow time of one hour.

A second method for determining the reservoir permeability is by type curve matching with a graph of $\log q_D$ vs $\log t_D$. This method was described by Jacob and Lohman (1952). If q_{DM} is the value for q_D which coincides with the value q on the graph of $\log q$ vs $\log t$ overlaying the type curve, the permeability can be determined from:

$$k = q_{DM} \mu / [2\pi h q_{DM} (p_i - p_{wf})] \quad (2.33)$$

Likewise, the porosity can be determined from the time match points:

$$\phi = k t_M / (\mu c_t r_w^2 t_{DM}) \quad (2.34)$$

The type curve for q_D vs t_D does not take the skin effect into account. If a non-zero skin factor is present, the estimate for k by type curve matching will be accurate, but the estimate for ϕ will be in error. For positive skin factors, the following approximation can often be used:

$$\phi e^{-2s} \cong kt_M / (\mu c_t r_w^2 t_{DM}) \quad (2.35)$$

The methods described thus far for transient rate analysis of an unbounded reservoir are exactly analogous to the pressure transient analysis techniques. Still other analogous techniques can be derived. For instance, multiple rate testing is analogous to analysis of the rate response to multiple changes of the producing pressure. Fetkovich (1973) applied the ideas of Hurst (1943) to determine the rate response to a change in producing pressure. In a similar fashion, a step change in the flowing bottomhole pressure from p_{wf1} to p_{wf2} at time t_1 results in:

$$q(t) = 2\pi kh[(p_i - p_{wf1}) q_D(t) + (p_{wf1} - p_{wf2}) q_D(t-t_1)] \quad (2.36)$$

For $t - t_1 \ll t_1$, $q(t) \cong q(t_1)$. A rearrangement of Eq. 2.36 results in:

$$q(t) - q(t_1) = \frac{2\pi kh}{\mu} (p_{wf1} - p_{wf2}) q_D(t-t_1) \quad (2.37)$$

Hence, a graph of $\log [q(t) - q(t_1)]$ vs $\log (t - t_1)$ can be matched with the q_D vs t_D type curve. Furthermore, a graph of $1/[q(t) - q(t_1)]$ vs $\log (t - t_1)$ can be examined for a semi-log straight line.

One difficulty with the analytical solutions for constant pressure (transient rate) production is that computed production rates very early in time may be unrealistically large. A realistic assumption might be that the initial flow rate for an instantaneous drop in the wellbore pressure must be equal to or less than some rate q_c possibly due to the critical flow phenomenon. Critical flow is the maximum possible rate of flow for a particular orifice, and is independent of the pressure drop across the orifice. The maximum rate is established when the flow velocity reaches the velocity of sound in the flowing fluid. Downstream changes in pressure will not propagate upstream, and the flowrate is a function of the upstream pressure only. For ideal gases, it is often shown that a pressure drop approximately half the upstream pressure will cause critical flow. Poettmann and Beck (1963) have shown that similar results may be obtained for multiphase flow of gas and oil. The existence of a critical orifice or flow restriction anywhere between the sand face and the surface could control the initial flow rate, and could prevent instantaneous establishment of an arbitrary constant bottomhole flowing pressure. If a particular bottomhole production pressure is specified, the result could be constant rate flow until the reservoir pressure at the sand face dropped to the desired value, and perhaps for a longer period of time depending upon the location of the critical choke. Then the rate would begin to

decline as the pressure is held constant. The mathematics needed to provide a solution for the rate decline after the initial constant flow is analogous to the mathematical solution of pressure buildup after constant pressure production presented in Section 3. Hence, this solution is discussed in Section 3.5.1.

2.3.2 Closed Bounded Reservoir

Fetkovich (1973) showed that one important effect of a closed boundary on constant pressure production is the generation of an exponential decline in the production rate at long times. This state was termed "exponential depletion". It is important in that this state must be the terminal state for any production condition.

The exponential depletion state can be derived from the dimensionless wellbore pressure function for constant-rate production after the onset of pseudo-steady state by use of Eq. 2.28. For pseudo-steady state for closed reservoirs produced at a constant rate Ramey and Cobb (1971) showed that:

$$p_{wD}(t_D) = 2\pi t_{DA} + \frac{1}{2} \ln \frac{4A}{\gamma C_A r_w^2} \quad (2.38)$$

Thus :

$$\begin{aligned}
\bar{q}_D(\ell) &= \frac{1}{\ell^2 P_{wD}(\ell)} & (2.28) \\
&= \frac{1}{\ell^2} \left(L \left\{ 2\pi t_{DA} + \frac{1}{2} \ell \ln \frac{4A}{\gamma C_A r_w^2} \right\} \right)^{-1} \\
&= \frac{1}{\ell^2} \left[\frac{2\pi}{\ell^2} + \left(\frac{1}{2} \ell \ln \frac{4A}{\gamma C_A r_w^2} \right) / \ell \right]^{-1} \\
\bar{q}_D(\ell) &= \frac{2}{\ell \ln \frac{4A}{\gamma C_A r_w^2}} \left[\frac{4\pi}{\ell \ln \frac{4A}{\gamma C_A r_w^2}} + \ell \right]^{-1}
\end{aligned}$$

and

$$q_D(t_D) = \frac{2}{\ell \ln \frac{4A}{\gamma C_A r_w^2}} \left[\exp \frac{-4\pi}{\ell \ln \frac{4A}{\gamma C_A r_w^2}} \right] \quad (2.40)$$

for $t_{DA} > (t_{pss})_D$.

where $(t_{pss})_D$ is the time required for development of true pseudo-steady state at the producing well for the constant rate case, and is dependent on the reservoir shape. See Earlougher and Ramey (1968). To allow for a skin factor, the effective wellbore radius $r_w' = r_w e^{-s}$ should be substituted for r_w .

For closed bounded circular reservoirs after the onset of exponential decline:

$$P_{wD} = 2\pi t_{DA} + \ell n r_{eD} - 3/4 \quad (2.41)$$

Following the same procedure as that used to demonstrate exponential decline for other reservoir shapes, for circular reservoirs:

$$q_D(t_D) = \frac{1}{\ln r_{eD} - \frac{3}{4}} \exp [-2\pi t_{DA} / (\ln r_{eD} - 3/4)] \quad (2.42)$$

for $t_{DA} \geq 0.1$

Fetkovich (1973) drew type curves for rate decline in closed bounded circular reservoirs which contained a slight error due to substitution of $1/2$ for the correct value of $3/4$ in Eq. 2.42. The Fetkovich type curves are reproduced in Fig. 2.3. Again, wellbore skin effects may be included by the substitution of $r'_w = r_w e^{-s}$ for r_w .

In the final depletion of an oil field, flow which has been at a constant rate eventually declines exponentially while the wellbore or wellhead pressure remains constant. This type of decline following constant rate production is slightly different, and is treated in Section 3.5.2.

An analogy for reservoir limit testing from constant rate production data exists for exponential rate decline. From Eqs. 2.40 and 2.11:

$$\ln q = \frac{-4\pi t_{DA}}{\ln \frac{4A}{\gamma C_A r_w^2}} + \ln \left[\frac{4\pi kh(p_i - P_{wf})}{\mu \ln \frac{4A}{\gamma C_A r_w^2}} \right] \quad (2.43)$$

Thus, a graph of $\log q$ vs t will have an intercept, q_{int}^* , and a slope, m , given by:

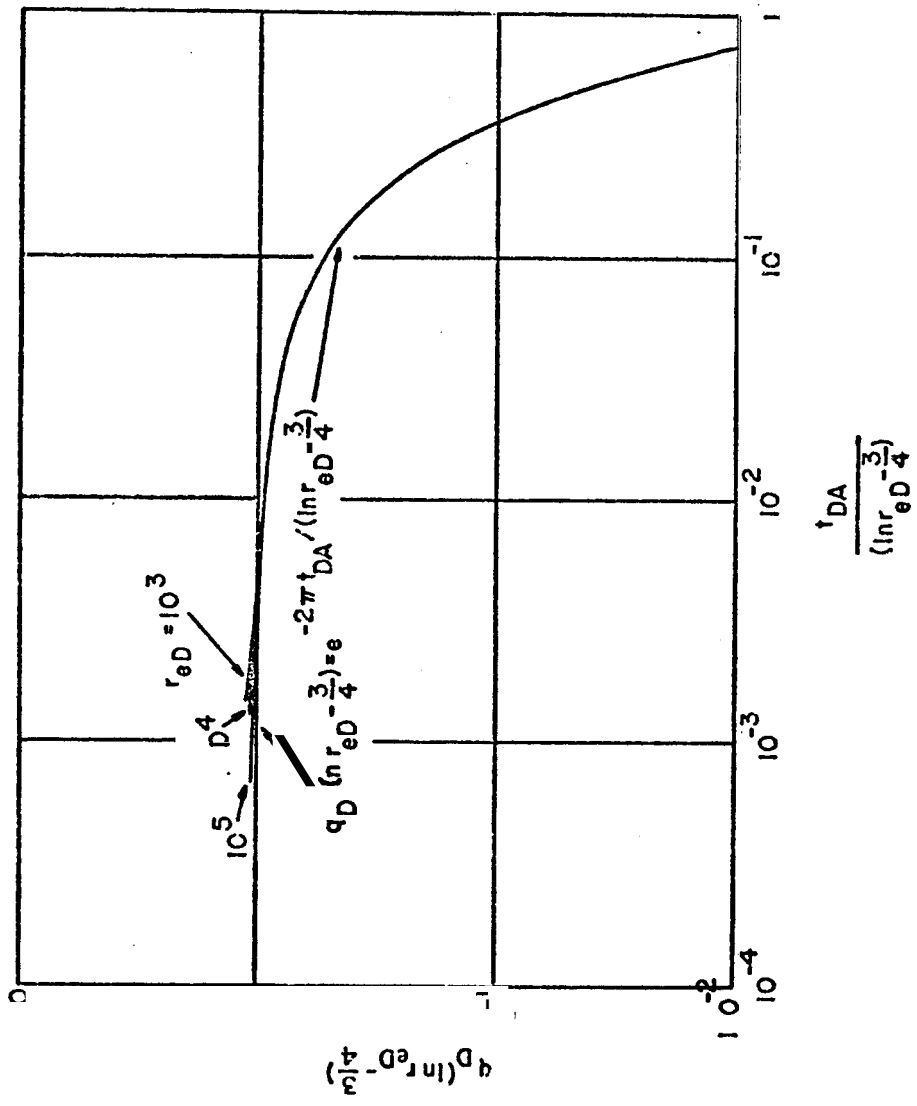


Figure 2 3: Dimensionless Flow Rate Functions for a Well Produced at a Constant Pressure from the Center of a Closed, Bounded, Circular Reservoir

$$q_{INT} = \frac{4\pi kh(p_i - p_{wf})}{\mu \ln \frac{4A}{\gamma C_A r_w^2}} \quad (2.44)$$

and :

$$m^* = \frac{4\pi k}{\phi \mu c_t A \ln \frac{4A}{\gamma C_A r_w^2}} \quad (2.45)$$

Solving for $\ln \left(\frac{4A}{\gamma C_A r_w^2} \right)$ in both equations and equating the resulting expressions:

$$A = \frac{q_{INT}^\mu}{m^* \phi c_t h (p_i - p_{wf})} \quad (2.46)$$

Then C_A can be estimated from either Eq. 2.44 or 2.45:

$$C_A = \frac{4A}{\gamma r_w^2} \exp[-4\pi kh(p_i - p_{wf})/q_{INT}^\mu] \quad (2.47)$$

$$C_A = \frac{4A}{\gamma r_w^2} \exp[-4\pi kh/m^* \phi \mu c_t A] \quad (2.48)$$

The Laplace space solution for cumulative production during the exponential rate decline period is determined from Eq. 2.25. The derivation is similar to the derivation of Eq. 2.40, and the result is that for closed, bounded reservoirs:

$$Q_D(t_D) = \frac{A}{2\pi r_w^2} \left[1 - \exp\left(-4\pi t_{DA} \ln \frac{A}{\gamma C_A r_w^2}\right) \right] \quad (2.49)$$

for $t_{DA} \geq t_{pssD}$. For circular reservoirs:

$$Q_D(t_D) = \frac{r_{eD}^2}{z} [1 - \exp(-2\pi t_{DA} / (\ln r_{eD} - 3/4))] \quad (2.50)$$

for $t_{DA} \geq 0.1$. A type curve graph of $\log(Q_D/r_{eD}^2)$ vs $\log t_D / (\ln r_{eD} - 3/4)$ for circular reservoirs is shown in Fig. 2.4.

2.3.3 Constant Pressure Bounded Circular- Reservoir

The solution for constant pressure production from a circular reservoir with constant pressure boundary involves the transition from the infinite acting rate function to true steady-state. The final value for the rate may be written immediately from the steady state rate equation for radial flow:⁵

$$q_o = \frac{1}{\ln r_{eD} + S} \quad (2.51)$$

Steady state flow occurs for $t_{DA} \geq \gamma/4 = 1/2.2458\pi$. This value was determined by equating the right hand side of Eq. 2.51 with the semi-log approximate solution for $1/q_D$, and solving for t_D . Fig. 2.5 is a graph of the solution for a constant pressure outer boundary.

This concludes the discussion of the solution for constant wellbore pressure production from a circular reser-

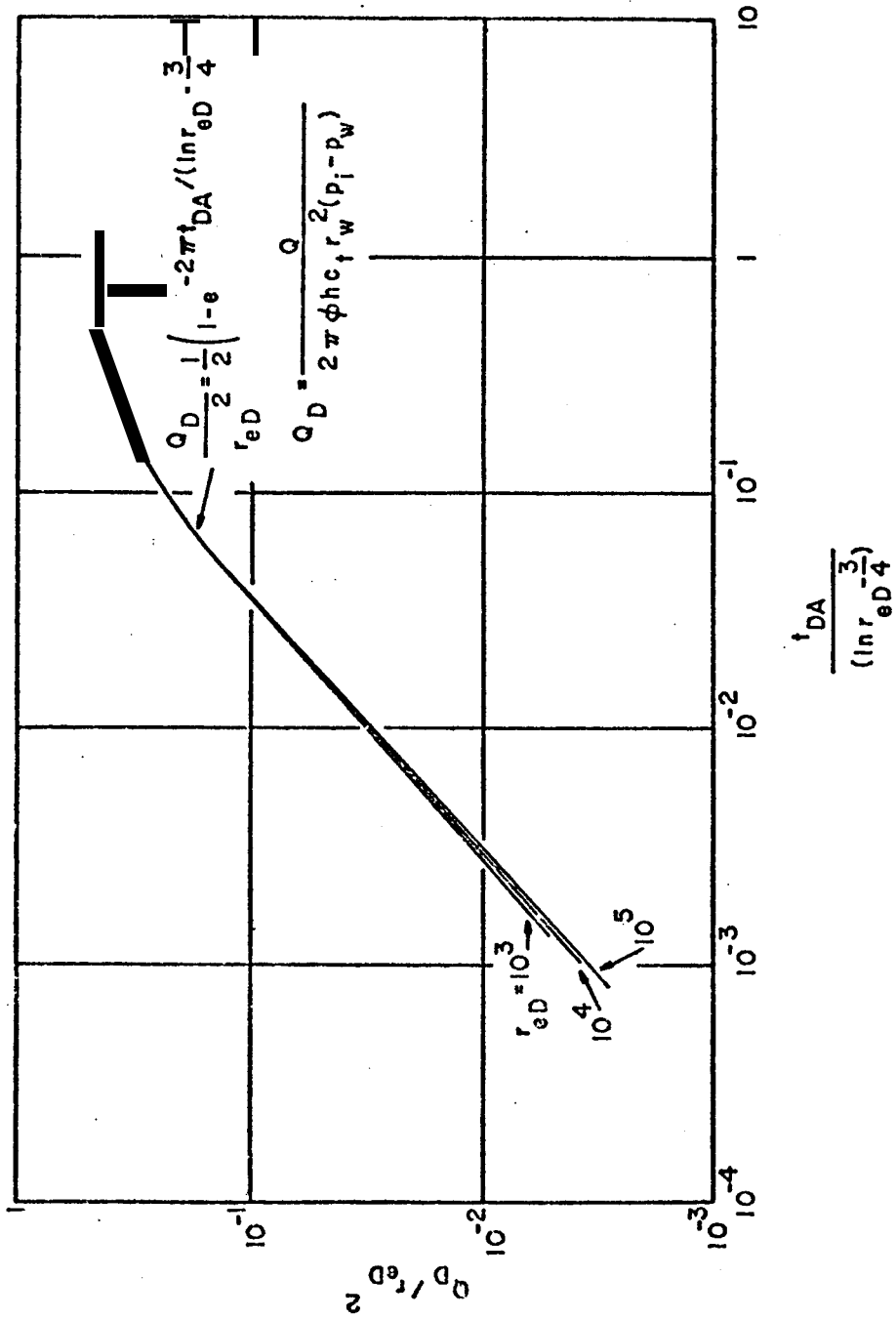


Figure 2.4: Dimensionless Cumulative Production for a Well Produced at a Constant Pressure from the Center of a Closed Bounded Circular Reservoir

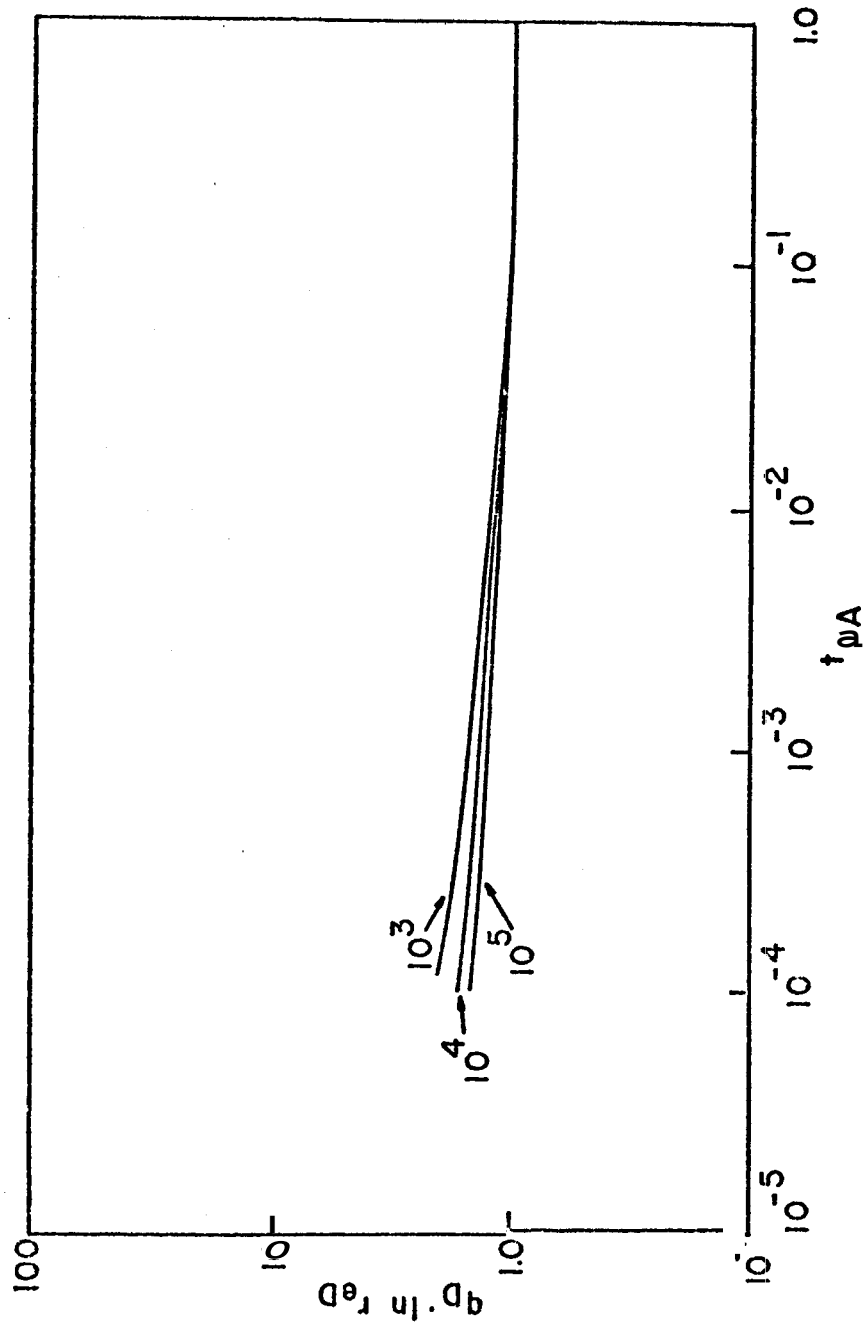


Figure 2 5: Dimensionless Flow Rate for a Well Produced at a Constant Pressure from the Center of a Constant Pressure Bounded Circular Reservoir

voir. In the next section, the theory is extended to solutions for constant pressure at the wellhead instead of at the sand face in the wellbore. The constant wellhead pressure problem is a simple extension of the constant wellbore pressure problem if the flow up the wellbore is laminar.

2.4 PRODUCTION AT CONSTANT WELLHEAD PRESSURE

Frequently reservoir fluids are produced with a constant pressure at the wellhead. Examples are production of fluids into a constant pressure separator and production of gas into a constant pressure pipeline. When the wellhead pressure is constant, the pressure drop in the wellbore due to flowing friction varies as a function of the flow rate, and hence, the wellbore sandface pressure is not constant. The solutions previously discussed are not directly valid for wells produced at constant wellhead pressure. In this section the solution for constant wellhead pressure production when the flow up the wellbore is laminar is derived. The resulting solution is a simple extension of the existing solutions.

Assuming negligible heat loss the mechanical energy balance in differential form for the flowing fluid in the wellbore is given by:

$$vdp + dH + \frac{UdU}{q_c} + dw_f = - dw_s \quad (2.52)$$

where v is specific volume, H is vertical distance, U is fluid velocity, W_f is frictional energy loss, and W_s is shaft work. Assuming in addition that the shaft work term and the kinetic energy term may be neglected, Eq. 2.52 becomes:

$$vdp = - dW_f - dH \quad (2.53)$$

The frictional energy loss is given by:

$$dW_f = \frac{4f_M U^2 dL}{2g_c D} \quad (2.54)$$

where L is the tubing length and D is the tubing diameter.

For one phase liquid flow in the tubing:

$$U = \frac{4q}{\pi D^2} \quad (2.55)$$

and the density, ρ , is approximately constant. Hence, the equation for the pressure drop in the wellbore for flowing liquid is given by:

$$p_{wf} - p_{tf} = \frac{4^2 f_M L \bar{\rho} q^2}{2g_c \pi^2 D^5} + H\bar{\rho} \quad (2.56)$$

where p_{wf} is the wellbore flowing pressure, $\bar{\rho}$ is the average density in the wellbore, and p_{tf} is the wellhead flowing pressure. For laminar flow in the wellbore, the Moody friction factor is given by:

$$4f_M = 64/N_{RE} \quad (2.57)$$

where $N_{RE} = 4q\rho/\pi\mu D$ is the Reynolds number. Thus, Eq. 2.56 becomes:

$$p_{wf} - p_{tf} = \frac{64Lq\mu}{2g_c \pi D^4} \quad (2.58)$$

The inner boundary condition is:

$$p(r_w, t) = p_{wf} + s \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} \quad (2.3)$$

Combining Eqs. 2.58 and 2.3:

$$p(r_w, t) = p_{tf} + \frac{64Lq\mu}{2g_c \pi D^4} + H\bar{\rho} + s \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} + \quad (2.59)$$

Redefine the following dimensionless groups:

$$p_D(r_D, t_D) = \frac{[p_i - p(r, t_D)]}{(p_i - p_{tf} - b)} \quad (2.60)$$

and:

$$q_D(t_D) = \frac{q(t)\mu}{2\pi kh(p_i - p_{tf} - b)} \quad (2.61)$$

where $b = H\bar{\rho}$. Finally let:

$$a = \frac{64khL}{q_c D^4} \quad (2.62)$$

Substitution of Eqs. 2.60-2.62 in 2.59 yields:

$$p(r_w, t) = p_{tf} + a(p_i - p_{tf} - b)q_D + b + s \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} + \quad (2.63)$$

Rearranging yields:

$$p_D(1, t_D) = 1 + (s+a) \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1} \quad (2.64)$$

Eq. 2.64 is exactly like Eq. 2.14, the dimensionless form of the inner boundary condition used previously. The solutions discussed in this chapter are therefore valid for constant wellhead pressure production with laminar flow in the wellbore, if the dimensionless variables are redefined as in the preceding. In particular, the transient rate response is identical except for an increase in the effective skin factor. Furthermore, substitution of typical values for the parameters in a indicates that a is typically less than 0.01; and hence, $s+a \approx s$.

In the case of fully turbulent flow in the wellbore, the friction factor depends only upon the relative roughness of the well pipe and would be a constant for a given case. In this case Eq. 2.53 applies with the friction factor constant.

$$p_{wf} - p_{tf} = a' q_D^2 (p_i - p_{tf} - b) + b \quad (2.65)$$

where:

$$a' = \frac{32 L \bar{\rho} f_M (kh)^2 (p_i - p_{tf} - b)}{\mu g_c D^5} \quad (2.66)$$

The inner boundary condition, Eq. 2.3, becomes:

$$p(r_w, t) = p_{tf} + a' q_D^2 (p_i - p_{tf} - b) + b + s \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} \quad (2.67)$$

Redefining dimensionless groups as before and rearranging yields:

$$p_D(1, t_D) = 1 + a' \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1}^2 + s \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1} \quad (2.68)$$

Although the problem could be resolved using this condition, it was beyond the objectives of this study to do so. The condition was one finding of the study and poses an interesting problem for future investigation. In the next section, the effect of wellbore storage is examined as a further extension of the constant wellhead pressure solution.

2.5 EFFECT OF WELLBORE STORAGE

A drop in the wellhead pressure, whether due to constant rate or constant pressure flow can cause fluid production from the wellbore itself independent of the formation. When the surface rate is constant, variable fluid production from the wellbore causes a variable rate at the sand face. For the constant rate case, the effect of wellbore storage is incorporated into the inner boundary condition through a material balance on the wellbore. The same procedure can be used to include wellbore storage for the case of constant pressure production. The derivation follows.

The isothermal compressibility of the wellbore fluid is defined by:

$$c_w = - \frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T \quad (2.69)$$

By the chain rule for differentiation:

$$c_w = - \frac{1}{V} \left(\frac{\partial V}{\partial t} \right)_T / \left(\frac{\partial p}{\partial t} \right)_T \quad (2.70)$$

Thus, the rate of fluid production from the wellbore volume, V_w , is:

$$q_w = \left(\frac{\partial V_w}{\partial t} \right)_T = - c_w V_w \frac{\partial p_{wf}}{\partial t} \quad (2.71)$$

V_w includes the volume of the wellbore, the annulus, and any additional volume of fluid connected with the wellbore which may be produced without changing the sand face pressure. The total surface fluid production rate, q_t , is the sum of the production rate from the wellbore volume, q_w , and the production rate from the sand face, q . Thus:

$$q_t = V_w c_w \frac{\partial p_{wf}}{\partial t} + \frac{2\pi kh}{\mu} \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} + \quad (2.72)$$

From Eq. 2.3:

$$p_{wf}(t) = p(r_w, t) - s \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} + \quad (2.3)$$

and :

$$\frac{dp_{wf}}{dt} = \left(\frac{\partial p}{\partial t} \right)_{r=r_w} + - s \frac{\partial}{\partial t} \left(r \frac{\partial p}{\partial r} \right)_{r=r_w} + \quad (2.73)$$

Defining p_D and q_D as in Section 2.4, and defining the dimensionless storage by:

$$C_D = \frac{v_w c_w}{2\pi\phi c_t h r_w^2} \quad (2.74)$$

the total dimensionless surface rate is:

$$q_{tD} = C_D \left[\left(\frac{\partial p_D}{\partial t_D} \right)_{r_D=1} + s \frac{\partial}{\partial t_D} \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1} + \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1} \right] \quad (2.75)$$

Taking the Laplace transformation of q_{tD} results in:

$$\bar{q}_{tD}(\ell) = \left[C_D \ell \bar{p}_D(1, \ell) + s \left(\frac{\partial \bar{p}_D}{\partial r_D} \right)_{r_D=1} + \frac{\partial \bar{p}_D}{\partial r_D} \right]_{r_D=1} \quad (2.76)$$

Substituting the solution for \bar{p}_D and $\left(\frac{\partial \bar{p}_D}{\partial r_D} \right)_{r=1}$ for the infinite system given in Table 2.1 with the skin factor adjusted to include wellbore friction pressure loss and rearranging results in:

$$\bar{q}_{tD}(\ell) = C_D \left[\frac{K_0(\sqrt{\ell}) + s\sqrt{\ell} K_1(\sqrt{\ell})}{K_0(\sqrt{\ell}) + (s+a)\sqrt{\ell} K_1(\sqrt{\ell})} \right] + \frac{K_1(\sqrt{\ell})}{\sqrt{\ell} [K_0(\sqrt{\ell}) + (s+a)\sqrt{\ell} K_1(\sqrt{\ell})]} \quad (2.77)$$

In the preceding section, comparison of s and $s+a$ indicated that they are approximately equal. Thus Eq. 2.77 reduces to:

$$\bar{q}_{tD}(\cdot) = c_D \frac{K_1(\sqrt{\ell})}{\sqrt{\ell} [K_0(\sqrt{\ell}) + s \sqrt{\ell} K_1(\sqrt{\ell})]} \quad (2.78)$$

This expression can be derived from the van Everdingen and Hurst (1949) equation discussed in Section 2.2 (Eq. 2.28).

The inverse transformation of the constant term in Eq. 2.78 is c_D multiplied by the Dirac delta, function, $\delta(t)$. (See Abramowitz and Stegun (1972), page 1029.) Thus, the theory implies an immediate unloading of the wellbore, and subsequent flow rates are unaffected by the wellbore storage effect. The lack of prolonged wellbore storage effects may be an advantage of constant pressure testing. However, if the initial flow rate is limited by a critical flow restriction, the wellbore storage effect may last for a longer period of time.

The final aspect of the problem of constant pressure production to be considered in this chapter is interference analysis. This topic is examined in the next section.

2.6 INTERFERENCE ANALYSIS

The well test analysis methods presented thus far in this work have concentrated on the behavior of the solutions at

the producing well. This section deals with the pressure variation in the reservoir away from the well. Interference analysis is a method for determining reservoir parameters by observing the pressure response or interference at a nearby non-producing well. For the constant rate case, Mueller and Witherspoon (1965) showed that the line source solution can be used to determine the pressure drop in the reservoir for $r_D > 25$, and that for $t_D/r_D^2 > 25$, the log approximation holds:

$$p_D(r_D, t_D) = \frac{1}{2} \left(\ln \frac{t_D}{r_D^2} + 0.80907 \right) \quad (2.79)^*$$

For zero storage, these approximations are valid even if a nonzero skin factor is present.

Interference analysis is more complicated when the production is at a constant pressure. The most obvious difficulties are shown in Fig. 2.6, a graph of p_D vs t_D/r_D^2 . The figure indicates that a different solution results for each value of r_D . Unlike the constant rate solution, the pressure distribution for constant pressure production does not correlate with the line source solution. Although the graph of p_D/q_D vs t_D/r_D^2 shown in Fig. 2.7 shows that for $t_D/r_D^2 \gtrsim 10^4$ the log approximation holds, this is not particularly useful. In order to make use of this property in well test analysis, the production rate must be known during the entire interference test. If the rate versus time data is

* In Eq. (2.79) p_D refers to the dimensionless pressure drop for constant rate production.

available, it can be analyzed directly, and the interference data does not, in general, produce additional information about the reservoir. Furthermore, for every nonzero skin factor, another family of curves results, as shown in Fig. 2.8.

Interference between flowing wells is also more complicated for constant pressure production. The method of imaging used to generate linear boundaries near a well requires superposition in time of constant rate solutions. When the rates are continuously varying, the derivation requires superposition in time and space. The method of superposition in time of continuously varying rate solutions is explained in Section 3. Hence, the topic of interference between flowing wells is revisited in Section 3.4.3.

This concludes the discussion of transient rate analysis for wells produced at constant pressure. In the next section pressure buildup solutions for wells produced at constant pressure are derived.

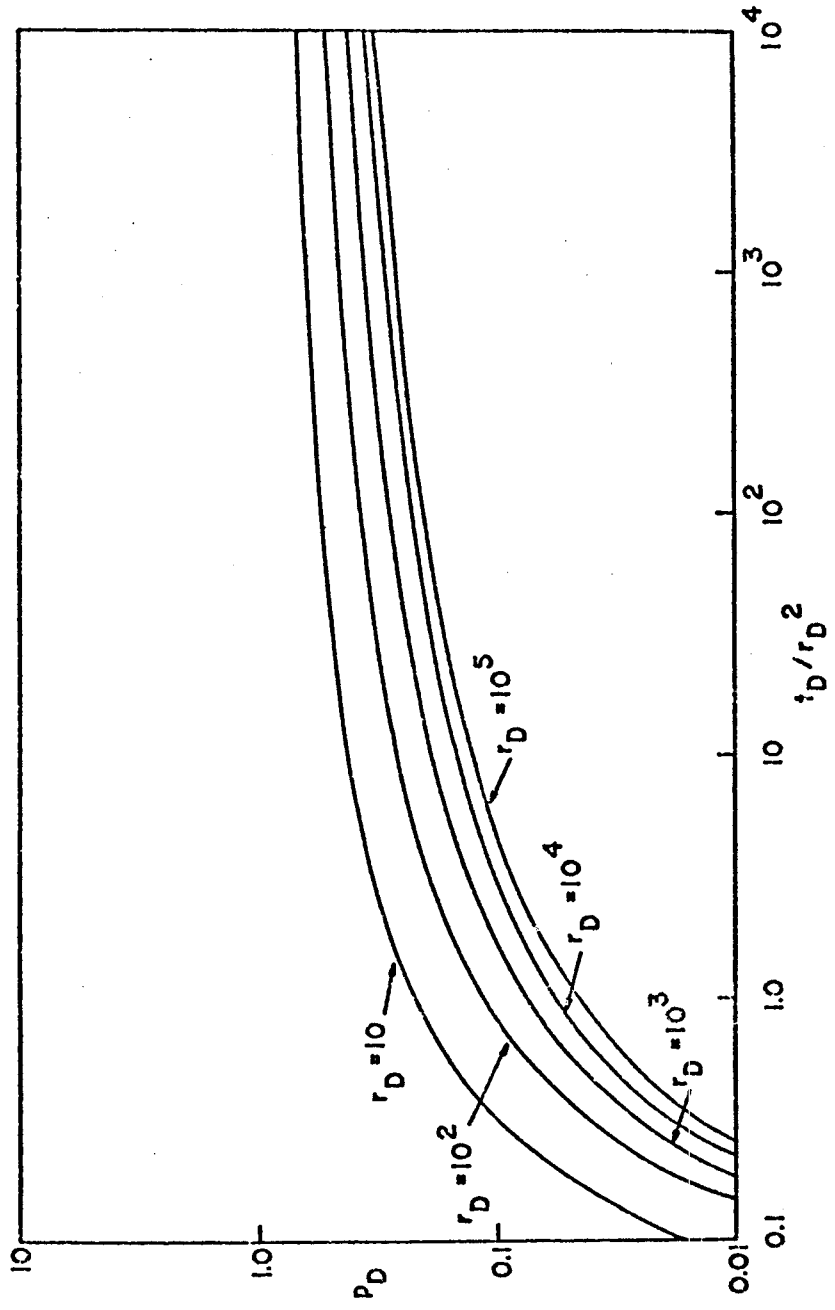


Figure 2.6: Dimensionless Pressure for Various Observation Well Locations for a Well Produced at Constant Pressure from an Unbounded Reservoir with Zero Skin

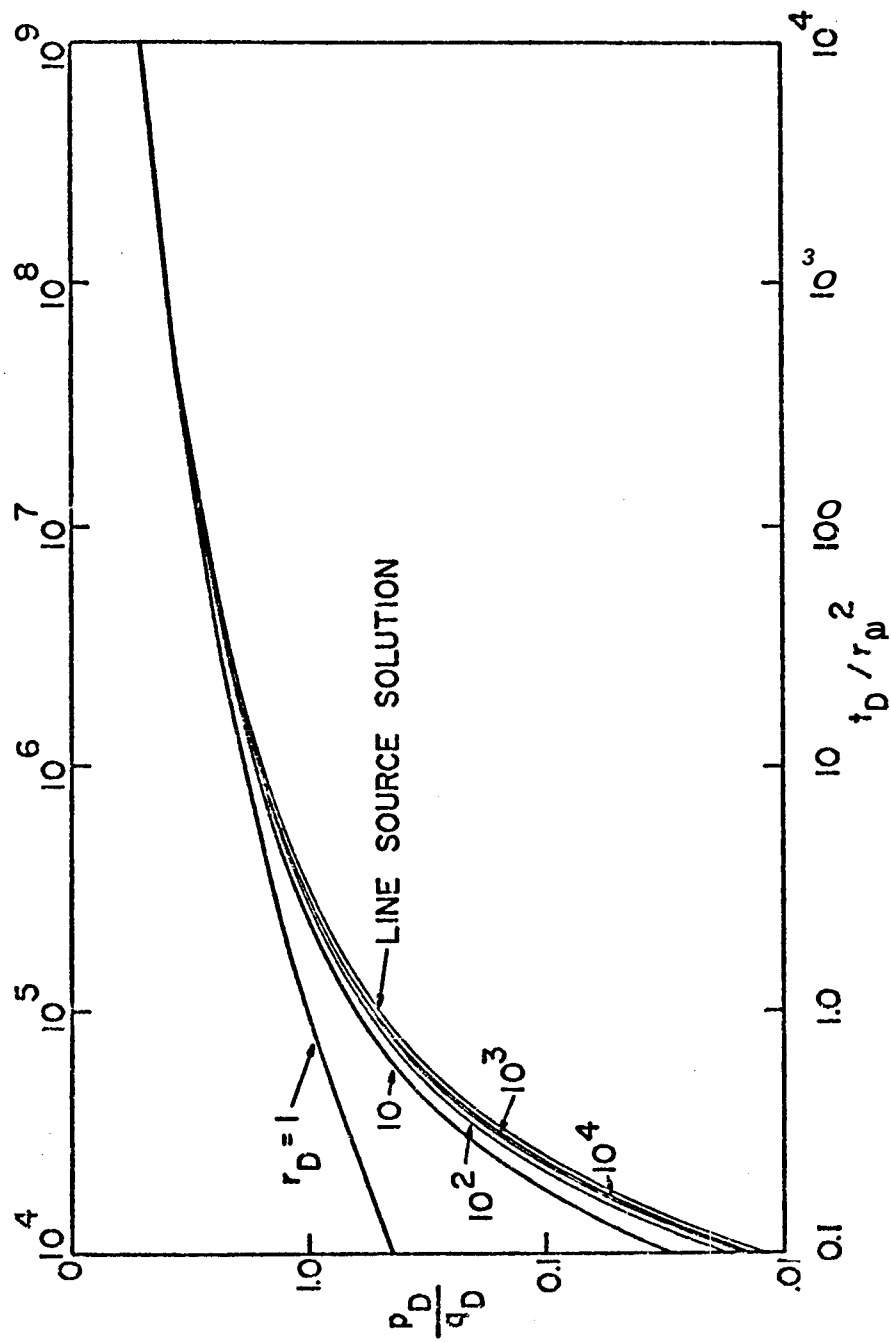


Figure 2.7: Normalized Graph of Dimensionless Pressure for Various Observation Well Locations for a Well Produced at a Constant Pressure from an Unbounded Reservoir with Zero Skin

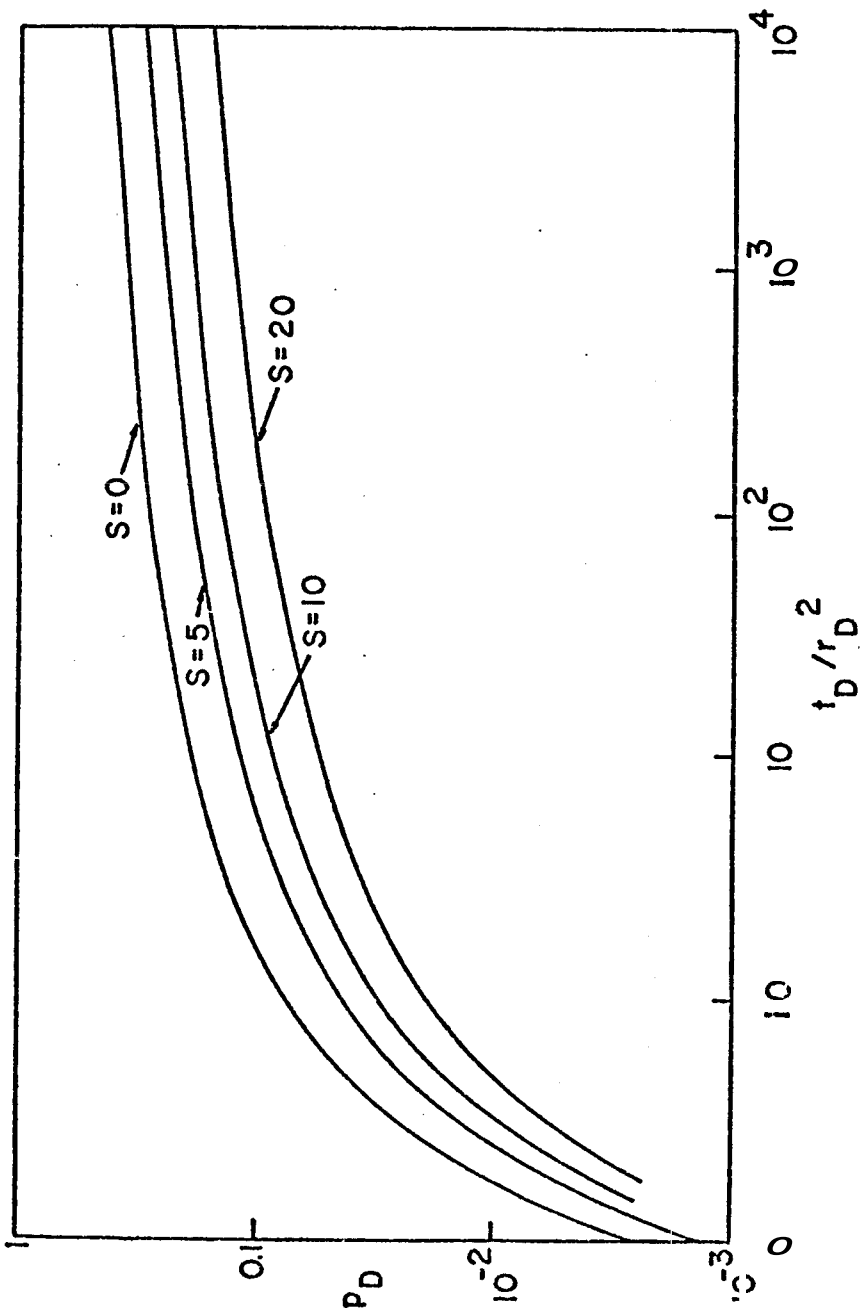


Figure 2 8: Dimensionless Pressure for an Observation Well Located at $r_w = 10^3$ for a Well Produced at a Constant Pressure from an Unbounded Reservoir with Various Nonzero Skin Values

SECTION 3

PRESSURE BUILDUP AFTER CONSTANT PRESSURE PRODUCTION

In Section 2 the transient rate response and pressure distributions for constant pressure production were discussed. Methods analogous to pressure drawdown analysis for constant rate well tests were provided. In this section, pressure buildup following constant pressure production is examined. Pressure buildup after constant rate production is a simpler problem to handle analytically, but through use of superposition in time of constant rate solutions, an integral expression for the pressure buildup after constant pressure production can be written. This method is explained in the Section 3.1. Section 3.2 reveals the solution for pressure buildup and how to apply conventional methods of pressure buildup analysis to wells produced at constant pressure. Methods are discussed for determination of wellbore storage and skin effect by type curve matching, Horner buildup analysis, and determination of average reservoir pressure. Section 3.3 discusses the practical limitations of the theory. Finally, three additional applications of the method of superposition in time of constant rate solutions are discussed in Section 3.4.

3.1 THEORETICAL EXPRESSION FOR PRESSURE BUILDUP

For a finite number of changes in production rate with each rate constant over a finite period in time, the pressure at the wellbore is given by

$$p_{wf}(t) = p_i - \frac{\mu}{2\pi kh} [q_0 p_{wD}(t) + (q_1 - q_0) p_{wD}(t - t_1) + \dots + (q_N - q_{N-1}) p_{wD}(t - t_N)] \quad (3.1)$$

where p_{wD} is the dimensionless pressure drop at the wellbore for unit constant rate production. This equation can be rewritten as the following:

$$p_{wf}(t) = p_i - \frac{\mu}{2\pi kh} \left\{ q_0 [p_{wD}(t) - p_{wD}(t - t_1)] + q_1 [p_{wD}(t - t_1) - p_{wD}(t - t_2)] + \dots + q_N [p_{wD}(t - t_{N-1}) - p_{wD}(t - t_N)] + q_N p_{wD}(t - t_N) \right\} \quad (3.2)$$

From Eq. 3.2 it is easily seen that for a continuously changing rate, $q(t)$,

$$p_{wf}(t) = p_i + \frac{\mu}{2\pi kh} \int_0^t q(\tau) p_{wD}'(t - \tau) d\tau \quad (3.3)$$

where the prime indicates the derivative with respect to time. If production is at constant pressure p_{wf} , Eq. 3.3 becomes:

$$1 = - \int_0^{t_D} q_D(\tau) p_{wD}'(t_D - \tau) d\tau \quad (3.4)$$

where q_D is the dimensionless flowrate defined by Eq. 2.11 in the preceding section, and t_D is dimensionless time. Referring again to Eq. 3.3, if production at constant pressure is changed to constant rate production after time t_p , the wellbore pressure at time t is given by:

$$p_{wf}(t) = p_i + \frac{\mu}{2\pi kh} \int_0^{\epsilon_p} q(\tau) p_{wD}'(t - \tau) d\tau - q_D(t_D) p_{wD}(t - t_p) \quad (3.5)$$

If the well is shut in, pressure buildup is exactly determined from:

$$p_{wf}(\Delta t) = p_i + \frac{\mu}{2\pi kh} \int_0^{\epsilon_p} q(\tau) p_{wD}'(t - \tau) d\tau \quad (3.6)$$

where Δt is the elapsed time after shut-in. The integral in Eq. 3.6 is difficult to evaluate because $q_D(0)$ is infinite. However, the equation can be written in a more easily evaluated form by using Eq. 3.4:

$$\frac{p_i - p_{ws}(\Delta t_D)}{p_i - p_{wf}} = - \int_0^{t_{pD} + \Delta t_D} q_D(\tau) p_{wD}'(t_{pD} + \Delta t_D - \tau) d\tau + \int_{t_{pD}}^{t_{pD} + \Delta t_D} q_D(\tau) p_{wD}'(t_{pD} + \Delta t_D - \tau) d\tau \quad (3.7)$$

or:

$$\frac{p_i - p_{ws}(\Delta t_D)}{p_i - p_{wf}} = 1 + \int_{t_{pD}}^{t_{pD} + \Delta t_D} q_D(\tau) p_{wD}'(t_{pD} + \Delta t_D - \tau) d\tau \quad (3.8)$$

Eq. 3.8 is general. The functions to be used for q_D and p_D can be chosen for any set of inner and outer boundary condi-

tions. Examination of the integration limits reveals that q_D is evaluated for late times ($t > t_p$) and p_{wD}' is evaluated beginning with time zero. Thus, phenomena such as wellbore storage, skin effect, or a fracture penetrated by the wellbore, should be included in the pressure function, while boundary effects will affect the rate function and, later in shut-in time, the pressure function as well.

Although the integral in Eq. 3.8 is similar to a convolution integral, it cannot be solved easily by Laplace transformation. However, Eq. 3.8 can be integrated numerically. Numerical evaluation of the integral is discussed in Appendix C.

3.2 ANALYSIS OF PRESSURE BUILDUP

The problem of pressure buildup after constant pressure production has received only limited attention in the literature. Methods of analysis have been suggested in both the petroleum and the groundwater literature,, but theoretical justification of the methods is almost nonexistent. Evaluation of the expression for pressure buildup given by Eq. 3.8 provides an exact solution which is used to determine methods of analysis which are theoretically valid.

Three periods of shut-in time are discussed: the early shut-in period, when wellbore effects dominate, the period when Horner buildup analysis applies, and the late time when outer boundary effects are evident.

3.2.1 Early Shut-in Time

For small shut-in periods, the rate function $q_D(\tau)$ is essentially constant for $t_{pD} < \tau < t_{pD} + \Delta t_D$. Hence, examination of Eq. 3.8 reveals that pressure recovery can be approximated accurately by:

$$\frac{p_i - p_{ws}(\Delta t)}{p_i - p_{wf}} \approx 1 - q_D(t_{pD}) p_{wD}(\Delta t_D) \quad (3.9)$$

whenever $q_D(t_{pD}) \approx q_D(t_{pD} + \Delta t_D)$. Dividing by $q_D(t_{pD})$ and rearranging results in:

$$\frac{p_{ws}(\Delta t) - p_{wf}}{q(t_p) \mu / 2\pi kh} = p_{wD}(\Delta t) \quad (3.10)$$

Thus, a log-log graph of $p_{ws}(\Delta t) - p_{wf}$ vs time can be compared to type curves of pressure drawdown for constant flow rate production. Effects of early transient behavior such as wellbore storage and skin effects, partial penetration, or the evidence of a fracture, can be analyzed using conventional type curve matching techniques.

3.2.2 Horner Buildup Analysis

According to the method by Horner (1951), buildup pressures may be graphed vs $\log \{(t + \Delta t) / \Delta t\}$ in order to produce a semilog straight line. The slope of the line is used to determine permeability from the equation:

$$k = \frac{q\mu \cdot \ln 10}{4\pi mh} \quad (3.11)$$

Horner suggested that for variable rate production prior to shut-in, the permeability should be calculated using Eq. 3.11 with q equal to the last established flow rate, q_f , and m determined from the slope of a graph of $p_{ws}(\Delta t)$ vs $\log [(t_p + \Delta t)/\Delta t]$, where $t_p^* = Q(t_p)/q(t_p)$. Jacob and Lohman (1952) graphed $p_{ws}(\Delta t)$ vs $\log [(t_p + \Delta t)/\Delta t]$ and calculated permeability from Eq. 3.17 with q equal to the average flow rate, instead of the last flow rate.

In the present work, several cases involving pressure buildup after constant pressure production for infinite, closed bounded, and constant-pressure bounded circular reservoirs were computed by numerical integration of Eq. 3.8. In every case, if there was a period of time when the pressure buildup was not dominated by boundary effects, the semilog straight line was present, and the slope produced the correct value for the permeability when the data were graphed according to Horner's method.

The following derivation shows that the Horner method of graphing buildup data will always result in the correct straight line, provided that early transient effects and late boundary effects are separated in time. Referring again to Eq. 3.9, we divide by $q_D(t_{pD} + \Delta t_D)$:

$$\frac{p_i - p_{wf}(\Delta t_D)}{(p_i - p_{wf})q_D(t_{pD} + \Delta t_D)} = \frac{1}{q_D(t_{pD} + \Delta t_D)} - p_{wD}(\Delta t_D) \quad (3.12)$$

When $10^4 \lesssim t_p \lesssim t_{pss}$, this can be written as:

$$\frac{p_i - p_{ws}(\Delta t_D)}{(p_i - p_{wf})q_D(t_{pD} + \Delta t_D)} \cong \frac{1}{2} [\ln(t_{pD} + \Delta t_D) + 0.80907] - p_{wD}(\Delta t_D) \quad (3.13)$$

For $At_D \geq 5$, the log approximation is valid for p_{wD} , and:

$$\frac{p_i - p_{wf}(\Delta t_D)}{(p_i - p_{wf})q_D(t_{pD} + \Delta t_D)} \cong \frac{1}{2} \ln[(t_{pD} + \Delta t_D)/\Delta t_D] \quad (3.14)$$

or:

$$p_{ws}(\Delta t) \sim p_i - \frac{q(t + \Delta t)}{4\pi kh} \ln[(t_p + \Delta t)/\Delta t] \quad (3.15)$$

Noting that $q(t_p) = q(t_p + \Delta t)$ for $At \ll t_p$, this expression is identical to the result for constant rate flow, except that if $q(t_p)$ were constant, t would be equal to the Horner corrected flow time, t_p^* . Hence, to produce the correct slope, t_p^* must be used.

At infinite shut-in time, the extrapolated pressure for Eq. 3.15 is p_i . Thus, the behavior of the Horner pressure buildup curve following constant pressure production that has not shown a boundary influence is identical to the constant rate case.

The Jacob and Lohman (1952) method of using the average rate prior to shut-in is justified by the following arguments. If the variation in q_D is small for $0 < t_D < t_{pD}$, then Eq. 3.6 may be approximated by the following:

For $At_D \geq 5$, the log approximation is valid for p_{wD} , and:

$$\frac{p_i - p_{ws}(\Delta t_D)}{p_i - p_{wf}} \approx \bar{q}_D(t_{pD}) \frac{1}{2} \ln[(t_{pD} + \Delta t_D)/\Delta t_D] \quad (3.16)$$

or:

$$p_{ws}(\Delta t) = p_i - \frac{\bar{q}_D}{4\pi h} \ln[(t_p + \Delta t)/\Delta t] \quad (3.17)$$

The last expression is identical to the result for constant rate flow except that \bar{q} is computed from $q(t_p)/t_p$. This method is equivalent to the Horner method as long as $t < t_{pss}$. Once exponential decline has begun, the approximation in Eq. 3.22 is no longer valid.

In the next section, boundary effects are considered. The Horner method is shown to be an effective means of analysis, even when boundary effects are evident prior to shut-in.

3.2.3 Outer Boundary Effects

When $t_p > t_{pss}$, the Horner method still produces a semi-log straight line for At sufficiently small, because $q_D(t_{pD})$ may be assumed to be constant. However, unlike in Eq. 3.15, the extrapolated pressure is not p_i , but, to use the conventional notation, p^* .

*

The equation for p is derived as follows. For the closed bounded reservoir, early enough in shut-in time that $t_p \gg \Delta t$ but late enough that $\frac{r_e^2}{k} > 100$:

$$\frac{p_i - p_{ws}(\Delta t_D)}{(p_i - p_{wf})q_D(t_{pD})} \cong \frac{1}{q_D(t_{pD})} - \frac{1}{2} (\ln \Delta t_D + 0.80907) \quad (3.18)$$

Adding and subtracting $\frac{1}{2} [\ln(t_{pD}^* + \Delta t_D) + 0.80907]$,

$$\begin{aligned} \frac{p_i - p_{ws}(\Delta t_D)}{(p_i - p_{wf})q_D(t_{pD})} &\cong \frac{1}{q_D(t_{pD})} - \frac{1}{2} [\ln(t_{pD}^* + \Delta t_D) + 0.80907] \\ &\quad + \frac{1}{2} \ln[(t_{pD}^* + \Delta t_D)/\Delta t_D] \\ &\cong \frac{1}{q_D(t_{pD})} = \frac{1}{2} (\ln t_{pD}^* + 0.80907) \\ &\quad + \frac{1}{2} \ln[(t_{pD}^* + \Delta t_D)/\Delta t_D] \end{aligned} \quad (3.19)$$

Rearranging:

$$p_{ws}(\Delta t) = p^* - \frac{q(t_p)\mu}{4\pi kh} \ln[(t_p^* + \Delta t)/\Delta t] \quad (3.20)$$

where:

$$\frac{p_i - p}{(p_i - p_{wf})q_D(t_{pD})} = \frac{1}{q_D(t_{pD})} - \frac{1}{2} (\ln t_{pD}^* + 0.80907) \quad (3.21)$$

Eq. 3.21 can be used to determine static pressure correction curves analogous to those derived by Matthews, Brons, and Hazebroek (1954) for pressure buildup after constant rate production. Referring to the definition of $q_D(t_D)$ in

Eq. 2.26, the average reservoir pressure at shut-in for a circular reservoir is given by:

$$\frac{p_i - \bar{p}(t_{pD})}{p_i - p_{wf}} = \frac{Q(t_{pD})\pi\phi c_t h r_e^2}{p_i - p_{wf}} = 2Q_D(t_{pD})/r_{eD}^2 \quad (3.22)$$

Hence, the departure of the extrapolated pressure, p^* , from the actual average reservoir pressure, \bar{p} , is given by:

$$\frac{2\pi kh(p^* - \bar{p})}{q_+} = \frac{2Q_D(t_{pD})}{q_D(t_{pD})r_{eD}^2} + \frac{1}{2} (\ln t_{pD} + 0.80907) \quad (3.23)$$

Substituting the exponential decline functions for Q_D and q_D , and recalling that $t_{pDA}^* = Q_D(t_{pD})/q(t_{pD})$, results in:

$$\frac{4\pi kh(p^* - \bar{p})}{q_{tP}\mu} = [\ln t_{pDA}^* + 3.45381] \quad (3.24)$$

This result is identical to the equation for the Mathews, Brons, and Hazebroek curves for determining the average pressure in a closed bounded circular reservoir produced at a constant rate for $t_{DA} > 0.1$. Fig. 3.1 is a graph of $4\pi kh(p^* - \bar{p})/q_{tP}\mu$ vs t_{pDA}^* .

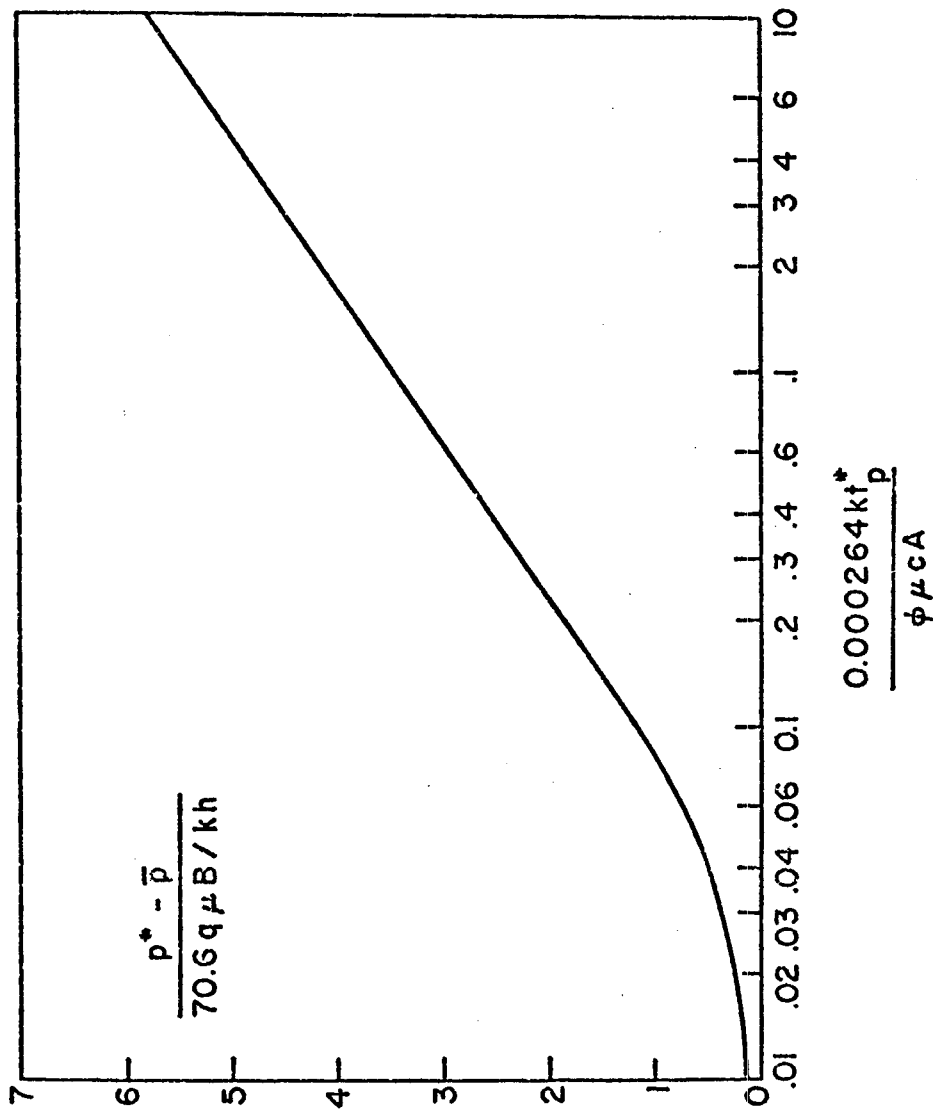


Figure 3.1: Graph of Mattheus-Brons-Hazebroek Static Pressure for a Well Produced at a Constant Pressure from a Closed, Bounded Circular Reservoir

3.3 PRACTICAL LIMITATIONS OF THE THEORY

In general, pressure buildup for wells produced at constant pressure can be analyzed as effectively as pressure buildup for wells produced at constant rate. Hence, specific limitations in the theory to be discussed in this section affect pressure buildup analysis after both constant pressure and constant rate flow. Nonetheless, to alert the reader to possible pitfalls in the analysis, three problems are discussed: a short flow time before shut-in, wellbore effects, and outer boundary effects. To avoid errors in the analysis of pressure buildup, the engineer needs to be aware of the approximate ranges of time for which the various methods apply.

Limitations in the application of the Horner method with adjusted flow time have been discussed by previous investigators including Clegg (19671, Odeh and Selig (19631, and Sandrea (1971). The reasons for differences between their conclusions and the results herein are considered in Section 3.4.4.

3.3.1 Short Flow Time Before Shut-in

If the production time before shut-in is very short, the wellbore pressure may return essentially to the initial reservoir pressure before the semi-log straight line develops. Such cases are shown in Fig. 3.2. For each of the three

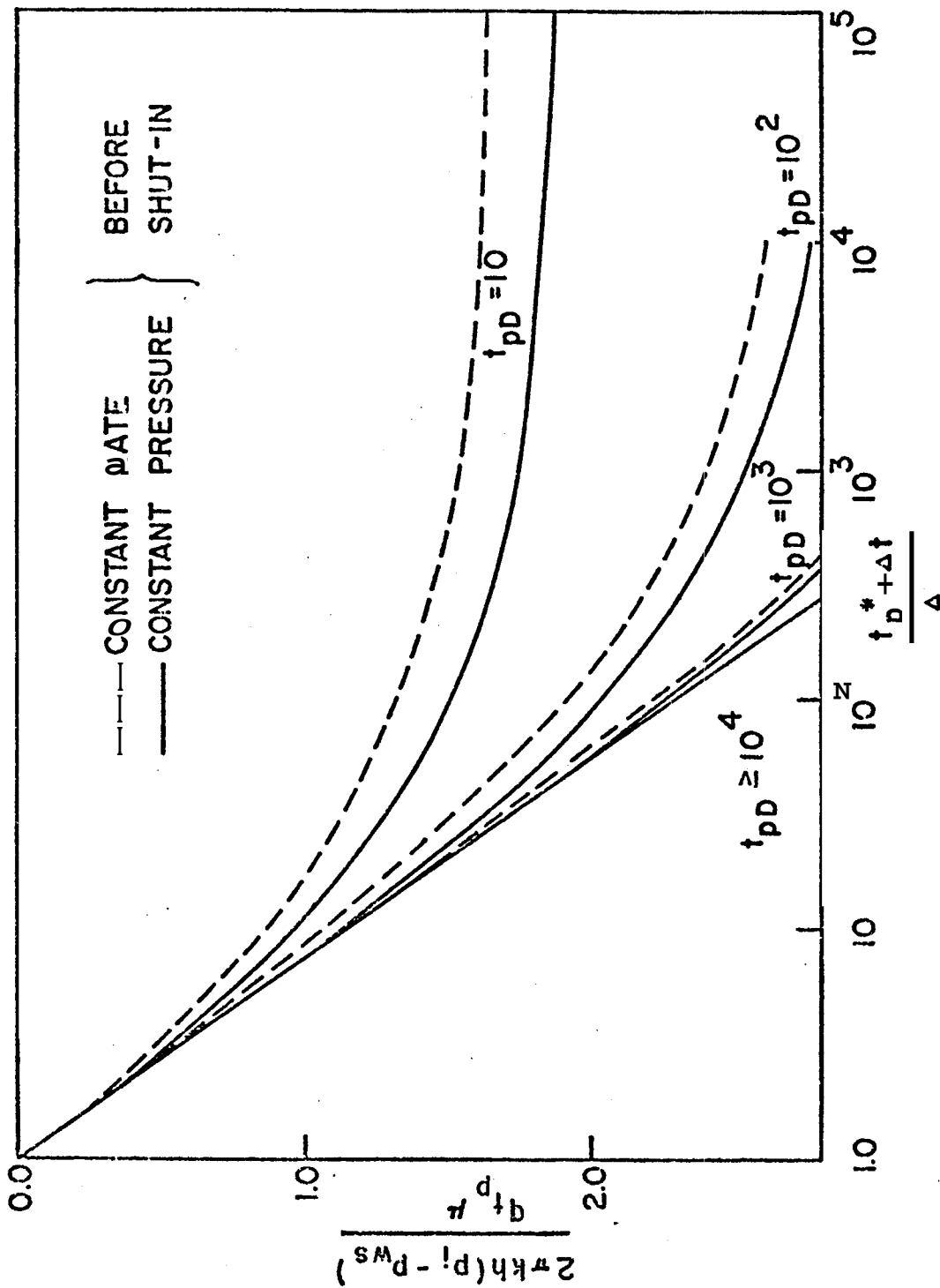


Figure 3.2: Examples of Horner Buildup Graphs for Very Short Flow Times for a Well Produced at a Constant Rate or a Constant Pressure Before Shut-In

flow times indicated, the Horner buildup graph failed to develop a semi-log straight line. However, as the dashed lines indicate, the problem **also** exists for each of these times for wells produced at constant rate. If the dimensionless flow time exceeds 10^4 , the correct semi-log straight line will develop for wells produced at constant rate or at constant pressure, provided that the semi-log straight line portion is not masked by wellbore and/or outer boundary effects.

3.3.2 Wellbore Effects

Earlougher (1977) showed schematically the effects on pressure buildup data of inner boundary effects such as wellbore storage, skin effect, and fracture effects for wells produced at constant rate. The same curves apply for wells produced at constant pressure, as long as $\Delta t \ll t_p$ for the duration of the effect. Such effects can greatly prolong the length of shut-in time required for the correct semi-log straight line to develop. For example, Chen and Brigham (1974) demonstrated that wellbore storage effects do not vanish until $\Delta t_D > 50C_D e^{.14s}$; and Earlougher estimated that the semilog straight line begins for $\Delta t_D > (60+3.5s)C_D$. Similarly, Earlougher indicated that effects of a fracture exist until the shut-in time exceeds a dimensionless fracture time, t_{xf_D} , of 3 for the infinite conductivity case, and 2 for the uniform flux case. Inner boundary effects should

be analyzed by type curve matching in accordance with Eq. 3.16.

3.3.3 Outer Boundary Effects

As mentioned in Section 3.3, if exponential rate decline, or constant rate production develops during the flow period, then the buildup curve will show the effects of an outer boundary, if the shut-in time is long enough. If there is a period of time between the end of the inner boundary effects and the start of the outer boundary effects, however, the correct semi-log straight line will develop, no matter how long the rate may have been declining exponentially, if $r_D > 10^3$. Care must be taken to choose the semi-log straight line from the correct portion of the buildup graph.

3.3.4 Comparison with Previous Studies

Results of this study indicate that the correct semi-log straight line will develop during the course of the pressure buildup after constant pressure production, provided that inner and outer boundary effects are separated in time. This conclusion is not in agreement with certain previous studies. In this section, we will attempt to explain the different results.

One such study was published by Clegg (1967). In his analytical solution, an approximation of the pressure dis-

tribution at the time of shut-in was used as an initial condition in the solution of Eq. 2.1. The inner boundary condition was specified as a zero flow rate; and the outer boundary was assumed to be infinite. The error in the initial condition is shown in Fig. 3.3. This error explains the qualitative differences between the Clegg approximate solution for pressure buildup and the solution herein.

Other pertinent studies are those by Odeh and Selig (1963), and Sandra (1971). These investigators concluded that the correct semi-log straight line would not develop when shut-in follows an exponentially declining production rate, particularly when the reservoir has undergone considerable depletion. Sandra attributed differences between the results of Horner and Odeh and Selig for new wells to the method used by Odeh and Selig to discretize and interpret the data. For old wells, Sandra concluded that the reservoir permeability would be underestimated and the static pressure overestimated by the Horner method. However, Sandra's model assumes exponential decline from the beginning of production with a finite initial rate. For a large reservoir radius, there is a long period of rate decline before the exponential decline period. Hence, the behavior of the old wells discussed by Sandra is not directly comparable with the results of the present study.

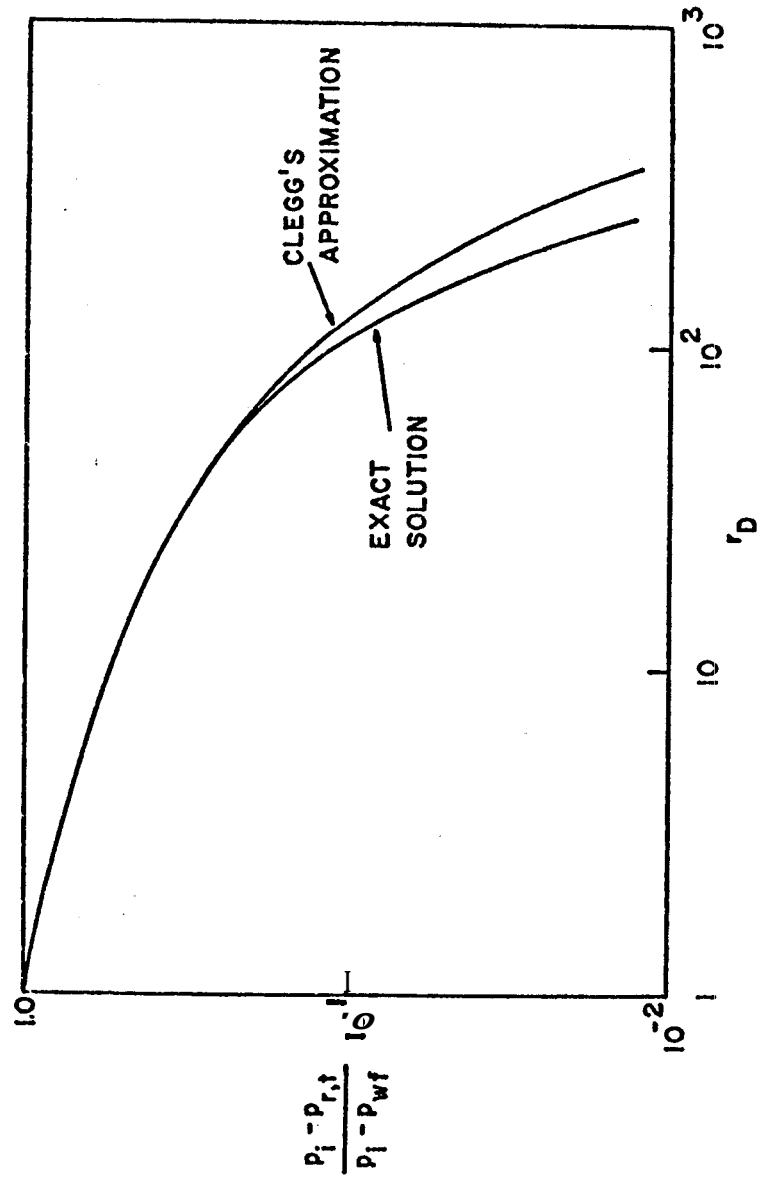


FIGURE 8: Graph of Clegg's Approximation and the Exact Solution for the Pressure Distribution for a Well Produced at a Constant Pressure from an Unbounded Circular Reservoir

3.4 FURTHER APPLICATIONS OF THE SOLUTION-TECHNIQUE

In this section the method of superposition in time is applied to three problems other than pressure buildup.

3.4.1 The Critical Flow Phenomenon

In Section 2.3.1 the possibility of a critical flow limited initial rate in the transient solution for wells produced at constant pressure **was** discussed. Using the superposition in time of solutions for step changes in the wellbore pressure, the rate as a function of time is given by:

$$q(t) = \frac{2\pi kh}{\mu} [(p_i - p_{wf_1})q_D(t) + (p_{wf_1} - p_{wf_2})q_D(t - t_1) + \dots + (p_{wf_{N-1}} - p_{wf_N})q_D(t - t_N)] \quad (3.25)$$

For a continuously changing pressure:

$$q(t) = \frac{-2\pi kh}{\mu} \int_0^t q_D(t - \tau) \frac{dp_{wf}}{d\tau}(\tau) d\tau \quad (3.26)$$

or:

$$\frac{q(t)}{q_i} = \int_0^t q_D(t - \tau) p_{wD}'(\tau) d\tau \quad (3.27)$$

If the initial rate is constant at q_c until the wellbore pressure reaches the pressure p_{wf} , then the rate as a function of the time after the onset of constant-pressure production is given by:

$$q(\Delta t) = \frac{-2\pi kh}{\mu} \int_0^{t_c} q_D(t - \tau) \frac{dp_{wf}}{d\tau}(\tau) d\tau \quad (3.28)$$

where t_c is the time elapsed during the constant rate production. Since q , and p_{wf} are specified conditions, the quantity $\frac{2\pi kh}{q_c \mu} (p_i - p_{wf}) = p_{wfD}$ is specified, although the value of p_{wf} is not, in general, known. The value of t_c is the time when $p_D = p_{wfD}$ determined from the p_D solution for constant rate production. If nonzero storage and/or skin are present, this will affect the value for t_c .

The initial value for $q(t)$ where $0 < t < t_c$ is given by $1/p_{wfD}$. When t is sufficiently large, $q(t - t_c) \approx q(t)$, and the following approximation holds:

$$q(\Delta t) = q_D(t) \cdot \frac{2\pi kh}{\mu} (p_i - p_{wf}) \quad (3.29)$$

or:

$$\begin{aligned} q_D(\Delta t_D) &= \frac{q(\Delta t_D)}{2\pi kh(p_i - p_{wf})} \\ &= q_D(t_D) \end{aligned} \quad (3.30)$$

Thus, the effect of the initial constant rate on the solution dies out in time, and hence, the analysis techniques already discussed become valid. Approximate solutions can be determined from Eq. 3.30 for specified p_{wf} , C_D , and s . Some solutions are graphed in Fig. 3.4.

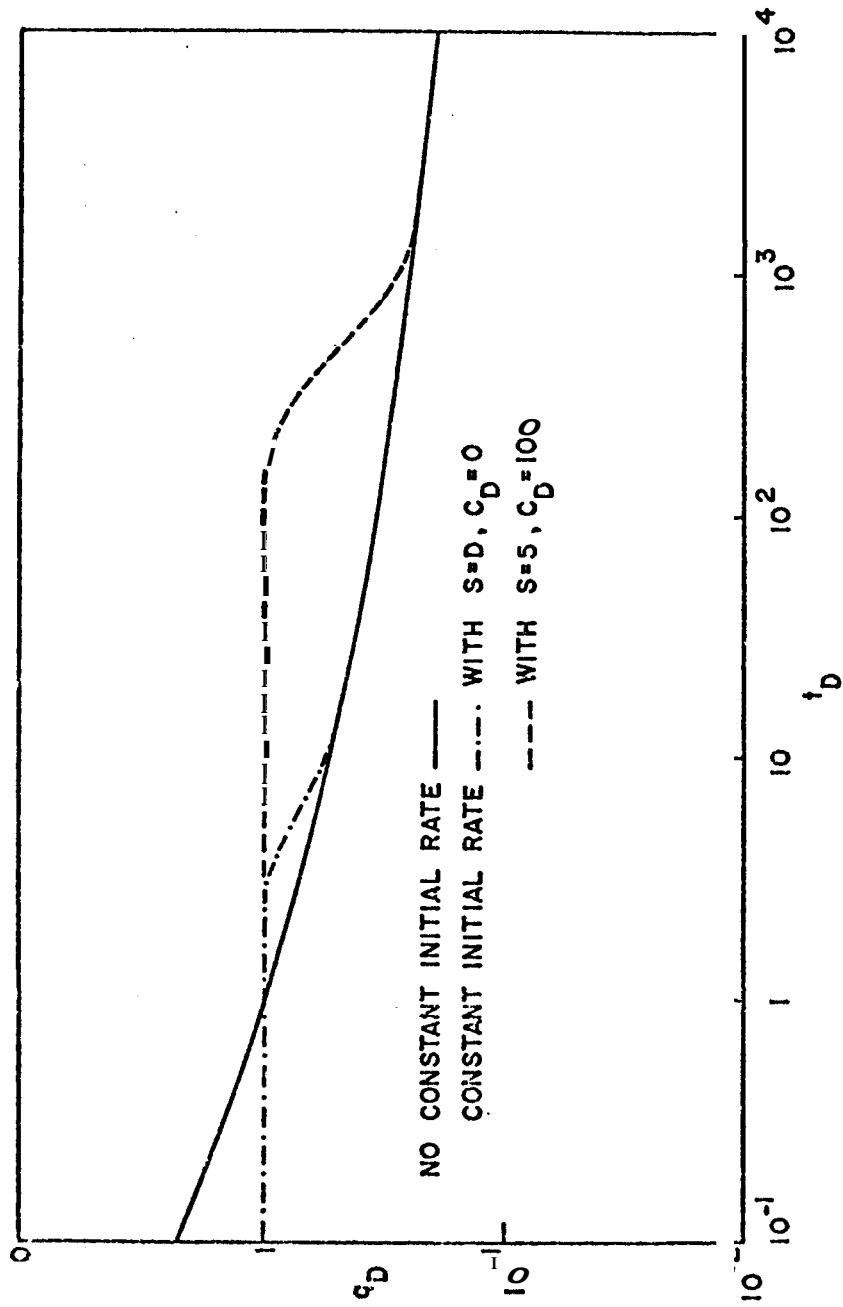


Figure 3.4: Approximate Transient Rate Solutions for Critical Flow Rate-Limited Production

3.4.2 Exponential Decline After Constant Rate Production

Often a well is produced at a constant rate until the reservoir has been nearly depleted, and the constant rate can no longer be maintained economically. Then the well is produced at the final pressure until it declines to some minimum allowable value. Assuming that the pressure decline has reached pseudo-steady state when constant pressure production begins, the expression for p_{wD} is given by Eq. 2.38. Hence:

$$\frac{dp_{wD}}{dt_D} = -2\pi r_w^2 / A \quad (3.31)$$

Referring to Eqs. 3.37 and 3.38:

$$\begin{aligned} \frac{q(\Delta t)}{q_c} &= \int_0^{t_c} q_D(t - \tau) p_{wD}'(\tau) d\tau \\ &= 1 - \int_{t_c}^t q_D(t - \tau) p_{wD}'(\tau) d\tau \end{aligned} \quad (3.32)$$

Substitution of Eq. 3.31 and the definition of q into the integral results in:

$$\begin{aligned} \frac{q(\Delta t)}{q_c} &= 1 - 2\pi r_w^2 / A \cdot \frac{\mu}{2\pi kh(p_i - p_{wf})} \cdot \frac{k}{\phi \mu C_t r_w^2} \\ &\quad \cdot \int_{t_c}^t q(t - \tau) d\tau \\ &= 1 - 2\pi \frac{r_w^2}{A} \cdot \frac{1}{2\pi \phi c_t h r_w^2 (p_i - p_{wf})} \int_0^{t-t_c} q(\tau) d\tau \end{aligned} \quad (3.33)$$

or:

$$\frac{q(\Delta t)}{q_c} = 1 - 2\pi Q_D (t - t_c) r_w^2 / A \quad (3.34)$$

For $(t-t_c)_{DA} > 0.1$, Eq. 2.50 may be substituted for $Q_D (t-t_c)$:

$$\begin{aligned} \frac{q}{q_c} &= 1 - \frac{2\pi r_w^2}{\pi r_e^2} \left[r_{eD}^2 (1 - e^{-2\pi(t-t_c)_{DA}/(\ln r_{eD}^{-3/4})}) \right] \\ &= e^{-2\pi(t-t_c)_{DA}/(\ln r_{eD}^{-3/4})} \end{aligned} \quad (3.35)$$

As noted before $q_c = 1/p_{wf}$ where p_{wf} is the final production pressure. Examination of Eq. 2.42 indicates that unlike the case in the last section in which the rate decline for a constant finite initial flow rate eventually matches the decline for constant pressure production for all time, in this case the rates are different for all time.

An example of two rate histories is shown in Fig 3.5. For a closed bounded circular reservoir of dimensionless radius $r_{eD} = 10^5$, curve A represents the production rates at a constant pressure p_{wf} for the entire production time. Curve B represents the production rates for constant-rate production, at $(q_c)_D = .025$, until the pressure in the wellbore declines to p_{wf} , and constant pressure production thereafter. Fig. 3.6 shows the cumulative production for the two rate histories. For this example, the skin factor was taken to be zero. Figures 3.7 and 3.8 show results for a positive

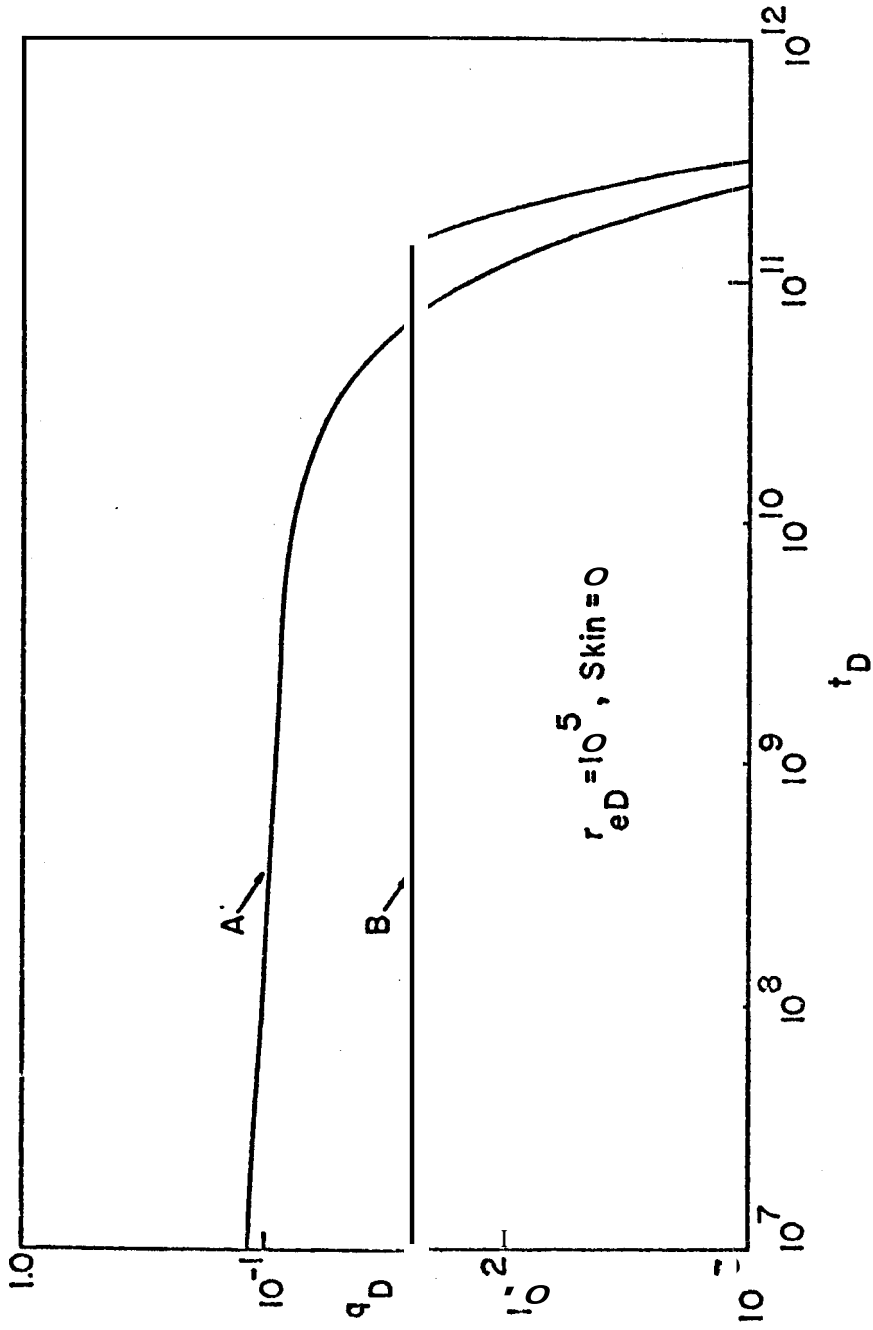


Figure 3.5: Two Rate Histories with Zero Skin

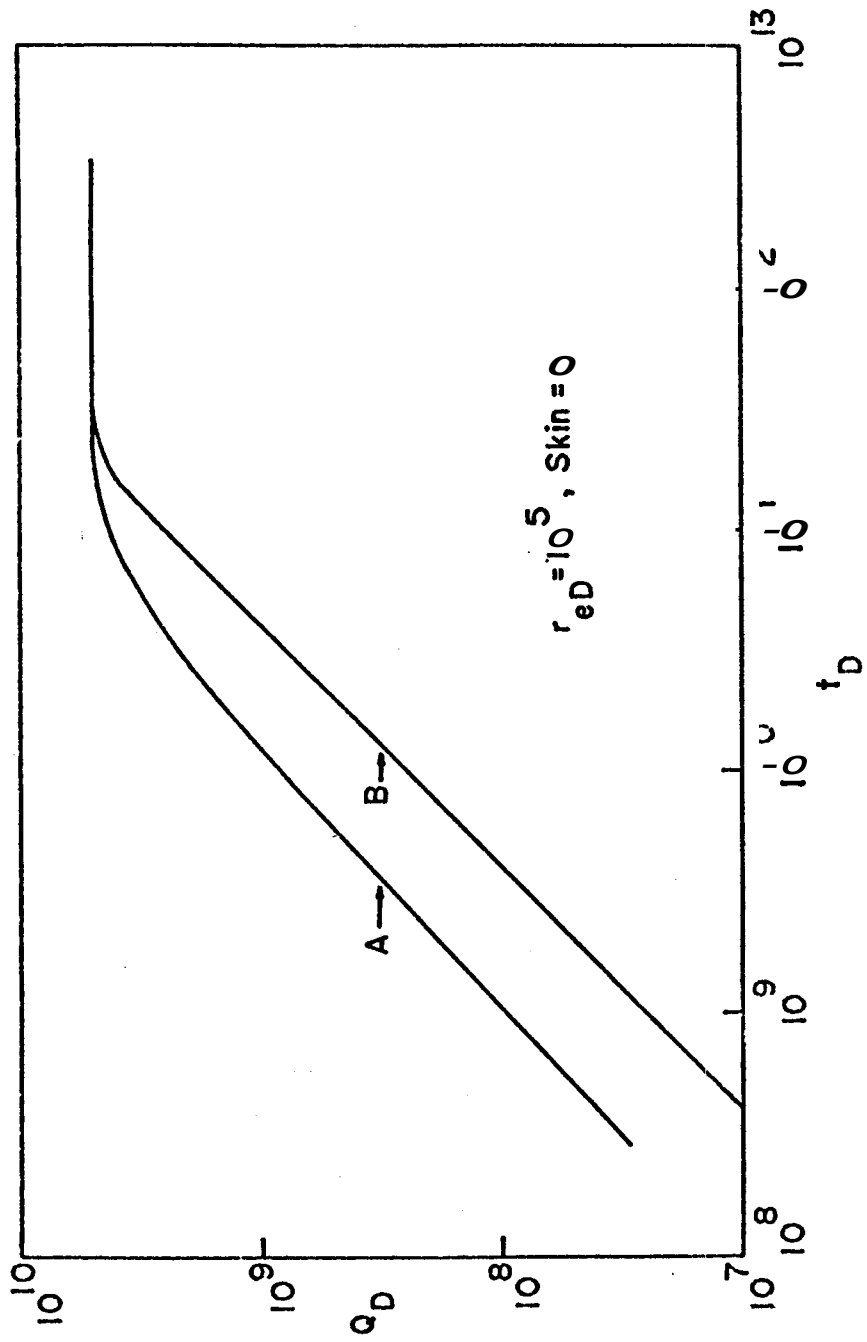


Figure 3.6: Cumulative Recovery for Rate Histories in Fig. 3.5

skin factor. The rate histories are compared in Fig. 3.7, and Fig. 3.8 shows the cumulative production.

3.4.3 Interference among Flowing Wells

The following derivation shows a general method for determining the pressure distribution and transient rate solutions for wells producing at constant pressures in interference with other wells producing at arbitrary constant rates or pressures. The pressure drop at any point (x, y) is given by:

$$p_i - p(x, y, t) = \sum_i \Delta p_i(t) \quad (3.36)$$

where Δp_i is the pressure drop due to the well at the point (x_i, y_i) produced at the rate q_i . If q_i is constant, then:

$$\Delta p_i(t) = p_D([(x_i - x)^2 + (y_i - y)^2]^{1/2}, t) \cdot \frac{q_i \mu}{2\pi kh} \quad (3.37)$$

(In this section, p_D refers to the dimensionless pressure drop for constant rate production. If the well at (x_i, y_i) is produced at a constant pressure:

$$\Delta p_j(t) = \frac{\mu}{2\pi kh} \int_0^t q_j(\tau) \frac{dp_D}{d\tau} ([x_j - x]^2 + [y_j - y]^2)^{1/2}, t - \tau) d\tau \quad (3.38)$$

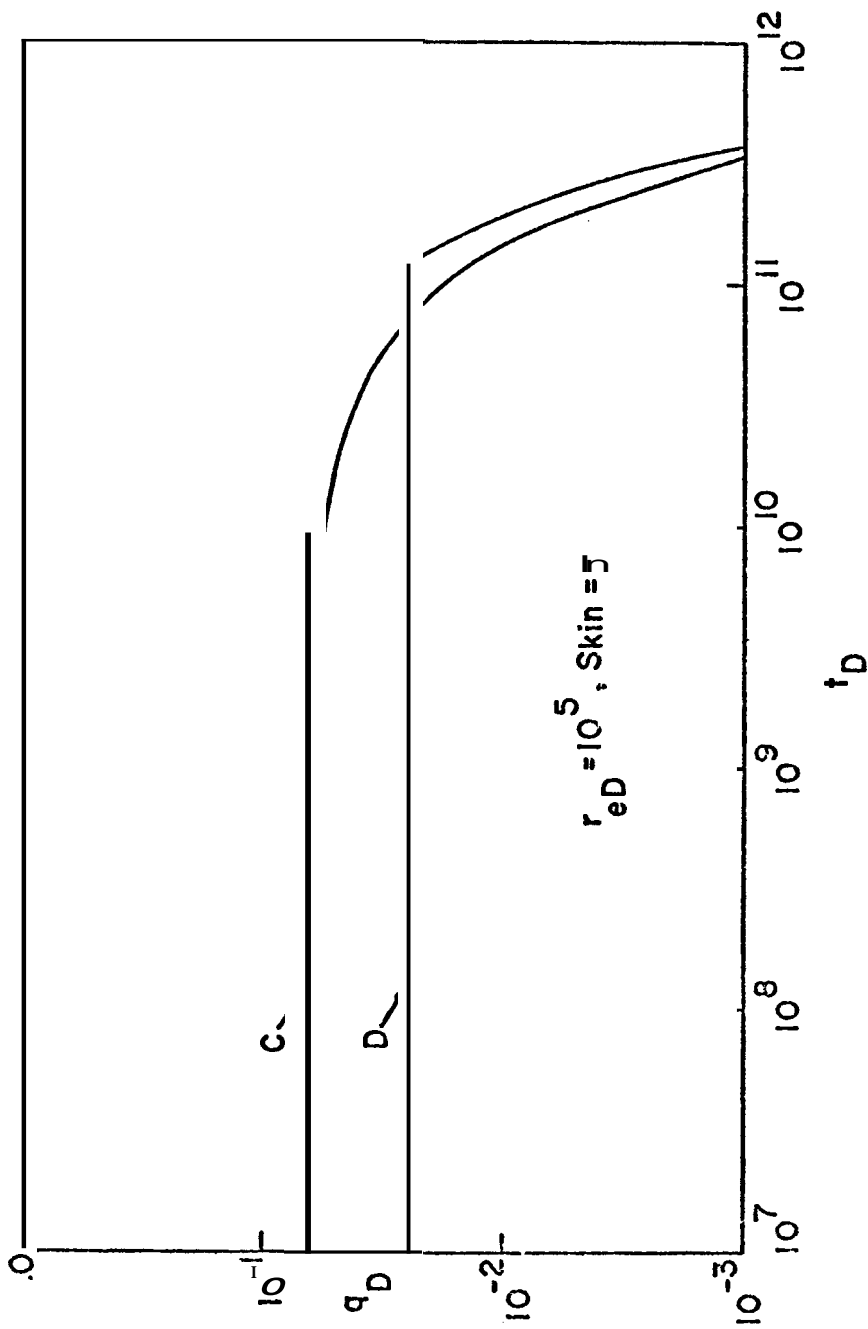


Figure 3.7: Two Rate Histories with Nonzero Skin

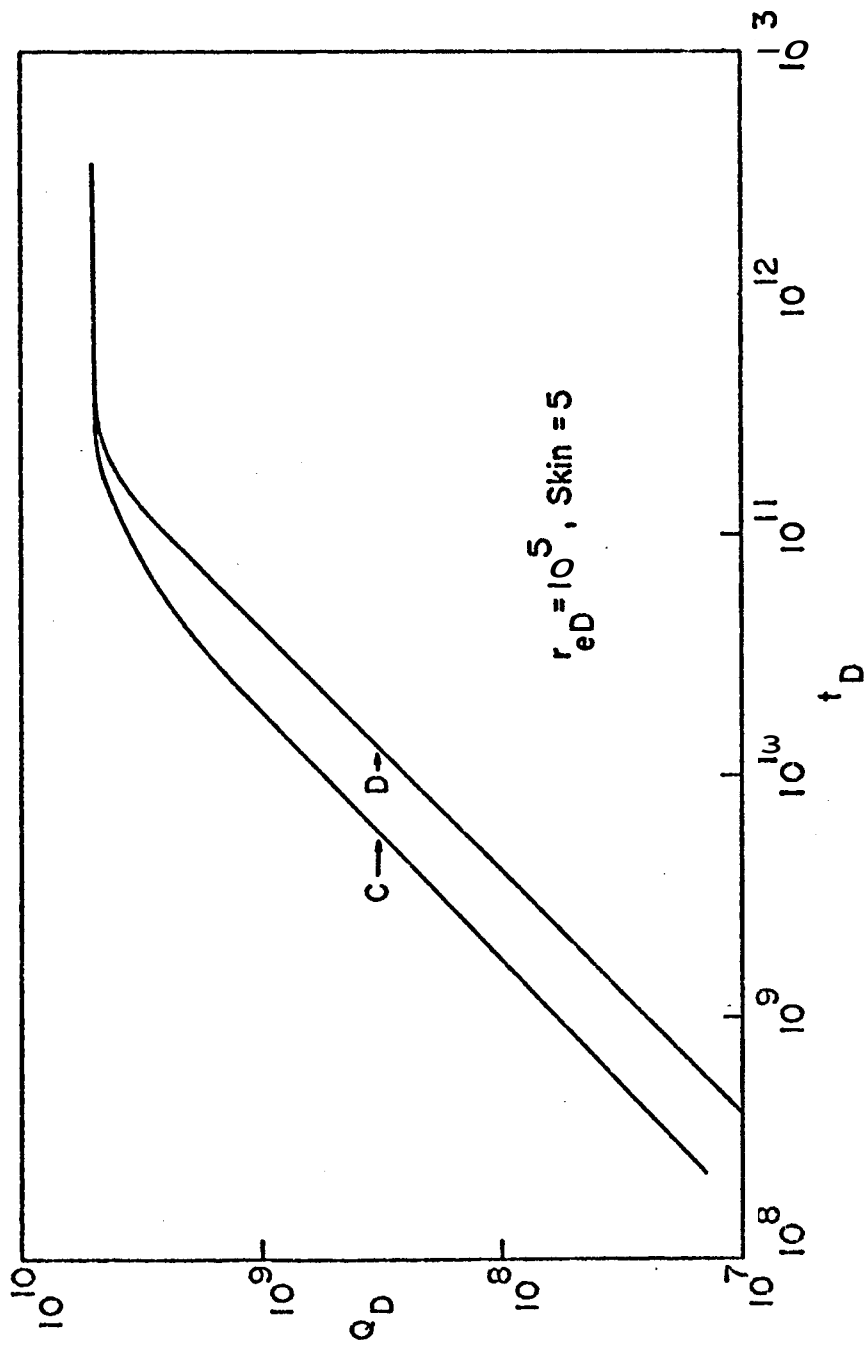


Figure 3.8: Cumulative Recovery for Rate Histories in Fig. 3.7

The rate functions, q_i , must be determined first; then the pressure distribution is computed using Eq. 3.35. To determine the rate functions, apply the Laplace transformation to the equations for the producing pressure at each constant pressure well:

$$\frac{p_i - \bar{p}(x_k, y_k)}{\ell} = \frac{\mu}{2\pi kh} \left\{ \sum_{i \neq k} q_j \bar{p}_D(r_{ik}, \ell) + \sum_{j \neq k} \bar{q}_j(\ell) \ell \bar{p}_D(r_{jk}, \ell) + \bar{q}_k(\ell) \ell \bar{p}_{wD}(\ell) \right\} \quad (3.39)$$

where $r_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$. The system of equations can be written in the form:

$$[\Delta \bar{p}]_N = [b]_{N \times N} \cdot [\bar{q}]_N \quad (3.40)$$

where:

$$\Delta \bar{p}_k = \frac{p_i - \bar{p}(x_k, y_k)}{\ell} - \frac{\mu}{2\pi kh} \sum_{i \neq k} \bar{p}_D(r_{ik}, \ell) q_i \quad (3.41)$$

and :

$$b_{kj} = \begin{cases} \frac{\mu}{2\pi kh} \bar{p}_D(r_{jk}, \ell) & k \neq j \\ \ell p_{wD}(\ell) & k = j \end{cases} \quad (3.42)$$

Once the rate functions are determined, the pressure distribution follows from Eq. 3.35. As an example, consider the case of two wells at a distance r_D , each produced at a constant pressure p_{wf} . Then:

$$p_i - p_{wf} = \frac{\mu}{2\pi kh} \int_0^t q(\tau) p'_{wD}(t-\tau) d\tau + \frac{\mu}{2\pi kh} \int_0^t q(\tau) p'_D(r_D, t-\tau) d\tau \quad (3.43)$$

or:

$$1 = \int_0^{t_D} q_D(\tau) p'_{wD}(t_D - \tau) + \int_0^{t_D} q_D(\tau) p'_D(r_D, t_D - \tau) d\tau \quad (3.44)$$

In Laplace space:

$$\frac{1}{\ell} = \ell \bar{q}_D(\ell) \bar{p}_{wD}(\ell) + \ell q_D(\ell) \bar{p}_D(r_D, \ell) \quad (3.45)$$

Solving for $\bar{q}_D(\ell)$:

$$\bar{q}_D(\ell) = \frac{1}{\ell^2 [\bar{p}_{wD}(\ell) + \bar{p}_D(r_D, \ell)]} \quad (3.46)$$

or

$$\bar{q}_D(\ell) = \frac{K_1(\sqrt{\ell})}{\sqrt{\ell} [K_0(\sqrt{\ell}) + s\sqrt{\ell} K_1(\sqrt{\ell}) + K_0(r_D \sqrt{\ell})]} \quad (3.47)$$

Using the Stehfest algorithm, a solution for $q_D(t_D)$ can be determined numerically.

The Laplace space solution for $[p_i - p(x, y, t)] / (p_i - p_{wf})$ is given by:

$$\frac{p_i - \bar{p}(x, y, t)}{p_i - p_{wf}} = \bar{q}_D(\ell) [\bar{p}_D(r_1, t) + \bar{p}_D(r_2, t)] \quad (3.48)$$

This concludes the section on the use of superposition in time of continuously varying rates as a method for generating solutions involving wells produced at constant pressure. The method is a powerful tool, and the solutions presented here are meant to suggest important ways in which this tool can be used.

SECTION 4

CONCLUSIONS

The solutions provided in this work show that well test analysis methods for wells produced at constant pressure provide the same information about the reservoir as is determined from the conventional methods derived for constant-rate production. For nearly every constant-rate well test method there is an analogous constant-pressure method. A notable exception is interference analysis. Methods for analyzing interference between producing wells are more complicated, and require additional study.

The transient rate analysis methods may be limited in their effectiveness by practical problems. The technology for measuring production rates is not nearly as advanced as the measurement of transient pressures. However, for the same reason, maintaining a constant wellhead pressure is more reliable than maintaining a constant rate. Pressure buildup following constant-pressure production is not technology bound, and appears to be a viable alternative method which avoids the necessity for establishing a constant rate for some length of time prior to shut-in.

In summary, the methods provided here include the following:

1. Determination of k and ϕe^{-2s} by type curve matching with a graph of $\log q_D$ vs $\log t$ for the infinite system
2. Determination of k and s from the semilog straight line in a graph of $1/q$ vs $\log t$
3. Determination of reservoir area and approximate shape from a graph of $\log q$ vs t after the onset of exponential decline
4. Analysis of transient rates when the wellhead pressure is constant
5. Determination of k and ϕe^{-2s} from an interference test by type curve matching with a graph of $\log p$ vs $\log t_D/r_D^2$ for the infinite system
6. Determination of C_D , s , x_f for fractures penetrated by the wellbore, and other inner boundary effects, by type curve matching of early pressure buildup data with conventional pressure transient solutions
7. Horner buildup analysis for wells produced at constant pressure

8. Analogous methods for Matthews, Brons, Hazebroek
determination of the static reservoir pressure

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NOMENCLATURE

A = area, L^2

C_A = shape factor

C_D = dimensionless wellbore storage coefficient, $\frac{V_w c_w}{2\pi\phi c_t h r_w^2}$

c_t = total compressibility, Lt^2/m

c_w = wellbore fluid compressibility, Lt^2/m

D = wellbore diameter, L

f_M = Moody friction factor

g_c = units conversion factor

h = reservoir thickness, L

H = wellbore vertical length, L

I_0, I_1 = Modified Bessel functions

k = reservoir absolute permeability, L^2

K_0, K_1 = Modified Bessel functions

λ = Laplace space variable

L = wellbore length, L

m = slope of Horner buildup graph, m/Lt^2

m_q = slope of $\frac{1}{q}$ vs $\log t$ graph for a constant-pressure test, t/L^3

m = slope of the $\log q$ vs t graph for a constant pressure test

N_{RE} = Reynold's number

p = pressure, m/Lt^2

p_D = dimensionless pressure ratio, $\frac{p_i - p_{r,t}}{p_i - p_{wf}}$

p_{wD} = dimensionless wellbore pressure, $2\pi kh(p_i - p_{wf})/q\mu$

p_i = initial reservoir pressure, m/Lt^2

p_{tf} = flowing wellhead pressure, m/Lt^2

p_{wf} = flowing bottom-hole pressure, m/Lt^2

- p_{wf} = bottom-hole pressure after shut-in, m/Lt^2
 p = extrapolated pressure on Horner buildup graph, m/Lt^2
 \bar{p} = volumetric average reservoir pressure, m/Lt^2
 q = production rate, L^3/t
 q_D = dimensionless production rate, $\frac{q\mu}{2\pi kh(p_i - p_{wf})}$
 q_c = constant initial flow rate, L^3/t
 $(q_c)_D$ = dimensionless constant initial flow rate, $\frac{q_c\mu}{2\pi kh(p_i - p_{wf})}$
 q_M = flow rate at match point for type curve matching, L^3/t
 $(q_D)_M$ = dimensionless flow rate at match point for type curve matching
 $\left(\frac{1}{q}\right)_{1hr}$ = ordinate value at 1 hour on straight-line graph of $\left(\frac{1}{q}\right)$ vs $\log t, t/L^3$
 Q = cumulative production, L^3
 Q_D = dimensionless cumulative production, $Q/[2\pi\phi c_t h r_w^2 (p_i - p_{wf})]$
 r_D = dimensionless radius, r/r_w
 r_e = reservoir radius, L
 r_{eD} = dimensionless reservoir radius, r_e/r_w
 r_w = wellbore radius, L
 r_w' = effective wellbore radius, $r_w e^{-s}$, L
 t = time
 t_D = dimensionless time, $\frac{k\epsilon}{\phi\mu c_t r_w^2}$
 t_{DA} = dimensionless time based on drainage area, $\frac{kt}{\phi\mu c_t r_w^2}$
 t_M = time at match point for type curve matching, t
 $(t_D)_M$ = dimensionless time at match point for type curve matching
 t_P = production time, t
 t_P = Horner corrected production time, t

- t_{pss} = time at the beginning of pseudo-steady state flow, t
 $(t_{pss})_D$ = dimensionless time at the beginning of pseudo-steady state flow
 At = shut-in time, t
 Δt_D = dimensionless shut-in time
 s = skin factor
 U = wellbore fluid velocity, L/t
 v = wellbore fluid specific volume, L³/m
 V_w = wellbore volume, L³
 W_f = wellbore friction energy loss, mL²/t²
 W_s = shaft work, mL²/t²
- γ = exponential of Euler's constant, $\gamma \approx 1.781$
 ϕ = porosity
 μ = fluid viscosity, m/Lt
 $\bar{\rho}$ = average wellbore fluid density, m/L³
 τ = variable of integration

APPENDIX A
UNITS CONVERSIONS

Variable	Darcy, SI Metric Units	English Units
t_D	$\frac{kt}{\phi\mu c_t r_w^2}$	$\frac{.000264 kt}{\phi\mu c_t r_w^2}$
q_D	$\frac{qB\mu}{2\pi kh(p_i - p_{wf})}$	$\frac{141.2 qB\mu}{2\pi kh(p_i - p_{wf})}$
Q_D	$Q/[2\pi\phi c_t hr_w^2(p_i - p_{wf})]$	$Q/[2\pi\phi c_t hr_w^2(p_i - p_{wf})]$
m	$\frac{.1832 qB\mu}{kh}$	$\frac{162.6 qB\mu}{kh}$
$\frac{kh(p^* - \bar{p})}{q_t B\mu} \cdot \alpha$	$a = 2\pi$	$\alpha = \frac{1}{141.2}$
c	atm^{-1}, Pa^{-1}	psi^{-1}
h	cm, m	ft
k	$darcy, m^2$	md
p	atm, Pa	psi
q	$cm^3/sec, m^3/sec$	$barrels/day$
r	cm, m	ft
t	sec, sec	hr
μ	$cp, Pa-sec$	cp

APPENDIX B
TABULATED SOLUTIONS

CONTENTS		page
INFINITE OUTER BOUNDARY		84
CLOSED OUTER BOUNDARY		88
CONSTANT PRESSURE OUTER BOUNDARY		100

Table B.1
 INFINITE OUTER BOUNDARY
 Skin = 0

t_D	Q_D	q_D	t_D	Q_D	q_D
1.00D-01	4.0435D-01	2.2489D 00	1.00D 04	2.1984D 03	1.9594D-01
2.00D-01	5.9863D-01	1.7153D 00	2.00D 04	4.0887D 03	1.8571D-01
3.00D-01	7.5642D-01	1.4764D 00	3.00D 04	5.8710D 03	1.7727D-01
4.00D-01	8.9636D-01	1.3326D 00	4.00D 04	7.6405D 03	1.7289D-01
5.00D-01	1.0244D 00	1.2336D 00	5.00D 04	9.3527D 03	1.6966D-01
6.00D-01	1.1439D 00	1.1601D 00	6.00D 04	1.1036D 04	1.6712D-01
7.00D-01	1.2569D 00	1.1025D 00	7.00D 04	1.2697D 04	1.6502D-01
8.00D-01	1.3648D 00	1.0550D 00	8.00D 04	1.4330D 04	1.6325D-01
9.00D-01	1.4684D 00	1.0169D 00	9.00D 04	1.5963D 04	1.6172D-01
1.00D 00	1.5684D 00	9.8383D-01	1.00D 05	1.7574D 04	1.6037D-01
2.00D 00	2.4455D 00	8.0063D-01	2.00D 05	3.3140D 04	1.5203D-01
3.00D 00	3.1789D 00	7.1624E-01	3.00D 05	4.8102D 04	1.4753D-01
4.00D 00	3.8805D 00	6.6443D-01	4.00D 05	6.2697D 04	1.4450E-01
5.00D 00	4.5339D 00	6.2821E-01	5.00D 05	7.7030D 04	1.4223D-01
6.00D 00	5.1480D 00	6.0091E-01	6.00D 05	9.1161D 04	1.4043D-01
7.00D 00	5.7377D 00	5.7930D-01	7.00D 05	1.0513D 05	1.3894D-01
8.00D 00	6.3079D 00	5.6160D-01	8.00D 05	1.1896D 05	1.3767E-01
9.00D 00	6.8619D 00	5.4671D-01	9.00D 05	1.3267D 05	1.3658D-01
1.00D 01	7.4021D 00	5.3394D-01	1.00D 06	1.4628D 05	1.3561D-01
2.00D 01	1.2321D 01	4.6116D-01	2.00D 06	2.7050D 05	1.2957D-01
3.00D 01	1.6742D 01	4.2612D-01	3.00D 06	4.0637D 05	1.2526D-01
4.00D 01	2.0386D 01	4.0399D-01	4.00D 06	5.3144D 05	1.2400E-01
5.00D 01	2.4645D 01	3.8620D-01	5.00D 06	6.5462D 05	1.2237E-01
6.00D 01	2.8662D 01	3.7609D-01	6.00D 06	7.7631D 05	1.2103E-01
7.00D 01	3.2373D 01	3.6638D-01	7.00D 06	8.9678D 05	1.1992D-01
8.00D 01	3.5796D 01	3.5833D-01	8.00D 06	1.0162D 06	1.1897D-01
9.00D 01	3.9544D 01	3.5149D-01	9.00D 06	1.1348D 05	1.1815D-01
1.00D 02	4.3029D 01	3.4557D-01	1.00D 07	1.2526D 06	1.1742E-01
2.00D 02	7.5593D 01	3.1081E-01	2.00D 07	2.4012D 06	1.1286E-01
3.00D 02	1.0573D 02	2.9335D-01	3.00D 07	3.5164D 06	1.1035D-01
4.00D 02	1.3447D 02	2.8204D-01	4.00D 07	4.6110D 06	1.0863E-01
5.00D 02	1.6224D 02	2.7382D-01	5.00D 07	5.6908D 06	1.0734E-01
6.00D 02	1.8930D 02	2.6744E-01	6.00D 07	6.7590D 06	1.0651E-01
7.00D 02	2.1578D 02	2.6226E-01	7.00D 07	7.8178D 06	1.0593E-01
8.00D 02	2.4172E 02	2.5792E-01	8.00D 07	8.8695D 06	1.0471E-01
9.00D 02	2.6739E 02	2.5421E-01	9.00D 07	9.9126D 06	1.0408E-01
1.00D 03	2.9264E 02	2.5097E-01	1.00D 08	1.0931E 07	1.0351E-01
2.00D 03	5.3254E 02	2.3151E-01	2.00D 08	2.1102E 07	9.9943E-02
3.00D 03	7.5862E 02	2.2142E-01	3.00D 08	3.0992E 07	9.7967E-02
4.00D 03	9.7651E 02	2.1477E-01	4.00D 08	4.0718E 07	9.6612E-02
5.00D 03	1.1863E 03	2.0996E-01	5.00D 08	5.0327E 07	9.5586E-02
6.00D 03	1.3767E 03	2.0607E-01	6.00D 08	5.9845E 07	9.4764E-02
7.00D 03	1.6011E 03	2.0287E-01	7.00D 08	6.9287E 07	9.4080E-02
8.00D 03	1.8026E 03	2.0022E-01	8.00D 08	7.8666E 07	9.3495E-02
9.00D 03	2.0012E 03	1.9794E-01	9.00D 08	8.7991E 07	9.2985E-02

INFINITE OUTER BOUNDARY
Skin = 5

t_D	Q_D	q_D	t_D	Q_D	q_D
1.000-01	1.91710-02	1.88050-01	1.000 04	1.04951 03	9.94640-02
2.000-01	3.77680-02	1.84150-01	2.000 04	2.02580 03	7.61650-02
3.000-01	5.60450-02	1.81460-01	3.000 04	2.97780 03	9.43340-02
4.000-01	7.40860-02	1.79380-01	4.000 04	3.91460 03	9.30760-02
5.000-01	9.19390-02	1.77670-01	5.000 04	4.84650 03	9.21240-02
6.000-01	1.09530-01	1.76200-01	6.000 04	5.75790 03	9.13890-02
7.000-01	1.27190-01	1.74920-01	7.000 04	6.66340 03	9.07230-02
8.000-01	1.44650-01	1.73780-01	8.000 04	7.57270 03	9.01190-02
9.000-01	1.61950-01	1.72760-01	9.000 04	8.47240 03	8.97040-02
1.000 00	1.79190-01	1.71830-01	1.000 05	9.36740 03	8.92840-02
2.000 00	3.47480-01	1.69370-01	2.000 05	1.81470 04	8.66140-02
3.000 00	5.10750-01	1.61400-01	3.000 05	2.67290 04	8.51240-02
4.000 00	6.70680-01	1.58540-01	4.000 05	3.51890 04	8.40960-02
5.000 00	8.28080-01	1.56310-01	5.000 05	4.35590 04	8.33190-02
6.000 00	9.83470-01	1.54490-01	6.000 05	5.18600 04	8.26930-02
7.000 00	1.13720 00	1.52950-01	7.000 05	6.01030 04	8.21710-02
8.000 00	1.28950 00	1.51620-01	8.000 05	6.82990 04	8.17240-02
9.000 00	1.44050 00	1.50460-01	9.000 05	7.64520 04	8.13330-02
1.000 01	1.59050 00	1.49420-01	1.000 06	8.45690 04	8.09870-02
2.000 01	3.04730 00	1.42730-01	2.000 06	1.64330 05	7.87830-02
3.000 01	4.45450 00	1.38970-01	3.000 06	2.42460 05	7.75480-02
4.000 01	5.83080 00	1.36390-01	4.000 06	3.19370 05	7.66940-02
5.000 01	7.18450 00	1.34420-01	5.000 06	3.95930 05	7.60450-02
6.000 01	8.52080 00	1.32850-01	6.000 06	4.71730 05	7.55230-02
7.000 01	9.84280 00	1.31550-01	7.000 06	5.47630 05	7.50870-02
8.000 01	1.11530 01	1.30440-01	8.000 06	6.21940 05	7.47140-02
9.000 01	1.24520 01	1.29470-01	9.000 06	6.96490 05	7.43870-02
1.000 02	1.37430 01	1.28620-01	1.000 07	7.70740 05	7.40980-02
2.000 02	2.63030 01	1.23240-01	2.000 07	1.50140 06	7.22470-02
3.000 02	3.84690 01	1.20280-01	3.000 07	2.21840 06	7.12060-02
4.000 02	5.03920 01	1.18260-01	4.000 07	2.92680 06	7.04860-02
5.000 02	6.21400 01	1.16740-01	5.000 07	3.62890 06	6.99370-02
6.000 02	7.37520 01	1.15520-01	6.000 07	4.32610 06	6.94250-02
7.000 02	8.52540 01	1.14510-01	7.000 07	5.01920 06	6.91260-02
8.000 02	9.66620 01	1.13650-01	8.000 07	5.70890 06	6.88090-02
9.000 02	1.07990 02	1.12900-01	9.000 07	6.39370 06	6.85320-02
1.000 03	1.19250 02	1.12240-01	1.000 08	7.07990 06	6.82860-02
2.000 03	2.29150 02	1.08060-01	2.000 08	1.38210 07	6.67100-02
3.000 03	3.35990 02	1.05760-01	3.000 08	2.04450 07	6.58220-02
4.000 03	4.40930 02	1.04190-01	4.000 08	2.69960 07	6.52060-02
5.000 03	5.44510 02	1.03000-01	5.000 08	3.34930 07	6.47360-02
6.000 03	6.47030 02	1.02040-01	6.000 08	3.99480 07	6.43570-02
7.000 03	7.48680 02	1.01250-01	7.000 08	4.63680 07	6.40390-02
8.000 03	8.49600 02	1.00570-01	8.000 08	5.27590 07	6.37670-02
9.000 03	9.49820 02	9.99860-02	9.000 08	5.91240 07	6.35290-02

INFINITE OUTER BOUNDARY

Skin = 10

t_D	QD	q_D	t_D	QD	q_D
1.00D-01	9.7896D-03	9.6936D-02	1.00D 04	6.8868D 02	6.6496D-02
2.00D-01	1.9429D-02	9.5901D-02	2.00D 04	1.3453D 03	6.5001D-02
3.00D-01	2.8983D-02	9.5176D-02	3.00D 04	1.9909D 03	6.4157D-02
4.00D-01	3.8472D-02	9.4607D-02	4.00D 04	2.6275D 03	6.3572D-02
5.00D-01	4.7910D-02	9.4137D-02	5.00D 04	3.2630D 03	6.3125D-02
6.00D-01	5.7305D-02	9.3730D-02	6.00D 04	3.8925D 03	6.2764D-02
7.00D-01	6.6661D-02	9.3372D-02	7.00D 04	4.5186D 03	6.2463D-02
8.00D-01	7.5995D-02	9.3052D-02	8.00D 04	5.1420D 03	6.2204D-02
9.00D-01	8.5275D-02	9.2762D-02	9.00D 04	5.7630D 03	6.1977D-02
1.00D 00	9.4539D-02	9.2496D-02	1.00D 05	6.3818D 03	6.1776D-02
2.00D 00	1.8602D-01	9.0619D-02	2.00D 05	1.2498D 04	6.0483D-02
3.00D 00	2.7602D-01	8.9431D-02	3.00D 05	1.8498D 04	5.9752D-02
4.00D 00	3.6501D-01	8.8557D-02	4.00D 05	2.4447D 04	5.9243D-02
5.00D 00	4.5322D-01	8.7864D-02	5.00D 05	3.0352D 04	5.8855D-02
6.00D 00	5.4080D-01	8.7292D-02	6.00D 05	3.6222D 04	5.8541D-02
7.00D 00	6.2785D-01	8.6803D-02	7.00D 05	4.2064D 04	5.8279D-02
8.00D 00	7.1445D-01	8.6378D-02	8.00D 05	4.7891D 04	5.8053D-02
9.00D 00	8.0065D-01	8.6001D-02	9.00D 05	5.3677D 04	5.7856D-02
1.00D 01	8.8649D-01	8.5663D-02	1.00D 06	5.9454D 04	5.7680D-02
2.00D 01	1.7308D 00	8.3434D-02	2.00D 06	1.1651D 05	5.6551D-02
3.00D 01	2.5535D 00	8.2139D-02	3.00D 06	1.7273D 05	5.5911D-02
4.00D 01	3.3750D 00	8.1230D-02	4.00D 06	2.2841D 05	5.5466D-02
5.00D 01	4.1838D 00	8.0532D-02	5.00D 06	2.8371D 05	5.5125D-02
6.00D 01	4.9863D 00	7.9967D-02	6.00D 06	3.3870D 05	5.4850D-02
7.00D 01	5.7837D 00	7.9493D-02	7.00D 06	3.9344D 05	5.4619D-02
8.00D 01	6.5766D 00	7.9095D-02	8.00D 06	4.4796D 05	5.4421D-02
9.00D 01	7.3658D 00	7.8729D-02	9.00D 06	5.0230D 05	5.4247D-02
1.00D 02	8.1515D 00	7.8411D-02	1.00D 07	5.5649D 05	5.4093D-02
2.00D 02	1.5879D 01	7.6375D-02	2.00D 07	1.0919D 06	5.3099D-02
3.00D 02	2.3456D 01	7.5224D-02	3.00D 07	1.6200D 06	5.2534D-02
4.00D 02	3.0938D 01	7.4427D-02	4.00D 07	2.1433D 06	5.2141D-02
5.00D 02	3.8350D 01	7.3819D-02	5.00D 07	2.6632D 06	5.1839D-02
6.00D 02	4.5707D 01	7.3329D-02	6.00D 07	3.1804D 06	5.1596D-02
7.00D 02	5.3020D 01	7.2920D-02	7.00D 07	3.6954D 06	5.1392D-02
8.00D 02	6.0295D 01	7.2568D-02	8.00D 07	4.2085D 06	5.1216D-02
9.00D 02	6.7537D 01	7.2261D-02	9.00D 07	4.7199D 06	5.1062D-02
1.00D 03	7.4750D 01	7.1989D-02	1.00D 08	5.2299D 06	5.0925D-02
2.00D 03	1.4577D 02	7.0244D-02	2.00D 08	1.0274D 07	5.0043D-02
3.00D 03	2.1550D 02	6.9261D-02	3.00D 08	1.5252D 07	4.9541D-02
4.00D 03	2.8441D 02	6.8580D-02	4.00D 08	2.0188D 07	4.9191D-02
5.00D 03	3.5273D 02	6.8060D-02	5.00D 08	2.5094D 07	4.8923D-02
6.00D 03	4.2055D 02	6.7642D-02	6.00D 08	2.9976D 07	4.8706D-02
7.00D 03	4.8805D 02	6.7292D-02	7.00D 08	3.4838D 07	4.8524D-02
8.00D 03	5.5520D 02	6.6992D-02	8.00D 08	3.9693D 07	4.8367D-02
9.00D 03	6.2207D 02	6.6729D-02	9.00D 08	4.4513D 07	4.8230D-02

INFINITE OUTER BOUNDARY
Skin = 20

t_D	Q_D	q_D	t_D	Q_D	q_D
1.000-01	4.94730-03	4.92240-02	1.000 04	4.07930 02	3.99570-02
2.000-01	9.05610-03	4.89590-02	2.000 04	8.04500 02	3.94120-02
3.000-01	1.47430-02	4.87700-02	3.000 04	1.19700 03	3.91000-02
4.000-01	1.96130-02	4.86310-02	4.000 04	1.58690 03	3.88610-02
5.000-01	2.44690-02	4.84970-02	5.000 04	1.97490 03	3.87140-02
6.000-01	2.93140-02	4.83900-02	6.000 04	2.36140 03	3.85770-02
7.000-01	3.41490-02	4.82950-02	7.000 04	2.74660 03	3.84630-02
8.000-01	3.89750-02	4.82100-02	8.000 04	3.13080 03	3.83650-02
9.000-01	4.37930-02	4.81320-02	9.000 04	3.51410 03	3.82780-02
1.000 00	4.86030-02	4.80610-02	1.000 05	3.89650 03	3.82010-02
2.000 00	9.63940-02	4.75540-02	2.000 05	7.68930 03	3.77030-02
3.000 00	1.43750-01	4.72270-02	3.000 05	1.14450 04	3.74170-02
4.000 00	1.90890-01	4.69340-02	4.000 05	1.51760 04	3.72170-02
5.000 00	2.37780-01	4.67900-02	5.000 05	1.89910 04	3.70630-02
6.000 00	2.84490-01	4.66880-02	6.000 05	2.28910 04	3.69390-02
7.000 00	3.31060-01	4.66090-02	7.000 05	2.62800 04	3.68330-02
8.000 00	3.77490-01	4.63690-02	8.000 05	2.99590 04	3.67430-02
9.000 00	4.23810-01	4.62600-02	9.000 05	3.36300 04	3.66640-02
1.000 01	4.70030-01	4.61620-02	1.000 06	3.72930 04	3.65930-02
2.000 01	9.28090-01	4.55100-02	2.000 06	7.36350 04	3.61350-02
3.000 01	1.38120 00	4.51230-02	3.000 06	1.09630 05	3.58720-02
4.000 01	1.83100 00	4.48430-02	4.000 06	1.45420 05	3.56830-02
5.000 01	2.27850 00	4.46350-02	5.000 06	1.81040 05	3.55470-02
6.000 01	2.72400 00	4.44610-02	6.000 06	2.16530 05	3.54320-02
7.000 01	3.16790 00	4.43140-02	7.000 06	2.51920 05	3.53360-02
8.000 01	3.61050 00	4.41890-02	8.000 06	2.87210 05	3.52520-02
9.000 01	4.05180 00	4.40760-02	9.000 06	3.22430 05	3.51790-02
1.000 02	4.49220 00	4.39770-02	1.000 07	3.57590 05	3.51140-02
2.000 02	8.85420 00	4.33230-02	2.000 07	7.06420 05	3.46920-02
3.000 02	1.31680 01	4.29560-02	3.000 07	1.05210 06	3.44500-02
4.000 02	1.74500 01	4.26940-02	4.000 07	1.39580 06	3.42810-02
5.000 02	2.17100 01	4.24930-02	5.000 07	1.73790 06	3.41500-02
6.000 02	2.59510 01	4.23300-02	6.000 07	2.07890 06	3.40440-02
7.000 02	3.01780 01	4.21930-02	7.000 07	2.41900 06	3.39590-02
8.000 02	3.43920 01	4.20760-02	8.000 07	2.75820 06	3.38780-02
9.000 02	3.85950 01	4.19720-02	9.000 07	3.09670 06	3.38110-02
1.000 03	4.27880 01	4.18800-02	1.000 08	3.43450 06	3.37510-02
2.000 03	8.43390 01	4.12820-02	2.000 08	6.78830 06	3.33610-02
3.000 03	1.25440 02	4.09410-02	3.000 08	1.01130 07	3.31370-02
4.000 03	1.66270 02	4.07010-02	4.000 08	1.34190 07	3.29800-02
5.000 03	2.06980 02	4.05180-02	5.000 08	1.67110 07	3.28590-02
6.000 03	2.47520 02	4.03690-02	6.000 08	1.99920 07	3.27610-02
7.000 03	2.87630 02	4.02440-02	7.000 08	2.32640 07	3.26780-02
8.000 03	3.27830 02	4.01360-02	8.000 08	2.65290 07	3.26070-02
9.000 03	3.67920 02	4.00410-02	9.000 08	2.97870 07	3.25450-02

Table B.2
CLOSED OUTER BOUNDARY

$r_{eD} = 20$			$r_{eD} = 200$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 02	4.29040 01	3.39300-01	1.000 04	2.19620 03	1.94930-01
2.000 02	7.34030 01	2.72610-01 *	2.000 04	4.04790 03	1.74130-01 *
3.000 02	9.79980 01	2.19510-01 *	3.000 04	5.68920 03	1.56170-01 *
4.000 02	1.17800 02	1.76680-01 *	4.000 04	7.17090 03	1.39990-01 *
5.000 02	1.33740 02	1.42450-01 *	5.000 04	8.50220 03	1.25560-01 *
6.000 02	1.46500 02	1.14720-01 *	6.000 04	9.69460 03	1.12500-01 *
7.000 02	1.54720 02	9.24480-02 *	7.000 04	1.07630 04	1.00790-01 *
8.000 02	1.64940 02	7.47080-02 *	8.000 04	1.17210 04	9.03360-02 *
9.000 02	1.71550 02	6.03910-02 *	9.000 04	1.25770 04	8.09740-02 *
1.000 03	1.76970 02	4.89060-02 *	1.000 05	1.33480 04	7.26380-02 *
2.000 03	1.96710 02	5.79030-03 *	2.000 05	1.77320 04	2.47220-02 *
3.000 03	1.99240 02	3.95770-05 *	3.000 05	1.92140 04	8.69740-03 *
			4.000 05	1.97430 04	3.02410-03 *
			5.000 05	1.99110 04	7.65520-04 *
			6.000 05	1.99940 04	1.46220-04 *

$r_{eD} = 50$			$r_{eD} = 500$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 03	2.89770 02	2.39210-01 *	1.000 05	1.74850 04	1.56430-01 *
2.000 03	5.01840 02	1.86400-01 *	2.000 05	3.20520 04	1.35390-01 *
3.000 03	6.87200 02	1.45030-01 *	3.000 05	4.46750 04	1.16980-01 *
4.000 03	7.96140 02	1.15080-01 *	4.000 05	5.53990 04	1.01130-01 *
5.000 03	8.95980 02	8.60670-02 *	5.000 05	6.50350 04	8.73310-02 *
6.000 03	9.73620 02	6.87540-02 *	6.000 05	7.31930 04	7.54930-02 *
7.000 03	1.03370 03	5.37410-02 *	7.000 05	8.02220 04	6.51960-02 *
8.000 03	1.08080 03	4.22270-02 *	8.000 05	8.63150 04	5.64640-02 *
9.000 03	1.11690 03	3.30110-02 *	9.000 05	9.13370 04	4.88030-02 *
1.000 04	1.14510 03	2.58420-02 *	1.000 06	9.60240 04	4.21420-02 *
2.000 04	1.24120 03	2.14890-03 *	2.000 06	1.18000 05	1.02220-02 *
			3.000 06	1.23330 05	2.43390-03 *
			4.000 06	1.26700 05	4.11390-04 *

$r_{eD} = 100$			$r_{eD} = 1000$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 02	4.30290 01	3.45570-01	1.000 05	1.75720 04	1.50360-01
2.000 02	7.55950 01	3.10810-01	2.000 05	3.31280 04	1.51860-01
3.000 02	1.05720 02	2.93350-01	3.000 05	4.80270 04	1.46370-01
4.000 02	1.34470 02	2.82040-01	4.000 05	6.24200 04	1.41580-01 *
5.000 02	1.62250 02	2.73810-01	5.000 05	7.63480 04	1.37040-01 *
6.000 02	1.89300 02	2.67420-01	6.000 05	8.98340 04	1.32690-01 *
7.000 02	2.15720 02	2.62230-01	7.000 05	1.02890 05	1.28460-01 *
8.000 02	2.41770 02	2.57900-01	8.000 05	1.15540 05	1.24390-01 *
9.000 02	2.67360 02	2.54190-01	9.000 05	1.27780 05	1.20430-01 *
1.000 03	2.92600 02	2.50970-01	1.000 06	1.39640 05	1.16570-01 *
2.000 03	5.32220 02	2.39970-01	2.000 06	2.39470 05	8.42860-02 *
3.000 03	7.55520 02	2.18150-01	3.000 06	3.11650 05	6.08980-02 *
4.000 03	9.69040 02	2.07010-01 *	4.000 06	3.63670 05	4.40890-02 *
5.000 03	1.17090 03	1.96600-01 *	5.000 06	4.01070 05	3.19890-02 *
6.000 03	1.36270 03	1.86770-01 *	6.000 06	4.28030 05	2.32890-02 *
7.000 03	1.54500 03	1.77410-01 *	7.000 06	4.47510 05	1.70010-02 *
8.000 03	1.71830 03	1.68540-01 *	8.000 06	4.61640 05	1.24350-02 *
9.000 03	1.88290 03	1.60080-01 *	9.000 06	4.71840 05	9.07350-03 *
1.000 04	2.03970 03	1.52010-01 *	1.000 07	4.79450 05	6.65970-03 *
2.000 04	3.23110 03	9.08240-02 *	2.000 07	4.99710 05	9.74310-04 *
3.000 04	3.83940 03	5.44070-02 *			
4.000 04	4.35810 03	3.27460-02 *			
5.000 04	4.81100 03	2.60230-02 *			
6.000 04	4.76260 03	1.21610-02 *			
7.000 04	4.87490 03	7.38400-03 *			
8.000 04	4.71270 03	4.44840-03 *			
9.000 04	4.94760 03	2.59740-03 *			
1.000 05	4.97320 03	1.56640-03 *			

* exponential rate decline

CLOSED OUTER BOUNDARY

Skin = 0

$r_{eD} = 2000$			$r_{eD} = 1 \times 10^4$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00E 05	1.7574E 04	1.6037E-01	1.00E 07	1.2525E 06	1.1742E-01
2.00E 05	3.3141E 04	1.5202E-01	2.00E 07	2.4005E 06	1.1278E-01
3.00E 05	4.8102E 04	1.4752E-01	3.00E 07	3.5126E 06	1.0974E-01
4.00E 05	6.2693E 04	1.4449E-01	4.00E 07	4.5968E 06	1.0712E-01
5.00E 05	7.7020E 04	1.4223E-01	5.00E 07	5.6552E 06	1.0460E-01
6.00E 05	9.1145E 04	1.4042E-01	6.00E 07	6.6888E 06	1.0217E-01
7.00E 05	1.0510E 05	1.3887E-01	7.00E 07	7.6984E 06	9.9786E-02
8.00E 05	1.1892E 05	1.3754E-01	8.00E 07	8.6845E 06	9.7466E-02
9.00E 05	1.3261E 05	1.3632E-01	9.00E 07	9.6478E 06	9.5195E-02
1.00E 06	1.4618E 05	1.3519E-01	1.00E 08	1.0588E 07	9.2969E-02
2.00E 06	2.7631E 05	1.2539E-01	2.00E 08	1.8831E 07	7.3411E-02
3.00E 06	3.9724E 05	1.1660E-01	3.00E 08	2.5439E 07	5.7921E-02
4.00E 06	5.0974E 05	1.0841E-01	4.00E 08	3.0616E 07	4.5721E-02
5.00E 06	6.1447E 05	1.0083E-01	5.00E 08	3.4690E 07	3.6109E-02
6.00E 06	7.1185E 05	9.3730E-02	6.00E 08	3.7895E 07	2.8559E-02
7.00E 06	8.0245E 05	8.7124E-02	7.00E 08	4.0416E 07	2.2621E-02
8.00E 06	8.8674E 05	8.0990E-02	8.00E 08	4.2401E 07	1.7942E-02
9.00E 06	9.6513E 05	7.5283E-02	9.00E 08	4.3959E 07	1.4229E-02
1.00E 07	1.0389E 06	6.9989E-02	1.00E 09	4.5201E 07	1.1330E-02
2.00E 07	1.5349E 06	3.3823E-02	2.00E 09	4.9556E 07	1.0964E-03
3.00E 07	1.7723E 06	1.6553E-02			
4.00E 07	1.8874E 06	8.2023E-03			
5.00E 07	1.9442E 06	4.0446E-03			
6.00E 07	1.9730E 06	1.9604E-03			
7.00E 07	1.9870E 06	8.4045E-04			
8.00E 07	1.9953E 06	3.4154E-04			
9.00E 07	1.9984E 06	3.8975E-05			

$r_{eD} = 5000$			$r_{eD} = 5 \times 10^4$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00E 06	1.4626E 05	1.3561E-01	1.00E 08	1.0951E 07	1.0351E-01
2.00E 06	2.7850E 05	1.2957E-01	2.00E 08	2.1102E 07	9.9939E-02
3.00E 06	4.0628E 05	1.2628E-01	3.00E 08	3.0939E 07	9.7967E-02
4.00E 06	5.3134E 05	1.2403E-01	4.00E 08	4.0713E 07	9.6604E-02
5.00E 06	6.5443E 05	1.2227E-01	5.00E 08	5.0316E 07	9.5532E-02
6.00E 06	7.7591E 05	1.2076E-01	6.00E 08	5.9821E 07	9.4611E-02
7.00E 06	8.9596E 05	1.1938E-01	7.00E 08	6.9239E 07	9.3772E-02
8.00E 06	1.0147E 06	1.1808E-01	8.00E 08	7.8576E 07	9.2980E-02
9.00E 06	1.1321E 06	1.1683E-01	9.00E 08	8.7835E 07	9.2217E-02
1.00E 07	1.2483E 06	1.1561E-01	1.00E 09	9.7019E 07	9.1474E-02
2.00E 07	2.3468E 06	1.0432E-01	2.00E 09	1.8493E 08	8.4491E-02
3.00E 07	3.3384E 06	9.4113E-02	3.00E 09	2.6615E 08	7.8039E-02
4.00E 07	4.2344E 06	8.4932E-02	4.00E 09	3.4124E 08	7.2094E-02
5.00E 07	5.0433E 06	7.6611E-02	5.00E 09	4.1043E 08	6.6505E-02
6.00E 07	5.7740E 06	6.9117E-02	6.00E 09	4.7480E 08	6.1505E-02
7.00E 07	6.4328E 06	6.2328E-02	7.00E 09	5.3408E 08	5.6788E-02
8.00E 07	7.0281E 06	5.6264E-02	8.00E 09	5.8892E 08	5.2470E-02
9.00E 07	7.5633E 06	5.0734E-02	9.00E 09	6.3948E 08	4.8440E-02
1.00E 08	8.0459E 06	4.5754E-02	1.00E 10	6.8619E 08	4.4718E-02
2.00E 08	1.0688E 07	1.6579E-02	2.00E 10	9.9443E 08	2.0282E-02
3.00E 08	1.1903E 07	6.1330E-03	3.00E 10	1.1323E 09	9.3372E-03
4.00E 08	1.2981E 07	2.3355E-03	4.00E 10	1.1951E 09	4.3295E-03
5.00E 08	1.2426E 07	7.4516E-04	5.00E 10	1.2247E 09	1.9934E-03
6.00E 08	1.2484E 07	1.4452E-04	6.00E 10	1.2389E 09	8.8065E-04
			7.00E 10	1.2462E 09	3.6523E-04
			8.00E 10	1.2490E 09	8.5455E-05

CLOSED OUTER BOUNDARY
Skin = 0

$$r_{eD} = 1 \times 10^5$$

t_D	QD	q_D
1.00E 08	1.0931E 07	1.0251E-03
2.00E 08	2.1107E 07	9.9943E-02
3.00E 08	3.0992E 07	9.7767E-02
4.00E 08	4.0719E 07	9.6612E-02
5.00E 08	5.0328E 07	9.5595E-02
6.00E 08	5.9846E 07	9.4762E-02
7.00E 08	6.9288E 07	9.4077E-02
8.00E 08	7.8665E 07	9.3491E-02
9.00E 08	8.7989E 07	9.2982E-02
1.00E 09	9.7264E 07	9.2531E-02
2.00E 09	1.8816E 08	8.9623E-02
3.00E 09	2.7680E 08	8.7722E-02
4.00E 09	3.6367E 08	8.6052E-02 *
5.00E 09	4.4891E 08	8.4456E-02 *
6.00E 09	5.3257E 08	8.2904E-02 *
7.00E 09	6.1469E 08	8.1380E-02 *
8.00E 09	6.9533E 08	7.9888E-02 *
9.00E 09	7.7445E 08	7.8420E-02 *
1.00E 10	8.5211E 08	7.6975E-02 *
2.00E 10	1.5551E 09	6.3931E-02 *
3.00E 10	2.1399E 09	5.3066E-02 *
4.00E 10	2.6260E 09	4.4050E-02 *
5.00E 10	3.0292E 09	3.6567E-02 *
6.00E 10	3.3634E 09	3.0375E-02 *
7.00E 10	3.6403E 09	2.5256E-02 *
8.00E 10	3.8690E 09	2.0996E-02 *
9.00E 10	4.0583E 09	1.7470E-02 *
1.00E 11	4.2155E 09	1.4560E-02 *
2.00E 11	4.8697E 09	2.3993E-03 *
3.00E 11	4.9822E 09	3.0237E-04 *

$$r_{eD} = 1 \times 10^6$$

t_D	QD	q_D
1.00E 10	8.7486E 08	8.3654E-02
2.00E 10	1.6983E 09	8.1304E-02
3.00E 10	2.5044E 09	7.9990E-02
4.00E 10	3.2977E 09	7.9082E-02
5.00E 10	4.0870E 09	7.8392E-02
6.00E 10	4.8682E 09	7.7837E-02
7.00E 10	5.6443E 09	7.7373E-02
8.00E 10	6.4161E 09	7.6976E-02
9.00E 10	7.1841E 09	7.6630E-02
1.00E 11	7.9488E 09	7.6323E-02
2.00E 11	1.5470E 10	7.4332E-02
3.00E 11	2.2835E 10	7.3027E-02
4.00E 11	3.0680E 10	7.1877E-02 *
5.00E 11	3.7211E 10	7.0777E-02 *
6.00E 11	4.4234E 10	6.9702E-02 *
7.00E 11	5.1150E 10	6.8645E-02 *
8.00E 11	5.7961E 10	6.7604E-02 *
9.00E 11	6.4668E 10	6.6581E-02 *
1.00E 12	7.1273E 10	6.5567E-02 *
2.00E 12	1.3209E 11	5.6268E-02 *
3.00E 12	1.8434E 11	4.8268E-02 *
4.00E 12	2.2924E 11	4.1406E-02 *
5.00E 12	2.6777E 11	3.5513E-02 *
6.00E 12	3.0081E 11	3.0466E-02 *
7.00E 12	3.2910E 11	2.6143E-02 *
8.00E 12	3.5332E 11	2.2442E-02 *
9.00E 12	3.7400E 11	1.9260E-02 *
1.00E 13	3.9176E 11	1.6559E-02 *
2.00E 13	4.7554E 11	3.7418E-03 *
3.00E 13	4.9463E 11	8.1472E-04 *
4.00E 13	4.9933E 11	1.0647E-04 *

$$r_{eD} = 5 \times 10^7$$

t_D	QD	q_D
1.00E 10	8.7487E 08	8.3654E-02
2.00E 10	1.6983E 09	8.1302E-02
3.00E 10	2.5043E 09	7.9989E-02
4.00E 10	3.2993E 09	7.9078E-02
5.00E 10	4.0963E 09	7.8358E-02
6.00E 10	4.8666E 09	7.7740E-02
7.00E 10	5.6412E 09	7.7176E-02
8.00E 10	6.4102E 09	7.6644E-02 *
9.00E 10	7.1741E 09	7.6130E-02 *
1.00E 11	7.9328E 09	7.5629E-02 *
2.00E 11	1.5254E 10	7.0890E-02 *
3.00E 11	2.2118E 10	6.6454E-02 *
4.00E 11	2.8555E 10	6.2297E-02 *
5.00E 11	3.4594E 10	5.8412E-02 *
6.00E 11	4.0256E 10	5.4743E-02 *
7.00E 11	4.5568E 10	5.1311E-02 *
8.00E 11	5.0550E 10	4.8091E-02 *
9.00E 11	5.5221E 10	4.5071E-02 *
1.00E 12	5.9602E 10	4.2242E-02 *
2.00E 12	9.0750E 10	2.2133E-02 *
3.00E 12	1.0696E 11	1.1693E-02 *
4.00E 12	1.1533E 11	6.2397E-03 *
5.00E 12	1.1982E 11	3.3494E-03 *
6.00E 12	1.2244E 11	1.7632E-03 *
7.00E 12	1.2358E 11	9.2902E-04 *
8.00E 12	1.2432E 11	4.6204E-04 *
9.00E 12	1.2460E 11	1.8229E-04 *
1.00E 13	1.2493E 11	4.8436E-05 *

CLOSED OUTER BOUNDARY

Skin = 5

$r_{eD} = 20$			$r_{eD} = 200$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 02	1.37550 01	1.28210-01	1.000 04	1.64710 03	9.92550-02
2.000 02	2.61010 01	1.19420-01	2.000 04	2.51470 03	9.40100-02 *
3.000 02	3.76410 01	1.11420-01	3.000 04	2.93050 03	8.92320-02 *
4.000 02	4.84160 01	1.04060-01	4.000 04	3.29920 03	8.46850-02 *
5.000 02	5.84860 01	9.71530-02	5.000 04	4.62530 03	8.03940-02 *
6.000 02	6.78860 01	9.06660-02	6.000 04	5.48900 03	7.62870-02 *
7.000 02	7.66620 01	8.45990-02	7.000 04	6.15220 03	7.23860-02 *
8.000 02	8.48630 01	7.89530-02	8.000 04	6.85940 03	6.86920-02 *
9.000 02	9.25190 01	7.36800-02	9.000 04	7.53040 03	6.51060-02 *
1.000 03	9.96670 01	6.87620-02	1.000 05	8.16760 03	6.18680-02 *
2.000 03	1.49360 02	3.45030-02	2.000 05	1.29950 04	3.66100-02 *
3.000 03	1.74000 02	1.74700-02	3.000 05	1.58360 04	2.17880-02 *
4.000 03	1.96480 02	9.02120-03	4.000 05	1.75120 04	1.30750-02 *
5.000 03	1.92720 02	4.57260-03	5.000 05	1.84930 04	7.81020-03 *
6.000 03	1.96060 02	2.33070-03	6.000 05	1.90930 04	4.76060-03 *
7.000 03	1.97770 02	1.09310-03	7.000 05	1.94450 04	2.80800-03 *
8.000 03	1.98740 02	4.72510-04	8.000 05	1.95810 04	1.72210-03 *
9.000 03	1.99110 02	9.09900-05	9.000 05	1.98160 04	9.80330-04 *
			1.000 06	1.98810 04	4.91660-04 *

$r_{eD} = 50$			$r_{eD} = 500$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 03	1.18860 02	1.10600-01 *	1.000 05	9.34450 03	8.83050-02 *
2.000 03	2.24190 02	1.00280-01 *	2.000 05	1.78440 04	8.18070-02 *
3.000 03	3.19740 02	9.09180-02 *	3.000 05	2.57190 04	7.57880-02 *
4.000 03	4.06560 02	8.24640-02 *	4.000 05	3.30210 04	7.02300-02 *
5.000 03	4.85200 02	7.47450-02 *	5.000 05	3.97890 04	6.50480-02 *
6.000 03	5.56650 02	6.77710-02 *	6.000 05	4.60660 04	6.02690-02 *
7.000 03	6.21420 02	6.14230-02 *	7.000 05	5.18820 04	5.58120-02 *
8.000 03	6.80230 02	5.57150-02 *	8.000 05	5.72820 04	5.17330-02 *
9.000 03	7.33350 02	5.04750-02 *	9.000 05	6.22750 04	4.79050-02 *
1.000 04	7.81510 02	4.57410-02 *	1.000 06	6.68940 04	4.43400-02 *
2.000 04	1.07200 03	1.73770-02 *	2.000 06	9.78860 04	2.07110-02 *
3.000 04	1.18070 03	6.73370-03 *	3.000 06	1.12160 05	9.80100-03 *
4.000 04	1.22220 03	2.53600-03 *	4.000 06	1.18850 05	4.86570-03 *
5.000 04	1.24050 03	9.83310-04 *	5.000 06	1.22080 05	2.23300-03 *
6.000 04	1.24680 03	2.65390-04 *	6.000 06	1.23650 05	1.02740-03 *
			7.000 06	1.24450 05	4.34720-04 *
			8.000 06	1.24840 05	1.48560-04 *

$r_{eD} = 100$			$r_{eD} = 1000$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 03	1.19240 02	1.12230-01	1.000 05	9.36710 03	8.92820-02
2.000 03	2.29090 02	1.07790-01	2.000 05	1.81430 04	8.65710-02
3.000 03	3.35540 02	1.05230-01	3.000 05	2.67080 04	8.47930-02
4.000 03	4.39640 02	1.02810-01 *	4.000 05	3.51090 04	8.32390-02 *
5.000 03	5.41260 02	1.00500-01 *	5.000 05	4.33570 04	8.17490-02 *
6.000 03	6.40620 02	9.82580-02 *	6.000 05	5.14580 04	8.03060-02 *
7.000 03	7.37760 02	9.60490-02 *	7.000 05	5.94140 04	7.89750-02 *
8.000 03	8.32750 02	9.39330-02 *	8.000 05	6.72310 04	7.74810-02 *
9.000 03	9.25620 02	9.18390-02 *	9.000 05	7.49080 04	7.61070-02 *
1.000 04	1.01640 03	8.97850-02 *	1.000 06	8.24470 04	7.47510-02 *
2.000 04	1.82120 03	7.16480-02 *	2.000 06	1.50900 05	6.24930-02 *
3.000 04	2.45430 03	5.71300-02 *	3.000 06	2.08270 05	5.22180-02 *
4.000 04	2.97700 03	4.95470-02 *	4.000 06	2.56250 05	4.36340-02 *
5.000 04	3.38640 03	3.63830-02 *	5.000 06	2.96320 05	3.64090-02 *
6.000 04	3.71090 03	2.90520-02 *	6.000 06	3.29770 05	3.04660-02 *
7.000 04	3.95970 03	2.32250-02 *	7.000 06	3.57640 05	2.59950-02 *
8.000 04	4.17410 03	1.86050-02 *	8.000 06	3.80860 05	2.13320-02 *
9.000 04	4.33720 03	1.49090-02 *	9.000 06	4.00170 05	1.78590-02 *
1.000 05	4.45910 03	1.20190-02 *	1.000 07	4.16340 05	1.49040-02 *
2.000 05	4.94150 03	1.30720-03 *	2.000 07	4.05250 05	2.64160-04 *
			3.000 07	4.97780 05	3.87950-04 *

CLOSED OUTER BOUNDARY
Skin = 5

$r_{eD} = 2000$

t_D	QD	q_D
1.00D 05	9.7674D 03	8.9204D-02
2.00D 05	1.8147D 04	8.6613D-02
3.00D 05	2.6730D 04	8.5122D-02
4.00D 05	3.5188D 04	8.4096D-02
5.00D 05	4.3556D 04	8.3318D-02
6.00D 05	5.1855D 04	8.2690D-02
7.00D 05	6.0095D 04	8.2157D-02
8.00D 05	6.8286D 04	8.1686D-02
9.00D 05	7.6432D 04	8.1257D-02
1.00D 06	8.4537D 04	8.0856D-02
2.00D 06	1.6361D 05	7.7393D-02 *
3.00D 06	2.3938D 05	7.4200D-02 *
4.00D 06	3.1202D 05	7.1139D-02 *
5.00D 06	3.8170D 05	6.8216D-02 *
6.00D 06	4.4849D 05	6.5395D-02 *
7.00D 06	5.1255D 05	6.2690D-02 *
8.00D 06	5.7399D 05	6.0100D-02 *
9.00D 06	6.3293D 05	5.7615D-02 *
1.00D 07	6.8747D 05	5.5235D-02 *
2.00D 07	1.1414D 06	3.6183D-02 *
3.00D 07	1.4367D 06	2.3735D-02 *
4.00D 07	1.6291D 06	1.5638D-02 *
5.00D 07	1.7544D 06	1.0343D-02 *
6.00D 07	1.8368D 06	6.8749D-03 *
7.00D 07	1.8909D 06	4.5517D-03 *
8.00D 07	1.9277D 06	3.0479D-03 *
9.00D 07	1.9518D 06	2.0043D-03 *
1.00D 08	1.9682D 06	1.3138D-03 *

$r_{eD} = 1x10^4$

t_D	QD	q_D
1.00D 07	7.7072D 05	7.4096D-02
2.00D 07	1.5012D 06	7.2218D-02
3.00D 07	2.2170D 06	7.0986D-02
4.00D 07	2.9213D 06	6.9901D-02 *
5.00D 07	3.6151D 06	6.8861D-02 *
6.00D 07	4.2985D 06	6.7847D-02 *
7.00D 07	4.9718D 06	6.6847D-02 *
8.00D 07	5.6352D 06	6.5865D-02 *
9.00D 07	6.2888D 06	6.4895D-02 *
1.00D 08	6.9327D 06	6.3936D-02 *
2.00D 08	1.2876D 07	5.5115D-02 *
3.00D 08	1.8005D 07	4.7493D-02 *
4.00D 08	2.2432D 07	4.0925D-02 *
5.00D 08	2.6248D 07	3.5239D-02 *
6.00D 08	2.9536D 07	3.0383D-02 *
7.00D 08	3.2365D 07	2.6187D-02 *
8.00D 08	3.4797D 07	2.2579D-02 *
9.00D 08	3.6884D 07	1.9462D-02 *
1.00D 09	3.8684D 07	1.6804D-02 *
2.00D 09	4.7333D 07	3.9600D-03 *
3.00D 09	4.9382D 07	9.1110D-04 *
4.00D 09	4.9906D 07	1.4250D-04 *

$r_{eD} = 5x10^4$

$r_{eD} = 5000$

t_D	QD	q_D
1.00D 06	8.4570D 04	8.0987D-02
2.00D 06	1.6433D 05	7.8780D-02
3.00D 06	2.4244D 05	7.7547D-02
4.00D 06	3.1953D 05	7.6690D-02
5.00D 06	3.9587D 05	7.6014D-02
6.00D 06	4.7158D 05	7.5431D-02
7.00D 06	5.4674D 05	7.4901D-02
8.00D 06	6.2139D 05	7.4400D-02 *
9.00D 06	6.9554D 05	7.3917D-02 *
1.00D 07	7.6922D 05	7.3446D-02 *
2.00D 07	1.4810D 06	6.8983D-02 *
3.00D 07	2.1494D 06	6.4792D-02 *
4.00D 07	2.7777D 06	6.0868D-02 *
5.00D 07	3.3680D 06	5.7169D-02 *
6.00D 07	3.9230D 06	5.3700D-02 *
7.00D 07	4.443D 06	5.0428D-02 *
8.00D 07	4.9348D 06	4.7376D-02 *
9.00D 07	5.3951D 06	4.4464D-02 *
1.00D 08	5.8276D 06	4.1768D-02 *
2.00D 08	8.9360D 06	2.3320D-02 *
3.00D 08	1.0580D 07	1.2019D-02 *
4.00D 08	1.1753D 07	6.5326D-03 *
5.00D 08	1.2929D 07	3.5742D-03 *
6.00D 08	1.2189D 07	1.9446D-03 *
7.00D 08	1.2359D 07	1.0407D-03 *
8.00D 08	1.2418D 07	5.3721D-04 *
9.00D 08	1.2459D 07	2.3230D-04 *
1.00D 09	1.2488D 07	8.6530D-05 *

t_D	QD	q_D
1.00D 08	7.0799D 06	6.8286D-02
2.00D 08	1.3821D 07	6.6709D-02
3.00D 08	2.0444D 07	6.5821D-02
4.00D 08	2.6994D 07	6.5202D-02
5.00D 08	3.3488D 07	6.4713D-02
6.00D 08	3.9938D 07	6.4292D-02
7.00D 08	4.6348D 07	6.3908D-02
8.00D 08	5.2720D 07	6.3545D-02 *
9.00D 08	5.9057D 07	6.3194D-02 *
1.00D 09	6.5359D 07	6.2853D-02 *
2.00D 09	1.2656D 08	5.9600D-02 *
3.00D 09	1.8459D 08	5.6517D-02 *
4.00D 09	2.3954D 08	5.3604D-02 *
5.00D 09	2.9185D 08	5.0831D-02 *
6.00D 09	3.4140D 08	4.8204D-02 *
7.00D 09	3.8840D 08	4.5707D-02 *
8.00D 09	4.3302D 08	4.3355D-02 *
9.00D 09	4.7532D 08	4.1102D-02 *
1.00D 10	5.1545D 08	3.8965D-02 *
2.00D 10	8.1856D 08	2.2893D-02 *
3.00D 10	9.9543D 08	1.3506D-02 *
4.00D 10	1.0986D 09	8.0194D-03 *
5.00D 10	1.1595D 09	4.7966D-03 *
6.00D 10	1.1957D 09	2.8782D-03 *
7.00D 10	1.2179D 09	1.7356D-03 *
8.00D 10	1.2309D 09	1.0139D-03 *
9.00D 10	1.2389D 09	5.7250D-04 *
1.00D 11	1.2448D 09	3.4051D-04 *

CLOSED OUTER BOUNDARY

Skin = 5

$$r_{eD} = 1 \times 10^5$$

t_D	QD	q_D
1.00D 09	6.5466D 07	6.3316E-02
2.00D 09	1.2803D 08	6.1940E-02
3.00D 09	1.0949D 08	6.1034E-02
4.00D 09	2.5017D 08	6.0236E-02 *
5.00D 09	3.0986D 08	5.9469E-02 *
6.00D 09	3.6905D 08	5.8720E-02 *
7.00D 09	4.2739D 08	5.7986E-02 *
8.00D 09	4.6500D 08	5.7252E-02 *
9.00D 09	5.4187D 08	5.6531E-02 *
1.00D 10	5.9802D 08	5.5817E-02 *
2.00D 10	1.1222D 09	4.9171E-02 *
3.00D 10	1.5845D 09	4.3305E-02 *
4.00D 10	1.9921D 09	3.8137E-02 *
5.00D 10	2.3515D 09	3.3581E-02 *
6.00D 10	2.6681D 09	2.9571E-02 *
7.00D 10	2.9468D 09	2.6045E-02 *
8.00D 10	3.1917D 09	2.2934E-02 *
9.00D 10	3.4089D 09	2.0197E-02 *
1.00D 11	3.5943D 09	1.7800E-02 *
2.00D 11	4.5945D 09	5.1313E-03 *
3.00D 11	4.8809D 09	1.4940E-03 *
4.00D 11	4.9715D 09	4.2229E-04 *
5.00D 11	4.9959D 09	5.6351E-05 *

$$r_{eD} = 1 \times 10^6$$

t_D	QD	q_D
1.00D 10	6.0892D 08	5.9022E-02
2.00D 10	1.1925D 09	5.7841E-02
3.00D 10	1.7674D 09	5.7172E-02
4.00D 10	2.3368D 09	5.6706E-02
5.00D 10	2.9021D 09	5.6356E-02
6.00D 10	3.4642D 09	5.6062E-02
7.00D 10	4.0236D 09	5.5820E-02
8.00D 10	4.5808D 09	5.5613E-02
9.00D 10	5.1361D 09	5.5435E-02
1.00D 11	5.6960D 09	5.5271E-02
2.00D 11	1.1139D 10	5.4219E-02
3.00D 11	1.6044D 10	5.3226E-02
4.00D 11	2.1846D 10	5.2913E-02 *
5.00D 11	2.7128D 10	5.2326E-02 *
6.00D 11	3.2331D 10	5.1749E-02 *
7.00D 11	3.7477D 10	5.1180E-02 *
8.00D 11	4.2546D 10	5.0617E-02 *
9.00D 11	4.7599D 10	5.0062E-02 *
1.00D 12	5.2675D 10	4.9516E-02 *
2.00D 12	9.9438D 10	4.4325E-02 *
3.00D 12	1.4142E 11	3.9675E-02 *
4.00D 12	1.7905D 11	3.5514E-02 *
5.00D 12	2.1276D 11	3.1783E-02 *
6.00D 12	2.4295D 11	2.8445E-02 *
7.00D 12	2.6998D 11	2.5457E-02 *
8.00D 12	2.9415D 11	2.2783E-02 *
9.00D 12	3.1574D 11	2.0366E-02 *
1.00D 13	3.3507D 11	1.8253E-02 *
2.00D 13	4.4460D 11	6.1309E-03 *
3.00D 13	4.8095D 11	2.1029E-03 *
4.00D 13	4.9359D 11	7.0050E-04 *
5.00D 13	4.9834D 11	2.0504E-04 *
6.00D 13	4.9985D 11	1.5131E-05 *

$$r_{eD} = 5 \times 10^5$$

t_D	QD	q_D
1.00D 10	6.0383D 08	5.9022E-02
2.00D 10	1.1925D 09	5.7840E-02
3.00D 10	1.7673D 09	5.7171E-02
4.00D 10	2.3366D 09	5.6704E-02
5.00D 10	2.9017D 09	5.6333E-02
6.00D 10	3.4634D 09	5.6014E-02
7.00D 10	4.0221D 09	5.5723E-02
8.00D 10	4.5779D 09	5.5448E-02 *
9.00D 10	5.1311E 09	5.5183E-02 *
1.00D 11	5.6816D 09	5.4923E-02 *
2.00D 11	1.1048E 10	5.2448E-02 *
3.00D 11	1.6173D 10	5.0087E-02 *
4.00D 11	2.1067E 10	4.7835E-02 *
5.00D 11	2.5744E 10	4.5691E-02 *
6.00D 11	3.0210E 10	4.3630E-02 *
7.00D 11	3.4477E 10	4.1655E-02 *
8.00D 11	3.8554E 10	3.9788E-02 *
9.00D 11	4.2450E 10	3.7994E-02 *
1.00D 12	4.6171E 10	3.6201E-02 *
2.00D 12	7.5329E 10	2.2867E-02 *
3.00D 12	9.4629E 10	1.4440E-02 *
4.00D 12	1.0508D 11	9.1634E-03 *
5.00D 12	1.1222D 11	5.8518E-03 *
6.00D 12	1.1685D 11	3.7499E-03 *
7.00D 12	1.1978D 11	2.4087E-03 *
8.00D 12	1.2167D 11	1.5454E-03 *
9.00D 12	1.2287D 11	9.6747E-04 *
1.00D 13	1.2369D 11	6.0519E-04 *

CLOSED OUTER BOUNDARY
Skin = 10

$r_{eD} = 20$			$r_{eD} = 200$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00E 02	8.1484E 00	7.8277E-02	1.00E 04	6.6848E 02	6.6411E-07
2.00E 02	1.5616E 01	7.5934E-02	2.00E 04	1.3406E 03	6.4080E-02 *
3.00E 02	2.3151E 01	7.2631E-02	3.00E 04	1.9704E 03	6.1917E-02 *
4.00E 02	3.0217E 01	6.9152E-02	4.00E 04	2.5787E 03	5.9827E-02 *
5.00E 02	3.6794E 01	6.5697E-02	5.00E 04	3.1671E 03	5.7815E-02 *
6.00E 02	4.2900E 01	6.2335E-02	6.00E 04	3.7353E 03	5.5882E-02 *
7.00E 02	4.8745E 01	5.9174E-02	7.00E 04	4.2843E 03	5.3971E-02 *
8.00E 02	5.4345E 01	5.6228E-02	8.00E 04	4.8151E 03	5.2148E-02 *
9.00E 02	6.1597E 01	5.3370E-02	9.00E 04	5.3281E 03	5.0386E-02 *
1.00E 03	6.7041E 01	5.4111E-02	1.00E 05	5.8242E 03	4.8689E-02 *
2.00E 03	1.1158E 02	3.5905E-02	2.00E 05	9.9543E 03	3.4496E-02 *
3.00E 03	1.4166E 02	2.3835E-02	3.00E 05	1.2882E 04	2.4459E-02 *
4.00E 03	1.6057E 02	1.5927E-02	4.00E 05	1.4949E 04	1.7382E-02 *
5.00E 03	1.7337E 02	1.0630E-02	5.00E 05	1.6399E 04	1.2335E-02 *
6.00E 03	1.8193E 02	7.1576E-03	6.00E 05	1.7429E 04	8.8367E-03 *
7.00E 03	1.8763E 02	4.8060E-03	7.00E 05	1.8153E 04	6.2937E-03 *
8.00E 03	1.9151E 02	3.2425E-03	8.00E 05	1.8683E 04	4.5532E-03 *
9.00E 03	1.9494E 02	2.1452E-03	9.00E 05	1.9053E 04	3.2626E-03 *
1.00E 04	1.9581E 02	1.4194E-03	1.00E 06	1.9309E 04	2.3021E-03 *
			2.00E 06	2.0006E 04	8.6366E-06 *
$r_{eD} = 50$			$r_{eD} = 500$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00E 03	7.4696E 01	7.1371E-02 *	1.00E 05	6.3716E 03	6.1335E-02 *
2.00E 03	1.4383E 02	6.7159E-02 *	2.00E 05	1.2348E 04	5.8240E-02 *
3.00E 03	2.0877E 02	6.3190E-02 *	3.00E 05	1.8032E 04	5.5303E-02 *
4.00E 03	2.7031E 02	5.9482E-02 *	4.00E 05	2.3412E 04	5.2524E-02 *
5.00E 03	3.2804E 02	5.5967E-02 *	5.00E 05	2.8531E 04	4.9870E-02 *
6.00E 03	3.8241E 02	5.2670E-02 *	6.00E 05	3.3395E 04	4.7362E-02 *
7.00E 03	4.3360E 02	4.9554E-02 *	7.00E 05	3.8015E 04	4.4966E-02 *
8.00E 03	4.8184E 02	4.6644E-02 *	8.00E 05	4.2410E 04	4.2715E-02 *
9.00E 03	5.2717E 02	4.3872E-02 *	9.00E 05	4.6579E 04	4.0551E-02 *
1.00E 04	5.6986E 02	4.1265E-02 *	1.00E 06	5.0537E 04	3.8486E-02 *
2.00E 04	8.7973E 02	2.2468E-02 *	2.00E 06	8.0570E 04	2.9224E-02 *
3.00E 04	1.0468E 03	1.2314E-02 *	3.00E 06	9.8560E 04	1.3700E-02 *
4.00E 04	1.1369E 03	6.7761E-03 *	4.00E 06	1.0904E 05	8.2425E-03 *
5.00E 04	1.1674E 03	3.8191E-03 *	5.00E 06	1.1534E 05	4.9949E-03 *
6.00E 04	1.2148E 03	2.1045E-03 *	6.00E 06	1.1913E 05	3.0435E-03 *
7.00E 04	1.2303E 03	1.1414E-03 *	7.00E 06	1.2145E 05	1.8409E-03 *
8.00E 04	1.2395E 03	6.0470E-04 *	8.00E 06	1.2268E 05	1.1095E-03 *
9.00E 04	1.2444E 03	2.8460E-04 *	9.00E 06	1.2373E 05	6.3734E-04 *
1.00E 05	1.2484E 03	1.4265E-04 *	1.00E 07	1.2431E 05	3.6016E-04 *
$r_{eD} = 100$			$r_{eD} = 1000$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00E 03	7.4749E 01	7.1987E-02	1.00E 05	6.3817E 03	6.1775E-02
2.00E 03	1.4575E 02	7.0217E-02	2.00E 05	1.2466E 04	6.0464E-02
3.00E 03	2.1535E 02	6.9052E-02	3.00E 05	1.8488E 04	5.9601E-02
4.00E 03	2.8389E 02	6.8027E-02 *	4.00E 05	2.4410E 04	5.8840E-02 *
5.00E 03	3.5142E 02	6.7043E-02 *	5.00E 05	3.0257E 04	5.8109E-02 *
6.00E 03	4.1797E 02	6.6084E-02 *	6.00E 05	3.6031E 04	5.7395E-02 *
7.00E 03	4.8357E 02	6.5138E-02 *	7.00E 05	4.1754E 04	5.6689E-02 *
8.00E 03	5.4823E 02	6.4307E-02 *	8.00E 05	4.7367E 04	5.5995E-02 *
9.00E 03	6.1196E 02	6.3388E-02 *	9.00E 05	5.2931E 04	5.5307E-02 *
1.00E 04	6.7476E 02	6.2379E-02 *	1.00E 06	5.8425E 04	5.4624E-02 *
2.00E 04	1.2557E 03	5.4001E-02 *	2.00E 06	1.0960E 05	4.8270E-02 *
3.00E 04	1.7593E 03	4.6731E-02 *	3.00E 06	1.5526E 05	4.2643E-02 *
4.00E 04	2.1956E 03	4.0429E-02 *	4.00E 06	1.9545E 05	3.7673E-02 *
5.00E 04	2.5737E 03	3.4998E-02 *	5.00E 06	2.3100E 05	3.3272E-02 *
6.00E 04	2.9006E 03	3.0278E-02 *	6.00E 06	2.6241E 05	2.9353E-02 *
7.00E 04	3.1829E 03	2.6197E-02 *	7.00E 06	2.9013E 05	2.5964E-02 *
8.00E 04	3.4270E 03	2.2681E-02 *	8.00E 06	3.1453E 05	2.2957E-02 *
9.00E 04	3.6375E 03	1.9637E-02 *	9.00E 06	3.3618E 05	2.0258E-02 *
1.00E 05	3.8201E 03	1.7041E-02 *	1.00E 07	3.5524E 05	1.7912E-02 *
2.00E 05	4.7192E 03	4.1732E-03 *	2.00E 07	4.5699E 05	1.3213E-03 *
3.00E 05	4.9314E 03	1.9913E-03 *	3.00E 07	4.8698E 05	1.6050E-03 *
4.00E 05	4.9710E 03	2.1742E-04 *	4.00E 07	4.9642E 05	4.4569E-04 *
			5.00E 07	4.9951E 05	8.6115E-05 *

CLOSED OUTER BOUNDARY
Skin = 10

$r_{eD} = 2000$			$r_{eD} = 1 \times 10^4$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00D 05	6.3819D 03	6.1776D-02	1.00D 07	5.5647D 05	5.4077D-02
2.00D 05	1.2488D 04	6.0483D-02	2.00D 07	1.0918D 06	5.3084D-02
3.00D 05	1.8498D 04	5.9750D-02	3.00D 07	1.6192D 06	5.2419D-02
4.00D 05	2.4447D 04	5.9242D-02	4.00D 07	2.1404D 06	5.1833D-02
5.00D 05	3.0351D 04	5.8855D-02	5.00D 07	2.6559D 06	5.1269D-02
6.00D 05	3.6220D 04	5.8540D-02	6.00D 07	3.1658D 06	5.0717D-02
7.00D 05	4.2060D 04	5.8273D-02	7.00D 07	3.6702D 06	5.0170D-02
8.00D 05	4.7875D 04	5.8046D-02	8.00D 07	4.1691D 06	4.9631D-02
9.00D 05	5.3667D 04	5.7820D-02	9.00D 07	4.6626D 06	4.9097D-02
1.00D 06	5.9439D 04	5.7618D-02	1.00D 08	5.1508D 06	4.8567D-02
2.00D 06	1.1616D 05	5.5855D-02	2.00D 08	9.7531D 06	4.7584D-02
3.00D 06	1.7120D 05	5.4234D-02	3.00D 08	1.3886D 07	3.9105D-02
4.00D 06	2.2462D 05	5.2650D-02	4.00D 08	1.7598D 07	3.5087D-02
5.00D 06	2.7450D 05	5.1118D-02	5.00D 08	2.0933D 07	3.1476D-02
6.00D 06	3.2685D 05	4.9622D-02	6.00D 08	2.3926D 07	2.8237D-02
7.00D 06	3.7573D 05	4.8170D-02	7.00D 08	2.6612D 07	2.5330D-02
8.00D 06	4.2319D 05	4.6762D-02	8.00D 08	2.9021D 07	2.2723D-02
9.00D 06	4.6929D 05	4.5394D-02	9.00D 08	3.1177D 07	2.0378D-02
1.00D 07	5.1405D 05	4.4069D-02	1.00D 09	3.3112D 07	1.8291D-02
2.00D 07	8.9614D 05	3.2734D-02	2.00D 09	4.4199D 07	6.2824D-03
3.00D 07	1.1803D 06	2.4307D-02	3.00D 09	4.7960D 07	2.2043D-03
4.00D 07	1.3908D 06	1.8070D-02	4.00D 09	4.9304D 07	7.5592D-04
5.00D 07	1.5463D 06	1.3450D-02	5.00D 09	4.9805D 07	2.3277D-04
6.00D 07	1.6614D 06	1.0039D-02	6.00D 09	4.9973D 07	2.8205D-05
7.00D 07	1.7465D 06	7.4936D-03			
8.00D 07	1.8102D 06	5.6305D-03			
9.00D 07	1.8573D 06	4.2169D-03			
1.00D 08	1.8927D 06	3.1671D-03			
2.00D 08	1.9766D 06	1.2622D-04			
$r_{eD} = 5000$			$r_{eD} = 5 \times 10^4$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00D 06	5.9455D 04	5.7680D-02	1.00D 08	5.2300D 06	5.0925D-02
2.00D 06	1.1651D 05	5.6550D-02	2.00D 08	1.0774D 07	5.0042D-02
3.00D 06	1.7272D 05	5.5911D-02	3.00D 08	1.5251D 07	4.9541D-02
4.00D 06	2.2839D 05	5.5464D-02	4.00D 08	2.0167D 07	4.9189D-02
5.00D 06	2.8367D 05	5.5109D-02	5.00D 08	2.5092D 07	4.8911D-02
6.00D 06	3.3863D 05	5.4804D-02	6.00D 08	2.9970D 07	4.8670D-02
7.00D 06	3.9329D 05	5.4526D-02	7.00D 08	3.4827D 07	4.8451D-02
8.00D 06	4.4769D 05	5.4262D-02	8.00D 08	3.9661D 07	4.8244D-02
9.00D 06	5.0182D 05	5.4009D-02	9.00D 08	4.4476D 07	4.8044D-02
1.00D 07	5.5571D 05	5.3760D-02	1.00D 09	4.9271D 07	4.7848D-02
2.00D 07	1.0813D 06	5.1390D-02	2.00D 09	9.6170D 07	4.5976D-02
3.00D 07	1.5837D 06	4.9126D-02	3.00D 09	1.4123D 08	4.4178D-02
4.00D 07	2.0640D 06	4.6969D-02	4.00D 09	1.8453D 08	4.2456D-02
5.00D 07	2.5232D 06	4.4901D-02	5.00D 09	2.2614D 08	4.0796D-02
6.00D 07	2.9625D 06	4.2927D-02	6.00D 09	2.6615D 08	3.9205D-02
7.00D 07	3.3824D 06	4.1032D-02	7.00D 09	3.0459D 08	3.7670D-02
8.00D 07	3.7844D 06	3.9235D-02	8.00D 09	3.4157D 08	3.6205D-02
9.00D 07	4.1684D 06	3.7499D-02	9.00D 09	3.7709D 08	3.4784D-02
1.00D 08	4.5358D 06	3.5841D-02	1.00D 10	4.1123D 08	3.3419D-02
2.00D 08	7.4360D 06	2.8822D-02	2.00D 10	6.8764D 08	2.2410D-02
3.00D 08	9.2657D 06	1.4555D-02	3.00D 10	8.7262D 08	1.5037D-02
4.00D 08	1.0426D 07	9.3271D-03	4.00D 10	9.9597D 08	1.0118D-02
5.00D 08	1.1164D 07	6.0129D-03	5.00D 10	1.0782D 09	6.8415D-03
6.00D 08	1.1635D 07	3.8901D-03	6.00D 10	1.1334D 09	4.6456D-03
7.00D 08	1.1941D 07	2.5235D-03	7.00D 10	1.1709D 09	3.1741D-03
8.00D 08	1.2139D 07	1.6363D-03	8.00D 10	1.1960D 09	2.1539D-03
9.00D 08	1.2266D 07	1.0382D-03	9.00D 10	1.2151D 09	1.4570D-03
1.00D 09	1.2354D 07	6.5914D-04	1.03D 11	1.2258D 09	1.0083D-03

CLOSED OUTER BOUNDARY
Skin = 10

$r_{eD} = 1 \times 10^5$			$r_{eD} = 1 \times 10^6$		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 07	4.93300 07	4.61070-02	1.000 10	4.66910 09	4.55860-02
2.000 09	9.69940 07	4.73090-02	2.000 10	9.18760 08	4.48770-02
3.000 09	1.44030 08	4.67810-02	3.000 10	1.36540 09	4.44730-02
4.000 09	1.90580 08	4.63140-02 *	4.000 10	1.80860 09	4.41910-02
5.000 09	2.36670 08	4.58660-02 *	5.000 10	2.24960 09	4.39740-02
6.000 09	2.82310 08	4.54260-02 *	6.000 10	2.68850 09	4.37980-02
7.000 09	3.27510 08	4.49910-02 *	7.000 10	3.12580 09	4.36510-02
8.000 09	3.72280 08	4.45610-02 *	8.000 10	3.56170 09	4.35240-02
9.000 09	4.16620 08	4.41340-02 *	9.000 10	3.99640 09	4.34130-02
1.000 10	4.60530 08	4.37100-02 *	1.000 11	4.43010 09	4.33140-02
2.000 10	8.77170 09	3.96990-02 *	2.000 11	8.72550 09	4.26650-02
3.000 10	1.25570 09	3.60510-02 *	3.000 11	1.29700 10	4.22360-02
4.000 10	1.59990 09	3.27390-02 *	4.000 11	1.71750 10	4.18560-02 *
5.000 10	1.91270 09	2.97260-02 *	5.000 11	2.13420 10	4.14920-02 *
6.000 10	2.19700 09	2.69920-02 *	6.000 11	2.54730 10	4.11330-02 *
7.000 10	2.45530 09	2.45100-02 *	7.000 11	2.95680 10	4.07790-02 *
8.000 10	2.68970 09	2.22490-02 *	8.000 11	3.36280 10	4.04270-02 *
9.000 10	2.90230 09	2.01970-02 *	9.000 11	3.76530 10	4.00790-02 *
1.000 11	3.09340 09	1.83400-02 *	1.000 12	4.16420 10	3.97320-02 *
2.000 11	4.26650 09	7.05660-03 *	2.000 12	7.96930 10	3.64340-02 *
3.000 11	4.71110 09	2.77100-03 *	3.000 12	1.14590 11	3.34060-02 *
4.000 11	4.88910 09	1.10930-03 *	4.000 12	1.46620 11	3.06310-02 *
5.000 11	4.95820 09	4.05930-04 *	5.000 12	1.76020 11	2.80850-02 *
6.000 11	4.98630 09	1.13530-04 *	6.000 12	2.02990 11	2.57470-02 *
			7.000 12	2.27730 11	2.36040-02 *
			8.000 12	2.50420 11	2.16370-02 *
			9.000 12	2.71210 11	1.98290-02 *
			1.000 13	2.90280 11	1.81820-02 *
			2.000 13	4.11450 11	7.67580-03 *
			3.000 13	4.61900 11	3.29930-03 *
			4.000 13	4.83340 11	1.42900-03 *
			5.000 13	4.93120 11	6.11300-04 *
			6.000 13	4.97310 11	2.32360-04 *
			7.000 13	4.99520 11	8.19870-05 *

$r_{eD} = 5 \times 10^5$		
t_D	Q_D	q_D
1.000 10	4.66910 09	4.55860-02
2.000 10	9.18770 08	4.48760-02
3.000 10	1.36540 09	4.44730-02
4.000 10	1.80860 09	4.41890-02
5.000 10	2.24960 09	4.39640-02
6.000 10	2.68800 09	4.37700-02
7.000 10	3.12490 09	4.35930-02
8.000 10	3.56000 09	4.34260-02 *
9.000 10	3.99330 09	4.32640-02 *
1.000 11	4.42530 09	4.31060-02 *
2.000 11	8.65910 09	4.15880-02 *
3.000 11	1.37430 10	4.01270-02 *
4.000 11	1.66850 10	3.87190-02 *
5.000 11	2.04880 10	3.73640-02 *
6.000 11	2.41590 10	3.60490-02 *
7.000 11	2.76990 10	3.47830-02 *
8.000 11	3.11180 10	3.35600-02 *
9.000 11	3.44170 10	3.23800-02 *
1.000 12	3.76020 10	3.12410-02 *
2.000 12	6.39300 10	2.18310-02 *
3.000 12	8.23270 10	1.52550-02 *
4.000 12	9.51110 10	1.06410-02 *
5.000 12	1.04000 11	7.50650-03 *
6.000 12	1.10190 11	5.29200-03 *
7.000 12	1.14540 11	3.74400-03 *
8.000 12	1.17500 11	2.65550-03 *
9.000 12	1.19730 11	1.87190-03 *
1.000 13	1.21290 11	1.32900-03 *

CLOSED OUTER BOUNDARY

Skin = 20

$r_{eD} = 20$

t_D	Q_D	q_D
1.000 02	4.49120 00	4.39480-02
2.000 02	8.83330 00	4.29330-02
3.000 02	1.30770 01	4.19680-02
4.000 02	1.72260 01	4.10340-02
5.000 02	2.13840 01	4.01240-02
6.000 02	2.55500 01	3.92300-02
7.000 02	2.97270 01	3.83540-02
8.000 02	3.39190 01	3.75010-02
9.000 02	3.81270 01	3.66660-02
1.000 03	4.23540 01	3.58510-02
2.000 03	7.23890 01	2.86120-02
3.000 03	9.80800 01	2.28190-02
4.000 03	1.18600 02	1.82210-02
5.000 03	1.34900 02	1.45270-02
6.000 03	1.47870 02	1.16110-02
7.000 03	1.58200 02	9.28150-03
8.000 03	1.66430 02	7.43810-03
9.000 03	1.72930 02	5.95030-03
1.000 04	1.78130 02	4.77480-03
2.000 04	1.97200 02	5.33300-04
3.000 04	1.99330 02	9.98220-06

$r_{eD} = 200$

t_D	Q_D	q_D
1.000 04	4.07860 02	3.99290-02
2.000 04	8.02870 02	3.90900-02 *
3.000 04	1.18980 03	3.83020-02 *
4.000 04	1.56890 03	3.75300-02 *
5.000 04	1.94030 03	3.67770-02 *
6.000 04	2.30430 03	3.60350-02 *
7.000 04	2.66090 03	3.53070-02 *
8.000 04	3.01020 03	3.45950-02 *
9.000 04	3.35260 03	3.38980-02 *
1.000 05	3.68820 03	3.32170-02 *
2.000 05	6.69580 03	2.70870-02 *
3.000 05	9.15370 03	2.20910-02 *
4.000 05	1.11600 04	1.80150-02 *
5.000 05	1.27890 04	1.46690-02 *
6.000 05	1.41180 04	1.19810-02 *
7.000 05	1.51940 04	9.76650-03 *
8.000 05	1.60750 04	8.00430-03 *
9.000 05	1.67880 04	6.54680-03 *
1.000 06	1.73610 04	5.34250-03 *
2.000 06	1.96400 04	7.46850-04 *
3.000 06	1.99760 04	6.25620-05 *

$r_{eD} = 50$

t_D	Q_D	q_D
1.000 03	4.27420 01	4.16830-02 *
2.000 03	8.37090 01	4.02660-02 *
3.000 03	1.23260 02	3.88960-02 *
4.000 03	1.61510 02	3.75800-02 *
5.000 03	1.98430 02	3.63020-02 *
6.000 03	2.34110 02	3.50720-02 *
7.000 03	2.68590 02	3.38790-02 *
8.000 03	3.01920 02	3.27340-02 *
9.000 03	3.34100 02	3.16160-02 *
1.000 04	3.65200 02	3.05390-02 *
2.000 04	6.24110 02	2.16030-02 *
3.000 04	8.07180 02	1.52780-02 *
4.000 04	9.35860 02	1.08070-02 *
5.000 04	1.02690 03	7.70690-03 *
6.000 04	1.09070 03	5.48520-03 *
7.000 04	1.13580 03	3.91660-03 *
8.000 04	1.16810 03	2.80920-03 *
9.000 04	1.19110 03	2.00870-03 *

$r_{eD} = 500$

t_D	Q_D	q_D
1.000 05	3.89280 03	3.80410-02 *
2.000 05	7.63740 03	3.68620-02 *
3.000 05	1.12650 04	3.57210-02 *
4.000 05	1.47820 04	3.46200-02 *
5.000 05	1.81880 04	3.35470-02 *
6.000 05	2.14910 04	3.25120-02 *
7.000 05	2.46910 04	3.15040-02 *
8.000 05	2.77950 04	3.05370-02 *
9.000 05	3.08020 04	2.95900-02 *
1.000 06	3.37150 04	2.86680-02 *
2.000 06	5.83750 04	2.09280-02 *
3.000 06	7.63840 04	1.52700-02 *
4.000 06	8.94800 04	1.11530-02 *
5.000 06	9.89950 04	8.16760-03 *
6.000 06	1.05910 05	5.99590-03 *
7.000 06	1.10950 05	4.41520-03 *
8.000 06	1.14640 05	3.25960-03 *
9.000 06	1.17330 05	2.39930-03 *
1.000 07	1.19350 05	1.77580-03 *
2.000 07	1.24860 05	3.39830-05 *

$r_{eD} = 100$

t_D	Q_D	q_D
1.000 03	4.27370 01	4.18790-02
2.000 03	8.43320 01	4.12740-02
3.000 03	1.25400 02	4.08730-02
4.000 03	1.66100 02	4.05180-02 *
5.000 03	2.05440 02	4.01760-02 *
6.000 03	2.46450 02	3.98400-02 *
7.000 03	2.86120 02	3.95090-02 *
8.000 03	3.25460 02	3.91790-02 *
9.000 03	3.64470 02	3.88520-02 *
1.000 04	4.03150 02	3.85270-02 *
2.000 04	7.72650 02	3.54310-02 *
3.000 04	1.11250 03	3.25790-02 *
4.000 04	1.42520 03	2.99550-02 *
5.000 04	1.71320 03	2.75500-02 *
6.000 04	1.97800 03	2.53280-02 *
7.000 04	2.22170 03	2.32820-02 *
8.000 04	2.44580 03	2.14050-02 *
9.000 04	2.65180 03	1.96750-02 *
1.000 05	2.84130 03	1.81000-02 *
2.000 05	4.06260 03	7.88210-03 *
3.000 05	4.58690 03	3.48350-03 *
4.000 05	4.81790 03	1.56580-03 *
5.000 05	4.81770 03	6.66560-04 *
6.000 05	4.96730 03	2.84970-04 *
7.000 05	4.99030 03	1.00330-04 *

$r_{eD} = 1000$

t_D	Q_D	q_D
1.000 05	3.89650 03	3.82010-02
2.000 05	7.68870 03	3.75960-02
3.000 05	1.14410 04	3.73610-02
4.000 05	1.51620 04	3.70640-02 *
5.000 05	1.88550 04	3.67790-02 *
6.000 05	2.25190 04	3.64990-02 *
7.000 05	2.61540 04	3.62210-02 *
8.000 05	2.97620 04	3.59460-02 *
9.000 05	3.33430 04	3.56730-02 *
1.000 06	3.68950 04	3.54000-02 *
2.000 06	7.09690 04	3.27960-02 *
3.000 06	1.02540 05	3.03810-02 *
4.000 06	1.31810 05	2.81450-02 *
5.000 06	1.58940 05	2.60710-02 *
6.000 06	1.84090 05	2.41500-02 *
7.000 06	2.07400 05	2.23680-02 *
8.000 06	2.29010 05	2.07170-02 *
9.000 06	2.49010 05	1.91830-02 *
1.000 07	2.67560 05	1.77700-02 *
2.000 07	3.91510 05	8.28950-03 *
3.000 07	4.48710 05	3.91770-03 *
4.000 07	4.75520 05	1.87350-03 *
5.000 07	4.88430 05	8.97410-03 *
6.000 07	4.94650 05	4.08670-04 *
7.000 07	4.98020 05	1.84790-04 *
8.000 07	4.99230 05	4.49100-05 *

CLOSED OUTER BOUNDARY
Skin = 20

$r_{eD} = 2000$

t_D	Q_D	q_D
1.00D 06	3.7287D 04	3.6569D-02
2.00D 06	7.3499D 04	3.5866D-02 *
3.00D 06	1.0993D 05	3.5204D-02 *
4.00D 06	1.4390D 05	3.4555D-02 *
5.00D 06	1.7814D 05	3.3920D-02 *
6.00D 06	2.1173D 05	3.3294D-02 *
7.00D 06	2.4471D 05	3.2679D-02 *
8.00D 06	2.7707D 05	3.2076D-02 *
9.00D 06	3.0885D 05	3.1485D-02 *
1.00D 07	3.4004D 05	3.0905D-02 *
2.00D 07	6.2218D 05	2.5650D-02 *
3.00D 07	8.5677D 05	2.1283D-02 *
4.00D 07	1.0516D 06	1.7657D-02 *
5.00D 07	1.2132D 06	1.4647D-02 *
6.00D 07	1.3459D 06	1.2157D-02 *
7.00D 07	1.4575D 06	1.0089D-02 *
8.00D 07	1.5492D 06	8.3933D-03 *
9.00D 07	1.6248D 06	6.9778D-03 *
1.00D 08	1.6875D 06	5.8102D-03 *
2.00D 08	1.9487D 06	9.6258D-04 *
3.00D 08	1.9939D 06	1.2684D-04 *

$r_{eD} = 1 \times 10^4$

t_D	Q_D	q_D
1.00D 07	3.5758D 05	3.5114D-02
2.00D 07	7.0537D 05	3.4687D-02
3.00D 07	1.0518D 06	3.4403D-02
4.00D 07	1.3946D 06	3.4152D-02 *
5.00D 07	1.7349D 06	3.3911D-02 *
6.00D 07	2.0728D 06	3.3673D-02 *
7.00D 07	2.4084D 06	3.3437D-02 *
8.00D 07	2.7415D 06	3.3204D-02 *
9.00D 07	3.0724D 06	3.2971D-02 *
1.00D 08	3.4009D 06	3.2740D-02 *
2.00D 08	6.5618D 06	3.0520D-02 *
3.00D 08	9.5085D 06	2.8447D-02 *
4.00D 08	1.2257D 07	2.6518D-02 *
5.00D 08	1.4820D 07	2.4717D-02 *
6.00D 08	1.7211D 07	2.3039D-02 *
7.00D 08	1.9441D 07	2.1472D-02 *
8.00D 08	2.1521D 07	2.0011D-02 *
9.00D 08	2.3459D 07	1.8645D-02 *
1.00D 09	2.5267D 07	1.7378D-02 *
2.00D 09	3.7748D 07	8.6086D-03 *
3.00D 09	4.3858D 07	4.3127D-03 *
4.00D 09	4.6892D 07	2.1855D-03 *
5.00D 09	4.8430D 07	1.1141D-03 *
6.00D 09	4.9214D 07	5.5192D-04 *
7.00D 09	4.9651D 07	2.7498D-04 *
8.00D 09	4.9838D 07	1.0575D-04 *
9.00D 09	4.9982D 07	4.1144D-05 *

$r_{eD} = 5000$

t_D	Q_D	q_D
1.00D 06	3.7294D 04	3.6593D-02
2.00D 06	7.3635D 04	3.6134D-02
3.00D 06	1.0963D 05	3.5972D-02
4.00D 06	1.4541D 05	3.5687D-02
5.00D 06	1.8102D 05	3.5541D-02
6.00D 06	2.1650D 05	3.5414D-02
7.00D 06	2.5188D 05	3.5296D-02
8.00D 06	2.8710D 05	3.5189D-02 *
9.00D 06	3.2224D 05	3.5083D-02 *
1.00D 07	3.5723D 05	3.4980D-02 *
2.00D 07	7.0205D 05	3.3984D-02 *
3.00D 07	1.0370D 06	3.3018D-02 *
4.00D 07	1.3624D 06	3.2033D-02 *
5.00D 07	1.6786D 06	3.1171D-02 *
6.00D 07	1.9858D 06	3.0288D-02 *
7.00D 07	2.2843D 06	2.9426D-02 *
8.00D 07	2.5746D 06	2.8595D-02 *
9.00D 07	2.8563D 06	2.7780D-02 *
1.00D 08	3.1302D 06	2.6989D-02 *
2.00D 08	5.4799D 06	2.0224D-02 *
3.00D 08	7.2422D 06	1.5145D-02 *
4.00D 08	8.5591D 06	1.1349D-02 *
5.00D 08	9.5417D 06	8.5216D-03 *
6.00D 08	1.0274D 07	6.4107D-03 *
7.00D 08	1.0822D 07	4.8358D-03 *
8.00D 08	1.1233D 07	3.6565D-03 *
9.00D 08	1.1539D 07	2.7597D-03 *
1.00D 09	1.1774D 07	2.0936D-03 *
2.00D 09	1.2467D 07	8.7718D-04 *

$r_{eD} = 5 \times 10^4$

t_D	Q_D	q_D
1.00D 08	3.4345D 06	3.3751D-02
2.00D 08	6.7883D 06	3.3360D-02
3.00D 08	1.0113D 07	3.3137D-02
4.00D 08	1.3418D 07	3.2979D-02
5.00D 08	1.6710D 07	3.2853D-02
6.00D 08	1.9990D 07	3.2745D-02
7.00D 08	2.3260D 07	3.2646D-02
8.00D 08	2.6520D 07	3.2553D-02 *
9.00D 08	2.9771D 07	3.2462D-02 *
1.00D 09	3.3013D 07	3.2371D-02 *
2.00D 09	6.4958D 07	3.1525D-02 *
3.00D 09	9.6059D 07	3.0694D-02 *
4.00D 09	1.2634D 08	2.9890D-02 *
5.00D 09	1.5583D 08	2.9105D-02 *
6.00D 09	1.8455D 08	2.8342D-02 *
7.00D 09	2.1251D 08	2.7597D-02 *
8.00D 09	2.3975D 08	2.6876D-02 *
9.00D 09	2.6627D 08	2.6169D-02 *
1.00D 10	2.9209D 08	2.5479D-02 *
2.00D 10	5.1620D 08	1.9521D-02 *
3.00D 10	6.8812D 08	1.4946D-02 *
4.00D 10	8.1957D 08	1.1446D-02 *
5.00D 10	9.1991D 08	8.7284D-03 *
6.00D 10	9.9663D 08	6.7450D-03 *
7.00D 10	1.0550D 09	5.1993D-03 *
8.00D 10	1.0995D 09	4.0342D-03 *
9.00D 10	1.1336D 09	3.0885D-03 *
1.00D 11	1.1605D 09	2.4045D-03 *
2.00D 11	1.2430D 09	1.3170D-04 *

CLOSED OUTER BOUNDARY
Skin = 20

$$r_{eD} = 1 \times 10^5$$

t_D	Q_D	q_D
1.00D 09	3.3039D 07	3.2489D-02
2.00D 09	6.5326D 07	3.2122D-02
3.00D 09	9.7325D 07	3.1879D-02
4.00D 09	1.2910D 08	3.1664D-02 *
5.00D 09	1.6066D 08	3.1457D-02 *
6.00D 09	1.9202D 08	3.1253D-02 *
7.00D 09	2.2317D 08	3.1050D-02 *
8.00D 09	2.5411D 09	3.0850D-02 *
9.00D 09	2.8486D 08	3.0650D-02 *
1.00D 10	3.1540D 09	3.0451D-02 *
2.00D 10	6.1016D 08	2.8535D-02 *
3.00D 10	8.8638D 08	2.6738D-02 *
4.00D 10	1.1453D 09	2.5055D-02 *
5.00D 10	1.3881D 09	2.3478D-02 *
6.00D 10	1.6157D 09	2.1999D-02 *
7.00D 10	1.8292D 09	2.0614D-02 *
8.00D 10	2.0293D 09	1.9312D-02 *
9.00D 10	2.2168D 09	1.8091D-02 *
1.00D 11	2.3926D 09	1.6949D-02 *
2.00D 11	3.6396D 09	8.8397D-03 *
3.00D 11	4.2835D 09	4.6520D-03 *
4.00D 11	4.6205D 09	2.4895D-03 *
5.00D 11	4.7965D 09	1.3245D-03 *
6.00D 11	4.8913D 09	6.9635D-04 *
7.00D 11	4.9435D 09	3.5640D-04 *
8.00D 11	4.9694D 09	1.5246D-04 *
9.00D 11	4.9884D 09	7.0612D-05 *
1.00D 12	4.9976D 09	2.2351D-05 *

$$r_{eD} = 5 \times 10^5$$

t_D	Q_D	q_D
1.00D 10	3.1829D 08	3.1318D-02
2.00D 10	6.2964D 08	3.0981D-02
3.00D 10	9.3845D 08	3.0789D-02
4.00D 10	1.2456D 09	3.0652D-02
5.00D 10	1.5516D 09	3.0544D-02
6.00D 10	1.8566D 09	3.0450D-02
7.00D 10	2.1507D 09	3.0365D-02
8.00D 10	2.4640D 09	3.0284D-02 *
9.00D 10	2.7664D 09	3.0206D-02 *
1.00D 11	3.0681D 09	3.0130D-02 *
2.00D 11	6.0440D 09	2.9392D-02 *
3.00D 11	8.9468D 09	2.8675D-02 *
4.00D 11	1.1779D 10	2.7975D-02 *
5.00D 11	1.4541D 10	2.7295D-02 *
6.00D 11	1.7236D 10	2.6627D-02 *
7.00D 11	1.9866D 10	2.5977D-02 *
8.00D 11	2.2432D 10	2.5344D-02 *
9.00D 11	2.4935D 10	2.4724D-02 *
1.00D 12	2.7376D 10	2.4121D-02 *
2.00D 12	4.8781D 10	1.6834D-02 *
3.00D 12	6.5518D 10	1.4697D-02 *
4.00D 12	7.8571D 10	1.1470D-02 *
5.00D 12	8.8732D 10	8.9615D-03 *
6.00D 12	9.6629D 10	7.0099D-03 *
7.00D 12	1.0278D 11	5.4946D-03 *
8.00D 12	1.0754D 11	4.3159D-03 *
9.00D 12	1.1128D 11	3.3947D-03 *
1.00D 13	1.1421D 11	2.6668D-03 *
2.00D 13	1.2405D 11	2.2044D-04 *

$$r_{eD} = 1 \times 10^6$$

t_D	Q_D	q_D
1.00D 10	3.1829D 08	3.1318D-02
2.00D 10	6.2963D 08	3.0982D-02
3.00D 10	9.3847D 08	3.0789D-02
4.00D 10	1.2457D 09	3.0653D-02
5.00D 10	1.5517D 09	3.0548D-02
6.00D 10	1.8568D 09	3.0463D-02
7.00D 10	2.1611D 09	3.0392D-02
8.00D 10	2.4648D 09	3.0330D-02
9.00D 10	2.7678D 09	3.0276D-02
1.00D 11	3.0704D 09	3.0228D-02
2.00D 11	6.0758D 09	2.9911D-02
3.00D 11	9.0562D 09	2.9700D-02
4.00D 11	1.2017D 10	2.9514D-02 *
5.00D 11	1.4960D 10	2.9334D-02 *
6.00D 11	1.7864D 10	2.9157D-02 *
7.00D 11	2.0791D 10	2.8981D-02 *
8.00D 11	2.3680D 10	2.8806D-02 *
9.00D 11	2.6552D 10	2.8633D-02 *
1.00D 12	2.9406D 10	2.8460D-02 *
2.00D 12	5.7017D 10	2.6790D-02 *
3.00D 12	8.3007D 10	2.5217D-02 *
4.00D 12	1.0748E 11	2.3738D-02 *
5.00D 12	1.3053D 11	2.2344D-02 *
6.00D 12	1.5224D 11	2.1032D-02 *
7.00D 12	1.7268D 11	1.9797D-02 *
8.00D 12	1.9194D 11	1.8632D-02 *
9.00D 12	2.1006D 11	1.7533D-02 *
1.00D 13	2.2714D 11	1.6503D-02 *
2.00D 13	3.5113D 11	9.0032D-03 *
3.00D 13	4.1817D 11	4.9469D-03 *
4.00D 13	4.5463D 11	2.7451D-03 *
5.00D 13	4.7474D 11	1.5353D-03 *
6.00D 13	4.8591D 11	8.5170D-04 *
7.00D 13	4.9249D 11	4.7821D-04 *
8.00D 13	4.9587D 11	2.4293D-04 *
9.00D 13	4.9825D 11	1.2907D-04 *
1.00D 14	4.9923D 11	5.2291D-05 *

CONSTANT PRESSURE OUTER BOUNDARY

$$r_{eD} = 20$$

Skin = 0			Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 01	7.40200 00	5.33940-01	1.000 01	8.84490-01	8.56630-02
2.000 01	1.23210 01	4.61190-01	2.000 01	1.73000 00	8.34340-02
3.000 01	1.67440 01	4.26130-01	3.000 01	2.55840 00	8.21410-02
4.000 01	2.08700 01	4.03930-01	4.000 01	3.37520 00	8.12300-02
5.000 01	2.48450 01	3.88190-01	5.000 01	4.18410 00	8.05300-02
6.000 01	2.86760 01	3.76430-01	6.000 01	4.98680 00	7.99680-02
7.000 01	3.23850 01	3.67420-01	7.000 01	5.78420 00	7.95040-02
8.000 01	3.60200 01	3.60430-01	8.000 01	6.57740 00	7.91160-02
9.000 01	3.95920 01	3.54950-01	9.000 01	7.36690 00	7.87940-02
1.000 02	4.31150 01	3.50530-01	1.000 02	8.15350 00	7.85210-02
2.000 02	7.71510 01	3.35510-01	2.000 02	1.59270 01	7.72790-02
3.000 02	1.10610 02	3.33800-01	3.000 02	2.36390 01	7.70140-02
4.000 02	1.44000 02	3.33630-01	4.000 02	3.13390 01	7.69370-02
5.000 02	1.77390 02	3.33760-01	5.000 02	3.92360 01	7.69440-02
6.000 02	2.10760 02	3.33710-01	6.000 02	4.67310 01	7.69390-02
7.000 02	2.44150 02	3.33860-01	7.000 02	5.44280 01	7.69460-02
8.000 02	2.77560 02	3.33900-01	8.000 02	6.21240 01	7.69470-02
9.000 02	3.10930 02	3.33850-01	9.000 02	6.98200 01	7.69480-02

Skin = 5			Skin = 20		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 01	1.59050 00	1.49420-01	1.000 01	4.70030-01	4.61620-02
2.000 01	3.04720 00	1.42760-01	2.000 01	9.28070-01	4.55100-02
3.000 01	4.45450 00	1.38970-01	3.000 01	1.38120 00	4.51240-02
4.000 01	5.83120 00	1.36380-01	4.000 01	1.83110 00	4.48480-02
5.000 01	7.18540 00	1.34430-01	5.000 01	2.27260 00	4.46350-02
6.000 01	8.52190 00	1.33040-01	6.000 01	2.72410 00	4.44610-02
7.000 01	9.84430 00	1.31920-01	7.000 01	3.16810 00	4.43170-02
8.000 01	1.11950 01	1.30950-01	8.000 01	3.61070 00	4.41970-02
9.000 01	1.24560 01	1.29580-01	9.000 01	4.05220 00	4.40950-02
1.000 02	1.37490 01	1.28940-01	1.000 02	4.49270 00	4.40080-02
2.000 02	2.64410 01	1.25810-01	2.000 02	8.86840 00	4.36030-02
3.000 02	3.89850 01	1.25190-01	3.000 02	1.33240 01	4.35130-02
4.000 02	5.15010 01	1.25060-01	4.000 02	1.75740 01	4.34890-02
5.000 02	6.40110 01	1.25050-01	5.000 02	2.19240 01	4.34850-02
6.000 02	7.65180 01	1.25040-01	6.000 02	2.62730 01	4.34830-02
7.000 02	8.90290 01	1.25040-01	7.000 02	3.06230 01	4.34850-02
8.000 02	1.01540 02	1.25070-01	8.000 02	3.49750 01	4.34860-02
9.000 02	1.14050 02	1.25070-01	9.000 02	3.93210 01	4.34860-02

$$r_{eD} = 50$$

Skin = 0			Skin = 5		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 02	4.30280 01	3.45580-01	1.000 02	1.37430 01	1.28620-01
2.000 02	7.56030 01	3.10820-01	2.000 02	2.63040 01	1.23240-01
3.000 02	1.05750 02	2.93310-01	3.000 02	3.84730 01	1.20280-01
4.000 02	1.34500 02	2.82200-01	4.000 02	5.03990 01	1.18270-01
5.000 02	1.62310 02	2.74620-01	5.000 02	6.21500 01	1.16810-01
6.000 02	1.89470 02	2.69300-01	6.000 02	7.37730 01	1.15720-01
7.000 02	2.16190 02	2.65510-01	7.000 02	8.53000 01	1.14900-01
8.000 02	2.42560 02	2.62800-01	8.000 02	9.67540 01	1.14280-01
9.000 02	2.68720 02	2.60830-01	9.000 02	1.08150 02	1.13800-01
1.000 03	2.94720 02	2.59400-01	1.000 03	1.19510 02	1.13440-01
2.000 03	5.51510 02	2.58650-01	2.000 03	2.32170 02	1.12280-01
3.000 03	8.07240 02	2.58570-01	3.000 03	3.44430 02	1.12200-01
4.000 03	1.05230 03	2.58550-01	4.000 03	4.56640 02	1.12190-01
5.000 03	1.31860 03	2.58680-01	5.000 03	5.68890 02	1.12210-01
6.000 03	1.57420 03	2.58610-01	6.000 03	6.81160 02	1.12200 01
7.000 03	1.83000 03	2.58740-01	7.000 03	7.93360 02	1.12230-01
8.000 03	2.08530 03	2.58650-01	8.000 03	9.05510 02	1.12190-01
9.000 03	2.34150 03	2.58760-01	9.000 03	1.01780 03	1.12230-01

CONSTANT PRESSURE OUTER BOUNDARY

$r_{eD} = 50$

Skin = 10			Skin = 20		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 02	8.15150 00	7.84120-02	1.000 02	4.47210 00	4.39770-02
2.000 02	1.59800 01	7.63760-02	2.000 02	8.07430 00	4.33290-02
3.000 02	2.34880 01	7.52230-02	3.000 02	1.31680 01	4.29550-02
4.000 02	3.07400 01	7.44090-02	4.000 02	1.74510 01	4.26950-02
5.000 02	3.83540 01	7.38440-02	5.000 02	2.17110 01	4.25010-02
6.000 02	4.57150 01	7.34010-02	6.000 02	2.59540 01	4.23520-02
7.000 02	5.30370 01	7.30620-02	7.000 02	3.01830 01	4.22380-02
8.000 02	6.03250 01	7.28010-02	8.000 02	3.44020 01	4.21480-02
9.000 02	6.75980 01	7.25990-02	9.000 02	3.86140 01	4.20790-02
1.000 03	7.48490 01	7.24420-02	1.000 03	4.28190 01	4.20240-02
2.000 03	1.46940 02	7.19230-02	2.000 03	8.47210 01	4.19380-02
3.000 03	2.19860 02	7.19780-02	3.000 03	1.26560 02	4.18200-02
4.000 03	2.90740 02	7.18720-02	4.000 03	1.68380 02	4.16170-02
5.000 03	3.62640 02	7.18570-02	5.000 03	2.10210 02	4.18200-02
6.000 03	4.34530 02	7.18780-02	6.000 03	2.52030 02	4.18190-02
7.000 03	5.06430 02	7.18860-02	7.000 03	2.93970 02	4.18220-02
8.000 03	5.78350 02	7.18750-02	8.000 03	3.35890 02	4.18180-02
9.000 03	6.50220 02	7.18900-02	9.000 03	3.77820 02	4.18230-02

$r_{eD} = 100$

Skin = 0			Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 02	4.30290 01	3.45570-01	1.000 02	8.15150 00	7.84110-02
2.000 02	7.59590 01	3.10810-01	2.000 02	1.58790 01	7.63750-02
3.000 02	1.05730 02	2.93350-01	3.000 02	2.34880 01	7.52240-02
4.000 02	1.34470 02	2.82040-01	4.000 02	3.07380 01	7.44270-02
5.000 02	1.62240 02	2.73830-01	5.000 02	3.83490 01	7.38190-02
6.000 02	1.89300 02	2.67450-01	6.000 02	4.57070 01	7.33300-02
7.000 02	2.15780 02	2.62270-01	7.000 02	5.30200 01	7.29210-02
8.000 02	2.41800 02	2.57930-01	8.000 02	6.02960 01	7.25700-02
9.000 02	2.67420 02	2.54210-01	9.000 02	6.75390 01	7.22620-02
1.000 03	2.92690 02	2.50960-01	1.000 03	7.47530 01	7.19890-02
2.000 03	5.32710 02	2.31980-01	2.000 03	1.45780 02	7.02650-02
3.000 03	7.60030 02	2.24010-01	3.000 03	2.15580 02	6.94260-02
4.000 03	9.81670 02	2.20370-01	4.000 03	2.84760 02	6.89860-02
5.000 03	1.20130 03	2.18660-01	5.000 03	3.53610 02	6.87510-02
6.000 03	1.41780 03	2.17830-01	6.000 03	4.22300 02	6.86210-02
7.000 03	1.63720 03	2.17420-01	7.000 03	4.90890 02	6.85500-02
8.000 03	1.85460 03	2.17220-01	8.000 03	5.59430 02	6.85090-02
9.000 03	2.07190 03	2.17140-01	9.000 03	6.27950 02	6.84680-02

Skin = 5			Skin = 20		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 02	1.37430 01	1.28620-01	1.000 02	4.49220 00	4.39770-02
2.000 02	2.63030 01	1.23240-01	2.000 02	8.85420 00	4.33280-02
3.000 02	3.81690 01	1.20280-01	3.000 02	1.31680 01	4.29560-02
4.000 02	5.03920 01	1.18260-01	4.000 02	1.74500 01	4.26940-02
5.000 02	6.21400 01	1.16740-01	5.000 02	2.17100 01	4.24930-02
6.000 02	7.37520 01	1.15520-01	6.000 02	2.59510 01	4.23310-02
7.000 02	8.52540 01	1.14510-01	7.000 02	3.01780 01	4.21940-02
8.000 02	9.66640 01	1.13650-01	8.000 02	3.43920 01	4.20740-02
9.000 02	1.07990 02	1.12900-01	9.000 02	3.85930 01	4.19720-02
1.000 03	1.19250 02	1.12240-01	1.000 03	4.27890 01	4.18800-02
2.000 03	2.29180 02	1.08120-01	2.000 03	8.43440 01	4.17890-02
3.000 03	3.36230 02	1.05190-01	3.000 03	1.25470 02	4.07940-02
4.000 03	4.41820 02	1.03200-01	4.000 03	1.66300 02	4.08360-02
5.000 03	5.45730 02	1.01690-01	5.000 03	2.07170 02	4.07300-02
6.000 03	6.51270 02	1.00410-01	6.000 03	2.47900 02	4.06700-02
7.000 03	7.58540 02	1.00260-01	7.000 03	2.88090 02	4.06250-02
8.000 03	8.67360 02	1.00180-01	8.000 03	3.29760 02	4.05590-02
9.000 03	9.64660 02	1.00140-01	9.000 03	3.69930 02	4.05000-02

CONSTANT PRESSURE OUTER BOUNDARY

$r_{eD} = 200$

Skin = 0

t_D	Q_D	q_D
1.000 03	2.92640 02	2.50970-01
2.000 03	5.32530 02	2.31520-01
3.000 03	7.50650 02	2.21430-01
4.000 03	9.76660 02	2.14760-01
5.000 03	1.18900 03	2.09850-01
6.000 03	1.59690 03	2.06030-01
7.000 03	1.69130 03	2.02980-01
8.000 03	1.80310 03	2.00520-01
9.000 03	2.00250 03	1.98500-01
1.000 04	2.20000 03	1.96850-01
2.000 04	4.12400 03	1.90060-01
3.000 04	6.01780 03	1.88910-01
4.000 04	7.90450 03	1.88710-01
5.000 04	9.79430 03	1.89700-01
6.000 04	1.16820 04	1.88690-01
7.000 04	1.35700 04	1.88740-01
8.000 04	1.54580 04	1.88760-01
9.000 04	1.73450 04	1.88750-01

Skin = 10

t_D	Q_D	q_D
1.000 03	7.47500 01	7.19890-02
2.000 03	1.45770 02	7.02440-02
3.000 03	2.15500 02	6.92620-02
4.000 03	2.84420 02	6.85800-02
5.000 03	3.52750 02	6.80590-02
6.000 03	4.20510 02	6.76420-02
7.000 03	4.88090 02	6.72980-02
8.000 03	5.55250 02	6.70110-02
9.000 03	6.22140 02	6.67690-02
1.000 04	6.88910 02	6.65650-02
2.000 04	1.34850 03	6.56260-02
3.000 04	2.00360 03	6.54210-02
4.000 04	2.65770 03	6.53730-02
5.000 04	3.31150 03	6.53630-02
6.000 04	3.96530 03	6.53600-02
7.000 04	4.61910 03	6.53650-02
8.000 04	5.27290 03	6.53670-02
9.000 04	5.92660 03	6.53660-02

Skin = 5

t_D	Q_D	q_D
1.000 03	1.19250 02	1.12240-01
2.000 03	2.29140 02	1.08070-01
3.000 03	3.35990 02	1.05770-01
4.000 03	4.40960 02	1.04190-01
5.000 03	5.44560 02	1.02990-01
6.000 03	6.47100 02	1.02040-01
7.000 03	7.48770 02	1.01270-01
8.000 03	8.49710 02	1.00620-01
9.000 03	9.50060 02	1.00090-01
1.000 04	1.04990 03	9.96360-02
2.000 04	2.03340 03	9.76170-02
3.000 04	3.00710 03	9.72010-02
4.000 04	3.97800 03	9.71100-02
5.000 04	4.95010 03	9.70930-02
6.000 04	5.92120 03	9.70880-02
7.000 04	6.89260 03	9.71000-02
8.000 04	7.86380 03	9.71060-02
9.000 04	8.83490 03	9.71030-02

Skin = 20

t_D	Q_D	q_D
1.000 03	4.27880 01	4.18800-02
2.000 03	8.45390 01	4.12030-02
3.000 03	1.25450 02	4.09410-02
4.000 03	1.66270 02	4.07020-02
5.000 03	2.06800 02	4.05170-02
6.000 03	2.47330 02	4.03690-02
7.000 03	2.87650 02	4.02460-02
8.000 03	3.27850 02	4.01420-02
9.000 03	3.67950 02	4.00550-02
1.000 04	4.07970 02	3.99810-02
2.000 04	8.05610 02	3.96510-02
3.000 04	1.20150 03	3.95510-02
4.000 04	1.59700 03	3.95320-02
5.000 04	1.99250 03	3.95270-02
6.000 04	2.38770 03	3.95260-02
7.000 04	2.78300 03	3.95270-02
8.000 04	3.17840 03	3.95280-02
9.000 04	3.57370 03	3.95280-02

CONSTANT PRESSURE OUTER BOUNDARY

$r_{eD} = 500$

Skin = 0

t_D	Q_D	q_D
1.00E 04	2.1966E 03	1.9594E-01
2.00E 04	4.0899E 03	1.6571E-01
3.00E 04	5.8918E 03	1.7721E-01
4.00E 04	7.6412E 03	1.7291E-01
5.00E 04	9.3548E 03	1.6985E-01
6.00E 04	1.1041E 04	1.6761E-01
7.00E 04	1.2708E 04	1.6596E-01
8.00E 04	1.4360E 04	1.6472E-01
9.00E 04	1.6002E 04	1.6390E-01
1.00E 05	1.7633E 04	1.6310E-01
2.00E 05	3.3801E 04	1.6191E-01
3.00E 05	4.9906E 04	1.6093E-01
4.00E 05	6.5971E 04	1.6037E-01

Skin = 10

t_D	Q_D	q_D
1.00E 04	6.8858E 02	6.6476E-02
2.00E 04	1.3454E 03	6.5002E-02
3.00E 04	1.9910E 03	6.4157E-02
4.00E 04	2.6297E 03	6.3573E-02
5.00E 04	3.2633E 03	6.3142E-02
6.00E 04	3.8939E 03	6.2814E-02
7.00E 04	4.5199E 03	6.2563E-02
8.00E 04	5.1444E 03	6.2369E-02
9.00E 04	5.7673E 03	6.2218E-02
1.00E 05	6.3883E 03	6.2100E-02
2.00E 05	1.2573E 04	6.1797E-02
3.00E 05	1.8742E 04	6.1676E-02
4.00E 05	2.4919E 04	6.1666E-02

Skin = 5

t_D	Q_D	q_D
1.00E 04	1.0494E 03	9.9454E-02
2.00E 04	2.0259E 03	9.6168E-02
3.00E 04	2.9780E 03	9.4332E-02
4.00E 04	3.9150E 03	9.3080E-02
5.00E 04	4.8411E 03	9.2154E-02
6.00E 04	5.7592E 03	9.1475E-02
7.00E 04	6.6711E 03	9.0951E-02
8.00E 04	7.5783E 03	9.0550E-02
9.00E 04	8.4820E 03	9.0241E-02
1.00E 05	9.3836E 03	9.0003E-02
2.00E 05	1.8331E 04	8.9228E-02
3.00E 05	2.7252E 04	8.9162E-02
4.00E 05	3.6176E 04	8.9156E-02

Skin = 20

t_D	Q_D	q_D
1.00E 04	4.0792E 02	3.9957E-02
2.00E 04	8.0451E 02	3.9412E-02
3.00E 04	1.1970E 03	3.9100E-02
4.00E 04	1.5870E 03	3.8982E-02
5.00E 04	1.9750E 03	3.8719E-02
6.00E 04	2.3614E 03	3.8595E-02
7.00E 04	2.7471E 03	3.8499E-02
8.00E 04	3.1317E 03	3.8424E-02
9.00E 04	3.5156E 03	3.8366E-02
1.00E 05	3.8990E 03	3.8320E-02
2.00E 05	7.7208E 03	3.8182E-02
3.00E 05	1.1537E 04	3.8146E-02
4.00E 05	1.5352E 04	3.8144E-02

$r_{eD} = 1000$

Skin = 0

t_D	Q_D	q_D
1.00E 04	2.1986E 03	1.9594E-01
2.00E 04	4.0897E 03	1.8371E-01
3.00E 04	5.8909E 03	1.7722E-01
4.00E 04	7.6404E 03	1.7289E-01
5.00E 04	9.3526E 03	1.6967E-01
6.00E 04	1.1036E 04	1.6712E-01
7.00E 04	1.2697E 04	1.6503E-01
8.00E 04	1.4339E 04	1.6326E-01
9.00E 04	1.5964E 04	1.6172E-01
1.00E 05	1.7575E 04	1.6037E-01
2.00E 05	3.3147E 04	1.5216E-01
3.00E 05	4.8150E 04	1.4845E-01
4.00E 05	6.2889E 04	1.4663E-01
5.00E 05	7.7498E 04	1.4571E-01
6.00E 05	9.2044E 04	1.4524E-01
7.00E 05	1.0656E 05	1.4499E-01
8.00E 05	1.2105E 05	1.4486E-01
9.00E 05	1.3554E 05	1.4479E-01

Skin = 10

t_D	Q_D	q_D
1.00E 04	6.8868E 02	6.6493E-02
2.00E 04	1.3453E 03	6.5001E-02
3.00E 04	1.9909E 03	6.4157E-02
4.00E 04	2.6295E 03	6.3572E-02
5.00E 04	3.2630E 03	6.3125E-02
6.00E 04	3.8924E 03	6.2765E-02
7.00E 04	4.5187E 03	6.2464E-02
8.00E 04	5.1421E 03	6.2205E-02
9.00E 04	5.7631E 03	6.1978E-02
1.00E 05	6.3820E 03	6.1776E-02
2.00E 05	1.2489E 04	6.0498E-02
3.00E 05	1.8504E 04	5.9871E-02
4.00E 05	2.4472E 04	5.9540E-02
5.00E 05	3.0417E 04	5.9352E-02
6.00E 05	3.6348E 04	5.9263E-02
7.00E 05	4.2272E 04	5.9200E-02
8.00E 05	4.8192E 04	5.9176E-02
9.00E 05	5.4110E 04	5.9159E-02

Skin = 5

t_D	Q_D	q_D
1.00E 04	1.0498E 03	9.9464E-02
2.00E 04	2.0253E 03	9.6165E-02
3.00E 04	2.9777E 03	9.4334E-02
4.00E 04	3.9145E 03	9.3076E-02
5.00E 04	4.8405E 03	9.2124E-02
6.00E 04	5.7579E 03	9.1361E-02
7.00E 04	6.6693E 03	9.0725E-02
8.00E 04	7.5751E 03	9.0181E-02
9.00E 04	8.4727E 03	8.9705E-02
1.00E 05	9.3679E 03	8.9285E-02
2.00E 05	1.8145E 04	8.6648E-02
3.00E 05	2.6743E 04	8.5883E-02
4.00E 05	3.5246E 04	8.4729E-02
5.00E 05	4.3697E 04	8.4382E-02
6.00E 05	5.2125E 04	8.4194E-02
7.00E 05	6.0545E 04	8.4091E-02
8.00E 05	6.8949E 04	8.4032E-02
9.00E 05	7.7337E 04	8.4002E-02

Skin = 20

t_D	Q_D	q_D
1.00E 04	4.0793E 02	3.9957E-02
2.00E 04	8.0450E 02	3.9412E-02
3.00E 04	1.1970E 03	3.9100E-02
4.00E 04	1.5869E 03	3.8981E-02
5.00E 04	1.9749E 03	3.8714E-02
6.00E 04	2.3614E 03	3.8576E-02
7.00E 04	2.7465E 03	3.8464E-02
8.00E 04	3.1308E 03	3.8365E-02
9.00E 04	3.5141E 03	3.8279E-02
1.00E 05	3.8966E 03	3.8207E-02
2.00E 05	7.6897E 03	3.7709E-02
3.00E 05	1.1447E 04	3.7461E-02
4.00E 05	1.5186E 04	3.7380E-02
5.00E 05	1.8911E 04	3.7356E-02
6.00E 05	2.2639E 04	3.7215E-02
7.00E 05	2.6359E 04	3.7192E-02
8.00E 05	3.0079E 04	3.7179E-02
9.00E 05	3.3797E 04	3.7171E-02

CONSTANT PRESSURE OUTER BOUNDARY

Skin = 0			$r_{eD} = 2000$	Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D	
1.00E 04	2.1983E 03	1.9594E-01	1.00E 04	6.4868E 02	6.6496E-02	
2.00E 04	4.0687E 03	1.8371E-01	2.00E 04	1.3453E 03	6.5091E-02	
3.00E 04	5.8910E 03	1.7722E-01	3.00E 04	1.9909E 03	6.4157E-02	
4.00E 04	7.6405E 03	1.7289E-01	4.00E 04	2.6295E 03	6.3572E-02	
5.00E 04	9.3527E 03	1.6966E-01	5.00E 04	3.2630E 03	6.3125E-02	
6.00E 04	1.1036E 04	1.6712E-01	6.00E 04	3.8925E 03	6.2764E-02	
7.00E 04	1.2697E 04	1.6502E-01	7.00E 04	4.5186E 03	6.2463E-02	
8.00E 04	1.4338E 04	1.6325E-01	8.00E 04	5.1420E 03	6.2204E-02	
9.00E 04	1.5963E 04	1.6172E-01	9.00E 04	5.7630E 03	6.1977E-02	
1.00E 05	1.7573E 04	1.6037E-01	1.00E 05	6.3818E 03	6.1776E-02	
2.00E 05	3.3139E 04	1.5203E-01	2.00E 05	1.2489E 04	6.0484E-02	
3.00E 05	4.8102E 04	1.4754E-01	3.00E 05	1.8498E 04	5.9753E-02	
4.00E 05	6.2702E 04	1.4450E-01	4.00E 05	2.4448E 04	5.9244E-02	
5.00E 05	7.7039E 04	1.4222E-01	5.00E 05	3.0354E 04	5.8854E-02	
6.00E 05	9.1174E 04	1.4043E-01	6.00E 05	3.6224E 04	5.8541E-02	
7.00E 05	1.0514E 05	1.3897E-01	7.00E 05	4.2066E 04	5.8283E-02	
8.00E 05	1.1898E 05	1.3778E-01	8.00E 05	4.7885E 04	5.8067E-02	
9.00E 05	1.3271E 05	1.3679E-01	9.00E 05	5.3693E 04	5.7885E-02	
1.00E 06	1.4634E 05	1.3597E-01	1.00E 06	5.9463E 04	5.7731E-02	
2.00E 06	2.8000E 05	1.3239E-01	2.00E 06	1.1675E 05	5.7017E-02	
3.00E 06	4.1198E 05	1.3170E-01	3.00E 06	1.7367E 05	5.6858E-02	
4.00E 06	5.4364E 05	1.3156E-01	4.00E 06	2.3052E 05	5.6820E-02	
5.00E 06	6.7524E 05	1.3155E-01	5.00E 06	2.8735E 05	5.6813E-02	
6.00E 06	8.0681E 05	1.3154E-01	6.00E 06	3.4417E 05	5.6810E-02	
7.00E 06	9.3841E 05	1.3156E-01	7.00E 06	4.0100E 05	5.6813E-02	
8.00E 06	1.0700E 06	1.3157E-01	8.00E 06	4.5783E 05	5.6815E-02	
9.00E 06	1.2016E 06	1.3156E-01	9.00E 06	5.1465E 05	5.6815E-02	
Skin = 5			Skin = 20			
t_D	Q_D	q_D	t_D	Q_D	q_D	
1.00E 04	1.0493E 03	9.9464E-02	1.00E 04	4.0793E 02	3.9957E-02	
2.00E 04	2.0258E 03	9.6165E-02	2.00E 04	8.0450E 02	3.9412E-02	
3.00E 04	2.9780E 03	9.4334E-02	3.00E 04	1.1970E 03	3.9109E-02	
4.00E 04	3.9146E 03	9.3076E-02	4.00E 04	1.5869E 03	3.8881E-02	
5.00E 04	4.8405E 03	9.2124E-02	5.00E 04	1.9749E 03	3.8714E-02	
6.00E 04	5.7579E 03	9.1359E-02	6.00E 04	2.3614E 03	3.8577E-02	
7.00E 04	6.6684E 03	9.0723E-02	7.00E 04	2.7466E 03	3.8463E-02	
8.00E 04	7.5729E 03	9.0179E-02	8.00E 04	3.1308E 03	3.8365E-02	
9.00E 04	8.4724E 03	8.9704E-02	9.00E 04	3.5141E 03	3.8278E-02	
1.00E 05	9.3674E 03	8.9284E-02	1.00E 05	3.8965E 03	3.8201E-02	
2.00E 05	1.8147E 04	8.6614E-02	2.00E 05	7.6893E 03	3.7703E-02	
3.00E 05	2.6730E 04	8.5126E-02	3.00E 05	1.1445E 04	3.7417E-02	
4.00E 05	3.5191E 04	8.4099E-02	4.00E 05	1.5177E 04	3.7217E-02	
5.00E 05	4.3562E 04	8.3317E-02	5.00E 05	1.8891E 04	3.7062E-02	
6.00E 05	5.1864E 04	8.2692E-02	6.00E 05	2.2592E 04	3.6938E-02	
7.00E 05	6.0109E 04	8.2180E-02	7.00E 05	2.6281E 04	3.6835E-02	
8.00E 05	6.8306E 04	8.1754E-02	8.00E 05	2.9961E 04	3.6748E-02	
9.00E 05	7.6464E 04	8.1396E-02	9.00E 05	3.3632E 04	3.6675E-02	
1.00E 06	8.4588E 04	8.1095E-02	1.00E 06	3.7297E 04	3.6612E-02	
2.00E 06	1.6481E 05	7.9725E-02	2.00E 06	7.3728E 04	3.6318E-02	
3.00E 06	2.4357E 05	7.9433E-02	3.00E 06	1.1001E 05	3.6250E-02	
4.00E 06	3.2230E 05	7.9365E-02	4.00E 06	1.4626E 05	3.6233E-02	
5.00E 06	4.0116E 05	7.9354E-02	5.00E 06	1.8250E 05	3.6230E-02	
6.00E 06	4.8053E 05	7.9349E-02	6.00E 06	2.1873E 05	3.6229E-02	
7.00E 06	5.6191E 05	7.9350E-02	7.00E 06	2.5497E 05	3.6230E-02	
8.00E 06	6.4120E 05	7.9360E-02	8.00E 06	2.9121E 05	3.6231E-02	
9.00E 06	7.2069E 05	7.9359E-02	9.00E 06	3.2746E 05	3.6230E-02	

CONSTANT PRESSURE OUTER BOUNDARY

$$r_{eD} = 5000$$

Skin = 0			Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 05	1.25270 04	1.68570-01	1.000 05	6.30180 03	6.17760-02
2.000 05	3.31400 04	1.52030-01	2.000 05	1.24980 04	6.04830-02
3.000 05	4.81000 04	1.47530-01	3.000 05	1.84700 04	5.97320-02
4.000 05	6.26970 04	1.44500-01	4.000 05	2.44470 04	5.92430-02
5.000 05	7.70300 04	1.42230-01	5.000 05	3.03520 04	5.88550-02
6.000 05	9.11610 04	1.40430-01	6.000 05	3.62220 04	5.85410-02
7.000 05	1.05130 05	1.38940-01	7.000 05	4.20630 04	5.82790-02
8.000 05	1.18960 05	1.37670-01	8.000 05	4.78810 04	5.80530-02
9.000 05	1.32670 05	1.36580-01	9.000 05	5.36760 04	5.78560-02
1.000 06	1.46280 05	1.35610-01	1.000 06	5.94540 04	5.76800-02
2.000 06	2.76510 05	1.29580-01	2.000 06	1.16510 05	5.65520-02
3.000 06	4.06360 05	1.26280-01	3.000 06	1.72730 05	5.59110-02
4.000 06	5.31500 05	1.24030-01	4.000 06	2.28420 05	5.54570-02
5.000 06	6.54730 05	1.22470-01	5.000 06	2.83730 05	5.51580-02
6.000 06	7.76540 05	1.21250-01	6.000 06	3.38740 05	5.48970-02
7.000 06	8.97290 05	1.20350-01	7.000 06	3.93530 05	5.46940-02
8.000 06	1.01720 06	1.19660-01	8.000 06	4.48140 05	5.45450-02
9.000 06	1.13660 06	1.19140-01	9.000 06	5.02620 05	5.44290-02
1.000 07	1.25550 06	1.18740-01	1.000 07	5.57000 05	5.43380-02
2.000 07	2.43440 06	1.17490-01	2.000 07	1.09840 06	5.40320-02
3.000 07	3.60710 06	1.17400-01	3.000 07	1.63060 06	5.40020-02
4.000 07	4.78330 06	1.17390-01	4.000 07	2.17370 06	5.40000-02
5.000 07	5.95770 06	1.17410-01	5.000 07	2.71390 06	5.40040-02
6.000 07	7.13180 06	1.17460-01	6.000 07	3.25900 06	5.40020-02
7.000 07	8.30650 06	1.17430-01	7.000 07	3.79920 06	5.40090-02
8.000 07	9.47990 06	1.17390-01	8.000 07	4.33910 06	5.40090-02
9.000 07	1.06550 07	1.17430-01	9.000 07	4.87940 06	5.40090-02

Skin = 5			Skin = 20		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 05	9.36740 03	8.92840-02	1.000 05	3.89250 03	3.82010-02
2.000 05	1.81470 04	8.66140-02	2.000 05	7.66930 03	3.77030-02
3.000 05	2.67290 04	8.51240-02	3.000 05	1.14450 04	3.74170-02
4.000 05	3.51990 04	8.40980-02	4.000 05	1.51760 04	3.72170-02
5.000 05	4.35590 04	8.33190-02	5.000 05	1.88910 04	3.70630-02
6.000 05	5.18600 04	8.26930-02	6.000 05	2.25910 04	3.69380-02
7.000 05	6.01030 04	8.21700-02	7.000 05	2.62800 04	3.68330-02
8.000 05	6.82900 04	8.17230-02	8.000 05	2.99590 04	3.67430-02
9.000 05	7.64520 04	8.13330-02	9.000 05	3.36330 04	3.66540-02
1.000 06	8.45680 04	8.09870-02	1.000 06	3.72930 04	3.65930-02
2.000 06	1.64330 05	7.87840-02	2.000 06	7.36330 04	3.61360-02
3.000 06	2.42470 05	7.75460-02	3.000 06	1.09640 05	3.58720-02
4.000 06	3.19590 05	7.66960-02	4.000 06	1.45420 05	3.56890-02
5.000 06	3.95980 05	7.60710-02	5.000 06	1.81040 05	3.55520-02
6.000 06	4.71800 05	7.55980-02	6.000 06	2.16550 05	3.54470-02
7.000 06	5.47210 05	7.52370-02	7.000 06	2.51950 05	3.53650-02
8.000 06	6.22290 05	7.49590-02	8.000 06	2.87290 05	3.53020-02
9.000 06	6.97130 05	7.47440-02	9.000 06	3.22560 05	3.52530-02
1.000 07	7.71770 05	7.45770-02	1.000 07	3.57820 05	3.52140-02
2.000 07	1.50390 06	7.40250-02	2.000 07	7.09670 05	3.50900-02
3.000 07	2.25400 06	7.39760-02	3.000 07	1.05980 06	3.50660-02
4.000 07	2.99390 06	7.39720-02	4.000 07	1.41060 06	3.50650-02
5.000 07	3.73390 06	7.39800-02	5.000 07	1.76130 06	3.50660-02
6.000 07	4.47370 06	7.39770-02	6.000 07	2.11200 06	3.50660-02
7.000 07	5.21360 06	7.39680-02	7.000 07	2.46280 06	3.50680-02
8.000 07	5.95330 06	7.39730-02	8.000 07	2.81340 06	3.50650-02
9.000 07	6.69360 06	7.39880-02	9.000 07	3.16420 06	3.50680-02

CONSTANT PRESSURE OUTER BOUNDARY

$$r_{eD} = 1 \times 10^4$$

Skin = 0			Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00E 05	1.7573E 04	1.6037E-01	1.00E 05	6.3818E 03	6.1776E-02
2.00E 05	3.3140E 04	1.5293E-01	2.00E 05	1.2488E 04	6.0483E-02
3.00E 05	4.8102E 04	1.4753E-01	3.00E 05	1.8498E 04	5.9752E-02
4.00E 05	6.2697E 04	1.4450E-01	4.00E 05	2.4447E 04	5.9243E-02
5.00E 05	7.7030E 04	1.4223E-01	5.00E 05	3.0352E 04	5.8855E-02
6.00E 05	9.1161E 04	1.4043E-01	6.00E 05	3.6222E 04	5.8541E-02
7.00E 05	1.0513E 05	1.3894E-01	7.00E 05	4.2064E 04	5.8279E-02
8.00E 05	1.1896E 05	1.3767E-01	8.00E 05	4.7881E 04	5.8053E-02
9.00E 05	1.3267E 05	1.3658E-01	9.00E 05	5.3677E 04	5.7856E-02
1.00E 06	1.4628E 05	1.3561E-01	1.00E 06	5.9454E 04	5.7680E-02
2.00E 06	2.7850E 05	1.2957E-01	2.00E 06	1.1651E 05	5.6551E-02
3.00E 06	4.0631E 05	1.2628E-01	3.00E 06	1.7273E 05	5.5911E-02
4.00E 06	5.3143E 05	1.2405E-01	4.00E 06	2.2841E 05	5.5455E-02
5.00E 06	6.5461E 05	1.2237E-01	5.00E 06	2.8471E 05	5.5125E-02
6.00E 06	7.7630E 05	1.2103E-01	6.00E 06	3.3870E 05	5.4850E-02
7.00E 06	8.9679E 05	1.1992E-01	7.00E 06	3.9344E 05	5.4620E-02
8.00E 06	1.0163E 06	1.1897E-01	8.00E 06	4.4797E 05	5.4422E-02
9.00E 06	1.1348E 06	1.1815E-01	9.00E 06	5.0231E 05	5.4240E-02
1.00E 07	1.2527E 06	1.1742E-01	1.00E 07	5.5649E 05	5.4093E-02
2.00E 07	2.4016E 06	1.1292E-01	2.00E 07	1.0920E 06	5.3110E-02
3.00E 07	3.5189E 06	1.1081E-01	3.00E 07	1.6204E 06	5.2625E-02
4.00E 07	4.6205E 06	1.0974E-01	4.00E 07	2.1452E 06	5.2367E-02
5.00E 07	5.7156E 06	1.0919E-01	5.00E 07	2.6681E 06	5.2227E-02
6.00E 07	6.8054E 06	1.0889E-01	6.00E 07	3.1900E 06	5.2150E-02
7.00E 07	7.8934E 06	1.0874E-01	7.00E 07	3.7114E 06	5.2106E-02
8.00E 07	8.9808E 06	1.0865E-01	8.00E 07	4.2324E 06	5.2081E-02
9.00E 07	1.0067E 07	1.0860E-01	9.00E 07	4.7533E 06	5.2068E-02

Skin = 5			Skin = 20		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00E 05	9.3674E 03	8.9204E-02	1.00E 05	3.8945E 03	3.8201E-02
2.00E 05	1.8147E 04	8.6614E-02	2.00E 05	7.6893E 03	3.7703E-02
3.00E 05	2.6729E 04	8.5124E-02	3.00E 05	1.1445E 04	3.7417E-02
4.00E 05	3.5189E 04	8.4098E-02	4.00E 05	1.5176E 04	3.7217E-02
5.00E 05	4.3559E 04	8.3319E-02	5.00E 05	1.8891E 04	3.7063E-02
6.00E 05	5.1860E 04	8.2693E-02	6.00E 05	2.2591E 04	3.6938E-02
7.00E 05	6.0103E 04	8.2171E-02	7.00E 05	2.6280E 04	3.6836E-02
8.00E 05	6.8299E 04	8.1724E-02	8.00E 05	2.9959E 04	3.6743E-02
9.00E 05	7.6452E 04	8.1333E-02	9.00E 05	3.3630E 04	3.6664E-02
1.00E 06	8.4529E 04	8.0987E-02	1.00E 06	3.7293E 04	3.6593E-02
2.00E 06	1.6433E 05	7.8783E-02	2.00E 06	7.3635E 04	3.6135E-02
3.00E 06	2.4246E 05	7.7347E-02	3.00E 06	1.0963E 05	3.5872E-02
4.00E 06	3.1957E 05	7.6694E-02	4.00E 06	1.4542E 05	3.5688E-02
5.00E 06	3.9573E 05	7.6046E-02	5.00E 06	1.8103E 05	3.5547E-02
6.00E 06	4.7172E 05	7.5524E-02	6.00E 06	2.1653E 05	3.5432E-02
7.00E 06	5.4703E 05	7.5089E-02	7.00E 06	2.5192E 05	3.5336E-02
8.00E 06	6.2194E 05	7.4715E-02	8.00E 06	2.8722E 05	3.5253E-02
9.00E 06	6.9651E 05	7.4389E-02	9.00E 06	3.2244E 05	3.5180E-02
1.00E 07	7.7077E 05	7.4098E-02	1.00E 07	3.5759E 05	3.5115E-02
2.00E 07	1.5016E 06	7.2270E-02	2.00E 07	7.0646E 05	3.4697E-02
3.00E 07	2.2193E 06	7.1381E-02	3.00E 07	1.0523E 06	3.4488E-02
4.00E 07	2.9305E 06	7.0916E-02	4.00E 07	1.3966E 06	3.4375E-02
5.00E 07	3.6332E 06	7.0660E-02	5.00E 07	1.7400E 06	3.4313E-02
6.00E 07	4.3442E 06	7.0531E-02	6.00E 07	2.0830E 06	3.4278E-02
7.00E 07	5.0492E 06	7.0456E-02	7.00E 07	2.4257E 06	3.4259E-02
8.00E 07	5.7537E 06	7.0413E-02	8.00E 07	2.7663E 06	3.4247E-02
9.00E 07	6.4579E 06	7.0391E-02	9.00E 07	3.1100E 06	3.4241E-02

CONSTANT PRESSURE OUTER BOUNDARY

$$r_{eD} = 5 \times 10^4$$

Skin = 0			Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 07	1.25260 06	1.17420-01	1.000 07	5.56460 05	5.40930-02
2.000 07	2.40120 06	1.12060-01	2.000 07	1.09190 06	5.30990-02
3.000 07	3.51640 06	1.10350-01	3.000 07	1.62000 06	5.25310-02
4.000 07	4.61100 06	1.08630-01	4.000 07	2.14330 06	5.21410-02
5.000 07	5.69080 06	1.07340-01	5.000 07	2.66320 06	5.18390-02
6.000 07	6.75900 06	1.06310-01	6.000 07	3.18040 06	5.15960-02
7.000 07	7.81770 06	1.05450-01	7.000 07	3.69540 06	5.13920-02
8.000 07	8.86850 06	1.04710-01	8.000 07	4.20850 06	5.12160-02
9.000 07	9.91250 06	1.04080-01	9.000 07	4.71990 06	5.10620-02
1.000 08	1.09500 07	1.03510-01	1.000 08	5.22990 06	5.09260-02
2.000 08	2.11020 07	9.99460-02	2.000 08	1.02740 07	5.00440-02
3.000 08	3.09940 07	9.79650-02	3.000 08	1.52530 07	4.95410-02
4.000 08	4.07220 07	9.66160-02	4.000 08	2.01990 07	4.91920-02
5.000 08	5.03340 07	9.56300-02	5.000 08	2.50980 07	4.89330-02
6.000 08	5.98580 07	9.48900-02	6.000 08	2.99790 07	4.87330-02
7.000 08	6.93160 07	9.43270-02	7.000 08	3.48450 07	4.85820-02
8.000 08	7.87250 07	9.38970-02	8.000 08	3.96970 07	4.84640-02
9.000 08	8.80950 07	9.35660-02	9.000 08	4.45360 07	4.83710-02
1.000 09	9.74370 07	9.33110-02	1.000 09	4.93720 07	4.82990-02
2.000 09	1.90190 08	9.24850-02	2.000 09	9.75100 07	4.80540-02
3.000 09	2.82660 08	9.24160-02	3.000 09	1.45560 08	4.80300-02
4.000 09	3.75090 08	9.24120-02	4.000 09	1.93600 08	4.80280-02
5.000 09	4.67540 08	9.24240-02	5.000 09	2.41640 08	4.80310-02
6.000 09	5.59960 08	9.24180-02	6.000 09	2.89670 08	4.80300-02
7.000 09	6.52430 08	9.24360-02	7.000 09	3.37720 08	4.80340-02
8.000 09	7.44810 08	9.24130-02	8.000 09	3.85740 08	4.80280-02
9.000 09	8.37300 08	9.24350-02	9.000 09	4.33800 08	4.80340-02

Skin = 0			Skin = 20		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 07	7.70740 05	7.40980-02	1.000 07	3.57590 05	3.51140-02
2.000 07	1.50140 06	7.22470-02	2.000 07	7.06420 05	3.46920-02
3.000 07	2.21040 06	7.12060-02	3.000 07	1.05210 06	3.44500-02
4.000 07	2.92680 06	7.04860-02	4.000 07	1.39580 06	3.42810-02
5.000 07	3.62890 06	6.99370-02	5.000 07	1.73790 06	3.41500-02
6.000 07	4.32610 06	6.94950-02	6.000 07	2.07890 06	3.40440-02
7.000 07	5.01920 06	6.91260-02	7.000 07	2.41900 06	3.39550-02
8.000 07	5.70890 06	6.88090-02	8.000 07	2.75820 06	3.38780-02
9.000 07	6.39570 06	6.85320-02	9.000 07	3.09670 06	3.38110-02
1.000 08	7.07980 06	6.82960-02	1.000 08	3.43450 06	3.37510-02
2.000 08	1.38210 07	6.67120-02	2.000 08	6.78830 06	3.36810-02
3.000 08	2.04460 07	6.58210-02	3.000 08	1.01130 07	3.31370-02
4.000 08	2.69980 07	6.52070-02	4.000 08	1.34190 07	3.29800-02
5.000 08	3.34960 07	6.47330-02	5.000 08	1.67120 07	3.28630-02
6.000 08	3.99540 07	6.44090-02	6.000 08	1.99940 07	3.27730-02
7.000 08	4.63910 07	6.41450-02	7.000 08	2.32680 07	3.27040-02
8.000 08	5.27840 07	6.39410-02	8.000 08	2.65350 07	3.26490-02
9.000 08	5.91690 07	6.37830-02	9.000 08	2.97980 07	3.26070-02
1.000 09	6.55410 07	6.36600-02	1.000 09	3.30570 07	3.25730-02
2.000 09	1.28730 08	6.32480-02	2.000 09	6.55570 07	3.24580-02
3.000 09	1.92160 08	6.32090-02	3.000 09	9.80140 07	3.24460-02
4.000 09	2.55330 08	6.32060-02	4.000 09	1.30460 08	3.24450-02
5.000 09	3.18610 08	6.32120-02	5.000 09	1.62920 08	3.24470-02
6.000 09	3.81830 08	6.32100-02	6.000 09	1.95370 08	3.24460-02
7.000 09	4.45060 08	6.32100-02	7.000 09	2.27820 08	3.24480-02
8.000 09	5.08260 08	6.32070-02	8.000 09	2.60270 08	3.24450-02
9.000 09	5.71510 08	6.32180-02	9.000 09	2.92730 08	3.24480-02

CONSTANT PRESSURE OUTER BOUNDARY

$$r_{eD} = 1 \times 10^5$$

Skin = 0			Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 07	1.2526E 06	1.1742E-01	1.000 07	5.5646E 05	5.4073E-02
2.000 07	2.4012E 06	1.1766E-01	2.000 07	1.0919E 06	5.3097E-02
3.000 07	3.5163E 06	1.1835E-01	3.000 07	1.6200E 06	5.2534E-02
4.000 07	4.6110E 06	1.0863E-01	4.000 07	2.1433E 06	5.2141E-02
5.000 07	5.6908E 06	1.0734E-01	5.000 07	2.6647E 06	5.1839E-02
6.000 07	6.7590E 06	1.0631E-01	6.000 07	3.1804E 06	5.1595E-02
7.000 07	7.8175E 06	1.0545E-01	7.000 07	3.6934E 06	5.1392E-02
8.000 07	8.8686E 06	1.0471E-01	8.000 07	4.2085E 06	5.1216E-02
9.000 07	9.9126E 06	1.0408E-01	9.000 07	4.7197E 06	5.1062E-02
1.000 08	1.0957E 07	1.0351E-01	1.000 08	5.2277E 06	5.0925E-02
2.000 08	2.1102E 07	9.9943E-02	2.000 08	1.0274E 07	5.0045E-02
3.000 08	3.0991E 07	9.7967E-02	3.000 08	1.5252E 07	4.9541E-02
4.000 08	4.0718E 07	9.6512E-02	4.000 08	2.0188E 07	4.9171E-02
5.000 08	5.0327E 07	9.5587E-02	5.000 08	2.5076E 07	4.8905E-02
6.000 08	5.9844E 07	9.4766E-02	6.000 08	2.9976E 07	4.8705E-02
7.000 08	6.9287E 07	9.4082E-02	7.000 08	3.4839E 07	4.8524E-02
8.000 08	7.8568E 07	9.3497E-02	8.000 08	3.9683E 07	4.8368E-02
9.000 08	8.7994E 07	9.2987E-02	9.000 08	4.4514E 07	4.8231E-02
1.000 09	9.7272E 07	9.2534E-02	1.000 09	4.9332E 07	4.8108E-02
2.000 09	1.8822E 08	8.9707E-02	2.000 09	9.7011E 07	4.7329E-02
3.000 09	2.7717E 08	8.8533E-02	3.000 09	1.4413E 08	4.6942E-02
4.000 09	3.6512E 08	8.7655E-02	4.000 09	1.9075E 08	4.6736E-02
5.000 09	4.5257E 08	8.7266E-02	5.000 09	2.3753E 08	4.6625E-02
6.000 09	5.3975E 08	8.7083E-02	6.000 09	2.8422E 08	4.6561E-02
7.000 09	6.2679E 08	8.6977E-02	7.000 09	3.3077E 08	4.6526E-02
8.000 09	7.1375E 08	8.6915E-02	8.000 09	3.7729E 08	4.6505E-02
9.000 09	8.0068E 08	8.6884E-02	9.000 09	4.2380E 08	4.6494E-02

Skin = 5			Skin = 20		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 07	7.7074E 05	7.4098E-02	1.000 07	3.5759E 05	3.5114E-02
2.000 07	1.5514E 06	7.2247E-02	2.000 07	7.0642E 05	3.4692E-02
3.000 07	2.2184E 06	7.1206E-02	3.000 07	1.0521E 06	3.4450E-02
4.000 07	2.9266E 06	7.0483E-02	4.000 07	1.3959E 06	3.4281E-02
5.000 07	3.6289E 06	6.9937E-02	5.000 07	1.7377E 06	3.4150E-02
6.000 07	4.3261E 06	6.9495E-02	6.000 07	2.0787E 06	3.4044E-02
7.000 07	5.0192E 06	6.9126E-02	7.000 07	2.4190E 06	3.3955E-02
8.000 07	5.7089E 06	6.8809E-02	8.000 07	2.7582E 06	3.3876E-02
9.000 07	6.3957E 06	6.8532E-02	9.000 07	3.0967E 06	3.3811E-02
1.000 08	7.0799E 06	6.8296E-02	1.000 08	3.4343E 06	3.3751E-02
2.000 08	1.3821E 07	6.6710E-02	2.000 08	6.7883E 06	3.3661E-02
3.000 08	2.0445E 07	6.5822E-02	3.000 08	1.0113E 07	3.3137E-02
4.000 08	2.6996E 07	6.5206E-02	4.000 08	1.3419E 07	3.2980E-02
5.000 08	3.3493E 07	6.4736E-02	5.000 08	1.6711E 07	3.2859E-02
6.000 08	3.9949E 07	6.4357E-02	6.000 08	1.9992E 07	3.2761E-02
7.000 08	4.6368E 07	6.4041E-02	7.000 08	2.3264E 07	3.2673E-02
8.000 08	5.2760E 07	6.3769E-02	8.000 08	2.6529E 07	3.2607E-02
9.000 08	5.9126E 07	6.3530E-02	9.000 08	2.9787E 07	3.2545E-02
1.000 09	6.5470E 07	6.3318E-02	1.000 09	3.3040E 07	3.2489E-02
2.000 09	1.2804E 08	6.1977E-02	2.000 09	6.5334E 07	3.2131E-02
3.000 09	1.8966E 08	6.1319E-02	3.000 09	9.7366E 07	3.1951E-02
4.000 09	2.5078E 08	6.0920E-02	4.000 09	1.2927E 08	3.1854E-02
5.000 09	3.1165E 08	6.0785E-02	5.000 09	1.6109E 08	3.1801E-02
6.000 09	3.7238E 08	6.0682E-02	6.000 09	1.9283E 08	3.1771E-02
7.000 09	4.3304E 08	6.0625E-02	7.000 09	2.2465E 08	3.1756E-02
8.000 09	4.9365E 08	6.0599E-02	8.000 09	2.5640E 08	3.1744E-02
9.000 09	5.5426E 08	6.0574E-02	9.000 09	2.8815E 08	3.1739E-02

CONSTANT PRESSURE OUTER BOUNDARY

$r = .5 \times 10^5$

Skin = 0			Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 09	9.72678 07	9.28540-02	1.000 09	4.94310 07	4.81080-02
2.000 07	1.88200 08	8.94690-02	2.000 09	9.70040 07	4.73300-02
3.000 07	2.77030 08	8.80740-02	3.000 09	1.44070 08	4.58710-02
4.000 09	3.64550 08	8.69760-02	4.000 09	1.90600 08	4.65570-02
5.000 07	4.51090 08	8.61430-02	5.000 09	2.37240 08	4.63170-02
6.000 09	5.36900 08	8.54740-02	6.000 09	2.83460 08	4.61220-02
7.000 09	6.22090 08	8.49160-02	7.000 09	3.29510 08	4.59590-02
8.000 09	7.06720 08	8.44320-02	8.000 09	3.75400 08	4.58180-02
9.000 07	7.91010 08	8.40230-02	9.000 09	4.21160 08	4.56950-02
1.000 10	8.74850 08	8.36540-02	1.000 10	4.66810 08	4.55860-02
2.000 10	1.69830 09	8.13060-02	2.000 10	9.18770 08	4.48780-02
3.000 10	2.50450 09	7.99880-02	3.000 10	1.36550 09	4.44730-02
4.000 10	3.29990 09	7.90550-02	4.000 10	1.80800 09	4.41910-02
5.000 10	4.08740 09	7.84200-02	5.000 10	2.24970 09	4.39810-02
6.000 10	4.86900 09	7.79180-02	6.000 10	2.68880 09	4.38210-02
7.000 10	5.64610 09	7.75350-02	7.000 10	3.12630 09	4.36980-02
8.000 10	6.41990 09	7.72400-02	8.000 10	3.56280 07	4.36020-02
9.000 10	7.19100 09	7.70120-02	9.000 10	3.99840 09	4.35270-02
1.000 11	7.96010 09	7.68360-02	1.000 11	4.43340 09	4.34680-02
2.000 11	1.56050 10	7.62540-02	2.000 11	8.76710 07	4.32680-02
3.000 11	2.32290 10	7.62010-02	3.000 11	1.30930 10	4.32470-02
4.000 11	3.08500 10	7.62000-02	4.000 11	1.74190 10	4.32450-02
5.000 11	3.84720 10	7.62040-02	5.000 11	2.17440 10	4.32480-02
6.000 11	4.60930 10	7.62030-02	6.000 11	2.60700 10	4.32470-02
7.000 11	5.37170 10	7.62140-02	7.000 11	3.03960 10	4.32510-02
8.000 11	6.13360 10	7.62040-02	8.000 11	3.47210 10	4.32480-02
9.000 11	6.89610 10	7.62160-02	9.000 11	3.90470 10	4.32510-02

Skin = 5			Skin = 10		
t_D	Q_D	q_D	t_D	Q_D	q_D
1.000 09	6.54670 07	6.33180-02	1.000 09	3.30390 07	3.24890-02
2.000 09	1.25030 08	6.19600-02	2.000 09	6.53310 07	3.21270-02
3.000 09	1.89590 08	6.11930-02	3.000 09	9.73510 07	3.19190-02
4.000 09	2.50510 08	6.06600-02	4.000 09	1.29200 08	3.17740-02
5.000 09	3.10970 08	6.02530-02	5.000 09	1.60920 08	3.16610-02
6.000 09	3.71060 08	5.99240-02	6.000 09	1.92540 08	3.15700-02
7.000 09	4.30850 08	5.96490-02	7.000 09	2.24070 08	3.14940-02
8.000 09	4.90390 08	5.94130-02	8.000 09	2.55540 08	3.14280-02
9.000 09	5.49700 08	5.92060-02	9.000 09	2.86940 08	3.13700-02
1.000 10	6.08920 08	5.90220-02	1.000 10	3.18290 08	3.13320-02
2.000 10	1.19250 09	5.78420-02	2.000 10	6.29640 08	3.09820-02
3.000 10	1.76750 09	5.71710-02	3.000 10	9.38490 08	3.07830-02
4.000 10	2.33690 09	5.67070-02	4.000 10	1.24570 09	3.06530-02
5.000 10	2.90230 09	5.63630-02	5.000 10	1.55180 09	3.05520-02
6.000 10	3.46450 09	5.61010-02	6.000 10	1.85690 09	3.04740-02
7.000 10	4.02450 09	5.59000-02	7.000 10	2.16140 09	3.04140-02
8.000 10	4.58270 09	5.57440-02	8.000 10	2.46530 09	3.03670-02
9.000 10	5.13950 09	5.56230-02	9.000 10	2.76800 09	3.03300-02
1.000 11	5.69520 09	5.55280-02	1.000 11	3.07200 09	3.03010-02
2.000 11	1.12270 10	5.52100-02	2.000 11	6.09570 09	3.02010-02
3.000 11	1.67470 10	5.51790-02	3.000 11	9.11570 09	3.01910-02
4.000 11	2.22660 10	5.51770-02	4.000 11	1.21350 10	3.01900-02
5.000 11	2.77850 10	5.51800-02	5.000 11	1.51550 10	3.01910-02
6.000 11	3.33040 10	5.51790-02	6.000 11	1.81740 10	3.01910-02
7.000 11	3.88230 10	5.51540-02	7.000 11	2.11940 10	3.01920-02
8.000 11	4.43410 10	5.51300-02	8.000 11	2.42130 10	3.01910-02
9.000 11	4.98610 10	5.51060-02	9.000 11	2.72330 10	3.01930-02

CONSTANT PRESSURE OUTER BOUNDARY

$$r_{eD} = 1 \times 10^6$$

Skin = 0				Skin = 10	
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00D 07	9.7267D 07	9.2534D-02	1.00D 09	4.9331D 07	4.8108D-02
2.00D 07	1.8820D 08	8.9669D-02	2.00D 09	9.7004D 07	4.7320D-02
3.00D 07	2.7703D 08	8.8074D-02	3.00D 09	1.4407D 08	4.6871D-02
4.00D 07	3.6453D 08	8.6975D-02	4.00D 09	1.9080D 08	4.6557D-02
5.00D 07	4.5109D 08	8.6143D-02	5.00D 09	2.3724D 08	4.6317D-02
6.00D 07	5.3690D 08	8.5474D-02	6.00D 09	2.8346D 08	4.6122D-02
7.00D 07	6.2209D 08	8.4917D-02	7.00D 09	3.2951D 08	4.5959D-02
8.00D 07	7.0678D 08	8.4439D-02	8.00D 09	3.7540D 08	4.5818D-02
9.00D 07	7.9102D 08	8.4023D-02	9.00D 09	4.2116D 08	4.5695D-02
1.00D 10	8.7486D 08	8.3654D-02	1.00D 10	4.6681D 08	4.5586D-02
2.00D 10	1.6983D 09	8.1304D-02	2.00D 10	9.1876D 08	4.4877D-02
3.00D 10	2.5044D 09	7.9990D-02	3.00D 10	1.3654D 09	4.4473D-02
4.00D 10	3.2996D 09	7.9083D-02	4.00D 10	1.8087D 09	4.4191D-02
5.00D 10	4.0849D 09	7.8393D-02	5.00D 10	2.2496D 09	4.3974D-02
6.00D 10	4.8681D 09	7.7839D-02	6.00D 10	2.6885D 09	4.3799D-02
7.00D 10	5.6443D 09	7.7377D-02	7.00D 10	3.1258D 09	4.3520D-02
8.00D 10	6.4162D 09	7.6980D-02	8.00D 10	3.5617D 09	4.3525D-02
9.00D 10	7.1844D 09	7.6633D-02	9.00D 10	3.9965D 09	4.3414D-02
1.00D 11	7.9494D 09	7.6325D-02	1.00D 11	4.4302D 09	4.3315D-02
2.00D 11	1.5474D 10	7.4387D-02	2.00D 11	9.7269D 09	4.2681D-02
3.00D 11	2.2860D 10	7.3447D-02	3.00D 11	1.2978D 10	4.2366D-02
4.00D 11	3.0177D 10	7.2956D-02	4.00D 11	1.7205D 10	4.2197D-02
5.00D 11	3.7458D 10	7.2693D-02	5.00D 11	2.1420D 10	4.2105D-02
6.00D 11	4.4720D 10	7.2550D-02	6.00D 11	2.5628D 10	4.2053D-02
7.00D 11	5.1971D 10	7.2471D-02	7.00D 11	2.9832D 10	4.2024D-02
8.00D 11	5.9218D 10	7.2427D-02	8.00D 11	3.4035D 10	4.2008D-02
9.00D 11	6.6461D 10	7.2402D-02	9.00D 11	3.8236D 10	4.1998D-02

Skin = 5				Skin = 20	
t_D	Q_D	q_D	t_D	Q_D	q_D
1.00D 09	6.5467D 07	6.3318D-02	1.00D 09	3.3039D 07	3.2489D-02
2.00D 09	1.2803D 08	6.1960D-02	2.00D 09	6.5331D 07	3.2127D-02
3.00D 09	1.8959D 08	6.1193D-02	3.00D 09	9.7351D 07	3.1919D-02
4.00D 07	2.5051D 08	6.0660D-02	4.00D 09	1.2920D 08	3.1774D-02
5.00D 09	3.1097D 08	6.0253D-02	5.00D 09	1.6092D 08	3.1661D-02
6.00D 09	3.7106D 08	5.9924D-02	6.00D 09	1.9254D 08	3.1570D-02
7.00D 09	4.3083D 08	5.9649D-02	7.00D 09	2.2407D 08	3.1494D-02
8.00D 09	4.9039D 08	5.9413D-02	8.00D 09	2.5554D 08	3.1428D-02
9.00D 09	5.4970D 08	5.9206D-02	9.00D 09	2.8694D 08	3.1370D-02
1.00D 10	6.0882D 08	5.9022D-02	1.00D 10	3.1829D 08	3.1318D-02
2.00D 10	1.1925D 09	5.7841D-02	2.00D 10	6.2963D 08	3.0982D-02
3.00D 10	1.7674D 09	5.7172D-02	3.00D 10	9.3846D 08	3.0729D-02
4.00D 10	2.3367D 09	5.6706D-02	4.00D 10	1.2457D 09	3.0653D-02
5.00D 10	2.9020D 09	5.6350D-02	5.00D 10	1.5517D 09	3.0548D-02
6.00D 10	3.4641D 09	5.6063D-02	6.00D 10	1.8568D 09	3.0464D-02
7.00D 10	4.0236D 09	5.5822D-02	7.00D 10	2.1611D 09	3.0393D-02
8.00D 10	4.5809D 09	5.5615D-02	8.00D 10	2.4648D 09	3.0331D-02
9.00D 10	5.1362D 09	5.5434D-02	9.00D 10	2.7679D 09	3.0277D-02
1.00D 11	5.6899D 09	5.5272D-02	1.00D 11	3.0705D 09	3.0229D-02
2.00D 11	1.1160D 10	5.4246D-02	2.00D 11	6.0764D 09	2.9919D-02
3.00D 11	1.6557D 10	5.3740D-02	3.00D 11	9.0599D 09	2.9762D-02
4.00D 11	2.1916D 10	5.3472D-02	4.00D 11	1.2032D 10	2.9678D-02
5.00D 11	2.7255D 10	5.3326D-02	5.00D 11	1.4997D 10	2.9632D-02
6.00D 11	3.2594D 10	5.3246D-02	6.00D 11	1.7959D 10	2.9605D-02
7.00D 11	3.7907D 10	5.3200D-02	7.00D 11	2.0919D 10	2.9591D-02
8.00D 11	4.3227D 10	5.3175D-02	8.00D 11	2.3875D 10	2.9582D-02
9.00D 11	4.8544D 10	5.3161D-02	9.00D 11	2.6837D 10	2.9577D-02

APPENDIX C
COMPUTER PROGRAMS

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C      ANALYTICAL SOLUTIONS FOR CONSTANT WELLBORE PRESSURE
C
C      IMPLICIT REAL8 (A-H,O-Z)
COMMON/PARA/SKIN,RDIM,REFF,TFLOW
COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXR
COMMON/HB/G1,G2,G3,G4,G5
COMMON/VAR/QD(1000),TD(1000),TDX(100),AMODES(100)
CHARACTER*4 SIGN
DOUBLE PRECISION TFORM,TFORMA,TFORMB,BESKO,BESK1
EXTERNAL TFORM,TFORMA,TFORMB
CALL TRAPS (SO,10,100,10,10)
C      N = NUMBER OF TERMS ■ LAPLACE INVERTER
N=8
M=0
SIGN=' '
C
C      SOLUTION DESCR IPTION:
C      ICHART = 1 FOR QD VS TIS
C              2 FOR QD VS TUA
C              3 FOR INITIALLY CONSTANT RATE
C              4 FOR PD VS RD
C              6 FOR BUILDUP FROM SUPERPOSITION
C      ICHART=1
C      LIMITS FOR TD ARE 10**IXA TO 10**IXB
C      IXA=-1
C      IXB=9
C      NSOLN = 1 FOR INFINITE OUTER BOUNDARY
C              2 FOR NO-FLOW OUTER BOUNDARY
C              3 FOR CONSTANT PRESSURE OUTER BOUNDARY
C      NSOLN=1
C      NTIMES = NUMBER OF LOG CYCLES TO EVALUATE
C      NTIMES=(IXB-IXA)
C
C      PARAMETER VALUES:
C      SKIN = WELLBORE SKIN FACTOR
C      SKIN=0.
C      RDIM = DIMENSIONLESS RADIUS (1 .LE. RDIM .LE. REFF)
C      RDIM=1.0
C      REFF = DIMENSIONLESS RESERVOIR RADIUS (FOR FINITE RESERVOIR)
C      REFF=50.
C      TFLOW = FLOW TIME (FOR PRESSURE BUILDUP OR PD VS RD)
C      TFLOW=10.
C      NRD = NUMBER OF RADIAL LOG CYCLES
C      NRD=5
C      IF (ICHART .EQ. 6) GO TO 70
C      IF (ICHART .LT. 3) GO TO 5
C      IF (ICHART .EQ. 4) GO TO 30
C

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C      CALCULATE TIMES FOR EVALUATION.
C
5  T0=3.14159 *REFF*REFF*0.1
   TMULT=1.
   DLOGT=1./DFLOAT(NTIMES)
   IF ( ICHART .EQ. 2) TMULT=REFF*REFF
   DO 10 J=1,NTIMES
   DO 10 I=1,9
   K=I+(J-1)*9
   TD(K)=DFLOAT( D*10.** (IXA+J-1)
10  TD(X(K))=TD(K)*TMULT
11  CALL OUTFORM

C
C      CALCULATE QD.
C
   NT=9*NTIMES
   DO 20 I=1,NT
   TDI=TD(I)
C      * INDICATES EXPONENTIAL RATE DECLINE
   IF ((TDI .GT. T0) .AND. (NSOLN .EQ. 2)) SIGN='*'
   CALL LINU(TFORM,TD,TDI,QD,N,M)
   CALL LINU(TFORM,TD,QD,I,N,M)
   WRITE (6,300)TDX(I),QD,I,QD,SIGN
20  QD(I)=QDI
   IF (SKIN .EQ. 0.) GO TO 21
   IF (SKIN .EQ. 5.) GO TO 22
   IF (SKIN .EQ. 10.) GO TO 23
   IF (SKIN .EQ. 20.) GO TO 50
21  SKIN=5.
   GO TO 11
22  SKIN=10.
   GO TO 11
23  SKIN=20.
   GO TO 11
30  CALL OUTFORM

C
C      CALCULATE PD US RD FOR TU = TFLOW
C
   DO 40 J=1,NRD
   DO 40 I=1,9
   K=I+(J-1)*9
   RDIM=DFLOAT( D*10.** (J-1)
   CALL LINU(TFORM,TFLOW,PD,N,M)
40  WRITE (6,300) RDIM,PD
300 FORMAT (' ',1PE10.2,2X,2(1PE12.4,2X),A1)
   SO STOP
70  CALL OUTFORM
   CALL SPBU(N,M)
   STOP
   ENU

C
C
C
C      SUBROUTINE LINU(P,T,FA,N,M
C      *FLIC * REAL *8 (A-ti,0-Z)
C      COMMON/LPL/G(50),V(50),H(25),GZ(1)
C      DOUBLE PRECISION P

C
C      LINU (LAPLACE INVERTER) IS A FORTRAN TRANSLATION OF THE
C      ALGOL PROCEDURE GIVEN BY STEHFEST (1970). P IS THE LAPLACE
C      SPACE EXPRESSION TO BE NUMERICALLY INVERTED. T IS THE TIME
C      AT WHICH THE SOLUTION IS TO BE EVALUATED. FA IS THE VALUE
C      OF THE SOLUTION AT TIME T DETERMINED BY THE NUMERICAL INVER-
C      SION OF THE LAPLACE SPACE SOLUTION. N IS THE NUMBER OF TERMS
C      IN THE SUMMATION. [SEE STEHFEST (1970)]
C

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DLOGTW=.6931471805599453
IF (M .EQ. N) GO TO 100
C CALCULATE V ARRAY,
M=N
G(1)=1.
NH=N/2
DO 5 I=2,N
5 G(I)=G(I-1)*I
H(1)=2./G(NH-1)
DO 10 I=2,NH
FI=I
IF (I .EQ. NH) GO TO 8
H(I)=FI**NH*G(2*I)/(G(NH-I)*G(I)*G(I-1))
GO TO 10
8 H(I)=FI**NH*G(2*I)/(G(I)*G(I-1))
10 CONTINUE
SN=2*(NH-NH/2*2)-1
DO 50 I=1,N
V(I)=0.
K1=(I+1)/2
K2=I
IF (K2 .GT. NH) K2=NH
DO 40 K=K1,K2
IF (2*K-I .EQ. 0) GO TO 37
IF (I .EQ. K) GO TO 38
V(I)=V(I)+H(K)/(G(I-K)*G(2*K-I))
GO TO 40
37 V(I)=V(I)+H(K)/G(I-K)
GO TO 40
38 V(I)=V(I)+H(K)/G(2*K-I)
40 CONTINUE
V(I)=SN*V(I)
SN=-SN
50 CONTINUE
100 FA=0.
A=DLOGTW/T
DO 110 I=1,N
ARG=DFLQAT(I)*A
110 FA=FA+V(I)*F(ARG)
FA=A*FA
RETURN
END
DOUBLE PRECISION FUNCTION TFORM(S)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/PARA/SKIN,RDIM,REFF,TFLOW
COMMON/TSOLN/ICHART,NSOLN,ITYF,E,IXA,IXD
COMMON/HB/G1,G2,G3,G4,G5
DIMENSION ARG(3),XK(2,3),XI(2,3)
REAL A,X
DOUBLE PRECISION BESK0,BESK1
C
C TFORM CONTAINS THE LAPLACE TRANSFORMED SOLUTIONS FOR THE
C TRANSIENT RATE DECLINE FOR A WELL PRODUCED AT A CONSTANT
C PRESSURE FROM A CIRCULAR RESERVOIR. ALSO INCLUDED ARE THE
C SOLUTIONS FOR THE PRESSURE DISTRIBUTIONS.
C
S1=DSQRT(S)
ARG(1)=S1
ARG(2)=RDIM*S1
ARG(3)=REFF*S1
DO 10 J=1,3
XK(1,J)=BESK0(ARG(J))
XK(2,J)=BESK1(ARG(J))
A=ARG(J)
CALL BESI(A,0,X,IER)
XI(1,J)=X

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      CALL BESI(A,1,X,IER)
10  XI(2,J)=X
      IF (ICHART .EQ. 3) GO TO 60
      GO TO (20,30,40),NSOLN
C
C      SOLUTION FOR INFINITE OUTER BOUNDARY
C
20  DENOM=S*(XK(1,1)+SKIN*S1*XK(2,1))
      IF (DENOM .EQ. 0.) GO TO 70
      PD=XK(1,2)/DENOM
      QD=S1*XK(2,1)/DENOM
      GO TO 50
C
C      SOLUTION FOR NO-FLOW OUTER BOUNDARY
C
30  DENOM=S*((XK(2,3)*XI(1,1)+XK(1,1)*XI(2,3))
      1-SKIN*S1*(XK(2,3)*XI(2,1)-XK(2,1)*XI(2,3)))
      IF (DENOM .EQ. 0.) GO TO 70
      PD=(XK(2,3)*XI(1,2)+XK(1,2)*XI(2,3))/DENOM
      QD=S1*(XK(2,1)*XI(2,3)-XK(2,3)*XI(2,1))/DENOM
      GO TO 50
C
C      SOLUTION FOR CONSTANT PRESSURE OUTER BOUNDARY
C
40  DENOM=S*((-XK(1,3)*XI(1,1)+XK(1,1)*XI(1,3))
      1+SKIN*S1*(XK(1,3)*XI(2,1)+XK(2,1)*XI(1,3)))
      IF (DENOM .EQ. 0.) GO TO 70
      PD=(-XK(1,3)*XI(1,2)+XK(1,2)*XI(1,3))/DENOM
      QD=S1*(XK(2,1)*XI(1,3)+XK(1,3)*XI(2,1))/DENOM
50  TFORM=PD
      IF (RDIM .EQ. 1) TFORM=QD
      RETURN
60  TFORM=(-S1*XK(2,1)*G4/XK(1,1)+G2)/(S-G1)
      RETURN
70  TFORM=-1.
      RETURN
      END

```

Values for the required Bessel functions were obtained through use of BESKO and BESK1 from the FUNPACK PACKET and the internal routine, BESI, available on the IBM 360 168 at the Stanford Computer Facility, Stanford University.

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      SIJHROUTINE SPBU(N,M)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PARA/SKIN,RDIM,REFF,TFLOW
      COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXH
      COMMON/VAR/QD(1000),TD(1000),DTD(100),AMODES(100)
      DOUBLE PRECISION TFORM,TFORMA,TFORMB,BESKO,BESK1
      EXTERNAL TFORM,TFORMA,TFORMB
C
C      SPBU COMPUTES PRESSURE BUILDUP SOLUTIONS FOR A WELL PRODUCED
C      AT A CONSTANT PRESSURE PRIOR TO SHUT-IN USING SUPERPOSITION
C      OF CONTINUOUSLY VARYING CONSTANT RATE SOLUTIONS. THE TECHNIQUE
C      FOR APPROXIMATING THE RESULTING INTEGRAL IS TO DETERMINE TIME

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C   INTERVALS DURING WHICH THE RATE ONLY CHANGES BY A SET AMOUNT,
C   AND THEN APPROXIMATE THE RATE IN EACH SUCH INTERVAL BY A
C   CONSTANT RATE OVER THE INTERVAL, THIS RESULTS IN A SUM OF TERMS
C   CONSISTING OF A RATE TIMES A PRESSURE DIFFERENCE. THE SUM IS
C   THEN COMPUTED AS THE APPROXIMATION OF THE PRESSURE BUILDUP.
C
      TO=.1*3.1416*REFF*REFF
      T1=.05*REFF*REFF*(DLOG(REFF)-.75+SKIN)
C   KK1 AND K1 SPECIFY WHAT SHUT-IN TIMES ARE TO BE EVALUATED.
      KK1=4
      KK=3
      IF (NSOLN .EQ. 1) GO TO 2
      IF (TFLOW .GT. T1) GO TO 60
2   DO 1 K1=1, KK1
      DO 1 K=1, KK
      J=K+(K1-1)*KK
1   DTD(J)=TFLOW*10.**((K1-KK1)*2.**((K-1)
      NDT=KK1*KK
      DTF=DTD(NDT)+TFLOW
C   DELQ IS THE MAXIMUM VARIATION IN THE RATE FOR EACH SUB-INTERVAL
C   IN TIME REPRESENTED BY A TERM IN THE SUMMATION,
C   AN ARRAY OF TD AND QD VALUES ARE CREATED WITH THE SPECIFIED
C   DELQ VALUE.
      DELQ=.005
      DELQX=1000.*DELQ
      CALL QFORM(TFLOW, QD(1), N, M, T1)
      CALL LINV (TFORMA, TFLOW, CUM, N, M)
      TD(1)=TFLOW
      CALL QFORM(DTF, QD(2), N, M, T1)
      IF ((QD(1)-QD(2)) .GT. DELQX) GO TO 25
      TD(2)=DTF
      JK=1
      DO 20 J=2, 1000
      DO 10 I=1, 8
      IF (J .GT. (JK+1)) GO TO 30
      IF ((QD(J-1)-QD(J)) .LT. DELQ) GO TO 20
      JK=JK+1
      DO 5 K=J, JK
      L=JK-K+J+1
      TD(L)=TD(L-1)
5   QD(L)=QD(L-1)
      TD(J)=(TD(J-1)+TD(J))*5
10  CALL QFORM(TD(J), QD(J), N, M, T1)
20  CONTINUE
25  WRITE(6, 103)
      RETURN
30  JK=JK+1
      DO 50 I=1, NDT
      TT=DTD(NDT)+TFLOW
      T=TT-100.
      IF (T .LT. 1.D5) T=TT
      SUM=1.
      CALL PFORM(DDTD(NDT), PDM, N, M, T)
      DO 40 J=1, JK
      IF (T .LT. TD(J)) GO TO 43
      TP=TT-TD(J)
      CALL PFORM(TP, PDP, N, M, T)
      IF ((PDM-PDP) .GT. 0.) SUM=SUM-QD(J)*(PDM-PDP)
40  PDM=PDP
43  IF (TT .GE. 1.D5) GO TO 42
      SUM=(SUM-PDP*QD(J-1))/QD(1)
      GO TO 44
42  CALL QFORM(T, QDF, N, M, T1)
      JM=J
      IF (J .NE. 1) JM=J-1
      IF ((PDM-2.71) .GT. 0.) SUM=SUM-QD(JM)*(PDM-2.71)

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SUM=(SUM-2.71*QDF)/QB(1)
44 DTH=(CUM/QD(1)+DTD(I))/DTD(I)
50 WR WTE(6,100)SUM,DTD(I),DTH
WRITE (6,101) TFLOW,REFF,NSOLN
WRITE (6,104) (TD(I),QD(I),I=1,JK)
RETURN
60 WRITE (6,102)
RETURN
100 FORMAT(' ',3(E12.4,2X))
102 FORMAT(' EXPONENTIAL DECLINE IN PROGRESS')
103 FORMAT(' TOO MANY Q EVALUATIONS REQUIRED')
101 FORMAT (' TFLOW = ',E12.4,2X,' REFF = ',E12.4,2X,' NSOLN = ',I1)
104 FORMAT (' ',2(E12.4,2X))
END
DOUBLE PRECISION FUNCTION TFORMA(S)
IMPLICIT REAL*8 (A-H,O-Z)
DOUBLE PRECISION TFORM
EXTERNAL TFORM
C
C TFORMA IS THE LAPLACE SPACE SOLUTIONS FOR CUMULATIVE PRODUCTION
C
TFORMA=TFORM(S)/S
RETURN
END
DOUBLE PRECISION FUNCTION TFORMB(S)
IMPLICIT REAL*8 (A-H,O-Z)
DOUBLE PRECISION TFORM
EXTERNAL TFORM
C
C TFORMB IS THE LAPLACE SPACE SOLUTIONS FOR TRANSIENT WELLBORE
C PRESSURE WITH CONSTANT KATE PRODUCTION.
C
TFORMB=1./(S*S*TFORM(S))
RETURN
END
SUBROUTINE PFORM(T,P,N,M,TO)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/PARA/SKIN,RDIM,REFF,TFLOW
COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXB
DOUBLE PRECISION TFORM,TFORMA,TFORMB,BESK0,BESK1
EXTERNAL TFORM,TFORMA,TFORMB
C
C PFORM USES LIMITING FORMS OF THE WELLBORE PRESSURE SOLUTION
C FOR CONSTANT RATE PRODUCTION WHENEVER POSSIBLE.
C
NCASE=3
IF (T .LT. 0.01) GO TO 30
IF (T .LT. 100.) NCASE=1
IF (T .GT. TO) NCASE=2
GO TO (10,20,22),NCASE
20 GO TO (22,24,26),NSOLN
27, P=.5*(DLOG(T)+.80907)+SKIN
RETURN
24 P=DLOG(REFF)-.75+2.*T/(REFF*REFF)+SKIN
RETURN
26 P=DLOG(REFF)+SKIN
RETURN
10 CALL LINV(TFORMB,T,P,N,M)
RETURN
30 P=DSQRT(4.*T/3.1416)
RETURN
END
SUBROUTINE RFORM (T,R,N,M,T1)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/PARA/SKIN,RDIM,REFF,TFLOW
COMMON/TSOLN/ICHART,NSOLN,ITYPE,IXA,IXB

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DOUBLE PRECISION TFORM,TFORMA,TFORMB,BESKO,BESK1
EXTERNAL TFORM,TFORMA,TFORMB
C
C QFORM USES LIMITING FORMS FOR THE RATE DECLINE FOR CONSTANT
C PRESSURE PRODUCTION WHENEVER POSSIBLE.
C
      NCASE=1
      IF (T .LT. 5.004) NCASE=1
      IF (T .GT. T1) NCASE=2
20 GO TO (20,20,20),NCASE
22 Q=2./((DLOG(T)+SKIN)*.80907)
      RETURN
24 Q=DEXP(-.1*T/T1)/(DLOG(REFF)+.75*SKIN)
      RETURN
26 T2=2.*T1
      IF (T .LT. T2) GO TO 10
      Q=1./((DLOG(REFF)+SKIN)
      RETURN
10 CALL LINV(TFORM,T,Q,N,M)
      RETURN
      END
      SUBROUTINE OUTFORM
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PARA/SKIN,RDIM,REFF,TFLOW
      COMMON/TSQLN/ICHART,NSQLN,ITYPE,IXA,IXB
      COMMON/HB/G1,G2,G3,G4,G5
      COMMON/VAR/QD(1000),TD(1000),TDX(100),AMQDES(100)
      IF (NSQLN .EQ. 1) WRITE (6,100)
      IF (NSQLN .EQ. 2) WRITE (6,101)
      IF (NSQLN .EQ. 3) WRITE (6,102)
      GO TO (10,20,10,30,40),ICHART
10 WRITE (6,103) SKIN,RDIM
      IF (NSQLN .NE. 1) WRITE (6,110) REFF
      IF (RDIM .EQ. 1) WRITE (6,104)
      IF (RDIM .NE. 1) WRITE (6,105)
      RETURN
20 WRITE (6,103) SKIN,RDIM
      IF (NSQLN .NE. 1) WRITE (6,111) REFF
      IF (RDIM .EQ. 1) WRITE (6,106)
      IF (RDIM .NE. 1) WRITE (6,107)
      RETURN
30 WRITE (6,108) SKIN,TFLOW
      RETURN
40 WRITE (6,103) SKIN,RDIM
      RETURN
100 FORMAT ('UNBOUNDED RESERVOIR')
101 FORMAT ('CLOSED BOUNDED RESERVOIR')
102 FORMAT ('CONSTANT PRESSURE BOUNDED RESERVOIR')
103 FORMAT (' SKIN = ',F6.3,3X,'RD = ',E12.4)
104 FORMAT ('/,6X,'TD',11X,'QD')
105 FORMAT ('/,6X,'TD',11X,'PD')
106 FORMAT ('/,5X,'TDA',11X,'QD')
107 FORMAT ('/,5X,'TDA',11X,'PD')
108 FORMAT (' SKIN = ',F6.3,3X,'TD = ',E12.4,/,6X,'RD',11X,'PD')
110 FORMAT (' OUTER RADIUS, RD = ',E12.4)
      END

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