NEAR-WELL UPSCALING FOR
TWO AND THREE-PHASE FLOWS

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DOCTOR OF PHILOSOPHY

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Abstract

Near-well effects can have a strong impact on many subsurface flow processes. In oil production, because dissolved gas is released from the oil phase when the pressure falls below the bubble point, the detailed pressure field in the immediate vicinity of a production well can strongly impact gas (and thus oil) production. This effect is complicated by the interplay of fine-scale heterogeneity and multiphase flow physics and can be difficult to capture in coarse-grid simulations.

In this thesis, new upscaling (coarse-graining) procedures to capture such near-well effects in coarse-scale flow simulation models are developed and applied. The general methodology entails the use of preprocessing computations over near-well domains (referred to as local well models) for the determination of upscaled single-phase and multiphase near-well parameters. These parameters are computed by minimizing the mismatch between fine and coarse-scale flows over the local well model. Minimization is accomplished using a gradient-based optimization procedure, with gradients calculated through solution of adjoint equations. The boundary conditions applied on the local well model can impact the upscaled parameters, but these boundary conditions depend on the global flow and are not, therefore, known a priori. To circumvent this difficulty, an adaptive local-global procedure is applied. This entails performing a global coarse-scale simulation with initial estimates for well-block parameters. The resulting pressure and saturation fields are then interpolated onto the boundaries of the local well model to provide boundary conditions for the near-well upscaling computations. The overall upscaling methodology is developed first for oil-gas flows. The procedures are then extended to three-phase (oil-gas-water) systems.

The new upscaling techniques are applied to a variety of heterogeneous reservoir
models. Two different fluid models, specifically a typical black oil system and a heavy oil system, are considered. Extensive simulation results are presented for both two and three-phase flows. These results illustrate that the methods are able to accurately capture key near-well effects and to provide predictions for component production rates that are in close agreement with reference fine-scale results. The level of accuracy of the procedures is shown to be significantly higher than that of a standard approach which uses only upscaled single-phase flow parameters. The computational requirements for the methods depend mainly on the number of fine-grid cells in the local well models. For the examples considered in this thesis, speedups of about a factor of 3-5 relative to the reference fine-grid simulation model are typically achieved, though larger speedups can be readily obtained through use of smaller local well models or by selecting an optimized coarsening factor. An interpolation procedure is introduced which avoids the need for some upscaling computations. Use of this approach results in significantly more speedup if many coarse-scale simulations are performed.

Finally, the general approach is extended to enable the upscaling of unstructured fine-scale models to structured coarse-scale descriptions. The basic upscaling procedures applied for these computations are very similar to those used for upscaling structured models, though additional complexity arises because the simulation of highly detailed unstructured local well models, with resolution down to the scale of the wellbore, is required. Numerical results for oil-gas systems demonstrate that the method is able to provide coarse-scale results in close agreement with reference unstructured fine-scale solutions.
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Chapter 1

Introduction

Reservoir simulation is one of the most important tools in reservoir management. It is used for many purposes such as the prediction of the future performance of reservoirs, the evaluation of the effects of geological and engineering uncertainties, and the optimization of well locations, configurations and development scenarios. Flow simulators handle geological models with various scales of data derived from well logs, seismic data, and sedimentology. Geostatistics enable the integration of this information and can provide multiple geocellular models. Recent progress in computer hardware and software allows us to perform reservoir simulations faster. Geological models, however, often still contain many more grid blocks than reservoir simulators can handle in practical time frames. Thus techniques are required to enable realistic flow predictions to be made without performing simulation on the underlying geological model.

Although other approaches such as parallel computation can be applied, the reduction of the number of equations to be solved is a natural way to accomplish this. Upscaling designates the set of techniques that determine coarse-scale properties such that flow characteristics which would be obtained in fine-scale flow simulations can be computed using coarse-scale models. The use of upscaling procedures therefore acts to decrease the computational requirements for reservoir simulation studies.

There are different levels of scale handled in reservoir engineering, such as pore scale, core scale, geocellular scale, and flow simulation scale. Fluid flow is dominated
by various effects (convection, capillarity, gravity, etc.) and the dominant effects vary with scale. Upscaling acts to connect these scale gaps. In this study, we consider upscaling from the geocellular scale to the flow simulation scale, in which convection and/or gravity are the dominant effects.

The effects of fine-scale geology are particularly important in the near-well region. Near-well upscaling is an effective tool to link very detailed geocellular models with standard (coarse) flow simulation models. Our motivation in this work is to develop flexible and reliable near-well upscaling techniques that are applicable for scenarios involving complex near-well physics, such as dissolved gas and three-phase systems.

Near-well upscaling techniques developed to date do not fully account for well-block pressure differences between fine and coarse-scale models. Because the coarse-scale pressure resembles the average pressure over the fine cells corresponding to the coarse well block, steep changes in pressure within the block are not captured using standard procedures. Consider for example the case in which only the fine well-block pressure falls below the bubble point pressure. In such a case, coarse-scale models obtained by conventional upscaling techniques may perform very differently than the fine-scale model, particularly in predictions of gas flow rate, because the coarse well-block pressure is at all times above the bubble point pressure. To account for this and related phenomena in the upscaled model, new upscaling approaches are required. Such techniques will be developed in this study. A new feature of this work is that we apply gradient-based optimization techniques, with gradients computed using an adjoint procedure, in the upscaling computation.

1.1 Literature Review

Upscaling techniques can be classified in various ways, such as in terms of target parameters computed (rock properties such as permeability, rock-fluid properties such as relative permeability), reservoir regions (global, near-well), and domains employed for upscaling calculations (local, extended local, global). More detailed discussion and classifications for upscaling techniques can be seen in Durlofsky (2005). In this study, we focus on local-global, near-well upscaling approaches. We introduce both
single-phase and multiphase parameter upscaling. We now discuss previous work in related areas.

1.1.1 Single-phase upscaling

The general procedure of flow-based upscaling entails calculation of fluxes from fine simulation results and post-processing based on Darcy’s law. Since the flow characteristics are so different between the regions around wells, which show radial flow toward or away from the well, and regions away from wells, which may display nearly linear pressure fields, the upscaling techniques that are applied are also different. We therefore review each set of techniques separately.

Upscaling away from wells

Local upscaling methods involve only the fine cells corresponding to a single coarse-scale block. Boundary conditions applied in solving the flow equation can impact the accuracy of the resulting coarse-scale model. The simplest set of boundary conditions is constant pressure-no flow (Warren and Price, 1961), in which pressure is specified on the inlet and outlet and on the other boundaries no flow is specified. As a generalization, King and Mansfield (1999) proposed linear pressure boundary conditions. Durlofsky (1991) applied periodic boundary conditions to calculate full tensor coarse-scale permeability. The coarse-scale permeability tensors computed with periodic boundary conditions are always symmetric and positive definite.

Local upscaling does not use any permeability information outside of the target coarse-scale grid block. Holden and Lia (1992) suggested using an extended local region rather than that corresponding only to a single coarse block. Gomez-Hernandez and Journel (1994) also introduced additional fine cells around the target cell as a “skin” for their calculation of coarse permeability. Wen et al. (2003) compared purely local and extended local permeability upscaling procedures and showed that the extended local techniques generally provide better accuracy, as would be expected.

In strongly heterogeneous permeability fields, local and extended local upscaling may not provide satisfactory coarse-scale block permeability. White and Horne (1987)
used the entire domain and applied various boundary conditions to compute coarse-scale transmissibilities. Nielsen and Tveito (1998), Holden and Nielsen (2000), and Holden et al. (2000) also applied global boundary conditions and employed a least square method to minimize pressure and flow errors between fine and coarse-scale solutions. Zhang et al. (2008) used global fine-scale single-phase flow solutions with analytical relative permeability upscaling and showed the approach provided accurate coarse-scale simulation results for strongly heterogeneous reservoirs. They considered both two and three-phase systems.

Although global upscaling can provide a high degree of accuracy, it becomes expensive (and eventually impractical) for large reservoir models. Chen et al. (2003), Chen and Durlofsky (2006a), Gerritsen and Lambers (2008), Wen et al. (2006), and Chen et al. (2009) proposed local-global upscaling methods to address some of the limitations of previous approaches. Local-global approaches first construct an approximate coarse-scale model by standard local or extended local upscaling. Then, boundary conditions for the next upscaling step, using the global coarse-scale solution of the approximate model, are estimated. Extended local upscaling is then performed using these boundary conditions to determine coarse-scale transmissibility. This procedure is iterated until self-consistency is achieved, which usually requires just a few iterations. The method takes advantage of the efficiency of local upscaling and the accuracy of global upscaling. Chen et al. (2003) showed transmissibility upscaling is more accurate and robust than permeability upscaling. This finding is consistent with earlier studies (Romeu and Noetinger, 1995; Abbaszadeh and Koide, 1996). Chen and Li (2009) recently extended the local-global procedure for the upscaling of two-phase relative permeabilities. They showed that the coarse-scale parameters given by their approach provided comparable accuracy to that achieved using global two-phase upscaling, though the local-global approach was much more efficient.

**Upscaling in the near-well region**

In local and extended local methods, linear pressure fields are generally applied. The pressure distribution in the vicinity of the well is not linear, hence a more suitable
assumption is required for this region. Ding (1995) first proposed a numerical upscaling approach to compute coarse-scale well index and transmissibilities between the well block and adjacent blocks. In this approach, well-driven flow was introduced by applying appropriate boundary conditions. Durlofsky et al. (2000) identified an extended local region around the well for the fine-scale calculations required for near-well upscaling. Muggeridge et al. (2002) also used a reduced computational domain and demonstrated the effectiveness of the method for horizontal and deviated well problems. Zhang et al. (2008) used the global single-phase fine-scale solution to determine the coarse-scale well index with global transmissibilities and showed accurate coarse-scale simulation results for strongly heterogeneous reservoirs. Chen and Wu (2008) suggested an averaging approach and a flow-based approach that used flexible grids for well index upscaling. They concluded that the flow-based approach provided high degrees of accuracy in coarse-scale simulation, though the simple averaging approach provided reasonable accuracy and was very easy to implement. For the minimization of flow error between fine and coarse models, Mascarenhas and Durlofsky (2000) introduced a Gauss-Newton based optimization approach rather than a simple averaging technique. This required iterating on the coarse-scale parameters to achieve agreement between fine and coarse-scale simulation quantities.

1.1.2 Multiphase upscaling

Single-phase upscaling mainly addresses the pressure equation. It is therefore not always sufficient for accurate multiphase coarse-scale flow modeling. When transport phenomena are important, multiphase parameter upscaling may also be needed. Ekramm and Dale (1992) defined and distinguished two major approaches in averaging flow properties: dynamic-pseudo approaches and effective-property approaches. Dynamic pseudos are generated based on the fine-scale simulation results of multiphase flow in the whole (global) or a representative portion (local or extended local) of the model. On the other hand, the effective property approaches assume that the actual physical conditions can be approximated by limiting cases such as viscous-dominated and capillary-dominated flow, in which case simplified solution procedures can be
applied. It is often not clear, however, when or if these limiting cases are applicable for a given problem.

**Upscaling away from wells**

The most common approaches using dynamic pseudo functions are those of Kyte and Berry (1975) and Stone (1991). Kyte and Berry (1975) applied an "integrated" Darcy’s law to determine coarse-scale parameters. They averaged pressure and fluxes obtained by fine-scale simulation to provide coarse relative permeabilities. Stone (1991) introduced the direct use of total mobility and fractional flow into the upscaling procedure. For the cases where there is a significant effect of gravity in the simulation model, Darman et al. (1999) proposed the transmissibility-potential weighted method. Although a number of approaches have been proposed, the resulting pseudo relative permeabilities may not always reproduce fine-scale results.

For effective property approaches, Pickup and Sorbie (1996) proposed an upscaling technique for capillary and viscous-dominated two-phase flow. Stephen et al. (2001) suggested a criteria for the use of steady state upscaling based on the diffusion-convection-gravity equation. Lohne and Virnovsky (2006) extended the approach of Pickup and Sorbie (1996) to three-phase flow. They presented steady-state upscaling procedures for relative permeability and capillary pressure for three limiting cases (capillary limit, viscous limit, and vertical equilibrium). Though effective property approaches are efficient, they have the problem that the (assumed) boundary conditions used for local fine simulations impact the coarse-scale properties. This is also an issue for dynamic upscaling techniques when a local method is applied.

Thibeau et al. (1995) and Thibeau (1996) employed global coarse-scale models with fine blocks corresponding to the target coarse block for which pseudo functions are computed. This approach is able to incorporate global flow effects in the local boundary conditions. Wallstrom et al. (2002) introduced effective flux boundary conditions (EFBCs), which assign flux for each fine block on the boundary of the target coarse block based on the fine-cell and average or global permeability. Hui (2005) demonstrated the effectiveness of EFBCs by applying this approach to upscaling for miscible displacement problems. Chen and Durlofsky (2006b) also addressed
two-phase upscaling problems using pseudo relative permeabilities computed using EFBCs. They also applied a generalized convection-diffusion (GCD) model for this problem.

Upscaling in the near-well region

Emanuel and Cook (1974) and Woods and Khurana (1977) were the first researchers who proposed two-phase, near-well upscaling, though their main purpose was to reduce the dimension of 3D models to 2D and to reduce numerical dispersion for relatively simple models. Nakashima and Nomura (2004) proposed a semi-analytical approach for the coarse well block total mobility curve to correct anomalous injection pressure. Hui (2005) suggested a mobility correction function which separates coarse-grid effects and heterogeneity effects. Hui (2005) and Hui and Durlofsky (2005) introduced an optimization procedure for computing coarse-scale relative permeability for oil-water systems. Their goal was to match coarse and fine-scale flux at the well and at the boundaries between the well block and adjacent blocks. The optimization used gradients determined using numerical perturbation. Drawbacks of this approach are in efficiency and robustness. The required number of forward calculations increases as the number of optimization parameters increases. Also, the numerical perturbation and resulting gradients may be sensitive to the size of the perturbation. A more efficient and robust gradient computation is therefore required.

1.1.3 Multiscale modeling for the near-well region

Several studies have addressed near-well treatment in the context of multiscale modeling. Aarnes (2004) proposed a mixed multiscale finite element (MMSFE) method to incorporate the effect of scale heterogeneity. He demonstrated high accuracy for incompressible two-phase flow simulations. Wolfsteiner et al. (2006) applied a multiscale finite volume (MSFV) procedure and successfully modeled near-well flow by removing the well singularity from the multiscale solution. Jenny and Lunati (2009) extended the MSFV method to handle complex wells. Their approach entails additional basis functions to account for degrees of freedom (i.e., well pressure) under
rate constrained production. Juanes and Dub (2008) presented a variational MMSE method for accurate modeling of well driven flow. They defined boundary conditions for the localization of the subgrid problems and removed the singularities due to concentrated sources by introducing additional multiscale basis functions. Krogstad and Durlofsky (2009) linked wellbore flow and reservoir flow using a multiscale finite element formulation. None of these studies specifically targeted the modeling of dissolved gas problems.

1.1.4 Use of unstructured grids

Simulations based on unstructured (e.g., tetrahedral) grids can be used to study near-well behavior and to resolve detailed aspects of the well completion such as gravel packs and perforations. Consonni et al. (1993) addressed coning problems with vertical and horizontal wells using a control volume finite difference (CVFD) method. They applied local grid refinement with Voronoi grids for near-well regions. A Cartesian grid system was used for the rest of domain. Ding and Weill (1998) applied a triangular grid system for accurate flow modeling in the vicinity of the well and demonstrated improved accuracy relative to standard grid modeling. Ding (2004) presented a near-well upscaling for single-phase flow parameters for distorted grid systems (corner point geometry) and achieved accurate simulation results with coarse corner-point grids. Karimi-Fard and Durlofsky (2009) proposed the usage of 3D unstructured grid systems for the accurate modeling of Darcy-Forchheimer flow. They demonstrated a high degree of accuracy for a case with significant non-Darcy effects.

1.1.5 Adjoint approach

Our upscaling approach minimizes the mismatch between fine and coarse-scale flow results. This can be viewed as an optimization problem with appropriate constraints. In petroleum engineering, adjoint-based approaches have been used to provide gradients for minimization problems that arise within the context of history matching and production optimization. Some relevant references are as follows.
Chen et al. (1974) presented the first study to use the adjoint approach for efficient history matching. For a single-phase flow problem, they employed the adjoint procedure to overcome the excessive computational requirements of numerical gradients. Oliver (1996) also used adjoints to estimate permeability around the well using well test data. Li et al. (2003) presented a general formulation for sensitivity coefficient computation for history matching using an adjoint approach. Their approach is appropriate for use with a fully implicit three-phase black oil simulator.

For production optimization, Faithi and Ramirez (1985) demonstrated applicability of the adjoint approach to optimize enhanced oil recovery. Ramirez (1987) presented water flooding optimization using an adjoint approach. Mehos and Ramirez (1989) and Liu and Ramirez (1994) used adjoints to optimize miscible flooding and steam flooding processes. Brouwer and Jansen (2004) presented a procedure to determine optimal water flooding schemes using an adjoint approach. They demonstrated the maximization of the net present value (NPV) under several well operating constraints and compared pressure-constrained and rate-constrained production. Sarma et al. (2006) incorporated adjoint code into Stanford’s General Purpose Research Simulator. They applied adjoints for both production optimization and history matching. In the context of upscaling, our work appears to be first to incorporate adjoint procedures.

1.2 Overview of Thesis Work

In previous two-phase near-well upscaling (Hui, 2005; Hui and Durlofsky, 2005) it was assumed that there was no mass transfer between fluid phases and the pressure dependency of fluid properties was neglected. These limitations are likely to be problematic when dealing with general flow problems with mass transfer between phases, such as gas liberation or dissolution. The introduction of PVT parameters into the upscaling will lead to much more general procedures.

In previous work, minimization of the mismatch between fine and coarse-scale near-well flows was accomplished using numerically computed gradients (Mascarenhas and Durlofsky, 2000; Hui, 2005; Hui and Durlofsky, 2005). Here we introduce an
adjoint-based optimization approach. This is a very efficient way to compute the
gradient of the cost function as its cost is independent of the number of unknowns.
Linear or nonlinear constraints can be incorporated if needed.

Our approach involves two components: a near-well, single-phase parameter up-
scaling procedure which we designate NW1P, and a near-well, multiphase optimization
procedure referred to as NWMP. The upscaling calculations are performed on an
extended local well model extracted from a fine-scale geological model. The NW1P
procedure is based on the work of Mascarenhas and Durlofsky (2000). A well-driven,
single-phase flow problem is considered and the mismatch in steady-state flow rates
between the fine and coarse-scale models is minimized by optimizing well indices
and transmissibilities linking well blocks to non-well blocks. The NWMP procedure,
which determines upscaled mobility and PVT parameters (formation volume factors,
gas solubility), takes a similar approach. Here we minimize differences in well rates
of each component between the fine and coarse models for a well-driven, multiphase
flow problem. An adaptive local-global upscaling procedure is introduced to account
for global flow effects in the near-well upscaling calculations. The near-well upscaling
procedures are developed and applied for both oil-gas and oil-gas-water systems. We
also generalize the approach to provide structured coarse-scale models from highly
detailed unstructured fine-scale models.

1.3 Thesis Outline

This thesis proceeds as follows. In Chapter 2, we present the equations governing
two-phase oil-gas flow. The single-phase near-well upscaling method (NW1P) and
the two-phase near-well upscaling method (NWMP), both of which compute coarse-
model parameters using optimization procedures with gradients determined from the
solution of adjoint equations, are then described. Next we discuss the use of adaptive
local-global multiphase upscaling (ALGMP) for the determination of boundary con-
ditions used for the near-well upscaling. We then describe an interpolation approach
to estimate coarse-scale parameters for different well settings, which avoids the need
to apply the full procedure when well conditions are changed. Finally, some implementa-
tion issues involving the construction of a tabular representation of optimized
coarse-scale parameters and the smoothing of these parameters are discussed.

In Chapter 3, numerical results demonstrating the accuracy of the method and
the significant improvements it offers relative to existing techniques are presented.
We first present application examples of NWMP for homogeneous local well models
(LWM) using typical black oil and highly viscous heavy oil models. Then ALGMP
results for global heterogeneous reservoir models generated using sequential Gaussian
simulation (sGsim) are presented. Both fluid systems are considered. We also present
results using the interpolation approach. Some discussion of the computational re-
quirements of the method is also provided.

In Chapter 4, we first present the equations governing three-phase flow and ex-
tend the NWMP and ALGMP procedures described in Chapter 2 to the three-phase
flow case. Upscaling results for homogeneous LWMs and ALGMP results for het-
erogeneous reservoir models are then presented. Generalizations of the fluid systems
introduced in Chapter 3 are considered. These results demonstrate that the overall
procedure is able to provide accurate predictions for oil, gas and water production
rates for challenging problems.

In Chapter 5, we generalize the procedure to handle unstructured fine-grid sys-
tems, with very detailed near-well resolution. Several implementation issues are dis-
cussed. Numerical results for the typical black oil (oil-gas) system demonstrate that
ALGMP continues to perform well for these cases.

In Chapter 6, we summarize the major conclusions from this work and suggest
future research directions. In the Appendix, additional results for oil-gas systems are
presented.
Chapter 2

Development of a Near-well Adaptive Local-global Multiphase Upscaling Procedure

In this chapter we describe the oil-gas system and present our near-well upscaling procedure. The upscaling approach entails the use of a local well model (LWM) and the subsequent minimization of differences in fine and coarse-scale flow quantities over the LWM. An adaptive local-global procedure is introduced to provide boundary conditions for the LWM calculations. Detailed numerical results using the techniques developed here are presented in Chapter 3.

2.1 Governing Equations and Numerical Representation

We consider a porous medium containing two components, referred to as oil and gas, flowing in two phases, which are designated the oil phase and the gas phase. The oil component exists only in the oil phase, while the gas component can exist either in the gas phase or dissolved in the oil phase. The equations describing this two-phase, two-component porous media flow are formed by combining mass conservation with
Darcy’s law for the two phases. Neglecting capillary pressure effects, the oil and gas conservation equations are, respectively:

\[
\nabla \cdot \left[ \frac{\lambda_o}{B_o} k (\nabla p - \rho_o g \nabla D) \right] = \frac{\partial}{\partial t} \left( \phi \frac{S_o}{B_o} \right) + \tilde{q}_{w,o}, \tag{2.1}
\]

\[
\nabla \cdot \left[ \frac{\lambda_g}{B_g} k (\nabla p - \rho_g g \nabla D) + R_s \frac{\lambda_o}{B_o} k (\nabla p - \rho_o g \nabla D) \right] = \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] + \tilde{q}_{w,g}, \tag{2.2}
\]

where \( p \) is pressure, subscripts \( o \) and \( g \) indicate oil and gas phase, \( S_o \) and \( S_g \) are oil and gas saturation, \( B_o \) and \( B_g \) are oil and gas formation volume factor (defined as the ratio of the volume of phase \( j \), with \( j = o, g \), at reservoir conditions to the phase volume at reference or stock tank conditions), \( R_s \) is solution gas-oil ratio (which quantifies the volume of gas dissolved in a unit volume of the oil phase), \( \rho_o \) and \( \rho_g \) are oil and gas density, \( g \) is the gravitational acceleration, \( D \) is depth, \( \tilde{q}_{w,o} \) and \( \tilde{q}_{w,g} \) are source terms, \( \phi \) is porosity, and \( \mathbf{k} \) is the permeability tensor. The flow (e.g., \( \nabla \cdot [(\lambda_o/B_o)k\nabla p] \)), accumulation (e.g., \( \partial/\partial t[\phi S_o/B_o] \)), and source (e.g., \( \tilde{q}_{w,o} \)) terms in Eqs. 2.1 and 2.2 are of units time\(^{-1}\). The oil and gas phase mobilities (\( \lambda_o \) and \( \lambda_g \)) are defined as:

\[
\lambda_o = \frac{k_{ro}}{\mu_o}, \quad \lambda_g = \frac{k_{rg}}{\mu_g}, \tag{2.3}
\]

where \( k_{ro} \) and \( k_{rg} \) are oil and gas relative permeabilities and \( \mu_o \) and \( \mu_g \) are oil and gas viscosities. We note that capillary pressure effects could be readily included in our formulation, though typically they will not be important for the near-well flows considered here.

We solve Eqs. 2.1 and 2.2 on structured three-dimensional Cartesian grids using a standard finite volume procedure (see, e.g., Aziz and Settari, 1979 for details). The permeability field, which can be highly variable, is here taken to be a diagonal tensor with components \( k_x, k_y, \) and \( k_z \) (i.e., off-diagonal terms are neglected). Because the
grid is orthogonal and permeability is diagonal, standard two-point flux approximations can be applied. This means that the flux of phase \( j \) from block \( i \) to block \( i+1 \) can be expressed as:

\[
(f_j)_{i+1/2} = T_{i+1/2} \left( \frac{\lambda_j}{B_j} \right)_{i+1/2} \left( p_i - p_{i+1} \right),
\]

where \( (f_j)_{i+1/2} \) is the flux of phase \( j \) across the interface between blocks \( i \) and \( i+1 \) (designated \( i+1/2 \)), \( T_{i+1/2} \) is the interblock transmissibility, and \( p_i \) and \( p_{i+1} \) are the block pressures. Transmissibility (for flow in the \( x \)-direction) is given as \( (k_x)_{i+1/2} \Delta y \Delta z / \Delta x \), where \( (k_x)_{i+1/2} \) is computed as the weighted harmonic average of \( (k_x)_i \) and \( (k_x)_{i+1} \) and \( \Delta x, \Delta y \) and \( \Delta z \) are grid block dimensions. The \( (\lambda_j/B_j)_{i+1/2} \) term in Eq. 2.4 is upstream-weighted based on the flow direction.

For a well in block \( i \), oil and gas well flow rates (in units of volume/time), designated \( (q_{w,o})_i \) and \( (q_{w,g})_i \), are represented using the Peaceman (1978) well model:

\[
(q_{w,o})_i = (\bar{q}_{w,o})_i V_i = \left( \frac{\lambda_o}{B_o} \right)_i W_i (p_i - p_w),
\]

\[
(q_{w,g})_i = (\bar{q}_{w,g})_i V_i = \left( \frac{\lambda_g}{B_g} + R_s \frac{\lambda_o}{B_o} \right)_i W_i (p_i - p_w),
\]

where \( W_i \) is the well index, \( p_i \) is the well-block pressure, \( V_i \) is the volume of block \( i \), and \( p_w \) is the well pressure. For a fully penetrating vertical well, the well index is given as (Peaceman, 1978):

\[
W_i = \frac{2\pi \Delta z (k_x k_y)^{0.5}}{\ln (r_0/r_w)},
\]

where \( r_w \) is the wellbore radius and \( r_0 \) is the so-called equivalent well-block radius given by

\[
r_0 = 0.28 \frac{(k_y/k_x)^{0.5} (\Delta x)^2 + (k_x/k_y)^{0.5} (\Delta y)^2)^{0.5}}{(k_y/k_x)^{0.25} + (k_x/k_y)^{0.25}}.
\]

All quantities in Eqs. 2.7 and 2.8 are evaluated in block \( i \). These equations can be readily modified for use with horizontal wells that are aligned with the grid.
2.2 Upscaled Parameters and Local Well Model

The flow equations and numerical formulation presented above are appropriate for use with fully resolved systems, for which subgrid effects can be neglected. When flow simulations are performed on coarse scales, i.e., with large grid blocks, in which case subgrid heterogeneity in rock properties is present, equivalent or upscaled parameters must be applied. Using the * superscript to designate an upscaled parameter, we can write the coarse-scale equations in the same form as the fine-scale equations presented above, but with coarse-scale parameters \((T_w^*, \ W^*, \ \lambda_o^*, \ \lambda_g^*, \ B_o^*, \ B_g^*, \ R_s^*)\) in place of their fine-scale counterparts. It is the goal of upscaling procedures to provide accurate representations of these coarse-scale parameters. We note that, in some cases, the form of the coarse-scale equations can differ from that of the fine-scale equations (see for example Auriault et al., 2007; Durlofsky, 1998; Efendiev and Durlofsky, 2003). Although such models will not be considered here, it is important to note that our formulation can be applied with any coarse-model representation.

Our emphasis here is on upscaling in the near-well region. For this purpose, it will be useful to consider a relatively small local region around the target well and to perform upscaling computations over this domain. This region, shown in Figure 2.1, will be referred to as a local well model (LWM). The LWM comprises the region corresponding to the coarse well block or blocks and one or more rings of surrounding coarse blocks. Near-well upscaling computations entail solution of the governing equations over the fine-scale blocks contained within the LWM, followed by appropriate averaging and, in some cases, iteration on the coarse-scale parameters to force agreement between fine and coarse LWM results.

In all computations presented in this thesis, the LWM comprises the domain corresponding to a single ring of coarse blocks around the target well blocks unless otherwise specified. In some cases, however, improved accuracy can be achieved through use of either a larger LWM (e.g., two rings of coarse blocks) or an oriented (rectangular) LWM that reflects the correlation structure of the permeability field. This issue is considered further in the Appendix.

The near-well upscaling approach presented here employs a two-step procedure.
We first compute the so-called upscaled single-phase parameters ($T^*_w$ and $W^*$, where $T^*_w$ designates the transmissibility between coarse-scale well blocks and adjacent non-well blocks) and then the upscaled two-phase parameters. In our case the upscaled two-phase parameters include $\lambda^*_o$, $\lambda^*_g$, $B^*_o$, $B^*_g$ and $R^*_s$ in all coarse well blocks. Upscaled parameters for blocks that do not contain wells are also required. In this work, in all cases we apply extended local transmissibility upscaling to provide $T^*$ for non-well blocks (see Chen and Durlofsky, 2006a for a description of this procedure). Two-phase flow functions in non-well blocks are not upscaled; i.e., we retain the fine-scale forms. These parameters could also be upscaled if necessary, though for the primary production examples considered here, near-well effects dominate and this does not appear to be necessary.

Our procedures do not require wells to be located at grid-block centers or to be fully penetrating on either the fine or coarse-scale. The upscaled parameters computed by the method account directly for these effects. If the well is not centered and fully penetrating in fine-grid blocks, the fine-scale well model (as defined in Eqs. 2.7 and 2.8) and possibly one or more fine-scale transmissibilities would however require modification. We now describe the two near-well upscaling procedures in turn.

### 2.3 Near-well Single-phase Upscaling (NW1P)

#### 2.3.1 Governing equations and minimization problem

The first step of our near-well upscaling procedure entails the determination of the coarse-scale well index and the well-block transmissibilities. Because these quantities can be computed from single-phase flow considerations, they are referred to as upscaled single-phase flow parameters.

The incompressible single-phase flow equation (ignoring gravitational effects and taking $\mu = 1$) can be expressed in dimensionless form as:

$$\nabla \cdot (k \nabla p) = \tilde{q}_w,$$

where $\tilde{q}_w$ is the source term. The flow terms are discretized as discussed in Section
2. For a well in block $i$, the well flow rate is given by Eq. 2.5 with $\lambda_o = B_o = 1$.

A number of previous studies (e.g., Ding, 1995; Durlofsky et al., 2000; Mascarenhas and Durlofsky, 2000; Muggeridge et al., 2002; Chen and Wu, 2008) have applied single-phase near-well upscaling procedures and shown them to be very effective for capturing fine-scale flow effects in coarse-scale models. Even in cases where upscaled two-phase flow parameters are also required, the use of accurate $W^*$ and $T_w^*$ is an essential first step. We now describe the near-well single-phase upscaling, referred to as NW1P, applied in this work. This approach most closely follows the procedures applied in Hui (2005), Hui and Durlofsky (2005) and Mascarenhas and Durlofsky (2000), though here we introduce an adjoint procedure to compute the gradients required to minimize the discrepancy between coarse and (integrated) fine-scale results.

We proceed by solving Eq. 2.9 over the LWM. Flow is driven by an injection well in the central block, for which we specify $p_w = 1$. We set $p = 0$ at the boundaries of the LWM that are aligned with the well (i.e., the boundaries with normal pointing in the $x$ or $y$ direction for a vertical well) and no-flow boundary conditions on the
LWM faces that intersect the well (see Figure 2.2). The upscaled parameters are independent of the values used for these pressures because Eq. 2.9 is linear. Other boundary specifications are possible, and the boundary conditions can be iterated using a local-global procedure (e.g., Chen and Durlofsky, 2006a), though the use of \( p = 0 \) was shown to provide results of reasonable accuracy in previous studies.

The basic idea of NW1P is to minimize the mismatch between the fine and coarse-scale solutions of Eq. 2.9 over the LWM. Specifically, we consider the square differences between the integrated (or summed) fine-scale flow rates and the coarse-scale flow rates. This can be posed as the following optimization problem:

\[
L = \sum_{i=1}^{n_{wb}} [(q_{w_i} - \langle q_w \rangle_i)^2 + \sum_{k=1}^{n_f} [q_k^c - \langle q \rangle_k]^2],
\]

subject to the governing flow equations (Eq. 2.9). We express the discretized flow equations as \( g(x, u) = 0 \), where \( x \) denotes the system states (pressure in this case) and \( u \) the coarse-scale parameters we seek to determine. In Eq. 2.10, \( (q_{w_i}^c) \) is the flow rate into or out of the well in coarse well block \( i \), \( \langle q_w \rangle_i \) is the sum of the fine-scale well flow rates over all fine well blocks corresponding to coarse well block \( i \), and \( n_{wb} \) is the number of coarse well blocks. The quantity \( q_k^c \) is the flow rate across coarse face \( k \) (which is the face shared by coarse well block \( i \) and non-well block \( k \)), \( \langle q \rangle_k \) is the integrated fine-scale flow rate across fine faces corresponding to coarse face \( k \), and \( n_f^c \) is the total number of coarse faces that the coarse well blocks share with non-well blocks (e.g., for a vertical or horizontal grid-aligned well that does not extend through the entire 3D model, \( n_f^c = 4n_{wb} + 2 \)). The quantities \( (q_{w_i}^c) \) and \( q_k^c \) are determined from the coarse-scale solution of Eq. 2.9, while \( \langle q_w \rangle_i \) and \( \langle q \rangle_k \) are derived from the fine-scale solution. Eqs. 2.4 and 2.5, with \( \lambda_j = B_j = 1 \), are applied to compute these quantities.

Figure 2.3 depicts the LWM for a 2D problem, which consists of \( 3 \times 3 \) coarse cells. In the figure \( i \) designates the well block and \( k \) a neighboring block. Here \( n_{wb} = 1 \) and \( n_f^c = 4 \) (the four coarse interfaces are indicated by the thick solid lines). The quantities \( q_k^c \) and \( \langle q \rangle_k \) designate the flow rates through each of the four interfaces shown in the figure, as computed from the coarse and fine-scale solutions respectively.
The upscaled (optimization) parameters appearing in \( \mathbf{u} \) are the transmissibilities linking well blocks to adjacent non-well blocks, \( T^*_w \), and the well indices, \( W^* \). Thus \( \mathbf{u} \), which is of dimension \( n_f^c + n_{wb}^c \), is written as:

\[
\mathbf{u} = \begin{bmatrix} T^*_{w,1}, \ldots, T^*_{w,n_f^c}, W^*_1, \ldots, W^*_{n_{wb}^c} \end{bmatrix}^T.
\]

We will compute \( \mathbf{u} \) using a gradient-based optimization procedure. Initial estimates for the components of \( \mathbf{u} \) can be determined from appropriate averages of the fine-grid solution of Eq. 2.9:

\[
W^*_i = \frac{\langle q_w \rangle_i}{\langle p \rangle_i - p_w}, \quad T^*_{w,ik} = \frac{\langle q \rangle_k}{\langle p \rangle_i - \langle p \rangle_k},
\]

where \( \langle p \rangle_i \) is computed as the bulk-volume weighted average of the fine-grid pressure:

\[
\langle p \rangle_i = \frac{1}{V_i} \sum_{l=1}^{n_t} p_l v_l,
\]

where \( V_i \) is the bulk volume of coarse cell \( i \), \( n_t \) is the total number of fine cells in coarse cell \( i \), and \( v_l \) is the bulk volume of fine cell \( l \).

The use of upscaled parameters computed using Eq. 2.12 can give results of reasonable accuracy in some cases. However, in models with high degrees of heterogeneity in the near-well region the level of accuracy may degrade considerably (Mascarenhas and Durlofsky, 2000). For this reason we introduce an optimization procedure that seeks to minimize the cost function \( L \).

### 2.3.2 Adjoint method for gradient calculation

We apply a gradient-based procedure to minimize the cost function \( L \). In previous studies, \( \partial L / \partial \mathbf{u} \) was formed using a numerical finite difference procedure. This required \( n_f^c + n_{wb}^c + 1 \) coarse-scale simulations. In addition, with this approach the derivatives can be sensitive to the magnitude of the perturbation parameter. Here we apply an adjoint procedure, which requires only two simulations so it is much more efficient than numerical finite difference approaches. In addition, the adjoint
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Figure 2.2: Boundary conditions applied for NW1P upscaling

Figure 2.3: Schematic of local well model (LWM) in 2D
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technique does not entail numerical perturbation, so errors associated with this are eliminated. Adjoint approaches have been widely used for minimization problems arising in production optimization (see, e.g., Ramirez, 1987; Brouwer and Jansen, 2004; Sarma, 2006) and history matching (e.g., Chen et al., 1974; Li et al., 2003), but do not appear to have received much attention within the context of upscaling.

To apply the adjoint procedure, we define an augmented cost function, $J_A$, which includes the original cost function $L$ modified to include the single-phase flow equations ($g(x, u) = 0$) as a constraint:

$$J_A = L(x, u) + \lambda^T g(x, u),$$

where $\lambda$ represents the vector of Lagrange multipliers and the superscript $T$ designates transpose (these $\lambda$ are not to be confused with the oil and gas phase mobilities, $\lambda_o$ and $\lambda_g$). Because $g = 0$, any values can be chosen for $\lambda$.

Optimality of the original problem is achieved when the first variation of $J_A$ vanishes:

$$\delta J_A = 0 = \left( \frac{\partial L}{\partial x} + \lambda^T \frac{\partial g}{\partial x} \right) \delta x + \left( \frac{\partial L}{\partial u} + \lambda^T \frac{\partial g}{\partial u} \right) \delta u.$$  \hspace{1cm} (2.15)

As we are free to specify $\lambda$, it will be most convenient if we compute it such that the terms multiplying $\delta x$ become zero; i.e.,

$$\lambda^T = -\frac{\partial L}{\partial x} \left( \frac{\partial g}{\partial x} \right)^{-1}. $$  \hspace{1cm} (2.16)

Now the gradient of the augmented cost function can be expressed as:

$$\frac{\partial J_A}{\partial u} = \frac{\partial L}{\partial u} + \lambda^T \frac{\partial g}{\partial u},$$

with $\lambda$ given by Eq. 2.16.

This method provides the full gradient vector $\partial J_A/\partial u$ with only two linear solves (determination of $x$ by solving $g(x, u) = 0$ and solution of Eq. 2.16). Both of these solutions are over the coarse-scale LWM. In solving Eq. 2.9 over the LWM, coarse-scale transmissibilities between non-well blocks need be computed only once. This
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is accomplished using the extended local transmissibility upscaling. In addition, the computational effort to compute the gradients is independent of the number of optimization parameters. There are a number of gradient-based optimization algorithms that can be used to update $u$ given $\partial J_A/\partial u$. Here we use a sequential quadratic programming (SQP) algorithm (Schittkowski, 2006). Using Eq. 2.12 to provide the initial guess, typically only 2-4 iterations are required to drive the cost function (Eq. 2.10) to $10^{-4}$.

2.4 Near-well Multiphase Upscaling (NWMP)

For some classes of problems, accurate coarse-scale models can be generated using only the NW1P procedure described above. In other cases, however, such as those involving the injection of water into very low-mobility (high-viscosity) oil or primary production with significant dissolved gas, the determination of upscaled two-phase or multiphase well-block parameters may also be required. We now describe a procedure for this upscaling, referred to as NWMP, which, like NW1P, entails the minimization of a cost function using a gradient-based optimization procedure, with gradients determined using an adjoint procedure.

The parameters to be upscaled in the NWMP procedure are $\lambda_o$, $\lambda_g$, $B_o$, $B_g$ and $R_s$ within coarse well blocks. These functions depend on saturation and/or pressure. The determination of the upscaled functions will therefore be accomplished by simulating the dynamic problem (Eqs. 2.1 and 2.2), in which pressure and saturation evolve in time. The idea now is to simulate oil-gas flow over the LWM and to compute the upscaled parameters, which are now functions of coarse variables $p^c$ and $S^c_g$, such that the flow rates of oil and gas in the coarse model match the integrated flow rates from the fine model. Boundary conditions must be prescribed for the LWM, and the upscaled parameters can be sensitive to the boundary specification (in contrast to the upscaled single-phase near-well parameters, which are often rather insensitive to LWM boundary conditions). We will discuss a local-global treatment for these boundary conditions in Section 2.5. For now, we can assume that no-flow boundary conditions are applied.
2.4.1 Cost function and adjoint equations for NWMP

The cost function we seek to minimize in NWMP is as follows:

\[
J = \sum_{n=1}^{N} L^n (x^n, u),
\]

where:

\[
L^n (x^n, u) = \sum_{i=1}^{n_{wb}} \left[ \alpha_o \left( \left( \frac{q_{w,o}^n}{q_{w,o}^n} \right)_i - \langle q_{w,o}^n \rangle_i \right)^2 + \alpha_g \left( \left( \frac{q_{w,g}^n}{q_{w,g}^n} \right)_i - \langle q_{w,g}^n \rangle_i \right)^2 \right].
\]

Here \( n \) denotes time step, \( N \) is the total number of time steps in the coarse-scale LWM simulation, \( x \) refers to the dynamic states of the system (pressure and gas saturation in this case), \( u \) is the set of optimization parameters (upscaled two-phase flow functions, see below for specific forms), \( (q_{w,o}^n)_i \) is the well oil flow rate in coarse block \( i \) over time step \( n \), \( \langle q_{w,o}^n \rangle_i \) is the integrated (summed) well oil flow rate in fine blocks corresponding to coarse well block \( i \) over time step \( n \), and \( (q_{w,g}^n)_i \) and \( \langle q_{w,g}^n \rangle_i \) are analogous quantities for gas flow rate. These gas rates include both free and dissolved gas. The parameters \( \alpha_o \) and \( \alpha_g \) are weighting factors which can be used to emphasize agreement in one or the other component, though in all examples in this thesis we use \( \alpha_o = \alpha_g = 1 \). The flow rates are normalized because the magnitudes of the oil and gas flow rates can differ significantly. The fine-scale LWM simulations are performed using Stanford’s General Purpose Research Simulator, GPRS (Cao, 2002; Jiang, 2007).

The optimization problem can now be stated as follows:

\[
\min \left[ J = \sum_{n=1}^{N} L^n (x^n, u) \right],
\]
subject to:

\[ g^n(x^n, x^{n-1}, u) = 0, \quad \forall n \in (1, \ldots, N). \]  

(2.21)

The quantity \( g^n = 0 \) (with initial conditions) represents the dynamic system; i.e., Eqs. 2.1 and 2.2, which act as constraints in the optimization. The dependence of \( g^n \) on \( x^n \) indicates that an implicit procedure is used in the solution of the flow equations.

Proceeding as in Section 2.3.2, we define the augmented cost function \( J_A \) by combining Eqs. 2.20 and 2.21:

\[ J_A = \sum_{n=1}^{N} L^n (x^n, u) + \sum_{n=1}^{N} (\lambda^T)^n g^n \left( x^n, x^{n-1}, u \right), \]  

(2.22)

where \( \lambda \) again represents Lagrange multipliers, here defined for each unknown in each grid block at each time step. This augmented cost function is more complicated than that used to determine the single-phase parameters (Eq. 2.14) because the system is now dynamic, which means that we must now sum over time.

To determine the optimum \( u \), we again require that the first variation of the augmented cost function vanish, which gives:

\[
\delta J_A = 0 = \sum_{n=1}^{N-1} \left( \frac{\partial L^n}{\partial x^n} + (\lambda^T)^{n+1} \frac{\partial g^{n+1}}{\partial x^{n+1}} + (\lambda^T)^n \frac{\partial g^n}{\partial x^n} \right) \delta x^n + \\
\left( \frac{\partial L^N}{\partial x^N} + (\lambda^T)^N \frac{\partial g^N}{\partial x^N} \right) \delta x^N + \sum_{n=1}^{N} \left( \frac{\partial L^n}{\partial u} + (\lambda^T)^n \frac{\partial g^n}{\partial u} \right) \delta u.
\]  

(2.23)

By choosing \( \lambda \) such that terms multiplying \( \delta x^n \) and \( \delta x^N \) vanish, the required gradient \( (\partial J_A/\partial u) \) can be readily computed. Thus we require that

\[
(\lambda^T)^N = -\frac{\partial L^N}{\partial x^N} \left( \frac{\partial g^N}{\partial x^N} \right)^{-1},
\]  

(2.24)

\[
(\lambda^T)^n = -\left( \frac{\partial L^n}{\partial x^n} + (\lambda^T)^{n+1} \frac{\partial g^{n+1}}{\partial x^{n+1}} \right) \left( \frac{\partial g^n}{\partial x^n} \right)^{-1}, \quad \forall n \in (1, \ldots, N - 1).
\]  

(2.25)

After computing \( (\lambda^T)^N \) using Eq. 2.24, \( (\lambda^T)^n \) is obtained by solving Eq. 2.25 backward in time. We note that \( \partial g^n/\partial x^n \) is the Jacobian matrix of the (converged) forward
NEAR-WELL MULTIPHASE UPSCALING

model. This matrix is saved during the forward run and then used for the adjoint solution.

With \((\lambda^T)^n\) determined, we can now construct the gradient of the augmented cost function:

\[
\frac{\partial J_A}{\partial u} = \sum_{n=1}^{N} \left( \frac{\partial L^n}{\partial u} + (\lambda^T)^n \frac{\partial g^n}{\partial u} \right). 
\]  

(2.26)

Once this gradient is calculated, the optimization parameters (upscaled functions) can be updated using a gradient-based minimization algorithm. We again use an SQP algorithm (Schittkowski, 2006) and a line search to define the step size. In order to avoid unphysical parameter values, we limit the size of the update to \(\pm 10\%\) of the previous value for the parameter.

2.4.2 Representation of upscaled parameters and overall procedure

A great advantage of the adjoint formulation presented above is that the computational requirements for the construction of the gradient are independent of the number of optimization parameters (i.e., the dimension of \(u\)). This means that we can incorporate more parameters into the upscaling, which allows for significant flexibility in terms of the representation of the upscaled functions.

One approach is to prescribe functional forms for the upscaled variables; e.g., \(R_*^s = ap^c + b\). In this case the optimization provides the coefficients \(a\) and \(b\). We implemented various procedures of this type, but found that it was difficult to determine a priori the optimal functional forms for the upscaled parameters. This determination is complicated because the optimal form may be different for different parameters and may also depend on the specific system being simulated. We observed the best accuracy using a more exhaustive description of the upscaled parameters – specifically, a tabular representation. More details on the construction of these tables are given below. Then, during subsequent coarse-scale simulations, we use a table lookup to determine the upscaled parameter value.

In coarse-scale simulations, we represent \(B_o^*, B_g^*\) and \(R_*^s\) as functions of \(p^c\) only,
and $\lambda^*_o$ and $\lambda^*_g$ as functions of $p^c$ and $S^c_g$. It is useful to take the $\lambda^*_j$ to be functions of both variables because, in a simulation of primary production, well-block $S^c_g$ can first increase and then decrease, which could lead to nonuniqueness in the upscaled parameters if they were taken to depend only on $S^c_g$. Because coarse-scale pressure in this case decreases monotonically, $\lambda^*_j = \lambda^*_j(p^c, S^c_g)$ can be represented uniquely. Other representations could also be applied, though this does not appear to be necessary for the cases considered here. For the initial estimates for the upscaled parameters, we apply averaging techniques analogous to that defined by Eq. 2.12.

The overall NWMP workflow is depicted in Figure 2.4. The key steps in the NWMP algorithm are as follows.

1. Perform a fine-scale oil-gas flow simulation on the LWM. Calculate flow quantities required for cost function evaluation; e.g., $\langle q_{w,o} \rangle_i$.

2. Perform a coarse-scale simulation on the LWM using the current estimate for $\lambda^*_j$, $B^*_j$ and $R^*_s$. During the simulation, calculate and save, at each time step, quantities required for the adjoint calculation.

3. Evaluate the cost function and check for convergence. Proceed to step 4 if convergence is not achieved.

4. Calculate the gradient of the cost function using the adjoint method.

5. Update the coarse-scale parameters using a gradient-based optimization algorithm and return to step 2.

### 2.5 Adaptive Local-Global Multiphase Upscaling (ALGMP)

Upscaled parameters are in general dependent on the boundary conditions used in their determination, and the oil-gas well-block parameters computed here are no exception. Thus it is important to impose boundary conditions on the LWM that
will coincide (approximately) with the conditions that the region will encounter in an actual (large-scale) simulation. Complications arise, however, because even if we know the global boundary conditions that are to be imposed in the large-scale simulations, there is not an obvious way to determine LWM boundary conditions from the global boundary specifications without performing a global fine-scale simulation. As the intent of our upscaling procedure is to avoid global fine-scale simulations, we will need to introduce an approximate method for this determination.

We address this issue using a local-global method. Such approaches, developed in, e.g., Chen and Durlofsky (2006a), Gerritsen and Lambers (2008) and Chen and Li (2009), use global coarse-scale simulations to approximate the local boundary conditions used for upscaling computations. Most previous local-global procedures have been applied for the upscaling of single-phase flow parameters, though the recent work reported in Chen and Li (2009) successfully extends the general approach to oil-water systems. Here we develop and apply an adaptive local-global (ALG) technique for the oil-gas near-well problem. We use the term ‘adaptive’ to indicate that the
global coarse-scale simulations are performed using the actual boundary conditions applied in the large-scale simulation rather than a generic set of boundary conditions.

We now describe the ALG procedure applied here for the near-well oil-gas problem (the procedure is referred to as ALGMP). In order to compute the upscaled parameters, we require pressure and saturation boundary conditions over each LWM (multiple wells can be considered). These boundary conditions vary in both space and time. In the description below, for convenience we assume that the wells operate under fixed pressure. We further assume that all upscaled single-phase flow parameters, such as coarse-scale transmissibility throughout the domain and in the near-well region, as well as $W^*$ for all well blocks, have already been computed. The overall ALGMP algorithm, with reference to Figure 2.5, is as follows:

1. Perform a coarse-scale oil-gas flow simulation with specified pressures for each well. In the first iteration, fine-scale mobility and PVT parameters are applied. The gray region (3 $\times$ 3 coarse cells) in Figure 2.5 represents an LWM. During this flow simulation, save coarse-block pressures and saturations at each time step in blocks bordering the LWM (these blocks are marked with $\times$’s in Figure 2.5).

2. Determine boundary conditions (at edges indicated by solid circles in Figure 2.5) for the local fine-scale near-well problems by interpolating coarse-scale pressures and saturations saved in step 1. These boundary conditions must be determined at each time step. For this interpolation, we use the trilinear spatial interpolation and linear interpolation in time.

3. Perform NWMP upscaling using the boundary conditions defined in step 2 (our General Purpose Research Simulator has been modified to handle general LWM boundary conditions). Iterate the upscaling procedure until convergence. This results in new coarse-scale parameters for each well block in the model. This procedure must be applied for each LWM.

4. Repeat steps 1 to 3 either a specified number of times or until an appropriate measure of the change in the upscaled parameters or LWM flow rates is reduced
2.6 Coarse Models for Different Well Conditions

The procedure described thus far requires that the wellbore pressure $p_w$ be specified. It is quite likely, however, that this parameter may be varied in subsequent large-scale simulations, and it would be preferable to avoid performing the full upscaling procedure for each new well setting. We have observed that the upscaled functions vary approximately linearly with the specified $p_w$ (over appropriate pressure ranges),

Figure 2.5: LWM boundary conditions determined from adaptive local-global procedure
to a prescribed tolerance. Because we have observed that changes after the first iteration are consistently very small, we apply only two iterations in all cases. It is possible, however, that there are cases for which additional iterations will improve the accuracy of the coarse-scale model.

This workflow is depicted schematically in Figure 2.6.
which suggests the following simple procedure for treating different wellbore pressure specifications.

We first define the minimum \((p_w^0)\) and maximum \((p_w^1)\) well pressures over which simulations are to be performed. Then, ALGMP is applied twice, first with well pressures set to \(p_w^0\) and then with well pressures set to \(p_w^1\). The pressures \(p_w^0\) and \(p_w^1\) should both be either above or below the bubble point pressure. The well pressures are constant in time in each simulation. For cases with multiple wells, different \(p_w^0\) and \(p_w^1\) can be used for different wells as appropriate.

In the global coarse-scale simulation, we can now specify any \(p_w\) that is between \(p_w^0\) and \(p_w^1\). A simple interpolation is applied to determine all of the upscaled parameters; i.e., we set \(u_i = (1 - \beta)u_i^0 + \beta u_i^1\), where \(\beta = (p_w - p_w^0)/(p_w^1 - p_w^0)\), \(u_i\) is the coarse-scale parameter of interest (for well pressure \(p_w\)), and \(u_i^0\) and \(u_i^1\) are the coarse-scale parameter values for well pressures \(p_w^0\) and \(p_w^1\). This treatment is clearly approximate, but it will be shown to provide a reasonable level of accuracy in Section 3.2.3.
We now describe techniques for constructing tabular representations of the optimized coarse-scale functions and for smoothing these functions.

Upscaled mobilities are functions of pressure and one or more saturations. For specificity, here we consider $\lambda^g$, which is a function of $p^c$ and $S_g^c$. The $\lambda^g$ function determined from the ALGMP procedure can be viewed as a trajectory through $p^c-S_g^c-\lambda^g$ space. The projection of this curve into the $p^c-S_g^c$ plane is shown in Figure 2.7, where the heavy solid curve represents the trajectory followed during the ALGMP procedure. Complications arise because the trajectory followed during subsequent global coarse-scale simulations may not exactly follow this curve, yet $\lambda^g$ must still be assigned as a function of $p^c$ and $S_g^c$. The $p^c-S_g^c$ trajectory for the coarse-scale simulation is represented as the heavy dotted curve in Figure 2.7. The $\times$ on this curve represents the $p^c-S_g^c$ values for which we wish to compute $\lambda^g$.

Our approach is to build a table of $\lambda^g$ (which we here designate simply as $\lambda^*$) as a function of $p^c$ and $S_g^c$ (here designated $S^c$) and to update the points in the table as we iterate within the NWMP procedure. The table points are represented by the
solid black circles. For a point along the trajectory that falls between \( p^c_l \) and \( p^c_{l+1} \) and \( S^c_m \) and \( S^c_{m+1} \), where \( l \) is the table index for \( p^c \) and \( m \) is the index for \( S^c \), \( \lambda^* \) is determined by the following bilinear interpolation:

\[
\lambda^*(p^c, S^c) = (1-s)(1-t)\lambda^*_{l,m} + s(1-t)\lambda^*_{l+1,m} + (1-s)t\lambda^*_{l,m+1} + st\lambda^*_{l+1,m+1}.
\] (2.27)

Here \( \lambda^*_{l,m} \) is a tabular upscaled gas mobility value at \((p^c_l, S^c_m)\) and \( s \) and \( t \) are defined as \( s = (p^c - p^c_l)/(p^c_{l+1} - p^c_l) \) and \( t = (S^c - S^c_m)/(S^c_{m+1} - S^c_m) \), respectively.

In the computation on the coarse-scale LWM, we update the table values for mobility through use of the gradients computed during the adjoint solution. Recall that we determine \( \partial L^n / \partial \lambda^* \) and \( \partial g^n / \partial \lambda^* \) as part of the optimization procedure. We then construct \( \partial L^n / \partial \lambda^*_{l,m} \) and \( \partial g^n / \partial \lambda^*_{l,m} \) to give \( \partial J_A / \partial \lambda^*_{l,m} \) (via Eq. 2.26) as follows:

\[
\frac{\partial L^n}{\partial \lambda^*_{l,m}} = \frac{\partial L^n}{\partial \lambda^*(p^c, S^c)} \frac{\partial \lambda^*(p^c, S^c)}{\partial \lambda^*_{l,m}},
\] (2.28)

and

\[
\frac{\partial g^n}{\partial \lambda^*_{l,m}} = \frac{\partial g^n}{\partial \lambda^*(p^c, S^c)} \frac{\partial \lambda^*(p^c, S^c)}{\partial \lambda^*_{l,m}},
\] (2.29)

where \( \partial \lambda^*(p^c, S^c) / \partial \lambda^*_{l,m} \) is calculated using Eq. 2.27. Thus, during the course of the minimization, \( \partial J_A / \partial \lambda^*_{l,m} \) is used to optimize the table points \( (\lambda^*_{l,m}) \).

It is still possible, however, that in subsequent global coarse-scale simulations, the \( p^c - S^c \) trajectory will fall in regions that are not included in the table. This is illustrated in Figure 2.7, where the target point \((\times)\) does not fall within the range of four table points (i.e., the \( \times \) is not within a rectangle defined by four solid black circles). To handle such cases, the table must be extended using some type of extrapolation procedure. Our approach is as follows. For the point \( P \), we calculate the distance to the closest optimized point in both the pressure and saturation directions and then compute the mobility value at \( P \) using 

\[
\lambda^*_P = \frac{a\lambda^*_Q + b\lambda^*_R}{a + b},
\]

where \( \lambda^*_P \), \( \lambda^*_Q \) and \( \lambda^*_R \) are upscaled mobility values at points \( P \), \( Q \) and \( R \) and \( a \) and \( b \) are distances (see Figure 2.7). Additional table values are computed only over a limited range (bounded...
by the light dash-dot curves). This provides table values for the points shown as solid gray circles in Figure 2.7. Other procedures for extending the range of the table are of course also possible, though this approach has the advantage of not introducing any new local extremes into the $\lambda^*_{l,m}$ tabular values.

In some cases oscillations are observed in the global simulation solutions. These oscillations appear to derive from a lack of smoothness in the upscaled mobility functions. Such mobility functions can arise from our use of unconstrained point-by-point optimization in determining the $\lambda^*_j$. This problem can be remedied by applying a smoothing algorithm. Here we use a least squares linear polynomial fitting of the upscaled functions (Guest, 1961). The algorithm allows us to choose the number of points used for smoothing. We found that the use of five points provides reasonable non-oscillatory results for our cases. In general, the use of more points for smoothing acts to further reduce oscillations, but some accuracy may be lost; e.g., in gas production spikes.

2.8 Summary

In this chapter we presented techniques for upscaling single-phase and two-phase well-block parameters. The procedures are distinguished NW1P and NWMP. Both approaches entail the use of gradient-based optimization to minimize the mismatch of fine and coarse simulations over local well regions. An adaptive local-global approach was introduced to provide boundary conditions for the local well model.

In the next chapter we present extensive numerical results.
Chapter 3

Numerical Results for Near-well Upscaling Procedures

In the previous chapter, we developed a new adaptive local-global multiphase near-well upscaling procedure. In this chapter, we present numerical results for several different models. The high degree of accuracy of the method for the local well model and for the global reservoir domain is demonstrated.

3.1 Application of NWMP to Local Well Model

We now demonstrate the performance of the near-well multiphase upscaling approach (NWMP) over the local well model (LWM). We consider two fluid systems – a typical black oil and a highly viscous heavy oil. For these examples, we consider a 3D homogeneous LWM. The fine model contains $33 \times 33 \times 5 = 5,445$ cells. The fine-grid cell dimensions are $\Delta x = \Delta y = 150$ ft and $\Delta z = 1$ ft. A vertical production well is located in the center of the model and is completed in cells $i = 17, j = 17, k = 1 - 5$. The coarse LWM contains $3 \times 3 \times 1 = 9$ uniform cells. Thus the overall coarsening factor is 605. No-flow conditions are applied on the LWM boundaries. Near-well properties for the coarse-scale LWM are computed using the NW1P and NWMP procedures.
### 3.1.1 Example 1 – Typical black oil system

For this case we consider a black oil with a bubble point pressure \( p_b \) of 4,000 psi \((27.58 \times 10^6 \text{ Pa})\). The relative permeabilities are prescribed as \( k_{rg} = S_g^2 \) and \( k_{ro} = (1 - S_g)^2 \). Curves for \( B_o \), \( B_g \) and \( R_s \) are shown in Figure 3.1 (these are taken from the Ninth SPE Comparative Solution Project; Killough, 1995). The oil density is 37.5 lb/ft\(^3\) \((601 \text{ kg/m}^3\)). Oil and gas viscosities are prescribed to be 1 cp and 0.015 cp, respectively, and are taken to be constant. The (homogeneous) permeability and porosity are set to 10 md and 25%. Initial reservoir pressure is prescribed as 4,500 psi (which is greater than \( p_b \)) at the top of the model and initial gas saturation is set to 0. We prescribe \( p_w = 1,000 \text{ psi} \) and simulate for a time of 500 days.

![Figure 3.1: Input curves for typical black oil](image)

**Figure 3.1:** Input curves for typical black oil (upper left: relative permeability, upper right: oil formation volume factor, lower left: gas formation volume factor, lower right: solution gas-oil ratio)
Figure 3.2 compares the performance of the fine (reference) and coarse-scale models. Coarse models constructed using only single-phase parameter upscaling (NW1P) and models that use both single and two-phase parameter upscaling (NWMP) are considered. Results (at surface or stock tank conditions) are shown for oil rate, gas rate, well-block pressure and well-block saturation. To enable appropriate comparisons between fine and coarse models, we average (using pore-volume weighting) the fine-model results for well-block pressure and saturation over the fine cells corresponding to the coarse-scale well block.

It is apparent from Figure 3.2 that the coarse model generated using NW1P shows reasonable accuracy for oil rate, but significant error appears in gas rate. In the reference model, the well-block pressure very quickly falls below the bubble point and dissolved gas is liberated. This gas moves to the top layer of the model due to gravity. These effects are not captured by the NW1P model. The NWMP model, by contrast, reproduces the fine-scale flow behavior very accurately. Well-block pressure and saturation are also matched closely by the NWMP model.

Figure 3.3 presents the upscaled well-block functions. The input curves are depicted using solid lines and the optimized coarse-scale curves are displayed using dashed lines. The actual block pressure during the LWM simulation ranges from 3,340 to 4,500 psi. Linear extrapolation is used to define the functions down to 3,000 psi, as pressures below 3,340 psi may be encountered in subsequent (global) simulations.

The most obvious differences are in the oil and gas mobilities, with the oil mobility decreased and the gas mobility increased in the coarse-grid model relative to the fine-scale functions. Note that $\lambda_0^*$ and $\lambda_g^*$ are now taken to be functions of both saturation and pressure, though here we plot them versus gas saturation only, as this is the stronger dependence. There is a subtle but important difference in $R_s^*$ compared to $R_s$. The $R_s$ curve is constant for pressures greater than $p_b = 4,000$ psi, but the $R_s^*$ curve decreases from the constant value at a larger pressure. This larger pressure (4,433 psi in this case) can be viewed as a ‘pseudo’ bubble point pressure. This shift in bubble point pressure accounts for the fact that, although the coarse well-block pressure may be above the original bubble point pressure, the subgrid pressure
variation is such that some portion of this region should be below the original bubble point. Thus, free gas should be generated, and this is captured on the coarse grid through a shift in $p_b$. The $B_o$ and $B_g$ curves also show small changes relative to the original curves, which reflect the shift in bubble point pressure.

![Graphs showing oil rate, gas rate, well-block pressure, and well-block gas saturation over time.]

Figure 3.2: Upscaling results for LWM for typical black oil case (upper left: oil rate, upper right: gas rate, lower left: well-block pressure, lower right: well-block gas saturation)

### 3.1.2 Example 2 – Heavy oil system

The next numerical test is performed for a heavy oil system. The fluid and rock-fluid properties, shown in Figure 3.4, are from an actual South American reservoir. Note that the oil viscosity is $O(10^4)$ cp. The bubble point pressure is 870 psi ($6.00 \times 10^6$ Pa) and the oil density is 63.03 lb/ft$^3$ (1,010 kg/m$^3$). The permeability for this case is increased to 50 D. Porosity is kept at 25%. Initial pressure and gas saturation are
Figure 3.3: Upscaled well-block functions for typical black oil (upper left: mobility, upper right: oil formation volume factor, lower left: gas formation volume factor, lower right: solution gas-oil ratio)
1,000 psi (this is again higher than $p_b$) and 0. We prescribe a constant well pressure of 300 psi.

Figure 3.5 presents simulation results for the fine model (reference case) and coarse models generated using NW1P and NWMP upscaling. The NW1P model in this case shows inaccuracy for both oil and gas rates. The loss of accuracy in oil rate is due in part to the very large mobility ratio and to the significant variation in $\mu_o$ (and thus $\lambda_o$) with pressure, which is not represented accurately in the NW1P coarse model. NWMP, by contrast, computes $\lambda_o^*$ which captures this effect accurately. Thus, results for oil rate using NWMP agree closely with the fine-scale simulation, as do those for gas rate. Improvements in accuracy using NWMP are also evident for well-block pressure and gas saturation.

Figure 3.6 presents upscaled well-block functions for the heavy oil case. The fine-scale oil mobility, which is a function of both pressure and saturation, is depicted as a surface. The upscaled oil mobility, again a function of pressure and saturation, is displayed by the dashed line. Gas mobility stays quite small and is not shown. The coarse-scale oil mobility is larger than the fine-scale mobility, though it approaches the input surface as pressure decreases and gas saturation increases. As in the previous case, we observe a ‘pseudo’ bubble point pressure (evident in the $R_s^*$ curve) that is larger than the original $p_b$. This shift acts to reduce the significant discrepancy between the reference fine-scale and NW1P simulation results. The $B_o^*$ curve also reflects the shift in bubble point pressure.

These examples clearly demonstrate the ability of NWMP to accurately capture key near-well effects in local well models. In the following section, we consider global reservoir models.

### 3.2 Application of ALGMP for Global Simulations

We now apply adaptive local-global multiphase near-well upscaling (ALGMP) to global reservoir models. We consider a heterogeneous model containing $50 \times 50 \times 30 = 75,000$ fine cells. The dimensions of each fine grid block are $\Delta x = \Delta y = 100$ ft and
Figure 3.4: Input curves for heavy oil (top left: relative permeability, top right: oil formation volume factor, middle left: gas formation volume factor, middle right: solution gas-oil ratio, bottom left: oil viscosity, bottom right: gas viscosity)
Figure 3.5: Upscaling results for LWM for heavy oil case (upper left: oil rate, upper right: gas rate, lower left: well-block pressure, lower right: well-block gas saturation)
Figure 3.6: Upscaled well-block functions for heavy oil (upper left: oil mobility, upper right: oil formation volume factor, lower left: gas formation volume factor, lower right: solution gas-oil ratio)
\( \Delta z = 10 \text{ ft} \). The isotropic permeability field, shown in Figure 3.7 (top), is generated using sequential Gaussian simulation (sGsim) with a spherical variogram model (Deutsch and Journel, 1998). The dimensionless correlation lengths for permeability (i.e., correlation length divided by system length in the corresponding direction) are 0.6 in the \( x \) and \( y \)-directions and 0.1 in the \( z \)-direction. The square root of the variance of \( \ln k \) (\( \sigma_{\ln k} \)) is 1.5, which indicates that the model contains significant variability in \( k \). Production is from three horizontal wells, all oriented in the \( x \)-direction, of lengths 1,500 ft (well 1), 2,000 ft (well 2) and 1,500 ft (well 3). The well locations are shown in Figure 3.7 (bottom).

Figure 3.7: Heterogeneous permeability field (top) and well locations (bottom)
CHAPTER 3. NUMERICAL RESULTS

The coarse model is constructed by uniformly upscaling the fine model by a factor of 5 in the \( x \) and \( y \)-directions and a factor of 3 in the \( z \)-direction. Thus the coarse grid is of dimensions \( 10 \times 10 \times 10 = 1,000 \). For interfaces not involving well blocks, permeability is upscaled using an extended local transmissibility upscaling procedure.

### 3.2.1 Global simulations with typical black oil

We first consider the typical black oil system. The initial conditions, fluid properties and relative permeabilities are as described in Section 3.1.1. In this model, the pressures for wells 1, 2 and 3 are prescribed to be 1,800 psi, 2,200 psi and 2,000 psi, respectively.

Figures 3.8-3.10 present oil and gas production rates for each well for the various models. The reference fine-scale simulations are performed using GPRS (Cao, 2002; Jiang, 2007). Production rates for well 3 are lower than those for the other two wells because well 3 intersects a low permeability region of the model. The NW1P model provides accurate results for oil rate, except for well 3 at early times, though results for gas rate show significant inaccuracy for all wells. This is due to the inability of the NW1P model to properly represent gas being liberated from the oil phase. ALGMP, by contrast, provides accurate results for both quantities throughout the simulation.

Figure 3.11 shows the gas saturation distribution around well 1 (layer 3) at early and late times. The left images present the fine-scale results averaged onto the coarse grid, the center images the ALGMP results, and the right images the NW1P results. It is evident that the ALGMP model is able to capture the distribution in the vicinity of the well quite accurately, in contrast to the NW1P model, which underpredicts gas saturation, especially at early time.

### 3.2.2 Global simulations with heavy oil

We next apply ALGMP for the heavy oil system. The initial conditions, fluid and rock-fluid properties are as described in Section 3.1.2. For this case, the \( x \) and \( y \)-components of permeability are multiplied by 1,000 and the \( z \)-component is multiplied by 10. The bottom hole pressures for wells 1, 2, and 3 are prescribed as 200 psi, 250
Figure 3.8: Comparison of global simulation results for typical black oil case (well 1)
Figure 3.9: Comparison of global simulation results for typical black oil case (well 2)
Figure 3.10: Comparison of global simulation results for typical black oil case (well 3)
Figure 3.11: Gas saturation around well 1 for typical black oil case (left: fine average, middle: ALGMP, right: NW1P; top row: 5 days, bottom row: 500 days)
psi and 150 psi, respectively.

Figures 3.12-3.14 present simulation results for the three models. It is evident that, at early time, the NW1P upscaled models show inaccuracy in oil rate, predicting only about 50% of the initial reference oil production. At later times the NW1P results for oil rate are more accurate. The NW1P results for gas rate are, however, inaccurate over much of the simulation. The model fails to predict any free gas (because the well-block pressures are always above $p_b$), except for well 1 at late time.

The ALGMP model, by contrast, provides a high degree of accuracy for oil rate for all wells. The results for gas rate are generally accurate, though some discrepancies are evident (e.g., the peaks in gas rate are not captured perfectly). Nonetheless, these results are quite encouraging as they demonstrate that the ALGMP procedure is able to capture complex near-well behavior involving the interaction of a heterogeneous permeability field with detailed phase behavior.

Figure 3.15 shows the gas saturation for the averaged fine model and the coarse ALGMP and NW1P models at two different times. These results are for layer 7, in the vicinity of well 3. It is clear that the ALGMP model essentially reproduces the near-well saturation profile, while the NW1P model does not predict the presence of any free gas at either time.

In the Appendix (Section A.4), we present simulation results for various coarsening factors. There we show that ALGMP maintains a high degree of accuracy over a range of coarse-model dimensions for both the typical black oil and heavy oil models.

### 3.2.3 Application of interpolation approach

We now represent well-block properties using the interpolation described in Section 2.6. As discussed previously, the idea here is to perform two initial ALGMP upscaling computations, referred to as base cases, with $p_w$ set to the minimum ($p_w^0$) or maximum ($p_w^1$) well pressures of interest. Then, for subsequent global simulations, the upscaled properties are interpolated based on the prescribed $p_w$.

We consider the heavy oil model discussed in Section 3.2.2. For the two base cases, we set $p_w^0 = 100$ psi for all wells and $p_w^1 = 300$ psi for all wells. Simulations
Figure 3.12: Comparison of global simulation results for heavy oil case (well 1)
Figure 3.13: Comparison of global simulation results for heavy oil case (well 2)
Figure 3.14: Comparison of global simulation results for heavy oil case (well 3)
Figure 3.15: Gas saturation around well 3 for heavy oil case (left: fine average, middle: ALGMP, right: NW1P; top row: 180 days, bottom row: 500 days)
are then performed using different well pressures for each well, with $p_w$ as indicated in Table 3.1, with upscaled parameters determined using interpolation.

Table 3.1: Well pressure settings (psi) for the three cases

<table>
<thead>
<tr>
<th>case</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>well 1</td>
<td>220</td>
<td>180</td>
<td>250</td>
</tr>
<tr>
<td>well 2</td>
<td>240</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>well 3</td>
<td>120</td>
<td>260</td>
<td>200</td>
</tr>
</tbody>
</table>

We first compare the interpolation procedure with direct application of ALGMP, for which we use the actual well pressures. Figure 3.16 displays results for well 1 in case 3. It is evident that, as expected, ALGMP is overall slightly more accurate than the interpolation procedure. The results from the interpolation procedure are, nonetheless, very accurate for oil rate and capture the general trend for gas rate.

Rather than present well by well comparisons for the three cases, we display total production results, meaning we sum production over the three wells. Figures 3.17-3.19 show the field oil and gas rates for the three cases. We present reference fine-scale results, NW1P results, and interpolated ALGMP results. The high level of accuracy achieved using interpolated ALGMP and the significant improvement over NW1P are clearly evident for both oil and gas production. These results suggest that, when properly calibrated, the interpolation procedure is viable for use in large-scale simulations.

### 3.3 Discussion

In this chapter, the near-well upscaling approach was shown to provide coarse-scale simulation results, both at the LWM level and globally, that are in close agreement with fine-scale simulations and are significantly more accurate than results generated using only near-well single-phase parameter upscaling (NW1P). In contrast to standard two-phase upscaling techniques, our method determines coarse-scale PVT parameters in addition to mobility functions. As a result, the method is particularly well-suited for models in which the near-well pressure is below the bubble point ($p_b$).
Figure 3.16: Accuracy of interpolation method relative to ALGMP (case 3, well 1)
Figure 3.17: Global simulation results using interpolation approach for heavy oil (case 1)
Figure 3.18: Global simulation results using interpolation approach for heavy oil (case 2)
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Figure 3.19: Global simulation results using interpolation approach for heavy oil (case 3)
while the coarse-block pressure is above $p_b$. For several such cases, results for gas production were shown to coincide reasonably well with fine-scale reference solutions, while results using NW1P showed significant error. Thus the method is clearly able to represent effects that are not captured in standard coarse modeling procedures. We have also applied the ALGMP procedure to other reservoir models and have generalized the method to handle other well specifications (e.g., fixed production rate in addition to fixed well pressure). These results are presented in the Appendix.

Because our new ALGMP upscaling procedure does require some amount of preprocessing computation, it is important to assess its computational requirements relative to those for fine-scale simulations. ALGMP requires both fine (local) and coarse (local and global) simulations. A significant number of coarse-scale LWM simulations and adjoint equation solutions are required; specifically, a total of $N_{\text{wells}} \times N_{\text{NWMP}}^{\text{NWMP}}$, where $N_{\text{wells}}$ is the number of wells to be upscaled and $N_{\text{NWMP}}^{\text{NWMP}}$ is the average number of solutions required during the NWMP iterations. In our examples, in all cases we perform two ALG iterations. In the first ALG iteration, about 20 coarse-scale simulations and adjoint solutions are required for each LWM, while in the second iteration only 1-2 simulations are required. Thus, $N_{\text{NWMP}}^{\text{NWMP}} \approx 22$. These simulations are, however, very inexpensive, as the largest LWM considered here contains only $6 \times 3 \times 3 = 54$ cells. The two global coarse-scale simulations required by ALGMP are similarly very inexpensive relative to global fine-scale simulations.

The greatest cost of the overall procedure is associated with the fine-scale LWM solutions. For the 75,000-cell example cases, the three LWMs contain a total of 10,800 cells. Because we apply two iterations in the ALGMP procedure, the computational complexity of the LWM solutions relative to that of the fine-scale solution is approximately $2(10,800)/75,000$, or about 0.29. The computational requirements for ALGMP will likely be less, however, as the LWM simulations can in general use larger time steps than the full simulation. We note also that speedups can be readily improved by choosing an upscaling ratio that minimizes the overall computational complexity of the upscaling computations plus coarse-scale simulation. The upscaling computations also parallelize naturally, so if many cores are available, multiple LWM simulations can be performed simultaneously. In addition, if the reservoir is to
be simulated at many different $p_w$ specifications, then the savings will be significantly greater. Specifically, using the interpolation procedure, the upscaling overhead will double (upscaling computations are required for $p_w^0$ and $p_w^1$), but subsequent global simulations will be essentially free. We note finally that it may be possible to use smaller LWMs for the upscaling computations, and this should be investigated in future work. For single-phase flow problems, it was shown in Wen et al. (2006) that coarse-model accuracy could still be achieved using relatively small border regions in the upscaling computations.
Chapter 4

Near-well Upscaling for Three-phase Flow

In Chapters 2 and 3, we developed and applied a near-well upscaling procedure for oil-gas flows. In this chapter we extend the method to handle three-phase flow problems. The basic approach is quite similar to that for the oil-gas case, though the formulation is more complicated here because we also track the water component. Both the formulation and numerical results for three-phase systems are included in this chapter.

4.1 Three-phase Flow Equations

In this section we present the equations describing three-phase flow in porous formations. The material in this section is similar to that presented in Section 2.1 except here we consider three components and three phases, in contrast to the two-component, two-phase systems considered in Section 2.1. Some of the equations presented in Section 2.1 are repeated here for completeness.

We consider the flow of oil, gas and water components in a porous medium. The components exist in three phases, also referred to as oil, gas and water. In our model, the oil and water components reside only in their respective phases, while the gas component can exist either in the gas phase or in the oil phase as dissolved gas. The model could be readily generalized to allow additional component exchange between
phases if necessary. The equations for three-phase, three-component flow are derived by combining statements of mass conservation for each component with Darcy’s law. Neglecting capillary pressure effects, the oil, gas, and water conservation equations are given by:

\[
\nabla \cdot \left[ \frac{\lambda_o}{B_o} k (\nabla p - \rho_o g \nabla D) \right] = \frac{\partial}{\partial t} \left( \phi \frac{S_o}{B_o} \right) + \tilde{q}_{w,o}, \tag{4.1}
\]

\[
\nabla \cdot \left[ \frac{\lambda_g}{B_g} k (\nabla p - \rho_g g \nabla D) + R_s \frac{\lambda_o}{B_o} k (\nabla p - \rho_o g \nabla D) \right] \]
\[
= \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] + \tilde{q}_{w,g}, \tag{4.2}
\]

\[
\nabla \cdot \left[ \frac{\lambda_w}{B_w} k (\nabla p - \rho_w g \nabla D) \right] = \frac{\partial}{\partial t} \left( \phi \frac{S_w}{B_w} \right) + \tilde{q}_{w,w}. \tag{4.3}
\]

All quantities in Eqs. 4.1 and 4.2 are as defined in Section 2.1. In Eq. 4.3, \(S_w\) is water saturation, \(B_w\) is water formation volume factor, \(\rho_w\) is water density, and \(\tilde{q}_{w,w}\) is the water-component source term. Water mobility \((\lambda_w)\) is defined as:

\[
\lambda_w = \frac{k_{rw}}{\mu_w}, \tag{4.4}
\]

where \(k_{rw}\) is water relative permeability and \(\mu_w\) is water viscosity. In typical three-phase relative permeability models, the water and gas relative permeabilities are functions of their own saturations only (i.e., \(k_{rg} = k_{rg}(S_g)\)) while oil relative permeability depends on two saturations. There are several models for oil-phase relative permeability – here we use the segregation model (Cao, 2002), though it is straightforward to apply other models such as Stone I or Stone II.

As in our oil-gas model, we solve Eqs. 4.1, 4.2 and 4.3 on structured three-dimensional Cartesian grids using a standard finite volume procedure. Permeability is taken to be a diagonal tensor. For a well in block \(i\), oil, gas, and water well flow rates (in units of volume/time), designated \((q_{w,o})_i\), \((q_{w,g})_i\), and \((q_{w,w})_i\), are described
using the standard well model (Peaceman, 1978):

\begin{equation}
(q_{w,o})_i = (\tilde{q}_{w,o})_i V_i = \left(\frac{\lambda_o}{B_o}\right)_i W_i (p_i - p_w), \tag{4.5}
\end{equation}

\begin{equation}
(q_{w,g})_i = (\tilde{q}_{w,g})_i V_i = \left(\frac{\lambda_g}{B_g} + R_s \frac{\lambda_o}{B_o}\right)_i W_i (p_i - p_w), \tag{4.6}
\end{equation}

\begin{equation}
(q_{w,w})_i = (\tilde{q}_{w,w})_i V_i = \left(\frac{\lambda_w}{B_w}\right)_i W_i (p_i - p_w), \tag{4.7}
\end{equation}

where \(W_i\) is the well index defined in Eq. 2.7 (Section 2.1). Definitions of other terms in Eqs. 4.5, 4.6 and 4.7 are given in Section 2.1.

In highly-resolved models, where the effects of subgrid heterogeneity can be neglected, the representation provided above is appropriate. As was the case for two-phase flow, however, upscaled parameters may be needed for coarse models. For the three-phase case, the set of upscaled parameters is (where the * superscript again designates upscaled parameter) \(T^*, W^*, \lambda^*_o, \lambda^*_g, \lambda^*_w, B^*_o, B^*_g\) and \(R^*_s\). The only coarse-scale parameter here not considered for the two-phase flow case is \(\lambda^*_w\) (\(B^*_w\) is not required as \(B_w\) is only a weak function of pressure). For the coarse-scale parameters, we take \(B^*_o, B^*_g\) and \(R^*_s\) to be functions of \(p^c\) only, \(\lambda^*_g\) to be a function of \(p^c\) and \(S^c_g\), \(\lambda^*_w\) to be a function of \(p^c\) and \(S^c_w\), and \(\lambda^*_o\) to be a function of \(p^c, S^c_g,\) and \(S^c_w\). Here the superscript \(c\) designates a coarse-scale quantity.

As in the two-phase flow case, near-well upscaling for three-phase flow problems requires two steps – near-well single-phase upscaling (NW1P), in which coarse-scale well indices and well-block transmissibilities are computed, and near-well multiphase upscaling (NWMP), in which \(\lambda^*_j\) \((j = o,g,w)\), \(B^*_o, B^*_g\) and \(R^*_s\) are computed. All of these parameters are computed using a gradient-based optimization algorithm, with all simulations performed over the region corresponding to the local well model (LWM). Detailed descriptions of the LWM and the NW1P calculations are provided in Chapter 2.
4.2 Near-well Multiphase Upscaling (NWMP) for Three-phase Flow

We now describe a procedure for near-well upscaling for three-phase flow (NWMP). The general workflow for this approach is the same as that for two-phase flow upscaling and entails the determination of coarse-scale parameters to minimize an appropriate cost function.

4.2.1 Cost function and adjoint equations for three-phase flow

The cost function to be minimized in our three-phase flow upscaling procedure is as follows:

\[
J = \sum_{n=1}^{N} L^n(x^n, u),
\]

(4.8)

where

\[
L^n(x^n, u) = \sum_{i=1}^{n_{w}} \left[ \alpha_o \left( \frac{(q^c_{w,o})_i^n - \langle q_{w,o} \rangle_i^n}{\langle q_{w,o} \rangle_i^n} \right)^2 + \alpha_g \left( \frac{(q^c_{w,g})_i^n - \langle q_{w,g} \rangle_i^n}{\langle q_{w,g} \rangle_i^n} \right)^2 + \alpha_w \left( \frac{(q^c_{w,w})_i^n - \langle q_{w,w} \rangle_i^n}{\langle q_{w,w} \rangle_i^n} \right)^2 \right].
\]

(4.9)

Here each term represents the normalized mismatch in oil, gas and water well flow rates between the fine and coarse models. Relative to Eq. 2.19 for the two-phase case, a new term, representing normalized error in the water production rate, appears. In this term (the last term in Eq. 4.9), \((q^c_{w,w})_i^n\) is the well water flow rate in coarse block \(i\) over time step \(n\) and \(\langle q_{w,w} \rangle_i^n\) is the integrated (summed) well water flow rate in fine blocks corresponding to coarse well block \(i\) over time step \(n\). Here \(x\), which refers to the dynamic states of the system, includes pressure, gas saturation and water saturation while \(u\) is the set of optimization parameters (tabular values for \(\lambda_j^*\); \(B_o^*\), \(B_g^*\) and \(R_s^*\)). All other parameters appearing in Eq. 4.9 are described in Chapter 2.
Eq. 4.8 is to be minimized subject to the governing flow equations, which we again write as

\[ g^n(x^n, x^{n-1}, u) = 0, \quad \forall n \in (1, \ldots, N), \]  

(4.10)

where \( g \) is the residual of the flow equations. The minimization is achieved using a gradient-based optimization algorithm (Schittkowski, 2006) and the gradient under equality constraints Eq. (4.10) is generated using an adjoint procedure. Through application of this technique, the gradient of the augmented objective function with respect to the optimization parameters is given by

\[ \frac{\partial J_A}{\partial u} = \sum_{n=1}^{N} \left( \frac{\partial L^n}{\partial u} + (\lambda^T)^n \frac{\partial g^n}{\partial u} \right), \]  

(4.11)

where

\[ (\lambda^T)^N = -\frac{\partial L^N}{\partial x^N} \left( \frac{\partial g^N}{\partial x^N} \right)^{-1}, \]  

(4.12)

\[ (\lambda^T)^n = -\left( \frac{\partial L^n}{\partial x^n} + (\lambda^T)^{n+1} \frac{\partial g^{n+1}}{\partial x^n} \right) \left( \frac{\partial g^n}{\partial x^n} \right)^{-1}, \quad \forall n \in (1, \ldots, N - 1). \]  

(4.13)

Full details on the adjoint equations and definitions of the parameters are provided in Chapter 2.

The general NWMP workflow is identical to that used in the two-phase case (Figure 2.4). The key differences between that procedure and the three-phase procedure are the addition of water flow terms in the cost function, the inclusion of the water conservation equation (which impacts both the reservoir model and the adjoint formulation), and the introduction of upscaled water mobility in the set of optimization variables.
4.3 Adaptive Local-Global Multiphase Upscaling (ALGMP)

As discussed in Chapter 2, the coarse-scale parameters can be sensitive to the boundary conditions applied on the LWM. Therefore we seek to determine boundary conditions for the LWM which will be close to the conditions encountered in global flow simulations. Here we use an adaptive local-global multiphase upscaling procedure (ALGMP), which is a direct extension of the approach developed for two-phase flow (described in Section 2.5).

The first step of the procedure is a coarse-scale global three-phase flow simulation under initial and well conditions of interest. Fine-scale mobility and PVT parameters are used in this first step. The gray region (3 × 3 coarse cells) in Figure 4.1 (this figure is identical to Figure 2.5) represents the LWM. During this flow simulation, we save the coarse-block pressures and both gas and water saturations at each time step in blocks bordering the LWM (these blocks are marked with ×’s in Figure 4.1). These saved states are then interpolated to provide boundary conditions (at edges indicated by solid circles in Figure 4.1) which will be applied in the local fine-scale near-well flow simulation. For this interpolation, we again use a trilinear interpolation in space and a linear interpolation in time.

Using these interpolated boundary conditions, we perform the NWMP technique to determine the coarse-scale parameters. This procedure is applied for each LWM. The coarse-scale parameters thus computed for each well block are then used in the next coarse-scale global simulation. The overall procedure (global coarse-scale simulation, determination of LWM boundary conditions, calculation of upscaled parameters using NWMP) is repeated either a specified number of times or until the change in the upscaled parameters or LWM flow rates is reduced to a prescribed tolerance. As in the oil-gas case, here only two ALGMP iterations are required.
Figure 4.1: LWM boundary conditions determined from adaptive local-global procedure
4.3.1 Coarse models for different well conditions

In Chapter 2 we presented an interpolation technique which enabled us to avoid performing the full upscaling procedure for each set of well conditions. This approach can also be used for the three-phase case. We now briefly review the procedure.

We first define the minimum ($p_{0w}^i$) and maximum ($p_{1w}^i$) well pressures which bound the range of interest. These pressures should both be either above or below the bubble point pressure. We then apply ALGMP twice – first using $p_{0w}^i$ and then using $p_{1w}^i$. In both simulations, $p_w$ is constant in time.

In subsequent global coarse-scale simulations, we can now specify any $p_w$ that is between $p_{0w}^i$ and $p_{1w}^i$. The following interpolation is applied to define all upscaled parameters: $u_i = (1 - \beta)u_{0i}^i + \beta u_{1i}^i$, where $\beta = (p_w - p_{0w}^i)/(p_{1w}^i - p_{0w}^i)$, $u_i$ is the coarse-scale parameter of interest (for well pressure $p_w$), and $u_{0i}^i$ and $u_{1i}^i$ are the coarse-scale parameter values for well pressures $p_{0w}^i$ and $p_{1w}^i$. This approach is applied for all upscaled oil, gas and water parameters.

4.4 Application of NWMP to Local Well Model

In this section, we present an application of the near-well multiphase upscaling approach (NWMP) over the local well model (LWM). We specify a similar local well reservoir model to that used in Chapter 3. To represent a very extensive bottom aquifer, a layer of cells with large thickness is added to the bottom of the model. The model is three-dimensional and has a homogeneous permeability of 10 md and a constant porosity of 0.25. The fine model contains $33 \times 33 \times (5 + 1) = 6,534$ cells. The fine-grid reservoir cell dimensions are $\Delta x = \Delta y = 150$ ft and $\Delta z = 1$ ft. For the bottom layer (aquifer), we set $\Delta z = 1,000$ ft. A horizontal production well oriented in the $x$-direction is located in the center of the model; the well is completed in cells $i = 12 - 22$, $j = 17$, $k = 3$. The LWM and the well are shown in Figure 4.2 (the bottom aquifer layer is not shown).

We consider a typical black oil with a bubble point pressure ($p_b$) of 4,000 psi ($27.58 \times 10^6$ Pa). This is the same fluid property model as was considered in Chapter 3,
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with the PVT and fluid properties taken from Killough (1995). The oil density is 37.5 lb/ft³ (601 kg/m³) and the water density is 63.0 lb/ft³ (1009 kg/m³). Oil, gas, and water viscosities are 3 cp, 0.015 cp, and 0.96 cp, respectively, and are taken to be constant. The relative permeabilities are prescribed as 

\[ k_{rg} = (S^n_g)^2, \quad k_{rw} = (S^n_w)^2, \]

\[ k_{rog} = (1 - S^n_g)^2 \quad \text{and} \quad k_{row} = (1 - S^n_w)^2, \]

where \( S^n_g \) and \( S^n_w \) are normalized gas and water saturations given by:

\[ S^n_g = \frac{S_g - S_{gc}}{1 - S_{or} - S_{gc}}, \quad S^n_w = \frac{S_w - S_{wc}}{1 - S_{or} - S_{wc}}. \] (4.14)

Here we use \( S_{gc} = 0, S_{wc} = 0.1, \) and \( S_{or} = 0.1. \) These relative permeability functions differ from those used by Killough (1995). Curves for the relative permeabilities and the fluid model are shown in Figure 4.3 (the oil-gas fluid and rock-fluid properties are the same as shown in Figure 3.1). Initial reservoir pressure is prescribed as 6,000 psi (which is greater than \( p_b \)) at the top of the model. We set \( p_w = 1,400 \) psi and simulate for a time of 500 days.

The coarse LWM contains \( 3 \times 3 \times (1 + 1) = 18 \) cells. Thus the overall coarsening factor is 363. No-flow conditions are applied on the LWM boundaries. Well-block properties for the coarse-scale LWM are computed using the NW1P and NWMP procedures.

Figure 4.4 compares the performance of the fine (reference) and coarse-scale models. Coarse models using only upscaled near-well single-phase parameters (NW1P) and models using parameters from both single and multiphase upscaling (NWMP) are shown along with the fine-scale solution. Results (at surface or stock tank conditions) are shown for oil rate, gas rate, water rate, well-block pressure, well-block gas saturation, and well-block water saturation. For well-block pressure and saturations, fine-model results are averaged (using pore-volume weighting) over the fine cells corresponding to the coarse-scale well block to enable direct comparisons.

It is apparent from Figure 4.4 that the coarse model generated using NW1P provides results for all quantities that differ significantly from those of the fine-scale simulation. This clearly demonstrates how NW1P can fail to capture near-well physics when phase behavior effects are important. In the NW1P model, the water rate is
Figure 4.2: Homogeneous local well model (top) and well location (bottom)
Figure 4.3: Input curves for typical black oil (upper left: relative permeability, upper right: oil formation volume factor, lower left: gas formation volume factor, lower right: solution gas-oil ratio)
much lower than in the reference fine-scale model. In the reference model, although the averaged water saturation is lower, the fine-scale well block saturation is higher than the NW1P well-block saturation. The NW1P model underpredicts total liquid rate even though oil rate is overpredicted. This in turn leads to overprediction of well-block pressure in the NW1P model. Essentially all of the gas produced in the NW1P model is dissolved gas (note that $S^c_g$ is zero in this model). The gas rate is thus overpredicted because oil rate is overpredicted.

By contrast, the NWMP model reproduces the fine-scale results with a reasonable degree of accuracy. Oil, gas, and water production rates show close agreement with the reference solution (recall that mismatches in these quantities are minimized in the NWMP optimization procedure). Well-block pressure is however somewhat less accurate, while gas saturation shows more significant discrepancies. It should be observed, however, that coarse-block gas saturation is very low throughout the entire LWM simulation ($S^c_g \sim O(10^{-5})$).

To confirm that there are three phases actually flowing in the coarse well block, we computed the production rate of free gas (the results for gas flow rate in Figure 4.4 include free and dissolved gas). At 400 days, the oil, free gas, and water rates for the NWMP model are 1171 stb/d, 34.8 Mscf/d, and 13,500 stb/d, respectively. Thus the well block is indeed experiencing three-phase flow, and this is accurately represented in the NWMP model.

Figure 4.5 presents the upscaled well-block properties. The fine-scale functions are shown with solid lines and the upscaled functions are indicated by the dashed lines. Although mobilities are functions of both pressure and gas and/or water saturation, the graphs present them as a function of only a single saturation, as this is the key dependency. The actual block pressure during the LWM simulation ranges from 3,881 to 4,500 psi. The upscaled parameters are extrapolated to 3,800 psi so they cover the range that may be encountered in subsequent global simulations.

As was observed in the oil-gas two-phase case, the bubble point pressure is shifted to higher pressure (5,307 psi) from the original $p_b$ of 4,000 psi, and $R^*_g$, $B^*_o$ and $B^*_g$ follow this shift (note that the pressure range is far from the original bubble point pressure). This PVT correction is necessary to enable the coarse-scale model to
capture near-well gas evolution. Differences between fine and coarse-scale curves are also seen in the mobilities. As the use of the fine-scale water mobility (in NW1P) was seen to lead to low water production, the optimized NWMP $\lambda^w$ curve is well above the fine-scale curve.

We now consider another simulation over the LWM. Here we specify different well conditions, namely a constant oil production rate of 1,400 stb/d subject to a minimum wellbore pressure of 1,000 psi. The initial pressure in this case is 4,200 psi at the top of the LWM. All other system parameters are the same as in the previous example.

Results for this case are displayed in Figure 4.6. Note that we show results here for well pressure rather than well-block pressure because rate is specified. The NW1P results for gas rate, water rate, and well-block pressure are quite inaccurate. In fact, the NW1P model does not predict any free gas, in disagreement with the reference solution. For the NWMP model, results for all quantities are in essential agreement with the fine-scale simulation. As in the previous example, three-phase flow in the well block is observed. For example, at 400 days, oil, free gas, and water rates for the NWMP model are 1,376 stb/d, 862 Mscf/d, and 1,301 stb/d, respectively.

These results demonstrate that NWMP is able to accurately capture near-well effects for three-phase flow. We next apply ALGMP to large-scale simulations.

### 4.5 Use of ALGMP for Global Simulations

We now apply the overall procedure – adaptive local-global multiphase near-well upscaling (ALGMP) – to three-phase global reservoir simulation models.

#### 4.5.1 Global simulations with typical black oil

In this example we use the basic reservoir model and fluid property data from the Ninth SPE Comparative Solution Project (Killough, 1995). The reservoir, shown in Figure 4.7 (top), is heterogeneous and has a dip of 10°. The model contains $24 \times 25 \times 15 = 9,000$ fine cells. The dimensions of each fine-grid block are $\Delta x = \Delta y = 300$ ft; $\Delta z$ is variable (Killough, 1995). Production here is via two horizontal
Figure 4.4: Upscaling results for LWM for typical black oil case (upper left: oil rate, upper right: gas rate, middle left: water rate, middle right: well block pressure, lower left: well block gas saturation, lower right: well block water saturation)
Figure 4.5: Upscaled well-block functions for typical black oil (upper left: oil mobility, upper right: gas mobility, middle left: water mobility, middle right: oil formation volume factor, lower left: gas formation volume factor, lower right: solution gas-oil ratio)
Figure 4.6: Upscaling results for LWM for typical black oil case with fixed oil rate of 1,400 stb/d (upper left: oil rate, upper right: gas rate, middle left: water rate, middle right: well pressure, lower left: well block gas saturation, lower right: well block water saturation)
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wells (this is different than in the original problem specification), both of length 1,500 ft as shown in Figure 4.7 (bottom). The oil-water contact, which is close to the toe of well 2, is at a depth of 9,950 ft.

The coarse model is generated by uniformly coarsening the fine model by a factor of 5 in all directions (except for the first coarse block in the $x$-direction, which contains only 4 fine cells). The coarse grid is therefore of dimensions $5 \times 5 \times 3 = 75$. As in the oil-gas examples, an extended local transmissibility upscaling procedure is applied to determine coarse-scale transmissibilities for grid interfaces that do not include well blocks (NW1P is applied for interfaces connecting well blocks to non-well blocks). The initial pressure is 4,500 psi at the top of the reservoir and fluid properties are the same as described in Section 4.4. Relative permeabilities, shown in Figure 4.8, are these given by Killough (1995). We also have $S_{gc} = 0$, $S_{wc} = 0.151$, and $S_{or} = 0.113$.

In this model, the wellbore pressures are prescribed to be 1,000 psi (constant in time). Figures 4.9 and 4.10 compare oil, gas, and water production rates for wells 1 and 2 for the various models. It is evident from Figure 4.9 that both the NW1P and ALGMP models provide close agreement with the reference results for well 1. The accuracy of NW1P for this well may be due to the fact that it is far from the aquifer and thus does not produce any water (or experience three-phase flow). For well 2, by contrast, which is close to the aquifer, the water rate in the NW1P model is very inaccurate. The early water breakthrough and high water rate are however represented accurately by the ALGMP model. This effect is captured largely through $\lambda^{*}_{w}$. Improved accuracy using ALGMP is also observed for gas rate. For well 2, the oil, free gas and water rates at 300 days for the ALGMP model are 15,430 stb/d, 8,216 Mscf/d, and 2,098 stb/d, respectively.

4.5.2 Global simulations with heavy oil

We next apply ALGMP to the heavy oil system. The reservoir model contains $50 \times 50 \times 31 = 77,500$ fine cells. The bottom layer represents an aquifer (the top of this layer represents the oil-water contact). The dimensions of each fine cell are $\Delta x = \Delta y = 180$ ft and $\Delta z = 10$ ft for reservoir blocks and $\Delta z = 1,000$ ft for
Figure 4.7: Heterogeneous permeability field (top) and well locations and oil-water contact (OWC) (bottom)
the aquifer layer. The isotropic permeability field, shown in Figure 4.11 (top, note that the bottom aquifer layer is not shown), was generated using sequential Gaussian simulation (sGsim) with a spherical variogram model (Deutsch and Journel, 1998). The dimensionless correlation lengths for permeability are 0.6 in the $x$ and $y$-directions and 0.1 in the $z$-direction. The square root of the variance of $\ln k$ ($\sigma_{\ln k}$) is 1.5. The coarse model is constructed by uniformly upscaling the fine model by a factor of 5 in the $x$ and $y$-directions and a factor of 3 in the $z$-direction for the upper 30 layers. The aquifer layer is kept as a separate layer. Thus the coarse grid contains $10 \times 10 \times (10 + 1) = 1,100$ blocks. Note that the reservoir portion of this model is identical to that used in Section 3.2 (Figure 3.7 top).

Production is from three horizontal wells, of lengths 800 ft (well 1), 1,600 ft (well 2) and 800 ft (well 3). The well locations are shown in Figure 4.11 (bottom). Initial reservoir pressure is 1,000 psi at the top of reservoir. Prescribed constant well pressures of 300 psi (well 1), 150 psi (well 2), and 250 psi (well 3) are applied.

The fluid and rock-fluid properties for oil and gas are the same as were used in the oil-gas case in Chapter 3. Recall that the oil viscosity is $O(10^4)$ cp, the bubble point pressure is 870 psi ($6.00 \times 10^6$ Pa), the oil density is 63.03 lb/ft$^3$ (1,010 kg/m$^3$), and $S_{gc} = 0$, $S_{wc} = 0.12$ and $S_{or} = 0.24$. The relative permeability curves and the fluid properties for oil, gas and water are shown in Figure 4.12.
Figure 4.9: Comparison of global simulation results for typical black oil case (well 1)
Figure 4.10: Comparison of global simulation results for typical black oil case (well 2)
Figure 4.11: Heterogeneous permeability field (top) and well locations (bottom)
Prior to performing the global simulations, upscaled multiphase flow functions are computed for all well blocks. In Figure 4.13 we present the upscaled properties for well 1 (dashed curves) along with the input (solid) curves. Note that the input functions are presented differently here than in Figure 4.12 (e.g., mobility is shown here rather than relative permeability and viscosity) to allow direct comparison with the upscaled functions. The upscaled functions are plotted only in terms of their primary dependencies; for example, $\lambda^*_w$ is a function of both $S^c_w$ and $p^c$, but it is presented as a function of $S^c_w$ only since the $p^c$ dependence is much weaker. Significant shifts in the coarse-scale mobility curves relative to the input curves are evident in Figure 4.13. In addition, the apparent bubble point has shifted to higher pressures. This was also observed in the oil-gas case and reflects the impact of subgrid pressure variation, as discussed in Section 3.3.

Figures 4.14-4.16 display simulation results for wells 1-3 for the fine (reference) and coarse-scale models. Coarse models using only upscaled near-well single-phase parameters (NW1P) and models using parameters from both single and multiphase upscaling (NWMP) are shown along with the fine-scale solution. Results at surface or stock tank conditions are shown for oil rate, gas rate and water rate.

For well 1 (Figure 4.14), NW1P shows reasonable results for oil rate (except at early times) and moderate error for water rate. Significant errors are apparent for gas rate until about 100 days. This error occurs because little if any of the dissolved gas is liberated in the NW1P model, in contrast to the reference fine-scale solution. The ALGMP model displays a high degree of accuracy for oil, gas and water rates over the entire simulation. For well 2 (Figure 4.15), the NW1P results again show errors for all quantities, particularly gas rate. The ALGMP results for well 2 continue to be very accurate. Consistent accuracy is also observed for oil, gas and water rates for well 3 (Figure 4.16) using ALGMP. Note that the production rates for this well are significantly lower than they are for wells 1 and 2.

In this example, all three wells experience three-phase flow (in the ALGMP model) over the course of the simulation. For example, oil, free gas and water rates at 400 days for the ALGMP model are 164 stb/d, 10.4 Mscf/d, and 183 stb/d for well 1; 275 stb/d, 11.2 Mscf/d, and 13.0 stb/d for well 2; and 58.5 stb/d, 4.72 Mscf/d, and 0.08
Figure 4.12: Input curves for heavy oil (top left: relative permeability for oil and gas, top right: relative permeability for oil and water, second row left: oil formation volume factor, second row right: gas formation volume factor, third row left: solution gas-oil ratio, third row right: oil viscosity, bottom: gas viscosity)
Figure 4.13: Upscaled well-block functions for well 1 (upper left: oil mobility, upper right: gas mobility, middle left: water mobility, middle right: oil formation volume factor, lower left: gas formation volume factor, lower right: solution gas-oil ratio).
Figure 4.14: Comparison of global simulation results for heavy oil case (well 1)
Figure 4.15: Comparison of global simulation results for heavy oil case (well 2)
Figure 4.16: Comparison of global simulation results for heavy oil case (well 3)
Figure 4.17: Gas saturation around well 1 at 30 days for heavy oil case (left: fine average, middle: ALGMP, right: NW1P)

stb/d for well 3.

Figure 4.17 shows the gas saturation around well 1 (in layer 8) for the three models at a time of 30 days. Well completions are shown by the dashed lines. The reference fine-scale results are here averaged onto the coarse grid (using a pore-volume weighting) to enable direct comparison with the coarse NW1P and ALGMP results. The underprediction of gas saturation in well blocks in the NW1P model is clearly evident, as is the accuracy of the ALGMP model.

As discussed in Section 3.3, the main computational cost for ALGMP is in the fine-scale near-well solutions. The other computations all involve inexpensive coarse-scale (local and global) simulations. Thus, the speedup provided by the ALGMP method relative to the fine-scale simulation is approximately one half the ratio of the number of fine cells in the global model to the number of fine cells in the LWMs (the factor of one half appears because we perform two ALGMP iterations). For the problems considered here, this corresponds to about a factor of 5 speedup.

4.5.3 Results using interpolation approach

We now apply the interpolation approach described in Section 4.3.1 to model wells subject to different $p_w$. The properties used here are those of the typical black oil model discussed in Section 4.5.1.

The fine-grid reservoir model is a $60 \times 110 \times 12$ portion of the model developed by
Christie and Blunt (2001), referred to as SPE 10. The portion considered here, shown in Figure 4.18 (top), is the upper 12 layers of the northern half of the reservoir. A thick layer representing the aquifer is included at the bottom of the model (this layer is not shown in Figure 4.18). The number of fine-grid blocks is thus $60 \times 110 \times (12 + 1) = 85,800$ cells. The dimensions of each of these cells are $\Delta x = 200$ ft, $\Delta y = 100$ ft and $\Delta z = 20$ ft. For the aquifer layer we set $\Delta z = 1,000$ ft. The coarse model is constructed by uniformly upscaling the fine model by a factor of 5 in the $x$ and $y$-directions and a factor of 3 in the $z$-direction for the 12 reservoir layers (the aquifer layer is maintained), so the coarse grid is of dimensions $12 \times 22 \times (4 + 1) = 1,320$ blocks.

Production is from three horizontal wells, shown in Figure 4.18 (bottom). The lengths of wells 1, 2 and 3 are 1,000 ft, 2,000 ft, and 500 ft, respectively. For the first base case we set $p_{w}^{0} = 500$ psi for all wells, while for the second base case we set $p_{w}^{1} = 2,000$ psi for all wells. Coarse-scale simulations are then performed using different well pressures for each well, with upscaled well-block functions interpolated from the base case results. The $p_{w}$ settings for the three cases considered are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Well</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>550</td>
<td>1,250</td>
<td>1,850</td>
</tr>
<tr>
<td>2</td>
<td>650</td>
<td>1,300</td>
<td>1,750</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>1,200</td>
<td>1,950</td>
</tr>
</tbody>
</table>

We now present coarse-scale simulation results using the interpolation procedure along with those of the fine-scale simulation model and the NW1P coarse-scale model. For these results, total field production rates (i.e., the sum of the rates from the three wells) are presented rather than individual well rates. Figures 4.19-4.21 display the field oil, gas and water rates for the three cases. It is evident that the interpolated ALGMP procedure provides close agreement with the fine-scale simulation results for all quantities in all three cases. This is in contrast to the NW1P results, which show significant inaccuracies. Comparable levels of agreement between interpolated
Figure 4.18: Heterogeneous permeability field (portion of SPE 10, top) and well locations (bottom)
ALGMP and fine-scale simulations are observed in the well-by-well predictions. We note that, in all three cases, three-phase flow occurs at some points in the simulation. For example, for well 3 in case 3 in the ALGMP simulation, at 400 days we observe an oil rate of 808 stb/d, a free gas rate of 648 Mscf/d, and a water rate of 24.4 stb/d.

These results demonstrate the applicability of the interpolated ALGMP procedure for the upscaling of large models to be simulated under a variety of well conditions. This capability may be highly useful in practical simulation studies.

4.6 Summary

In this chapter, we extended the ALGMP approach for two-phase oil and gas flow to three-phase, oil, gas, and water flow.

The coarse-scale parameters to be optimized in the three-phase upscaling are the mobility of oil, gas, and water, the formation volume factor of oil and gas, and the solution gas-oil ratio. As in the two-phase upscaling, we estimate boundary conditions applied to the LWM using the local-global approach. Numerical results for both the LWM and for heterogeneous global models demonstrate the accuracy of the three-phase upscaling procedure. The method was shown to be very well suited for capturing water production from aquifers in the vicinity of horizontal wells.
Figure 4.19: Global simulation results using interpolation approach (case 1)
Figure 4.20: Global simulation results using interpolation approach (case 2)
Figure 4.21: Global simulation results using interpolation approach (case 3)
Chapter 5

Near-well Upscaling for Unstructured Grids

In Chapters 2-4, we developed and applied a near-well upscaling procedure for structured grid systems. In this chapter we extend the method to upscale unstructured fine-scale systems to structured coarse-scale systems. We consider the oil-gas case. The basic approach is very similar to the technique developed in Chapter 2, though the implementation details differ because the LWM is unstructured. In this chapter, the procedure is described and results are presented. The method is shown to perform well for the cases considered.

5.1 Flow Simulation using Unstructured Grids

5.1.1 General modeling procedure

We use a finite volume approach, which is based on a two-point flux approximation (TPFA) (Karimi-Fard et al., 2004), to perform the fine-scale flow simulations. GPRS is used for all of the unstructured grid simulations. Since GPRS is based on a connectivity list, we need to provide it with the control volume connections and the associated transmissibilities. GPRS also requires pore volumes and cell-center depths. Due to the general shape of the control volumes, the grid can display varying degrees
of nonorthogonality. The TPFA method, however, requires orthogonal systems to be strictly accurate. Multipoint flux approximation (MPFA) procedures are required for nonorthogonal systems. The use of TPFA with general grids will therefore introduce some error into our modeling, though, for isotropic permeability fields, Karimi-Fard (2008) has shown that the overall approach can still be expected to provide reasonable accuracy (all permeability fields considered in this chapter are isotropic). We note that the near-well upscaling method developed here is also compatible with the use of an MPFA procedure for the LWM computations.

In this work we use tetrahedral control volumes, but the following descriptions are applicable for any control volume shape. Flow between adjacent control volumes is given by:

\[ q_{12} = T_{12}(p_2 - p_1), \]  

where \( q_{12} \) is flow rate between control volumes 1 and 2, \( p \) is pressure, and \( T_{12} \) is the transmissibility given by Karimi-Fard et al. (2004) as:

\[ T_{12} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}, \]  

\[ \alpha_i = \frac{k_i A}{D_i \cdot n_i \cdot f_i}. \]  

Figure 5.1 illustrates the transmissibility calculation for the two-dimensional case; the three-dimensional case is analogous. In the figure, \( C_i \) is the centroid of control volume \( i \) and \( C_0 \) is the centroid of the interface between the two control volumes. In Eq. 5.3, \( k_i \) is the isotropic permeability of control volume \( i \), \( A \) is the area of the shared interface, \( D_i \) is the distance between \( C_0 \) and \( C_i \), \( n_i \) and \( f_i \) are the unit normals to the interface and the unit vector in the direction from \( C_0 \) to \( C_i \), respectively. For further discussion of the TPFA treatment of unstructured models in GPRS, see Karimi-Fard et al. (2004).
5.1.2 Representation of wells

In the fine-scale unstructured grid simulations, the well components are modeled using Darcy’s law (i.e., as porous media). In the well models used here, we include tubing and gravel pack. More complex well models, involving fractured and cased-hole wells (with perforations), have been developed and simulated by Karimi-Fard and Durlofsky (2009), though such cases will not be considered here.

Our representation of the wellbore and the near-well region is shown in Figure 5.2. Consistent with the values used by Karimi-Fard and Durlofsky (2009), cells representing the tubing are specified to be of permeability $10^9$ md, while cells representing gravel are of permeability $10^5$ md. Production is specified to occur from the topmost tetrahedron in the tubing portion of the domain, for which we specify a very large well index of $10^9$ mD-ft. Because the tubing cells are of very high permeability, the pressure is expected to be essentially constant in the tubing. We confirmed that this is indeed the case for the examples presented later in this chapter.
5.1.3 Overall upscaling procedure

As discussed in Chapter 2, once boundary conditions for the NWMP computation are determined and the fine-scale LWM simulation has been conducted, the coarse-scale LWM flow simulations are performed. The coarse-scale LWM simulations are independent of the fine-scale grid system. Thus the overall upscaling procedure described in Chapter 2 is quite general and can be applied to any type of fine-scale grid system.

Complications arise, however, since local and extended local upscaling computations must be performed on unstructured grids. We now describe the determination of upscaled transmissibility for non-well blocks. See Figure 5.3 for a schematic. We first extract all of the fine-scale cells whose centroids fall within the two coarse blocks sharing the interface of interest (we will calculate the upscaled transmissibility for this interface). In Figure 5.3, the dots show the outermost control volumes included in this upscaling computation. We solve the single-phase steady-state pressure equation with constant-pressure and no-flow boundary conditions. In contrast to structured grid systems, the boundary conditions applied to each cell are not immediately obvious due to the complex shape of the solution domain. To determine the appropriate...
boundary condition, we find the closest coarse-block boundary to each triangular face falling on the boundary of the local unstructured domain. The boundary condition used in the solution of the pressure equation is assigned based on the closest coarse-block boundary. An example of the resulting boundary conditions is shown in Figure 5.3 (red segments are constant pressure and blue segments are no-flow). Flow rates at fine-cell faces corresponding to the coarse-block interface (shown by the green line) are calculated using Eq. 5.1. The coarse-scale transmissibility is then computed using:

$$T^* = \frac{\langle q \rangle}{\langle p \rangle_i - \langle p \rangle_{i+1}}$$  \hspace{1cm} (5.4)

where $\langle p \rangle$ is computed as the bulk-volume weighted average of the fine-grid pressure over the target coarse block and $\langle q \rangle$ is the integrated fine-scale flow rate across the coarse-block interface.

Figure 5.3: Schematic illustrating the upscaling of transmissibility for non-well blocks in unstructured system

Near-well single-phase flow upscaling (NW1P), applied to determine $W^*$ and $T^*_w$, is performed analogously. Consider the LWM shown in Figure 5.4. We apply constant pressure $p = 0$ at the outer boundary (indicated by the red line) and $p = 1$ at
the central well. The steady-state single-phase pressure equation is then solved and the integrated fine-scale flow rates are used in the NW1P optimization. The same LWM is also used for the near-well multiphase upscaling (NWMP), though for this computation the boundary conditions vary in both space and time.

The ALGMP procedure is performed following these single-phase upscaling computations. As described in Chapter 2, the fine-scale mobility and PVT parameters are used for the initial coarse-scale global simulation. The global pressure and saturation are then interpolated to provide boundary conditions for the NWMP computations. Linear interpolation in time is again used here. As shown in Figure 5.4, the boundaries of the LWM are not on a line in 2D cases or a plane in 3D cases. For the determination of the boundary conditions, we therefore calculate the centroid locations of the outermost faces of each LWM cell and then apply trilinear interpolation for these points. Iteration then proceeds as in the structured grid case. As was the case for structured models, only two iterations are used here.

5.2 Application of NWMP for Unstructured Grid Systems

We first apply our near-well multiphase upscaling procedure (NWMP) to a small local well model (LWM). The LWM is of dimensions 150 ft in the $x$ and $y$-directions and 50 ft in the $z$-direction. The fine-scale LWM contains 6,596 tetrahedral cells. Homogeneous permeability and porosity are specified ($k = 10$ md and $\phi = 0.25$). A vertical production well located in the center of the model is completed from 15 ft to 35 ft depth from the top of the model. The open-hole radius is 1.5 ft and the tubing diameter is 0.32 ft. The space between the open hole and the tubing is filled with a gravel pack. The initial pressure is 4,500 psi at the top of the reservoir and there is no initial gas. The simulation is performed for 500 days. Pressure at the boundary of the LWM is prescribed to decrease linearly in time from 4,500 psi to 4,000 psi (bubble point pressure) over the course of the simulation. Zero gas saturation is also specified on the LWM boundaries. In this example we use the PVT property data from the
Figure 5.4: Single-phase near-well upscaling (NW1P) for unstructured systems
Ninth SPE Comparative Solution Project (Killough, 1995), used also in Section 3.1.1. The relative permeabilities are also the same as those used in Section 3.1.1 (the curves are shown in Figure 3.1).

The fine-scale unstructured model is upscaled to a structured coarse-scale model containing $3 \times 3 \times 1$ uniform cells ($\Delta x = \Delta y = 50$ ft and $\Delta z = 50$ ft for each coarse cell) using the NWMP procedure. Figure 5.5 compares oil and gas rate, well-block pressure and gas saturation for the reference (fine unstructured) and the coarse (upscaled using NW1P and NWMP) models. The model upscaled using NW1P displays errors in both oil and gas rates. The NWMP results, by contrast, are in close agreement with the reference fine-scale simulation for all quantities.

![Figure 5.5: Simulation results for unstructured fine model and structured coarse NW1P and NWMP models](image)

Figure 5.5: Simulation results for unstructured fine model and structured coarse NW1P and NWMP models

Figure 5.6 shows the upscaled PVT parameters along with the input (fine-scale) curves. The bubble point pressure is seen to shift to higher values (4,998 psi in this
case), consistent with the behavior observed in Chapters 3 and 4. The other upscaled PVT parameters follow this shift. Though not shown here, the oil mobility curve is similar to the fine-scale curve while the gas mobility curve shifts to lower values.

![Graphs showing upscaled well-block functions for typical black oil](image)

Figure 5.6: Upscaled well-block functions for typical black oil (upper left: oil formation volume factor, upper right: gas formation volume factor, lower: solution gas-oil ratio)

### 5.3 Use of ALGMP for Global Simulations with Unstructured Grid Systems

#### 5.3.1 Global simulations with typical black oil

We consider the same black oil model used above in Section 5.2. The global fine-scale model contains 75,415 tetrahedral control volumes (Figure 5.7). The dimensions of
the model are 5,000 ft in the horizontal directions and 300 ft in the vertical direction. The permeability field, shown in Figure 5.8, is based on the same underlying permeability model as was used in Section 3.2. For each tetrahedron in the unstructured grid, we find the grid block in the $50 \times 50 \times 30$ fine-scale structured model in which the centroid of the tetrahedron falls. The tetrahedral cell is then assigned this permeability value. The coarse grid used here is of dimensions $10 \times 10 \times 3$.

The initial pressure is 4,500 psi at the top of the reservoir. Production in this case is from two vertical wells, both of which are 100 ft in length (see Figure 5.7). Wells 1 and 2 are prescribed to produce at constant oil rates of 500 stb/d and 300 stb/d, respectively. Both wells are subject to a minimum well pressure of 2,500 psi.

![Diagram of a well configuration with labeled permeability values](image)

Figure 5.7: Unstructured fine-scale model showing well locations and near-well detail

Figures 5.9 and 5.10 compare oil and gas production rates and well pressure for wells 1 and 2 for the various models. For both wells, NW1P overpredicts the constant production period (for well 2, the difference from the reference model is about 100 days). The NW1P results for gas production also show noticeable errors. The AL-GMP model is very accurate for all quantities for both wells. This demonstrates the
applicability of the overall procedure for upscaling fine-scale globally unstructured models to structured coarse-scale representations.

5.3.2 Global simulations with heavy oil

In the following two examples we use fluid data and rock-fluid properties from the South American heavy oil first described in Section 3.1.2. Curves for these properties are shown in Figure 3.4.

Horizontal well case

The permeability field used in this example, shown in Figure 5.8, is the same as that considered in Section 5.3.1. The tetrahedral grid system is not identical to that used above since the well is in a different location, and the well location strongly impacts the grid structure. The fine model in this case contains 64,704 control volumes (Figure 5.11). Production here is from a horizontal well that is 500 ft in length. The well is oriented at an angle of 20° from the $y$-axis. The coarse grid is again of dimensions $10 \times 10 \times 3$. The initial pressure is 1,000 psi at the top of the reservoir.
Figure 5.9: Comparison of global simulation results (unstructured fine grid system) for typical black oil case (well 1)
Figure 5.10: Comparison of global simulation results (unstructured fine grid system) for typical black oil case (well 2)
We prescribe a constant well pressure of 300 psi. The simulation is performed for a period of 250 days.

Figure 5.11: Unstructured fine-scale model with horizontal well

Figure 5.12 shows oil and gas production rates for the three models. For oil rate both NW1P and ALGMP display close agreement with the fine-scale simulation. The NW1P results for gas production show substantial errors. These errors are largely eliminated through use of ALGMP.

**Slanted well case**

In this example (Figure 5.13), we use a portion of the permeability field presented in the Tenth SPE Comparative Solution Project (Christie and Blunt, 2001). This is the same permeability model used in Section 4.5.3, except here the permeability is isotropic (we use the horizontal permeability given by Christie and Blunt, 2001). The dimensions of the reservoir are 3,000 ft in the horizontal directions and 260 ft in the vertical direction (these are different from the dimensions used in Section 4.5.3).
Figure 5.12: Comparison of global simulation results (unstructured fine grid system with horizontal well) for heavy oil case
The fine model in this case contains 77,763 cells. There is a single slanted well in the center of the model that is 600 ft in length (see Figure 5.14). The coarse grid contains $10 \times 11 \times 3$ blocks. The initial pressure is 1,000 psi at the top of the reservoir. We prescribe a constant well pressure of 250 psi and simulate for a period of 250 days.

![Heterogeneous permeability field for slanted well case](image)

Figure 5.13: Heterogeneous permeability field for slanted well case

Figure 5.15 compares oil and gas production rates for this case. For the first 100 days, the NW1P results for oil rate show some error, though close agreement with the fine model is achieved for the latter part of the simulation. Substantial error in the NW1P results is evident in the gas rate prediction. Consistent with the first two examples, ALGMP provides a generally high degree of accuracy for both oil and gas production rates. There is, however, some discrepancy in the peak gas rate between ALGMP and the fine-scale results.

5.4 Discussion

In this chapter, we extended the ALGMP approach to handle unstructured fine-grid systems. The general methodology is essentially the same as in structured grid cases,
though the use of unstructured fine-grid models does lead to additional complications. Numerical results at the level of the LWM and for large heterogeneous models demonstrate that the procedure provides accurate coarse-grid results for the problems considered.

The simulation of large unstructured models with high degrees of near-well resolution is very time consuming. The fine-scale LWM computations, which are by far the most expensive portion of the ALGMP procedure, are similarly expensive as they contain the same degree of near-well resolution. For the three examples presented here, the computation times for the global fine-scale simulations varied from 42 to 52 hours on a PC with Intel Core 2 Duo CPU E6750 2.66 GHz and 1.95 GB RAM. The ALGMP procedure (two iterations) required from 16 to 19 hours, which is a significant fraction of the fine-scale simulation. Future effort should be focused on reducing the simulation times of these highly resolved models. It will also be useful to investigate the use of smaller LWMs for the upscaling computations. Finally, we note that the interpolation procedure described in Section 2.6 could be applied to the unstructured case. This should lead to large reductions in computation time if many simulations with different well conditions are to be performed.
Figure 5.15: Comparison of global simulation results (unstructured fine grid system with slanted well) for heavy oil case
Chapter 6

Conclusions and Future Work

The key developments and findings presented in this thesis can be summarized as follows:

- Gradient-based optimization procedures, with gradients determined using adjoint methods, were developed for the computation of upscaled well-block parameters for multiphase flow problems. These procedures were developed for both oil-gas and three-phase (oil-gas-water) systems. The methods provide coarse well-block mobility functions, formation volume factors, and solution gas-oil ratio, in addition to well index and transmissibilities. The upscaled functions are determined by minimizing the mismatch between integrated (summed) fine-scale component flow rates and analogous coarse-scale quantities over a local region (referred to as a local well model or LWM) surrounding the well.

- The boundary conditions used for the LWM can significantly impact the upscaled parameters. These boundary conditions are determined in this work using an adaptive local-global procedure. This entails performing a global coarse-scale simulation, using initial estimates for well-block parameters. LWM boundary conditions are then determined by interpolating the global coarse-scale solution, and the well-block parameters are recomputed. In numerical tests, it was determined that only two iterations of this procedure are required to generate accurate coarse-scale parameters.
• The overall method is very flexible in terms of the form of the coarse-scale models and the representation of the coarse-scale parameters. In the applications presented here, the coarse models are of the same form as the fine-scale models, but this is not a requirement of the method. In addition, because adjoint procedures are applied to determine the necessary gradients, the computational requirements for the upscaling are largely independent of the number of parameters used to describe the upscaled functions. In this work, tabular representations of the upscaled functions were used, with the table entries determined by the optimization procedure.

• An interpolation procedure was introduced to avoid the need to perform the upscaling computations for each set of well specifications. Using this approach, two (base case) sets of upscaling computations are required, but subsequent coarse simulations, with well settings that fall within the range of the base cases, can be run very inexpensively; i.e., without any pre-processing calculations.

• Extensive numerical simulations for local well domains and global models, for both two and three-phase flow systems, demonstrated the accuracy of the methods. Various heterogeneous reservoir models were considered in conjunction with a typical black oil system and a heavy oil system. The overall procedures were shown to be significantly more accurate, in general, than using only upscaled single-phase near-well parameters (upscaled well index and well-block transmissibilities). Substantial improvements in accuracy were observed in many cases for gas flow rates. This is due in part to the fact that standard coarse models often provide inaccurate near-well pressure solutions, which results in errors in gas evolution. This effect is captured in the new upscaling procedures. More accurate prediction of water breakthrough was also observed in some three-phase flow cases.

• The computational requirements of the upscaling method depend largely on the number of wells in the model and on the number of fine-scale cells in the local well models. For the examples considered here, these computations correspond to about 20-30% of the fine-scale computational demand. The upscaling
computations can be reduced through use of smaller LWMs, careful selection of coarse-grid dimensions, or parallelization. Use of the interpolation procedure can significantly reduce the computational requirements of the overall method, for both two and three-phase flow problems, for cases in which many coarse-scale simulations are to be run.

- The procedure was extended to enable the upscaling of unstructured fine-scale models to structured coarse-scale models. In these cases, the unstructured description resolves features down to the scale of the wellbore. Numerical results for oil-gas flow in such systems demonstrated the ability of the method to capture detailed near-well effects in structured coarse-scale models.

We suggest the following topics for future research in this general area:

- In the existing implementation, the fine-scale local well model computations are performed in GPRS, though a standalone research code is used for the coarse-scale computations. The standalone code is used because GPRS cannot currently handle multiple PVT regions, which are required because the upscaled well-block PVT properties (such as formation volume factors and solution gas-oil ratio) differ from those in non-well blocks. We suggest that GPRS be extended to accommodate multiple PVT regions. This will enable GPRS to be used for all of the fine and coarse-scale computations required by the procedures developed in this thesis.

- In this work we considered only black oil representations for fluid properties. It will be useful to extend the methods developed here to compositional systems. This will enable the accurate coarse-scale representation of, for example, condensate systems. Near-well effects such as condensate dropout can be difficult to capture in coarse-scale models of such systems, so the development of near-well upscaling procedures could be quite useful.

- The computational demands for the two-phase and three-phase upscaling procedures are largely dependent on the size of the local well models (LWMs). It
will therefore be useful to assess the impact of the size and shape of the LWM on the accuracy of the upscaled model (this can be done for different styles of permeability heterogeneity). It may also be possible to use different sized LWMs for the two ALGMP iterations.

- In the implementation applied here, extended local procedures were used for the upscaling of single-phase flow parameters (i.e., for the computation of upscaled well index and upscaled transmissibilities). Near-well multiphase flow parameters, by contrast, were computed using a local-global procedure (ALGMP). We recommend integrating the ALGMP near-well upscaling procedure with a local-global (or fully global) technique for the determination of upscaled single-phase flow parameters. The incorporation of upscaled two or three-phase flow parameters in non-well blocks may also be useful for some cases. These capabilities may provide enhanced accuracy for highly heterogeneous permeability fields, particularly in the case of significant fluid injection.

- Further development of the overall procedure for unstructured grid systems is necessary to handle more complex well models. Only relatively simple completion models (e.g., gravel pack) were considered here because of the difficulties encountered in simulating the unstructured fine-scale local well model for more complicated cases (such as those involving perforations or hydraulic fractures). Successfully addressing this issue will require improvements in the efficiency and robustness of GPRS for unstructured near-well models and also the determination of the minimum necessary size of the local well model.

- In this work, only Darcy flow was considered. In practical cases, non-Darcy effects may be important in the near-well region. Therefore we recommend incorporating non-Darcy effects into the overall upscaling procedure.
Appendix A

Additional Applications of ALGMP for Oil-Gas Simulations

In Chapter 3 we presented global simulation results for adaptive local-global multiphase near-well upscaling (ALGMP) with wellbore pressure specified. The ALGMP procedure can also be applied with well rate specified. In addition, new wells can be introduced during the course of the simulation. In this appendix we present results for such cases. A challenging application, involving use of ALGMP with a highly heterogeneous reservoir model, is also considered. Finally, we demonstrate the consistent accuracy of ALGMP for varying degrees of coarsening. All examples in this appendix involve oil-gas systems.

A.1 Global Simulations under Specified Production Rate

In reservoir simulation, production is generally prescribed by specifying either wellbore pressures or the rate of a particular phase (or total liquid rate etc.). We now present examples with oil rate specified.

We use the same reservoir model as in Section 3.2. The model, shown in Figure 3.7 (top), contains $50 \times 50 \times 30 = 75,000$ fine cells. There are three horizontal wells (see
Figure 3.7, bottom), all oriented in the $x$-direction, of lengths 1,500 ft (well 1), 2,000 ft (well 2) and 1,500 ft (well 3). The coarse-scale model configuration is also identical to that considered in Section 3.2. All upscaling procedures are as described in Chapter 2.

A.1.1 Typical black oil case

Here we use the fluid model described by Killough (1995) – the typical black oil used in Section 3.1.1. All model properties are as described in Section 3.1.1 (see Figure 3.1 for input curves). The bubble point pressure ($p_b$) of the fluid is 4,000 psi and the initial reservoir pressure is 4,800 psi. We prescribe oil production rates of 1,300 stb/d (well 1), 550 stb/d (well 2), and 150 stb/d (well 3) subject to a minimum wellbore pressure of 1,000 psi for all wells. The simulation is run for 500 days.

Coarse-scale simulation results for ALGMP and NW1P are compared to fine-scale simulation results in Figures A.1-A.3. These figures display results for oil and gas production rates and wellbore pressure (not well-block pressure) for the various models. The NW1P model provides results of reasonable accuracy for this example, though some discrepancy is evident for gas rate for all three wells and well pressure for well 3 (note that well 3 reaches the minimum wellbore pressure at relatively early time). The ALGMP results are of high accuracy in all cases. This example demonstrates that the ALGMP procedure can be used for a variety of well specifications.

A.1.2 Heavy oil case

We next consider the heavy oil used in Section 3.1.2. Recall that the $x$ and $y$-direction permeability components are multiplied by 1,000 and the $z$-component is multiplied by 10 for this case. The oil viscosity is $O(10^4)$ cp, the bubble point pressure is 870 psi, and the oil density is 63.03 lb/ft$^3$. The fluid and rock-fluid properties are shown in Figure 3.4.

Initial reservoir pressure is prescribed as 1,000 psi (greater than $p_b$) at the top of the model and there is no initial gas saturation. We prescribe oil production rates of 100 stb/d (well 1), 25 stb/d (well 2), and 6 stb/d (well 3) with a minimum wellbore pressure of 300 psi for all wells. Simulation time is 500 days.
Figure A.1: Global simulation results of typical black oil system for specified rate case (well 1)
Figure A.2: Global simulation results of typical black oil system for specified rate case (well 2)
Figure A.3: Global simulation results of typical black oil system for specified rate case (well 3)
Figures A.4-A.6 compare oil and gas production rates and wellbore pressures for the various models. For this case the NW1P model displays significant errors in gas rate and for wellbore pressure at early time for all three wells. These errors are essentially eliminated using ALGMP (though some error is still noticeable for gas rate for well 3), again demonstrating that the method can be applied for different well specifications.

A.2 Global Simulations with New Wells Added

It is common in flow simulations for new wells to be introduced during the course of the simulation. We now test the ALGMP procedure for such cases. The reservoir and fluid models used here (typical black oil and heavy oil models) are the same as were used in Section A.1. The well locations, problem specifications and coarse-scale model configurations are also identical except where otherwise indicated.

A.2.1 Typical black oil case

For this case we specify oil production rates of 1,300 stb/d (well 1), 550 stb/d (well 2), and 150 stb/d (well 3) with a minimum wellbore pressure of 1,000 psi for all wells. Well 2, however, does not begin producing until 500 days after the start of the simulation (wells 1 and 3 produce over the entire simulation). The simulation is run for 1,000 days.

Figures A.7-A.9 compare oil and gas production rates and wellbore pressure for the three models. For wells 1 and 2, both NW1P and ALGMP provide results of reasonable accuracy, though ALGMP is the more accurate (some error is evident in gas rate for well 1). Errors for well 3 using the NW1P model are more substantial; these errors are essentially eliminated through use of the ALGMP procedure.

A.2.2 Heavy oil case

For this case we specify maximum oil production rates of 100 stb/d (well 1), 25 stb/d (well 2), and 6 stb/d (well 3) with a minimum wellbore pressure of 300 psi for all
Figure A.4: Global simulation results of heavy oil system for specified rate case (well 1)
Figure A.5: Global simulation results of heavy oil system for specified rate case (well 2)
Figure A.6: Global simulation results of heavy oil system for specified rate case (well 3)
Figure A.7: Global simulation results of typical black oil system with new well added at 500 days (well 1)
Figure A.8: Global simulation results of typical black oil system with new well added at 500 days (well 2)
Figure A.9: Global simulation results of typical black oil system with new well added at 500 days (well 3)
APPENDIX. ADDITIONAL RESULTS

wells. Total simulation time is 1,000 days; well 2 does not start producing until day 500.

Results are shown in Figures A.10-A.12. The general level of accuracy of both models is lower in this case than for the typical black oil case. The NW1P model displays fairly significant errors for all wells. ALGMP provides very accurate results for all quantities for well 1. For wells 2 and 3, oil rate and wellbore pressure predicted by the ALGMP model are quite accurate, though the results display some inaccuracy for gas rate (though these gas rates are smaller than those for well 1). In any event, the ALGMP results represent a substantial improvement over those from the NW1P model. These results, along with those presented above for the typical black oil case, demonstrate the range of applicability of the ALGMP approach.

A.3 Results for a Highly Heterogeneous Permeability Field

Our results for a variety of cases have demonstrated that local well models (LWMs) comprising one ring of coarse blocks around each well (see Figure 2.3) generally provide adequate accuracy for the ALGMP procedure. Here we demonstrate that, for more challenging highly heterogeneous models, improved accuracy can be achieved by increasing the size of the LWM used in the ALGMP procedure.

We consider a channelized system adapted from the permeability field described by Christie and Blunt (2001) (the SPE 10 model). We extract the northern half of the model and consider layers 36-60. This gives a fine model that is $60 \times 110 \times 25$ ($165,000$ cells). Each fine-grid block is of dimensions $\Delta x = 20$ ft, $\Delta y = 10$ ft and $\Delta z = 2$ ft. The SPE 10 permeability field is anisotropic; see Christie and Blunt (2001) for details. The horizontal permeability is shown in Figure A.13 (top). We introduce three horizontal wells (see Figure A.13, bottom) – well 1 is oriented in the $y$-direction and wells 2 and 3 are oriented in the $x$-direction. The wells have completion lengths of 150 ft (well 1), 300 ft (well 2) and 300 ft (well 3).

We use the typical black oil model considered above. The bubble point pressure
Figure A.10: Global simulation results of heavy oil system with new well added at 500 days (well 1)
Figure A.11: Global simulation results of heavy oil system with new well added at 500 days (well 2)
Figure A.12: Global simulation results of heavy oil system with new well added at 500 days (well 3)
Figure A.13: Permeability field for horizontal directions (top) and well locations (bottom) for highly heterogeneous model (this model is a portion of SPE 10)
is 4,000 psi. Oil production rates of 500 stb/d are prescribed, along with a minimum wellbore pressure of 2,000 psi for each well. The simulation is run for 500 days.

The model is uniformly upscaled by a factor of 5 in all directions, which gives a coarse model containing 1,320 blocks. Coarse models using the ALGMP procedure are generated using LWMs containing one ring of coarse cells around well blocks (this is the procedure used in all previous results presented in this thesis – these results are designated ALGMP(1)) and two rings of coarse cells around well blocks (these results are designated ALGMP(2)).

Figures A.14-A.16 compare results for the reference fine-scale model, the NW1P model, and the ALGMP(1) and ALGMP(2) models. All three coarse models perform reasonably well for this case. For well 1, the ALGMP procedures provide more accurate results than NW1P for both oil rate and gas rate, while for well 2 the three coarse models are of comparable accuracy. For well 3, however, ALGMP(2) shows clear improvement in accuracy compared to ALGMP(1) for oil rate and gas rate. This illustrates that coarse-scale simulation results can be impacted by the size of the LWM. It may be useful to consider this general issue further in future work, as it may be possible to use smaller LWMs, or LWMs with dimensions and orientation based on the fine-scale permeability field, for some cases.

**A.4 Results for Various Degrees of Coarsening**

In the examples presented in this thesis, we generated upscaled results (NWMP or ALGMP) for a single coarse-grid model corresponding to a particular coarsening factor (or upscaling ratio). Here we present results for a range of coarsening factors. This allows us to assess both the performance of ALGMP and the variation of the upscaled functions for a series of coarse models.

For this study, we use a heterogeneous reservoir model, shown in Figure A.17 (top), containing $45 \times 45 \times 30 = 60,750$ fine cells. The permeability field was generated using sequential Gaussian simulation (sGsim) with a spherical variogram model. The dimensionless correlation lengths for permeability and the square root of the variance of $\ln k$ are the same as in Section 3.2 (0.6 in the horizontal direction, 0.1 in the
Figure A.14: Global simulation results for typical black oil and highly heterogeneous reservoir model (well 1)
Figure A.15: Global simulation results for typical black oil and highly heterogeneous reservoir model (well 2)
Figure A.16: Global simulation results for typical black oil and highly heterogeneous reservoir model (well 3)
vertical direction, $\sigma_{\ln k} = 1.5$). A horizontal well of length 333 ft, oriented in the $x$-direction, is placed in the center of the model (Figure A.17, bottom). We upscaled this model to coarse models of dimensions $15 \times 15 \times 10 = 2,250$, $9 \times 9 \times 10 = 810$, and $5 \times 5 \times 10 = 250$. These models correspond to areal coarsening factors of 3, 5 and 9. Note that there are 10 blocks in the $z$-direction in all three coarse models. All of the upscaling procedures applied here are as described in Chapter 2. We consider both the typical black oil and heavy oil fluid systems.

Figure A.17: Permeability field (top) and well locations (bottom)
APPENDIX. ADDITIONAL RESULTS

A.4.1 Typical black oil case

Here we use the fluid model described by Killough (1995), which is the typical black oil used in Section 3.1.1. All other model parameters including the relative permeability curves (Figure 3.1) are the same as in Section 3.1.1, except where otherwise specified. The bubble point pressure is 4,000 psi and the initial reservoir pressure is 4,500 psi at the top of the model. We prescribe an oil production rate of 300 stb/d with a minimum wellbore pressure of 1,000 psi. The model is simulated for 500 days.

Simulation results for the fine and coarse models are displayed in Figure A.18. Oil and gas production rates and wellbore pressure for seven different models are presented. The solid symbols correspond to NW1P models and the open symbols to ALGMP models. The NW1P results show significant error and only slight improvement as the coarse model is refined. The ALGMP models are essentially unaffected by the degree of coarsening and consistently provide accurate simulation results.

Figure A.19 shows the $R^*_s$ curves for the three coarse-scale ALGMP models. The pseudo bubble point pressure ($p^*_b$) associated with each curve, which is the pressure where the $R^*_s$ curve reaches a constant value, is displayed in Figure A.20. We see that $p^*_b$ increases as the coarsening factor increases, as would be expected.

A.4.2 Heavy oil case

We next assess the impact of coarsening factor for the heavy oil system described in Section 3.1.2. The fluid model and relative permeabilities are the same as in Section 3.1.2. The bubble point pressure is 870 psi and the initial pressure at the top of the reservoir is 900 psi. The simulation is performed with a constant wellbore pressure of 150 psi for a period of 500 days.

Figure A.21 displays the simulation results for the seven models. The NW1P models are inaccurate, and significant improvement in accuracy is not observed with increasing refinement for these models. The ALGMP models are again consistently accurate. The $R^*_s$ and $p^*_b$ results are shown in Figures A.22 and A.23. As seen in the previous example, $p^*_b$ increases as the coarsening factor increases.

The results presented in this section demonstrate that the accuracy of the ALGMP
Figure A.18: Global simulation results for typical black oil for various degrees of coarsening
Figure A.19: $R^*_s$ for various ALGMP models for typical black oil system

Figure A.20: $p^*_b$ as a function of coarsening factor for typical black oil system
model is essentially independent of the level of coarsening, at least for the cases considered. This is an encouraging finding as it suggests that extensive numerical experimentation is not required to establish a reliable ALGMP model.
Figure A.21: Global simulation results for heavy oil for various degrees of coarsening
Figure A.22: $R_s^\ast$ for various ALGMP models for heavy oil system

Figure A.23: $p_b^\ast$ as a function of coarsening factor for heavy oil system
Nomenclature

Abbreviations

ALGMP  adaptive local-global multiphase upscaling
GPRS   general purpose research simulator
LWM    local well model
NW1P   near-well single-phase upscaling
NWMP   near-well multiphase upscaling
OWC    oil-water contact
SQP    sequential quadratic programming
TPFA   two-point flux approximation

Variables

⟨ ⟩  averaged or integrated fine-scale quantity

$B_j$  formation volume factor of phase $j$

$B_j^*$ upscaled formation volume factor of phase $j$

$D$  depth

$f_j$  flux of phase $j$ across interface

$g$  dynamic flow model
**NOMENCLATURE**

\( g \) gravitational acceleration

\( J \) cost function

\( J_A \) augmented cost function

\( k \) absolute permeability tensor

\( k_x, k_y, k_z \) diagonal components of permeability tensor

\( k_{rj} \) relative permeability of phase \( j \)

\( L \) cost function

\( n_j \) total number of coarse faces that the coarse well blocks share with non-well blocks

\( n_t \) total number of fine cells in coarse cell

\( n_{wb} \) number of well blocks

\( p \) pressure

\( p_b \) bubble point pressure

\( p_b^* \) pseudo bubble point pressure

\( p_i \) pressure of grid block \( i \)

\( p_w \) well pressure

\( \bar{q}_{w,j} \) well rate of phase \( j \) (units of time\(^{-1}\))

\( q_{w,j} \) volumetric well rate of phase \( j \) (units of volume/time)

\( r_0 \) equivalent well-block radius

\( R_s \) solution gas-oil ratio

\( R_s^* \) upscaled solution gas-oil ratio
NOMENCLATURE

\( r_w \) wellbore radius

\( S_j \) saturation of phase \( j \)

\( T \) transmissibility

\( t \) time

\( T^* \) upscaled transmissibility

\( T_w^* \) upscaled transmissibility between coarse-scale well blocks and adjacent non-well blocks

\( u \) vector of control parameters

\( V \) bulk volume of grid-block

\( v \) bulk volume of fine cell

\( W \) well index

\( W^* \) upscaled well index

\( x \) vector of state variables

**Greek**

\( \alpha_j \) weighting factor in cost function

\( \beta \) interpolation parameter

\( \lambda \) vector of Lagrange multipliers

\( \lambda_j \) mobility of phase \( j \)

\( \lambda_j^* \) upscaled mobility of phase \( j \)

\( \mu_j \) viscosity of phase \( j \)

\( \phi \) porosity
NOMENCLATURE

\( \rho_j \)  density of phase \( j \)

\( \Delta x, \Delta y, \Delta z \)  grid block dimensions

**Superscripts**

*  upscaled quantity

\( c \)  coarse

\( N \)  total number of time steps

\( n \)  time step

\( T \)  transpose

**Subscripts**

\( f \)  face of coarse well block

\( g \)  gas

\( j \)  phase

\( o \)  oil

\( w \)  water

\( wb \)  well block
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