UPSCALING OF MULTIPHASE FLOW PARAMETERS FOR MODELING NEAR-WELL AND MISCIBLE DISPLACEMENTS

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF PETROLEUM ENGINEERING AND THE COMMITTEE OF GRADUATE STUDIES OF STANFORD UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Mun-Hong (Robin) Hui
August 2005
I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy

___________________________________
Dr. Louis J. Durlofsky
(Principal Advisor)

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy

___________________________________
Dr. Khalid Aziz

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy

___________________________________
Dr. Hamdi A. Tchelepi

Approved for the University Committee on Graduate Studies.
Abstract

Because fine-scale, geocellular descriptions of heterogeneous reservoirs are typically too detailed for direct flow simulation, the need to generate coarse-scale reservoir models through upscaling persists. In this work, we develop and apply two-phase upscaling techniques for modeling well-driven, high-mobility-ratio, immiscible displacements and first-contact miscible displacements. These displacements are relevant to widely-used improved oil recovery processes such as waterflooding and immiscible or miscible gas injection.

For first-contact miscible displacements, we present a novel upscaling technique that comprises two key components: effective flux boundary conditions (EFBCs) and the extended Todd-Longstaff with upscaled relative permeabilities \( k^*_ij \) or ETLU formulation. EFBCs, which account approximately for the effect of the global flow field on the local upscaling problems, mitigate inaccuracies introduced by standard procedures (e.g., premature breakthrough). The ETLU formulation modifies the computation of effective fluid properties and \( k^*_ij \) such that bypassed oil that is immobile and unavailable for mixing is properly treated.

Using synthetic 2D fields with varying permeability correlation lengths, we demonstrate that our miscible upscaling technique is effective for partially layered systems. Accurate coarse-scale simulation predictions and significant computational savings are obtained. By quantifying the upscaling errors, we show that the technique is more accurate than standard upscaling techniques over a wide range of coarsening factors and for
different heterogeneity structures. By considering multiple production scenarios and permeability realizations, the accuracy and speedup from the technique are also demonstrated for synthetic 3D systems. The upscaling procedure is then applied to a miscible gas injection field study involving a nonuniform 3D fine grid. It is found that the original fine grid must be refined areally to achieve numerical convergence. A near-well treatment is also incorporated in the methodology. We evaluate our upscaling technique by considering various realistic miscible displacement scenarios. The overall approach provides a very close reproduction of the converged (refined) fine-scale solutions. Overall speedup factors of about two orders of magnitude are obtained. Our technique is thus shown to be useful for practical studies of miscible displacements.

High mobility ratios are often encountered in improved oil recovery processes due to high oil viscosities. A new two-phase upscaling approach for modeling well-driven, high-mobility-ratio displacements is presented. For a local fine region around the well, we apply near-well, single-phase (NW1P) and two-phase (NW2P) procedures. The coarse-scale well indices ($W^*$), wellblock transmissibilities ($T^*$), and relative permeabilities ($k^*_{ij}$) are determined such that the fine- and coarse-scale flow rates are in agreement. Away from wells, the $k^*_{ij}$ for each coarse block are computed by imposing EFBCs.

The performance of this upscaling approach is assessed by considering multiple realizations of synthetic 3D models with varying correlation structures and degrees of spatial variability, as well as different fluid mobility contrasts and production scenarios. Our overall approach consistently yields accurate coarse-scale simulation results. By quantifying the upscaling errors, it is shown that each additional component of our approach (NW1P, NW2P, EFBC $k^*_{ij}$) contributes to increased accuracy of the coarse models. The upscaling components are not always all needed, e.g., the two-phase procedures (NW2P, EFBC $k^*_{ij}$) are more important for injectors than producers and when the mobility ratio is high.
Acknowledgements

At the outset of my pursuit of this doctorate, it was clear to me that this endeavor ought not be purely academic in nature but should encompass the broad enrichment offered by this institution. In the course of the past four and a half years, in the quest for this holistic experience, I took many an interesting class, attended numerous scintillating seminars, gave more presentations than I care to count, and also indulged in the pleasures of life such as traveling. With great satisfaction, I conclude that I have grown in so many respects through this journey—academically, professionally, and personally.

First and foremost, I must thank my academic advisor, Dr. Louis J. Durlofsky for his patient and insightful guidance in regards with my academic, research, and professional directions. I was initially discouraged by his generous list of “suggested modifications” for my write-ups and presentation slides, but I soon realized the value of his unwavering emphasis on precise and clear phrasing. Through him, I learnt that scientific rigor and a practical approach to solving problems need not be mutually exclusive. His dedication to his work and his meticulous sense of conduct have been truly inspiring. I will, no doubt, be imbued with his influence in my professional development in years to come.

I am very grateful to Dr. Khalid Aziz, Dr. Hamdi Tchelepi, and Dr. Kristian Jessen for being part of my defense committee and for offering their constructive remarks to improve my dissertation. I would also like to thank Dr. Atilla Aydin for chairing the defense committee and exposing me to the exciting field of fractures and faults.
It is fitting to acknowledge the financial support from the industrial affiliates of the SUPRI-HW/B consortia and the U.S. Department of Energy (under contract no. DE-AC26-99BC15213). I also owe a large measure of gratitude to Chevron Energy Technology Company, where I spent three thoroughly rewarding summer internships—for the fruitful research, professional development, and importantly, gainful post-graduation employment. A significant portion of this dissertation work was performed during these stints, in collaboration with Dr. Dengen Zhou and Dr. Xian-Huan Wen. I must thank Dr. Alan Bernath for so readily lending a hand in solving myriad technical and logistical issues. The useful comments from Dr. Jairam Kamath and Dr. Kaveh Dehghani are also gratefully acknowledged.

I am also indebted to Dr. Yuguang Chen, Dr. Mohammad Karimi, and Dr. Bradley Mallison for many meaningful discussions. Yuanlin Jiang deserves special mention for his assistance in optimizing my upscaling/simulation code. The faculty, administrators, and students from the PE department must be credited for a very rewarding PhD experience. I would also like to thank all my friends, who have so deeply enriched my life for the past few years.

Ultimately, I dedicate this manuscript to my father, mother, sister (Josephine Hui), and brothers (Harvey Hui, Daniel Hui, and Simon Hui) for their love and understanding, and for being the source of strength and inspiration that made this journey worthwhile. Last but certainly not least, I want to thank my best friend, Markus Rauschenberger, for constantly reminding me to cherish the joys of life, every day. My heartfelt gratitude goes out to every single one of you I mentioned above for your encouragement, support, and love, without which this achievement would be meaningless, if not impossible.
# Table of Contents

Abstract................................................................................................................................... v

Acknowledgements ..............................................................................................................vii

Table of Contents .................................................................................................................. ix

List of Figures......................................................................................................................xiii

Chapter 1  Introduction.................................................................................................... 1

1.1  First-contact miscible displacements ....................................................................... 3

1.1.1  Literature review.................................................................................................. 4

1.1.2  Proposed approach.............................................................................................. 8

1.2  Well-driven, high-mobility-ratio displacements...................................................... 9

1.2.1  Literature review................................................................................................ 10

1.2.2  Proposed approach............................................................................................ 14

1.3  Dissertation outline ................................................................................................ 15

Chapter 2  A new upscaling technique for first-contact miscible displacements..... 19

2.1  Introduction............................................................................................................ 19

2.2  Extended Todd-Longstaff (ETLU) formulation .................................................... 21

2.3  Effective flux boundary conditions (EFBCs) ........................................................ 28

2.4  Results and discussion (2D cases) ......................................................................... 33

2.4.1  Evaluation for varying correlation structure..................................................... 33

2.4.2  Evaluation for varying coarsening factor.......................................................... 41

2.4.3  Interplay of heterogeneity and numerical dispersion ....................................... 51
2.5 Application to 3D systems ................................................................................................. 54
  2.5.1 Evaluation for varying production scenario .............................................................. 55
  2.5.2 Evaluation for multiple realizations ........................................................................ 59
2.6 Concluding remarks ........................................................................................................... 62

Chapter 3 Field application of miscible upscaling technique ........................................... 65

3.1 Introduction ....................................................................................................................... 65
3.2 Description of field problem ............................................................................................ 66
3.3 Areal grid refinement study for a cross section ............................................................... 68
3.4 Cross-sectional model (10 × 858) .................................................................................. 73
3.5 Cross-sectional model (5 × 858) ................................................................................... 77
3.6 Sector model (15 × 15 × 466) ......................................................................................... 81
3.7 Sector model (7 × 7 × 180) ............................................................................................ 85
3.8 Concluding remarks ........................................................................................................ 88

Chapter 4 Near-well upscaling for high-mobility-ratio displacements ....................... 89

4.1 Introduction ....................................................................................................................... 89
4.2 Near-well, single-phase upscaling (NW1P) .................................................................. 91
4.3 Near-well, two-phase upscaling (NW2P) ..................................................................... 100
4.4 Effective flux boundary conditions ............................................................................... 105
4.5 Semianalytical subgrid radial well model ..................................................................... 106
  4.5.1 Introduction ............................................................................................................ 106
  4.5.2 Theory .................................................................................................................... 107
  4.5.3 Illustrative examples .............................................................................................. 115
  4.5.4 Summary ............................................................................................................... 120
4.6 Results and discussion ................................................................................................. 120
  4.6.1 Description of flow problem studied ................................................................. 121
  4.6.2 Horizontal injector local well model (\(M^{0} = 100\)) ........................................... 125
List of Figures

Figure 2-1 – Averaged solvent fractional-flow curve showing the nonzero $S_{orh}^*$ ................................24
Figure 2-2 – Upscaled relative permeabilities have better behavior using nonzero $S_{orh}^*$ .......27
Figure 2-3 – Flux-attenuation behavior of EFBCs for a circular inclusion in a 2D domain . 30
Figure 2-4 – A local upscaling problem illustrating the principle of flux attenuation in
EFBCs ........................................................................................................................................31
Figure 2-5 – Synthetic Fields A through D ($\mu_{ln} = 3, \sigma_{ln}^2 = 3, l_z = 0.1$) .........................34
Figure 2-6 – Effects of using different boundary conditions and saturation-averaging
schemes ......................................................................................................................................37
Figure 2-7 – Typical EFBC $k_{ij}^*$ from different saturation-averaging schemes .................39
Figure 2-8 – Solvent saturation at breakthrough for Field B; coarse model from proposed
upscaling technique .................................................................................................................40
Figure 2-9 – Synthetic Field E ($\mu_{ln} = 3, \sigma_{ln}^2 = 5, l_x = 0.5, l_z = 0.01$) .................................42
Figure 2-10 – Oil-cut curves from various methods (Field B) .................................................45
Figure 2-11 – Fractional oil-cut and breakthrough-time errors from various methods
(Field B) .........................................................................................................................................46
Figure 2-12 – The fine (400 $\times$ 100) and nonuniform coarse (48 $\times$ 40) models of Field B .... 47
Figure 2-13 – Typical EFBC $k_{ij}^*$ of one coarse block for two different fields; Field E is
more heterogeneous than Field B ..........................................................................................47
Figure 2-14 – Oil-cut curves from various methods (Field E)..................................................50
Figure 2-15 – Fractional oil-cut and breakthrough-time errors from various methods (Field E) .......................................................................................................................... 51
Figure 2-16 – Breakthrough-time prediction using $k^*$ only .................................................. 53
Figure 2-17 – Synthetic 3D Field F with uniform fine grid ($50 \times 50 \times 25$) .......................... 54
Figure 2-18 – Simulation flow results for 3D Field F; line-drive displacement ....................... 56
Figure 2-19 – Simulation flow results for 3D Field F; edge-to-edge displacement .............. 56
Figure 2-20 – Simulation flow results for 3D Field F; corner-to-corner displacement ......... 57
Figure 2-21 – Simulation flow results for 3D Field F; five-spot displacement ................. 57
Figure 2-22 – Fractional oil-cut and breakthrough-time errors from EFBC $k_{ij}^r$ and $k^*$ only for different production scenarios (Field F) .......................................................... 58
Figure 2-23 – One realization of synthetic 3D Field G with uniform fine grid ($55 \times 55 \times 25$) ........................................................................................................................................ 60
Figure 2-24 – Simulation results for a realization of 3D Field G; edge-to-edge displacement ...................................................................................................................... 61
Figure 2-25 – Fractional oil-cut and breakthrough-time errors from EFBC $k_{ij}^r$ and $k^*$ only for different realizations of Field G ........................................................................ 61
Figure 3-1 – Geocellular model of the real field ($98 \times 104 \times 858$); vertically exaggerated by a factor of 5 ...................................................................................................................... 67
Figure 3-2 – Fine-scale cross section ($10 \times 858$) extracted from the field model ............... 69
Figure 3-3 – Areal grid refinement results for the $10 \times 858$ model shown in Figure 3-2 ...... 71
Figure 3-4 – Simulation results from different methods for the $10 \times 858$ model shown in Figure 3-2; line-drive displacement .................................................................................. 75
Figure 3-5 – Solvent-saturation maps just after solvent breakthrough (the refined $90 \times 858$ model versus the $10 \times 27$ model from our miscible upscaling technique) ...... 76
Figure 3-6 – Cross-sectional ($5 \times 858$) model extracted from the field model in Figure 3-1 77
Figure 3-7 – Simulation results from different methods for the 5 × 858 model shown in
Figure 3-6; line-drive displacement ................................................................. 79

Figure 3-8 – Solvent-saturation maps just after solvent breakthrough (the refined 25 × 858
model versus the 5 × 27 model from our miscible upscaling technique) .......... 80

Figure 3-9 – A 3D (15 × 15 × 466) sector model extracted from the full-field model in
Figure 3-1 ......................................................................................................... 82

Figure 3-10 – Simulation results from different methods for the 15 × 15 × 466 sector
model in Figure 3-9; five-spot pattern displacement ..................................... 84

Figure 3-11 – A 3D (7 × 7 × 180) sector model extracted from the full-field model in
Figure 3-1 ......................................................................................................... 85

Figure 3-12 – Simulation results for the 7 × 7 × 180 sector model in Figure 3-11; five-
spot pattern displacement ............................................................................. 87

Figure 4-1 – Local well model showing the fine grid (green), coarse grid (blue), and
coarse wellblock (orange) ................................................................................ 94

Figure 4-2 – Illustration of a volume-equivalent radial region............................ 109

Figure 4-3 – Straight-line relative permeabilities ............................................. 111

Figure 4-4 – Fractional flow and dimensionless wave velocity of water ............... 111

Figure 4-5 – Comparison of original and averaged total-mobility curves ............. 113

Figure 4-6 – Analytically computed mobility correction function $m\left(\overline{S_w}\right)$ for an injection
crude oil well block in a homogeneous permeability field ........................... 114

Figure 4-7 – Effects of analytically computed $m\left(\overline{S_w}\right)$ (Figure 4-6) on the optimized $k_{ij}^*$
from the NW2P procedure (homogeneous permeability field) ..................... 116

Figure 4-8 – LWM of a binary channel reservoir with water injector at center; $k_{\text{high}} =$
50 mD (yellow), $k_{\text{low}} = 5$ mD (green) .......................................................... 117

Figure 4-9 – Effects of analytically computed $m\left(\overline{S_w}\right)$ (Figure 4-6) on the optimized $k_{ij}^*$
from the NW2P procedure (heterogeneous permeability field) ..................... 117
Figure 4-10 – Numerically computed mobility correction function, \(m\left(S_w^c\right)\) for an injection coarse wellblock in a heterogeneous permeability field .............. 118

Figure 4-11 – Effects of numerically computed \(m\left(S_w^c\right)\) (Figure 4-10) on the optimized \(k_{ij}^*\) from the NW2P procedure (heterogeneous permeability field) ..................... 119

Figure 4-12 – Power-law fine-scale relative permeabilities .................................................. 123

Figure 4-13 – Production scenarios: (a) horizontal wells, and (b) five-spot pattern. Coarse grid shown in (b) to emphasize low areal resolution ......................... 124

Figure 4-14 – Local well model (LWM) for a horizontal injector .............................. 125

Figure 4-15 – Injector LWM flow results; \(M^0 = 100, \mu_{in} = \sigma_{in}^2 = 3.0, l_x = l_y = 0.5, l_z = 0.1126\)

Figure 4-16 – Optimized \(k_{ij}^*\) for a coarse injection wellblock from the NW2P procedure compared with input relative permeabilities .................................................. 128

Figure 4-17 – Producer LWM flow results; \(M^0 = 100, \mu_{in} = 3.0, \sigma_{in}^2 = 1.0, l_x = l_y = 0.5, l_z = 0.1\)............................................................. 129

Figure 4-18 – Optimized \(k_{ij}^*\) for a coarse production wellblock from the NW2P procedure compared with input relative permeabilities ............................................. 130

Figure 4-19 – Global simulation flow results; horizontal wells, \(M^0 = 100, \mu_{in} = \sigma_{in}^2 = 3.0, l_x = l_y = 0.5, l_z = 0.1\)............................................................. 132

Figure 4-20 – Individual producer total flow rate; horizontal wells, \(M^0 = 100, \mu_{in} = \sigma_{in}^2 = 3.0, l_x = l_y = 0.5, l_z = 0.1\)............................................................. 133

Figure 4-21 – Composite error as a function of spatial variability; horizontal wells, \(M^0 = 100, l_x = l_y = 0.5\)............................................................. 134

Figure 4-22 – Average errors in individual producer variables as a function of spatial variability; horizontal wells, \(M^0 = 100, l_x = l_y = 0.5\)............................................................. 136

Figure 4-23 – Average error in individual producer water cut (dimensional time) as a function of spatial variability; horizontal wells, \(M^0 = 100, l_x = l_y = 0.5\) .... 137
Figure 4-24 – Errors as a function of areal correlation length; horizontal wells, $M^0 = 100$, $\sigma_{in}^2 = 3.0$.......................................................................................................................... 139

Figure 4-25 – Errors as a function of spatial variability; horizontal wells, $M^0 = 100$, $l_x = l_y = 0.5$..................................................................................................................... 140

Figure 4-26 – Global flow results; horizontal wells, $M^0 = 10$, $\mu_{in} = \sigma_{in}^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$......................................................................................................................... 143

Figure 4-27 – Composite error as a function of spatial variability; horizontal wells, $M^0 = 10$, $l_x = l_y = 0.5$......................................................................................................................... 144

Figure 4-28 – Errors as a function of mobility ratio; horizontal wells, $\mu_{in} = \sigma_{in}^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$......................................................................................................................... 145

Figure 4-29 – Global simulation flow results; five-spot pattern, $M^0 = 100$, $\mu_{in} = \sigma_{in}^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$......................................................................................................................... 147

Figure 4-30 – Individual producer total flow rate; five-spot pattern, $M^0 = 100$, $\mu_{in} = \sigma_{in}^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$......................................................................................................................... 148

Figure 4-31 – Errors as a function of spatial variability; five-spot pattern, $M^0 = 100$, $l_x = l_y = 0.5$......................................................................................................................... 149

Figure 4-32 – Errors as a function of areal correlation length; five-spot pattern, $M^0 = 100$, $\sigma_{in}^2 = 3.0$......................................................................................................................... 150

Figure 4-33 – Global flow results; five-spot pattern, $M^0 = 10$, $\mu_{in} = \sigma_{in}^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$......................................................................................................................... 152

Figure 4-34 – Composite error as a function of spatial variability; five-spot pattern, $M^0 = 10$, $l_x = l_y = 0.5$......................................................................................................................... 153

Figure A-1 – Impact of using directional $k_{ij}$ in a corner-to-corner, cross-sectional problem (Field B of Figure 2-5).............................................................................................................. 184

Figure A-2 – Error comparison to show the effects of employing directional $k_{ij}$..................... 185

Figure A-3 – Cross section of a real field (Field X).............................................................................. 187
Figure A-4 – Oil-cut curves from EFBC $k_{ij}^*$ (Field X) ........................................................ 188
Figure A-5 – Oil-cut curves from standard $k_{ij}^*$ (Field X) .......................................................... 188
Figure A-6 – Oil-cut curves from $k^*$ only (Field X) ....................................................................... 189
Figure A-7 – Fractional oil-cut and breakthrough errors from various methods (Field X). 190
Figure A-8 – Various simulation flow results for Field X (EFBC $k_{ij}^*$) ........................................... 191
Figure A-9 – Oil-production curves for Field X (EFBC $k_{ij}^*$ and $k^*$ only) ................................. 192
Figure A-10 – Solvent-saturation maps at breakthrough for Field X (15 x 27 model
          obtained using EFBC $k_{ij}^*$) .................................................................................................. 192
Figure B-1 – Results of areal grid refinement for an oil-water displacement in the
          10×858 cross-sectional model shown in Figure 3-2 .............................................................. 194
Chapter 1

Introduction

The field of upscaling is motivated by the need to coarsen fine-scale, geocellular descriptions into coarse-scale models suitable for fast and accurate reservoir simulations. Efficient flow calculations are essential as a typical reservoir engineering study may include numerous simulations for history matching and optimization, the investigation of different well configurations for performance prediction, and the assessment of uncertainties using multiple geostatistical realizations.

Although advances in numerical solution techniques (Cao and Aziz, 2002), parallel computing (Chien et al., 1997), streamline methods (Thiele et al., 1997), and adaptive grid refinement (Nilsson et al., 2005) allow for the simulation of more detailed models, upscaling is still essential. This is due in part to the fact that geological models continue to grow in size—these models typically contain about two orders of magnitude more gridblocks than reservoir simulators can handle in a practical time frame. Numerous past researchers have investigated many aspects of the upscaling problem and proposed a wide variety of upscaling methods (see reviews in Christie, 1996; Farmer, 2002; Durlofsky, 2003). These approaches differ in various respects, and one broad classification is based on whether the upscaled parameters are single-phase (e.g., upscaled absolute per-
meabilities, upscaled transmissibilities) or two-phase (e.g., upscaled relative permeabilities, dispersion coefficient). As discussed in this chapter, other aspects of upscaling that have been studied include the boundary conditions imposed (pressure, flux, effective flux); upscaling domains considered (local, extended local, global); the types of grids employed (nonuniform, flow-based, unstructured); and the nature of displacement process (immiscible, miscible, compositional). Nevertheless, there are still many significant facets of the field that have not been fully explored. This dissertation focuses on some important areas, namely the upscaling of two-phase flow processes in which the physics is complicated by factors such as near-well effects and first-contact miscibility. The primary objective of this work is the development and evaluation of new upscaling approaches that enable the accurate and efficient coarse-scale simulations of such displacements.

This chapter proceeds as follows. In Section 1.1, we motivate the need to upscale first-contact miscible displacements and review past approaches for modeling such displacements. The limitations of previous miscible upscaling approaches are highlighted and the overview of a novel upscaling technique that addresses these limitations is described. In Section 1.2, the upscaling of well-driven, high-mobility-ratio displacements is introduced. Here, a literature survey of existing approaches that are pertinent to this upscaling problem is presented and unresolved issues are discussed. A new near-well, two-phase upscaling approach is proposed for the proper modeling of this type of displacement.
1.1 First-contact miscible displacements

Under proper conditions in a miscible gas injection process, the interfacial tension between the injected gas and the reservoir oil can be eliminated and a single miscible phase is formed. The displacement is termed first-contact miscible when the displacing solvent and displaced oil mix in all proportions to form a single phase. In contrast, in a multicontact miscible process, miscibility is achieved through the mass transfer of components between the oil and solvent phases as a result of repeated contact. If miscibility occurs, significantly lower residual oil saturations and increased local displacement efficiencies typically result (Stalkup, 1983). For this reason, the ultimate oil recoveries from miscible displacements can be substantially higher than those from immiscible fluid-injection processes such as waterflooding. In oil fields with significant amounts of associated gas but no viable market for natural gas, limited gas-handling capacity can become a bottleneck for oil production. In this scenario, miscible gas injection also represents a promising approach.

In a miscible gas displacement, channeling and viscous fingering occur due to heterogeneities and the adverse mobility ratio between the injected gas/solvent and oil (Tchelepi and Orr, 1994). Density contrasts between the fluids can also lead to gravity segregation and tonguing. These fine-scale effects need to be properly captured for an accurate estimation of displacement efficiency but the grid resolution required to represent them explicitly is extremely high. For this reason, most field-scale simulations of miscible processes are performed on grids that do not resolve important physical effects. In order to obtain meaningful simulation results, coarse-scale reservoir models must therefore account for these subgrid phenomena implicitly. The goal of this portion of the
dissertation is to develop an accurate and efficient upscaling technique to account for subgrid heterogeneity effects in first-contact miscible displacements.

1.1.1 Literature review

Approaches to modeling miscible and near-miscible displacements can be broadly divided into two categories: first-contact miscible or multicontact miscible.

For multicontact miscible processes, for which there are complications such as complex phase behavior and the development of miscibility, fully-compositional simulations are generally required. However, fully-compositional simulations tend to be very computationally intensive, especially if a large number of components must be included (Newley and Merrill, 1991). Compositional streamline techniques (Thiele et al., 1997; Christie and Clifford, 1998; Crane, 2000; Jessen and Orr, 2002) have been proposed to efficiently perform fully-compositional simulations, and these may eventually allow for fine-scale, fully-compositional simulations in cases of practical interest. At the present time, however, computational issues associated with capillary pressure, gravity (Jessen and Orr, 2004), compressibility, streamline updating, nonuniform initial conditions, and mapping errors (Mallison et al., 2004) remain active areas of research. Recently, Nilsson et al. (2005) proposed a computationally-efficient adaptive mesh refinement technique that selectively and dynamically resolves the grid where important flow processes are occurring, e.g., the displacement front. However, this technique is still under development and is not yet ready for practical applications.

When the displacement is first-contact miscible, the limited-compositional (LC) approach (e.g., Koval, 1963; Todd and Longstaff, 1972; Fayers, 1988) is typically adopted since it is substantially more efficient (no flash calculations are required). The
1.1. **FIRST-CONTACT MISCIBLE DISPLACEMENTS**

LC formulation employs the (immiscible) black-oil simulation framework to represent oil and solvent as distinct pseudophases even though they form a single phase. By modifying the fluid properties (e.g., viscosities) of the pseudophases, partial mixing of fluids within a gridblock due to viscous fingering is implicitly accounted for (in an approximate sense). Early LC models (e.g., Koval, 1963; Todd and Longstaff, 1972) primarily focused on accounting for the viscous fingers observed in laboratory experiments.

The issue of upscaling a first-contact miscible displacement from geocellular, fine-scale models to reservoir-simulation, coarse-scale models was not specifically addressed in the early LC models. Straight-line relative permeabilities for the hydrocarbon pseudophases were suggested based on the argument that the pseudophases do not interfere with each other’s flow. The residual oil saturation in a gridblock subjected to miscible flooding was routinely assumed to be zero. While these assumptions may be appropriate for the relatively homogeneous media used in experiments, they are inappropriate for coarse simulation gridblocks that possess a significant degree of subgrid heterogeneity. Over the scale of a coarse gridblock (typically tens or hundreds of feet in size), it is inevitable that some oil is bypassed during miscible flooding. This coarse-block residual oil saturation (designated $S_{orb}$) is neither mobile nor mixed by the injected solvent. The computation of effective fluid properties and upscaled relative permeabilities ($k_r^*$) for the coarse block should account for this $S_{orb}$. We call this idea the $S_{orb}$ concept.

Zhang and Sorbie (1995) proposed a LC upscaling approach that employs $k_r^*$ to better capture the effects of permeability heterogeneity. They were mainly concerned with matching first-contact miscible experimental results and therefore considered relatively homogeneous cases. Christie *et al.* (1995) applied a renormalization-based, LC upscaling method to compute pseudo functions (calculated using standard boundary conditions) for use in first-contact miscible, water-alternating-gas (WAG) floods. Applying the pseudo
functions to a mildly heterogeneous case (variance of natural logarithm of permeability of 1.0), they obtained simulation results that were more accurate than those from the use of upscaled absolute permeabilities only. None of the LC upscaling approaches discussed above accounted for $S_{orb}^*$, which is likely to be important for highly heterogeneous systems.

Fully-compositional models have traditionally neglected the effects of viscous fingering and assumed complete mixing within each gridblock (Stalkup, 1983). This assumption can lead to optimistic oil recovery predictions. The first empirical model of compositional viscous fingering was proposed by Blunt et al. (1994). In analogy with the use of pseudo relative permeabilities $k_{ij}^*$ to modify the flow of phases, Barker and Fayers (1994) introduced the use of transport coefficients ($\alpha$-factors) to modify the flow of components out of a gridblock as a means of capturing subgrid effects in fully-compositional simulations. Rubin et al. (1993) applied $\alpha$-factors (in conjunction with $k_{ij}^*$) to a fully-compositional case and showed that fine-grid simulation results could be reproduced for essentially homogeneous cases. Thibeau et al. (1995) also investigated the use of $\alpha$-factors and $k_{ij}^*$ in a 2D, completely layered model and introduced an extended local upscaling procedure called Dual Scale Simulations. Jerauld (1998) presented a 3D case study in which the fine-scale, fully-compositional results for a multicontact miscible WAG process were predicted with reasonable accuracy through the use of $k_{ij}^*$ in a coarse-scale LC model.

As was the case with LC approaches, the fully-compositional techniques involving pseudoization discussed above did not include the $S_{orb}^*$ concept. For fully-compositional processes, the amount of oil represented by $S_{orb}^*$ should also be excluded when performing flash calculations. It should be noted that recently some researchers have attempted to account for bypassed oil saturations in fully-compositional simulations (Coats et al.,
1.1. FIRST-CONTACT MISCIBLE DISPLACEMENTS

2004; Barker et al., 2005). However, the general applicability of these approaches remains to be demonstrated since these formulations tend to be quite complex and computational costs are potentially substantial.

Thus, there are several important issues that remain unresolved in the context of upscaling first-contact miscible displacements for reservoir-simulation purposes. An accurate coarse simulation model needs to account for the existence of bypassed oil within gridblocks (i.e., $S_{orb}^*$. Further, the $k_{ij}^*$ for the coarse grid can be very different from the straight-line relative permeabilities used for the fine grid. The proper modeling of these effects will be important when upscaling the highly heterogeneous systems typically encountered in practical field studies. Previous studies mostly considered relatively mild degrees of heterogeneity, so the performance of existing miscible upscaling techniques for more challenging models remains to be assessed. Finally, most existing miscible upscaling approaches that compute purely local pseudo functions imposed standard (constant pressure-no flow) boundary conditions during the upscaling calculations. These boundary conditions are known to result in pessimistic coarse-scale, reservoir-simulation predictions in many cases (see for example, “REV” results in Thibeau et al., 1995).

It is clearly desirable to apply more appropriate local boundary conditions in the upscaling calculations such that the coarse models are not biased. The theory of effective flux boundary conditions (EFBCs) was formulated by Wallstrom et al. (2002a) to prescribe more appropriate boundary conditions for the purely local upscaling calculation of $k_{ij}^*$. Using EFBCs, approximate (analytical) global flow information is introduced to improve the estimation of the local flow field. EFBCs appropriately attenuate the flux through high permeability streaks that span the coarse blocks but do not span the global system. The standard boundary conditions commonly employed in upscaling do not give this flux-attenuation behavior. Rather, using standard boundary conditions, the flux
through any high permeability feature that spans the coarse block will be large even though the feature is discontinuous at the global scale. The resulting overestimation of the upscaled relative permeability of the injected phase leads to coarse-scale models that predict early breakthrough. EFBCs rectify this pessimistic bias and introduce a degree of self-consistency between the global flow field and the local boundary conditions employed during upscaling. The effectiveness of EFBCs has been demonstrated in a variety of immiscible displacement calculations (Wallstrom et al., 2002b; Chen, 2005). Other aspects of coarse-grid $k_{ij}^*$ calculation are discussed (within the context of well-driven, high-mobility-ratio displacements) in Section 1.2.

1.1.2 Proposed approach

Our proposed miscible upscaling approach is designed to address the issues highlighted in the above discussion. We present a novel upscaling technique for the fast and accurate coarse-scale simulation of first-contact miscible processes. The method has two key components: the use of EFBCs for the calculation of upscaled relative permeabilities ($k_{ij}^*$) and the extended Todd-Longstaff with $k_{ij}^*$ (ETLU) formulation.

As described in the previous section, EFBCs, by accounting approximately for the effects of the global flow field on the local computation of $k_{ij}^*$, mitigate the pessimistic bias introduced by standard procedures (e.g., premature breakthrough). The application and detailed testing of EFBCs for miscible displacements is presented for the first time in this work (see also Hui et al., 2004). We reiterate that none of the previous limited- and fully-compositional approaches entailing $k_{ij}^*$ employed specialized boundary conditions such as EFBCs. Our ETLU formulation is based on the well-known LC approach proposed by Todd and Longstaff (1972), which is applicable to first-contact miscible dis-
1.2. **WELL-DRIVEN, HIGH-MOBILITY-RATIO DISPLACEMENTS**

placements. As indicated above, in the original formulation, each gridblock is assumed to have no bypassed oil saturation and straight-line relative permeabilities are used to characterize the flow of oil and solvent. In the extended formulation, we account for the fact that each upscaled coarse block contains a bypassed oil saturation $S_{orb}^*$, which is immobile and unavailable for mixing. The magnitude of $S_{orb}^*$ is determined by the degree of subgrid heterogeneity within the coarse block. We exclude $S_{orb}^*$ from the calculations of the effective viscosities and prevent the flow of oil for oil saturations less than $S_{orb}^*$.

By applying EFBCs on each coarse-block region and performing a local fine-scale simulation, the $k_{ij}^*$ for the coarse block are computed. These $k_{ij}^*$ represent the flow of oil and solvent out of the coarse block such that the overall flow behavior of the fine-scale region that corresponds to the target coarse block is reproduced. Without the $S_{orb}^*$ concept, the $k_{ij}^*$ can display a high level of nonmonotonicity, which may pose numerical difficulties for some simulators. By properly representing $S_{orb}^*$, the behavior of the $k_{ij}^*$ is improved.

**1.2 Well-driven, high-mobility-ratio displacements**

In recent decades, the world’s oil consumption rate has persistently outpaced the rate of new discoveries. As a result, the role of improved-oil-recovery processes to provide petroleum from existing reservoirs has been gaining prominence (Hirsch *et al.*, 2005). Waterflooding persists as the single most used improved-oil-recovery process because water/brine is commonly and cheaply available (Gulick and McCain, 2003). In addition, immiscible gas injection in some cases offers an attractive means of utilizing produced gas to maintain or increase oil production. High mobility ratios are often encountered in these processes because the displaced oil can be much more viscous than the injected
water or gas (e.g., Adams, 1982; Ko and Domier, 1995). For these reasons, the subject of high-mobility-ratio, two-phase flow displacement is highly pertinent to the petroleum industry.

Wells are the tools for exerting control over reservoir flow processes. They account for most of the pressure drop and constitute the major driving force for fluid flow. In the context of reservoir simulation, an accurate well model is needed to numerically relate the well to the gridblock in which it is completed (i.e., the wellblock). As a result, a coarse-scale simulation model will not provide an accurate representation of the fine-scale reference model unless the near-well region is upscaled appropriately (Aziz et al., 1999). A robust near-well upscaling technique is important for the successful modeling, planning, and drilling of the increasingly popular but complicated and expensive nonconventional wells (Wolfsteiner et al., 2000). Accurate and efficient well models are also a prerequisite for the full exploitation of real-time well data and well trajectory optimization schemes (Yeten et al., 2003). Thus, our motivation is to develop a fast and accurate upscaling technique for modeling well-driven, high-mobility-ratio displacements.

1.2.1 Literature review

In the past, many upscaling techniques were directed toward the computation of upscaled absolute permeabilities (designated $k^*$) (Wen and Gómez-Hernández, 1996; Renard and de Marsily, 1997). These approaches assume a linear, constant-gradient pressure field in the form of periodic (Durlofsky, 1991), linearly varying or constant pressure-no flow boundary conditions. In this work, we will refer to these upscaling techniques as standard $k^*$ (or $k^*$ only) approaches. Contrary to these assumptions, the flow in the near-well region is radial and the pressure field does not have a constant gradient. Further, it has
1.2. **WELL-DRIVEN, HIGH-MOBILITY-RATIO DISPLACEMENTS**

been established that the use of standard $k^*$ to calculate the well indices (Peaceman, 1983) in coarse-scale models can lead to large inaccuracies in flow results (Ding, 1995; Durlofsky *et al.*, 2000; Mascarenhas and Durlofsky, 2000). Although the approach of transmissibility ($T^*$) upscaling (e.g., White and Horne, 1987) was shown to be more accurate than standard $k^*$ approaches, the linear flow assumption is still inappropriate. Clearly, the proper upscaling of the near-well region requires a treatment specially designed to account for the radial nature of well-driven flow.

One possible approach for an accurate near-well treatment is the use of local grid refinement around the well (Göktaş and Ertekin, 1999). This method may be expensive, however, as more blocks are included in the simulation and small time steps may be required due to the introduction of small blocks in the high-flow region.

In the past, a variety of analytical near-well upscaling methods have been proposed. Lin (1995) addressed the issue of partial penetration by computing the coarse-scale well index using a fine-scale flow simulation that has reached quasi-steady-state conditions. Lin’s formulation was based on analytical results for homogeneous reservoirs. Chen *et al.* (1995a, 1995b) proposed an alternative well model for cases in which the Peaceman equation is not applicable, for instance deviated wells. Their method converts a heterogeneous model into a homogeneous but anisotropic system by computing a pseudo skin factor using the analytical solution for pseudo-steady-state flow. Soeriawinata *et al.* (1997) derived analytical expressions for upscaled permeabilities in the near-well region based on an incomplete-layer upscaling procedure and steady-state, radial-flow averaging laws. The analytical approaches discussed above are effective and efficient in some cases but they rely on simplifying assumptions. Thus, for highly heterogeneous reservoirs, these approaches are likely to be less accurate than numerical techniques.
The first numerical near-well upscaling technique was proposed by Ding (1995). He applied a global fine-scale solution of a single-phase, steady-state, incompressible, well-driven flow problem to determine the upscaled well index ($W_t^*$) and wellblock transmissibilities ($T_w^*$). Durlofsky et al. (2000) extended the method to 3D cases involving vertical wells and employed a smaller (extended local) fine-scale domain to compute the upscaled parameters. Their near-well method performed better than standard approaches in many cases even though they used a relatively small domain for the calculation of near-well parameters. In a similar but independent work, Muggeridge et al. (2002) also extended Ding’s method and considered a reduced computational domain. They found the near-well, single-phase upscaling approach to be quite effective for a variety of 2D and 3D problems involving partially penetrating and non-vertical wells. Mascarenhas and Durlofsky (2000) further extended the near-well methodology by introducing an efficient Gauss-Newton optimization procedure to force agreement between the coarse- and fine-scale flow rates. They observed a significant improvement over the standard $k^*$ approach in a 3D, three-phase flow problem involving a horizontal producer. Recently, Zhang et al. (2005) presented a fully global upscaling technique that calculates $W_t^*$ and transmissibilities and showed that it could provide accurate results for highly heterogeneous reservoirs.

While the single-phase upscaling techniques described above often give satisfactory results, their effectiveness deteriorates with extreme levels of coarsening (two orders of magnitude or more). Single-phase upscaling will also become inadequate when multiphase effects are dominant, for instance in a high-mobility-ratio displacement. In such cases, two-phase upscaling techniques may also be needed. In the context of near-well upscaling, this implies that, apart from the upscaled single-phase flow parameters ($W_t^*$ and $T_w^*$), two-phase parameters (e.g., pseudo functions) may also be required. Numerous investigators have proposed two-phase upscaling approaches involving pseudo functions.
(see literature reviews in Barker and Dupouy, 1999; Darman et al., 2002; and the evaluation of pseudoization approaches in Cao and Aziz, 1999). These methods can be divided into three broad categories: gravity-dominated vertical equilibrium (Coats et al., 1971), capillary equilibrium (Lemouzy et al., 1993) and dynamic pseudo functions (calculated from a simulation run; Jacks et al., 1973).

In the near-well region, viscous forces are important, so dynamic pseudo functions can be expected to be most applicable. There are two general types of dynamic pseudo functions—those based on directly upscaling Darcy’s law (Kyte and Berry, 1975; Guzman et al., 1999)—and those that use an average total mobility formulation (Stone, 1991; Christie et al., 1995). Perhaps the strongest criticism of dynamic pseudo functions is their process dependency (Barker and Thibeau, 1997), i.e., they may only be applicable for the boundary conditions (e.g., well locations and rates) imposed during the upscaling calculations and hence may not be appropriate for a wide range of flow scenarios. Nevertheless, in the near-well region, the prevailing conditions are better known since they will be largely determined by the wells. In addition, in the case of injectors, the near-well conditions are largely unaffected by distant wells or boundary conditions (Ding, 1995).

Of the large body of work addressing the calculation of pseudo functions, relatively little consideration has been given to flow in the near-well region. Instead, most of the previous approaches are formulated based on linear flow assumptions and are thus not suitable for use in the near-well region. Previously, Emanuel and Cook (1974) and Woods and Khurana (1977) computed pseudo functions based on the upscaling of Darcy’s law in the near-well region in order to reduce the dimensionality of the flow problem. These efforts were directed at accounting for numerical dispersion in the coarse grid and considered relatively simple reservoir descriptions (homogeneous or purely layered). As such, they were not specifically designed to capture the effects of subgrid
heterogeneity. To our knowledge, there has yet not been an in-depth investigation into the upscaling of near-well, two-phase flow parameters for high-mobility-ratio displacements in heterogeneous reservoirs.

1.2.2 Proposed approach

We initially modified two of the popular dynamic pseudoization methods (Kyte and Berry, 1975; Stone, 1991) to account for well-driven flow but found that they do not provide satisfactory results for the high-mobility-ratio cases of interest. Instead, we propose a new upscaling technique that comprises two components: a near-well, single-phase (NW1P) procedure and a near-well, two-phase (NW2P) procedure. The upscaling calculations are performed on an extended local well model extracted from the global simulation grid.

The NW1P procedure is based on the work of Mascarenhas and Durlofsky (2000). A well-driven, single-phase flow problem is imposed and the mismatch in steady-state flow rates between the fine and coarse models is minimized by optimizing $W^*$ and $T^*$. The NW2P procedure determines upscaled relative permeabilities ($k_{ij}^*$) by minimizing the differences in both the oil and water flow rates between the fine and coarse models for a well-driven, two-phase flow problem. In the past, some investigators have adopted an optimization approach in the context of two-phase upscaling (Killough and Foster, 1979; Tan, 1995), but without a focus on the near-well region. Johnson et al. (1982) proposed a two-phase optimization method for the near-well region but their method was only applied to completely layered models and was not used in conjunction with any near-well, single-phase upscaling technique, as in our approach. The two-phase flow phenomena in regions away from wells must also be upscaled in some cases to achieve global accuracy.
For this upscaling, we apply effective flux boundary conditions (EFBCs) for the calculation of the $k_{ij}^*$ for each coarse block. EFBCs form a key component of our miscible upscaling technique and were described in Section 1.1.

The optimized $k_{ij}^*$ from our NW2P procedure often exhibit a nonmonotonic behavior, which is caused by coarse-grid mobility effects. This behavior was also observed by Hewett et al. (1998) in 1D, homogeneous cases. To address the issue in the near-well problem, we formulate a semianalytical, subgrid model to compute a mobility correction function $m$ for the well from the radial form of the Buckley-Leverett solution (Buckley and Leverett, 1942). A similar but independent approach was proposed by Nakasha and Nomura (2004), but important differences exist between our approach and theirs, which will be discussed in Section 4.5. By isolating the effects of heterogeneity and coarse-grid mobility, and capturing these through the optimized $k_{ij}^*$ and $m$ respectively, a more robust upscaling approach may be obtained. Along somewhat similar lines, Mugggeridge (1991) attempted to isolate the effects of heterogeneity and numerical dispersion using a multi-stage approach. Her approach, however, had no analytical component and did not address the near-well problem.

1.3 Dissertation outline

In this chapter, we motivated the need to develop upscaling techniques to model two-phase displacements complicated by high-mobility-ratio, near-well effects and miscibility. A literature review of the existing upscaling approaches for modeling these types of displacements was presented. The limitations of past approaches were highlighted and unresolved issues were identified. We outlined our overall strategies for the development
CHAPTER 1. INTRODUCTION

of upscaling techniques that will enable the accurate and efficient simulation of these displacements.

The organization of this dissertation is as follows. In Chapter 2, the formulation of a novel technique for upscaling first-contact miscible displacements is presented. Different boundary conditions and saturation-averaging schemes will be compared. We will evaluate our miscible upscaling technique on synthetic 2D fields for varying correlation lengths and coarsening factors, as well as on synthetic 3D fields for different production scenarios and multiple permeability realizations. The upscaling errors of our approach will be quantified and compared with those from other standard upscaling methods. Computational efficiency issues will also be discussed.

In Chapter 3, the miscible upscaling technique is applied to a miscible gas injection field study involving 3D, nonuniform grids and realistic production scenarios. A near-well treatment (NW1P or local grid refinement) is also incorporated into the methodology. Results from an areal grid refinement study will be described. The practical usefulness of our approach is clearly demonstrated based on the level of accuracy and overall speedup that it provides. It should be noted that the miscible upscaling work (presented in Chapters 2 and 3) was performed in collaboration with Dr. Degen Zhou and Dr. Xian-Huan Wen from Chevron Energy Technology Company in San Ramon, California. The miscible upscaling code we used was modified from code for the immiscible case developed at the Los Alamos National Laboratory (Wallstrom et al., 2002a and 2002b). Early tests of this code were performed by S. Jákupsstovù from the Technical University of Denmark. The geocellular models of real fields considered in Chapter 3 were generated by Chevron geologists and engineers.

In Chapter 4, a new near-well two-phase upscaling methodology based on both single-phase and two-phase optimization procedures will be described. We evaluate our
near-well approach by considering multiple realizations of 3D synthetic fields with varying correlation structures and degrees of spatial variability, as well as different fluid mobility contrasts and well configurations. Again, the accuracy and computational efficiency provided by different upscaling techniques will be quantified and compared. Finally, in Chapter 5, we summarize the major conclusions from this work and recommend possible future research directions. Additional results that support and augment our discussions in Chapters 2 and 3 are included in Appendices A and B.
Chapter 2
A new upscaling technique for first-contact miscible displacements

2.1 Introduction

In many oil fields with significant amounts of associated gas, miscible gas injection is a potentially attractive recovery method because it can yield high local displacement efficiencies and may also offer a solution for gas handling. For an accurate estimation of the displacement efficiency, complex phenomena like channeling and viscous fingering may need to be represented. When the displacing gas and displaced oil are miscible in all proportions, the displacement is termed first-contact miscible (Stalkup, 1983). For such a displacement, the limited-compositional (LC) formulation (e.g., Todd and Longstaff, 1972) is often adopted due to its computational efficiency. The LC formulation allows the simulator to model miscibility within a black-oil framework and empirically accounts for viscous fingering by modifying the fluid properties of the hydrocarbon pseudophases. However, because fine-scale LC simulations are still computationally demanding, there remains a clear need for a robust miscible upscaling technique within the LC context.
In this work, we present a novel upscaling procedure for the fast and accurate coarse-scale simulation of first-contact miscible displacements. Our method is an LC approach that has two key components: the use of EFBCs (Wallstrom et al., 2002a) for the calculation of upscaled (pseudo) relative permeabilities \( k_{ij}^* \) and the extended Todd-Longstaff with \( k_{ij}^* \) (ETLU) formulation. EFBCs incorporate approximate global flow information into the local upscaling calculations and appropriately suppress the flux through high-permeability streaks that are not continuous throughout the domain. As a result, EFBCs mitigate the problem of premature breakthrough of injected fluid in coarse-scale simulations, which can occur because of the overestimation of flux that results from the use of standard boundary conditions. Our ETLU formulation extends the Todd-Longstaff method (Todd and Longstaff, 1972) by accounting for the fact that, within reservoir-simulation length scales, there exists an amount of oil that is practically immobile and not available for mixing \( S_{orb}^* \). The computation of effective fluid properties and upscaled relative permeabilities, therefore, should not include the oil saturation represented by \( S_{orb}^* \). This concept leads to the improved behavior of the \( k_{ij}^* \).

This chapter will proceed as follows. We first describe the two main components of our miscible upscaling methodology: the extended Todd-Longstaff with \( k_{ij}^* \) (ETLU) formulation (Section 2.2) and the use of effective flux boundary conditions (EFBCs) for the calculation of \( k_{ij}^* \) (Section 2.3). To simplify these descriptions, the development here is for a 2D \((x-z)\) system, though the method has also been implemented in three dimensions (for the 3D formulation, see Hui et al., 2005a). We next present the numerical results obtained from the application of the method to a variety of synthetic 2D fields in Section 2.4. Here, we will evaluate the impact of varying the correlation structure of the fields and compare the use of standard boundary conditions against EFBCs in terms of how accurately the coarse-scale simulation results match the fine-scale solutions. For
different heterogeneity structures and a wide range of coarsening factors, we also compare the proposed method to other commonly used techniques such as standard $k_r^e$, single-phase upscaling, and nonuniform coarsening. The upscaling errors as well as speedup factors are quantified to gauge the accuracy and efficiency of the various methods. In addition, we will discuss how the interplay between numerical dispersion and heterogeneity effects explains the trends of breakthrough-time errors. The application of the method to synthetic 3D models will be presented in Section 2.5, where different well configurations and multiple permeability realizations are investigated. In Section 2.6, we will summarize the key observations. Application of the method to a field case is presented in Chapter 3. The work presented in this chapter has been published (Hui et al., 2004; Hui et al., 2005b) but we include additional results and discussion here.

All flow simulations presented in this chapter were run using Chevron’s CHEARS simulator while the upscaling calculations were performed using a research upscaling code. We employ a fully-implicit formulation for the global simulations and an IMPES procedure for the upscaling computations. For these simulations and upscaling computations, we used either a LINUX machine with a Pentium4 1.5 GHz CPU or an IBM machine with a Power4 1.3 GHz CPU.

### 2.2 Extended Todd-Longstaff (ETLU) formulation

In first-contact miscible displacements, the displaced fluid (oil) and displacing fluid (gas/solvent) form a single miscible phase in all proportions (Stalkup, 1983). The Todd-Longstaff method (Todd and Longstaff, 1972) is a limited-compositional (LC) approach widely used for the modeling of this type of flow process. The LC formulation modifies the black-oil simulator such that the flow of oil and solvent can be modeled as separate
phases (pseudophases) even though only one phase (i.e., the mixture) exists. The mixing phenomenon in the hydrocarbon phase is modeled using a set of mixing rules to modify the effective fluid properties of the hydrocarbon pseudophases (oil and solvent).

In this chapter, we consider a uniform, structured, Cartesian grid for the reservoir model. The fine-scale permeability field, which is anisotropic and heterogeneous, is described by a diagonal permeability tensor $k$ with zero off-diagonal elements (i.e., $k_{ij} = 0$ if $i \neq j$). Both the fine and coarse grids are orthogonal and two-point flux approximations are used. Capillary effects do not exist in a miscible displacement. Thus, if we neglect gravity, the incompressible flow of the two pseudophases (oil and solvent) is governed by the following pressure and saturation equations:

$$\nabla \cdot (\mathbf{u}_t) = -q_t,$$  \hspace{1cm} (2.1)

$$\phi \frac{\partial S_s}{\partial t} + \nabla \cdot (f_s \mathbf{u}_s) = -q_s,$$  \hspace{1cm} (2.2)

where $\phi$ is porosity, $t$ is time, $S_s$ is solvent saturation, $q_t$ and $q_s$ are the total and solvent source/sink terms, and $\mathbf{u}_t, f_s,$ and $\lambda_t$ are the Darcy total velocity, fractional flow of solvent, and total mobility respectively, defined as:

$$\mathbf{u}_t = -k \lambda_t \nabla p,$$  \hspace{1cm} (2.3)

$$f_s = \frac{k_{rs} / \mu_{se}}{k_{rs} / \mu_{se} + k_{ro} / \mu_{oe}},$$  \hspace{1cm} (2.4)

$$\lambda_t = \frac{k_{rs}}{\mu_{se}} + \frac{k_{ro}}{\mu_{oe}},$$  \hspace{1cm} (2.5)

where $p$ is pressure, $k_{rs}$ and $k_{ro}$ are the solvent and oil relative permeabilities, and $\mu_{se}$ and $\mu_{oe}$ are the effective solvent and oil viscosities. Since pseudophases do not interfere with each other’s flow, their relative permeabilities are proportional to their respective saturations, i.e., $k_{ij} = S_j (j = o, s)$. Without capillary trapping, the bypassed oil saturation ($S_{orb}$) is
2.2. **EXTENDED TODD-LONGSTAFF (ETLU) FORMULATION**

The effective viscosity for pseudophase \( j \), \( \mu_{je} \), is typically assumed to be zero at the fine scale. The effective viscosity is computed with the following formula:

\[
\mu_{je} = \mu_j^{(1-\omega)} \mu_m^\omega,
\]

where \( \mu_j \) is the pure viscosity of the phase or pseudophase \( j \) and \( \omega \) is the empirical mixing parameter while \( \mu_m \) is the mixture viscosity given by the quarter-power mixing rule:

\[
\frac{1}{\mu_m^{1/4}} = \frac{S_t}{\mu_t^{1/4}} + 1 - S_t.
\]

The empirical mixing parameter \( \omega \) can be used to account for the effects of channeling and viscous fingering on the mixing of fluids. When \( \omega = 0 \), channeling and/or viscous fingering in the system are so severe that no mixing occurs, while \( \omega = 1 \) is applicable to a system with perfect mixing. For the fine-scale simulations, our assumption is that the resolution is high enough so that \( \omega = 1 \). In other words, it is assumed that there are no subgrid phenomena within each fine block.

Our coarse-scale model is an extension of the Todd-Longstaff method and is designed for use in an upscaling framework to allow accurate coarse-scale simulations of first-contact miscible displacements. An important idea in our extended formulation (ETLU) is the concept of a nonzero bypassed oil saturation within each coarse block, which is designated \( S_{arb} \). In miscible flooding, we can theoretically recover all of the oil within the rock by continuously injecting solvent, thus achieving zero bypassed oil saturation. However, for a miscible displacement in real media, full oil recovery may take an extremely long time because viscous fingering, channeling, and gravity segregation cause pockets of oil to be bypassed (Tchelepi and Orr, 1994). In practice, because of the cost of solvent, injection is stopped once the oil cut is too low to be economic. In the context of subgrid simulations in upscaling, the \( S_{arb} \) concept implies that each coarse block, typically tens of
feet in \( x \) and \( y \) or larger, should have a nonzero \( S_{orb}^* \). In other words, even if the miscible displacement efficiency is 100% and \( S_{orb} = 0 \) at the fine scale, \( S_{orb}^* \neq 0 \) at the coarse scale.

It is important to note that the assumptions of \( \omega = 1 \) and \( S_{orb} = 0 \) at the fine scale will be invalid if the fine grid is itself too coarse. In that case, an appropriate \( \omega (0 \leq \omega < 1) \) should be used. A value of 0.67 is often adopted in practice since Todd and Longstaff (1972) found good agreement with experimental results using it. A nonzero \( S_{orb} \) may also be needed. Again, in our fine-scale simulations, we apply \( \omega = 1 \) under the assumption that key permeability variations are resolved at this scale. If necessary, however, our upscaling methodology can easily account for \( \omega \neq 1 \) and/or \( S_{orb}^* \neq 0 \) for the fine model.

![Figure 2-1 – Averaged solvent fractional-flow curve showing the nonzero \( S_{orb}^* \)](image)

The magnitude of \( S_{orb}^* \) depends on the subgrid heterogeneity of the coarse block and the fluid properties. To illustrate this, consider the averaged fractional-flow curve for a coarse block that is initially full of oil (Figure 2-1). The flatness of the curve when the averaged solvent saturation is beyond approximately 0.6 indicates that the last 40% of the
oil is essentially unrecoverable. Thus, for this case, the practical limit is reached when the averaged solvent fractional flow is equal to 0.98 and $S^*_{orb} = 0.4$.

In our ETLU formulation, instead of using straight-line relative permeabilities (as are appropriate on the fine scale), we will compute the miscible upscaled relative permeabilities ($k^*_r$) using a local upscaling procedure. We incorporate the $S^*_{orb}$ concept into the computation of $k^*_r$ by excluding the oil saturation represented by $S^*_{orb}$ and defining the normalized solvent saturation $(S^*_S)$ as:

$$\langle S^*_S \rangle = \frac{\langle S^*_s \rangle}{1 - S^*_{orb}},$$

(2.8)

where $\langle S^*_s \rangle$ is averaged solvent saturation. We use a fractional-flow formulation (Stone, 1991) to compute the $k^*_r$ for each coarse block as a function of $S^*_S$:

$$k^*_r (\langle S^*_S \rangle) = f^*_j \lambda^*_r \mu^*_{je},$$

(2.9)

where $f^*_j$ and $\mu^*_{je}$ are the averaged fractional flow and coarse-scale effective viscosity for pseudophase $j$, respectively, while $\lambda^*_r$ is the averaged total mobility.

We consider a purely local domain in our upscaling approach, i.e., the calculations are performed on the fine-scale region corresponding to a single coarse block by subjecting the region to flow driven by appropriate boundary conditions. In this study, two kinds of boundary conditions (standard or effective flux) are considered (see Section 2.3). From the local fine-grid simulations, the $f^*_s$ and $\lambda^*_r$ curves are computed as weighted sums of the appropriate fine-block quantities along the outlet boundary of the coarse block. For a 2D ($x$-$z$) problem,

$$f^*_s = \frac{\sum_{i=1}^{m_z} \Delta x_k u_x,-1/2,k \sum_{n=1}^{m_z} \Delta z_k \left( u_x,-1/2,k + u_x,-1/2,k \right)}{\sum_{k=1}^{m_z} \Delta x_k \left( u_x,-1/2,k + u_x,-1/2,k \right)},$$

(2.10)
where \( u_{x,n,s} \), \( u_{o,n,o} \), \( k_{x,n,k} \), and \( \lambda_{x,n,k} \) are the solvent velocity, oil velocity, absolute permeability, and total mobility in the \( x \)-direction of the \( k^{th} \) block at the outlet face, respectively, while \( \Delta z_k \) is the gridblock size in \( z \). The quantity \( \langle S_s \rangle \) is computed either via a porosity-weighted volume average or outlet average of the fine-scale saturations. In their study of immiscible displacements, Wallstrom et al. (2002b) found the outlet-averaging scheme to be appropriate for the computation of \( r_j^* \). We will show in Section 2.4.1 that this scheme in fact leads to more accurate results in the cases we considered.

The quantity \( \mu_{je}^* \) in Eq. 2.9 is computed using Eqs. 2.6 and 2.7, with \( \langle S_m \rangle \) replacing \( S_s \). We do not use \( \omega \) (in Eq. 2.6) as an upscaling parameter because it does not impact the value of \( S_{orb}^* \) and thus cannot be used to model the nonzero \( S_{orb}^* \). This is because the upscaled mobility for pseudophase \( j \), \( k_j^* \langle \langle S_m \rangle \rangle / \mu_{je} \), is independent of \( \omega \) as is evident from Eq. 2.9. We note that values of \( \mu_s \) and \( \mu_o \) need to be estimated for our upscaling, and the ratio \( \mu_o / \mu_s \) used in the upscaling calculations should be close to that encountered in the actual simulation. In our study, even though the fluid viscosities depend on gridblock pressure, \( \mu_o / \mu_s \) is a weak function of pressure and only decreases from 3 to 2 over a 9,000 psi increase in pressure. If the fluid properties are strong functions of pressure, then the upscaling calculations may need to incorporate these functionalities.

The benefit of using the nonzero \( S_{orb}^* \) concept can be seen in Figure 2-2, where the \( k_{ij}^* \) for a typical coarse block are shown. We see that when \( S_{orb}^* = 0 \), the \( k_{ij}^* \) possess a non-monotonic, greater-than-end-point character, a behavior that has been observed by other investigators (Barker and Fayers, 1994; Zhang and Sorbie, 1995). This can pose numerical problems for the simulator and is an unnecessary complication because the oil saturation represented by \( S_{orb}^* \) is immobile and unavailable for mixing. When the \( S_{orb}^* \) concept is
2.2. **EXTENDED TODD-LONGSTAFF (ETLU) FORMULATION**

used, the curves are seen to be much better behaved—$k_{ij}^*$ is reduced over the entire range of averaged solvent saturation $\langle S_s \rangle$. This is because by using $\langle S_m \rangle$ instead of $\langle S_s \rangle$ in Eq. 2.7, lower values of $\mu_m$ are obtained, which in turn lead to smaller values of $\mu_{je}$ and $k_{ij}^*$ (see Eqs. 2.6 and 2.9). Since the oil saturation represented by $S_{\text{orb}}^*$ is unavailable for mixing, to include it in the computation of the mixture fluid properties (e.g., viscosities) is to consider more oil in the mixture than is physical.

![Figure 2-2](image_url) – Upscaled relative permeabilities have better behavior using nonzero $S_{\text{orb}}^*$

In short, we perform our miscible upscaling by first applying local boundary conditions on each coarse block to obtain the $f_j^*$ and $\chi^*$ curves. We then use the ETLU formulation to determine the values of $S_{\text{orb}}^*$, $\langle S_m \rangle$, and $\mu_{je}$ and, finally, compute $k_{ij}^*$. Each coarse block in the coarse-scale simulation thus has one set of $k_{ij}^*$ and an $S_{\text{orb}}^*$ value. If the storage of multiple sets of $k_{ij}^*$ poses memory or CPU issues due to the large number of coarse blocks, one can group $k_{ij}^*$ with similar characteristics into fewer sets using several different criteria (Christie, 1996). However, for all the cases that we have considered in this
work, where coarse models with as many as 10,000 blocks were treated, the use of a
distinct set of $k_{ij}^*$ for every coarse block did not lead to memory or CPU problems.

### 2.3 Effective flux boundary conditions (EFBCs)

The theory of EFBCs was developed by Wallstrom et al. (2002a) to improve the flux-
boundary conditions used in the purely local computation of upscaled relative permeabili-
ties ($k_{ij}^*$). Their application of EFBCs to obtain $k_{ij}^*$ for immiscible displacements produced
accurate coarse-scale results (Wallstrom et al., 2002b). The application of EFBCs for
miscible displacements was first considered by Hui et al. (2004).

By employing EFBCs, the flow field used in the purely local upscaling calculations
includes (approximately) some of the effects of the global flow field. The theory can be
motivated by considering an idealized problem involving an inclusion that is embedded
in an infinite background medium. The permeabilities of both the inclusion and the back-
ground ($k_I$ and $k_B$, respectively) are homogeneous and anisotropic with zero off-diagonal
values, i.e., $k_{ij} = k_{ji} = 0$ if $i \neq j$; $k_{ii} = k_i \neq 0$, $k_{Bi} = k_{Bi} = 0$. For a uniform background
velocity $u_B$, the velocity through the inclusion ($u_I$) can be determined analytically. This
velocity (along the $i^{th}$ axis of the inclusion) is given by the following expression (Wall-
strom et al., 2002a):

$$u_{ii} = R_i \left( \frac{k_{II}}{k_{Bi} + R_i - 1} \right) u_{Bi},$$  \hspace{1cm} (2.12)

where $R_i$ is the asymptotic flux ratio. For a 2D ($x$-$z$) system, the asymptotic flux ratios for
the different coordinate directions, designated $R_x^\prime$ and $R_z^\prime$, are given by:

$$R_x^\prime = 1 + A^\prime,$$  \hspace{1cm} (2.13)

$$R_z^\prime = 1 + A^{-1},$$  \hspace{1cm} (2.14)
where $A'$ is the effective aspect ratio of the inclusion, which is defined as follows:

$$A' = \frac{\lambda_x}{\lambda_z}.$$  \hspace{1cm} (2.15)

and $\lambda_i$ is the correlation length in the $i^{th}$ direction that has been rescaled so that the background permeability is isotropic. Designating $\lambda_i$ and $L_i$ as the dimensional correlation length and domain length in the $i^{th}$ direction respectively, the rescaled correlation and domain lengths are given by:

$$L'_x = L_x, \quad L'_z = L_z \sqrt{\frac{k_{Br}}{k_{Be}}},$$  \hspace{1cm} (2.16)

$$\lambda'_x = \lambda_x, \quad \lambda'_z = \lambda_z \sqrt{\frac{k_{Br}}{k_{Be}}}.$$  \hspace{1cm} (2.17)

For a circular or spherical inclusion ($\lambda_x = \lambda_y = \lambda_z$), $R' = d$, the spatial dimension, for all $i$. Wallstrom et al. (2002b) employed $R' = 2$ for the cross-sectional cases they considered even though $\lambda_x \neq \lambda_z$. This approximation was justified by noting that the $R'$ computed using Eq. 2.13 tends not to be too large ($R'$ is generally less than 7 in Wallstrom et al., 2002b) even for high-correlation-length ($\lambda_z$) systems. This is due in part to the fact that $k_{Br}$, the global effective permeability in the $z$-direction, is typically small compared with $k_{Be}$. This introduces a vertical stretching when the system is rescaled (Eq. 2.17), which means that elliptical inclusions (as might represent high-correlation-length features) become more circular in the rescaled system. In addition, the flux through high permeability regions will be attenuated during upscaling whether the computed $R'_x$ or a value of 2 is used for the asymptotic flux ratio in Eq. 2.12. In the present study, we also use $R'_i = d$. It may be more accurate in some cases if the noncircular (nonspherical in 3D) shape of inclusions is accounted for, and we will discuss this issue further for 3D cases in Section 2.5.
The important implication of Eq. 2.12 is that when the inclusion permeability is small compared with the background permeability ($k_{li}/k_{Bi} \ll 1$),

$$\frac{u_{li}}{u_{Bi}} \rightarrow \frac{k_{li}}{k_{Bi}} \frac{R_i}{R_i - 1},$$

(2.18)

i.e., the velocity through the inclusion scales linearly with $k_{li}$. Conversely, for a large inclusion permeability ($k_{li}/k_{Bi} \gg 1$), the velocity through the inclusion reaches an asymptotic value (i.e., $u_{li}/u_{Bi} \rightarrow R_i$), instead of continuing to increase with $k_{li}$. A plot of Eq. 2.12 for a circular inclusion in a 2D domain (Figure 2-3) demonstrates these behaviors.

![Figure 2-3 – Flux-attenuation behavior of EFBCs for a circular inclusion in a 2D domain](image)

To see the connection between the analytical result in Figure 2-3 and the choice of boundary conditions to use in the purely local upscaling of $k_{ij}^*$, consider a single coarse block and its corresponding local fine-scale region, as shown in Figure 2-4. The inclusion corresponds to any fine block on the boundary of the coarse block, while the background is the global domain surrounding the block. For the purely local computation of the $k_{ij}^*$...
for the coarse block, we need to specify either the pressure- or flux-boundary conditions for the fine blocks along its inlet and outlet edges, as well as saturation along its inlet edge. In the fairly typical case in which a permeability feature spans the coarse block but does not span the global domain, global fine-scale simulation results will display the qualitative trends of the analytical solution in Figure 2-3. Specifically, velocities through high-permeability regions reach some maximum beyond which they no longer increase with increasing local permeability (Christie et al., 2000). EFBCs are designed to capture this phenomenon. This introduces an important consistency between the global flow field and the boundary conditions used for upscaling.

![Figure 2-4 – A local upscaling problem illustrating the principle of flux attenuation in EFBCs](image)

In contrast, the flux-attenuation behavior seen in the analytical solution in Figure 2-3 will not be obtained if standard boundary conditions (constant pressures, such as \( p = 1 \) and \( p = 0 \), on opposite coarse-block boundaries) are used in the \( k^*_i \) upscaling calculations. In this case, the inclusion flux will instead scale close to linearly with \( k_i \) over the entire range of \( k_i/k_{li} \) (see dashed line in Figure 2-3). As a result, the impact of high-permeability streaks is systematically overestimated during upscaling. The resulting elevation of the upscaled relative permeability of the injected phase leads to coarse-scale
simulation predictions that are pessimistically biased. In other words, the use of standard boundary conditions in computing $k_{ij}^*$ may lead to coarse-scale models that predict early breakthrough. Chen (2005) demonstrated that standard boundary conditions also lead to an overestimation of the total flow rate.

In analogy with Eq. 2.12, EFBCs specify fluxes on the fine blocks along the inlet and outlet edges of the coarse-block boundaries. These fluxes account approximately for global effects. To compute $k_{ij}^*$ in the $x$-direction for a 2D ($x$-$z$) problem, the inlet and outlet dimensionless velocities are specified as follows (Wallstrom et al., 2002b):

\[
\begin{align*}
    u_{x,i/2,k} &= \frac{k_{x,i,k}}{k_{x,i,k} + k_{Bx}} , \quad u_{x,n_{x}+1/2,k} = \frac{k_{x,n_{x},k}}{k_{x,n_{x},k} + k_{Bx}}, \\
    u_{z,i/2} &= u_{z,n_{z}+1/2} = 0 ,
\end{align*}
\]  

(2.19)  

(2.20)

where $n_x$ and $n_z$ are the number of fine blocks comprising the coarse block in the $x$- and $z$-directions respectively. The background permeability $k_{Bx}$ is computed from a global single-phase, steady-state solution. In Eq. 2.19, the outlet velocities must in general be rescaled so that the sum of the inlet and outlet fluxes is equal. The saturation is specified to be constant (and equal to 1) along the inlet face.

Since the theory of EFBCs is an approximate way of incorporating nonlocal information into the subgrid model, it will become less applicable as its underlying assumptions are violated. There are a few features existing in actual flows in geological media that can lead to violations of these assumptions, but the most important for our purposes is the presence of permeability features with very large length scales. By considering how the disturbance in the pressure field caused by the inclusion decays with distance, Wallstrom et al. (2002a) obtained a parameter $N_{mf}$ for gauging the applicability of EFBCs:

\[
N_{mf} = \frac{\lambda_{\text{max}}}{L_{\text{min}}} ,
\]  

(2.21)
where, in two dimensions (x-z), \( L_{min} = \min\{ L_x, L_z \} \) and \( \lambda_{max} = \max\{ \lambda_x, \lambda_z \} \) (see Eqs. 2.16 and 2.17). EFBCs are expected to be applicable when \( N_{mf} \leq \sim 1 \). When \( N_{mf} \) is much greater than one, the system acts as though it is layered, and the use of standard local boundary conditions (rather than EFBCs) may be appropriate.

2.4 Results and discussion (2D cases)

We now evaluate the proposed miscible upscaling method for synthetic 2D fields by assessing its performance for varying correlation structure (Section 2.4.1) and coarsening factor (Section 2.4.2). In Section 2.4.3, we analyze the trends of breakthrough-time errors in the context of an interplay between numerical dispersion and heterogeneity effects.

2.4.1 Evaluation for varying correlation structure

In this section, we compare the different boundary conditions and saturation-averaging schemes for generating miscible upscaled relative permeabilities (\( k_{ij}' \)). We accomplish this by testing the different combinations on a set of synthetic fields with varying correlation lengths. Our intent here is to compare the effective flux boundary conditions (EFBCs) against the standard approach (constant pressure on inlet and outlet blocks). Another aspect investigated is the saturation-averaging scheme, which determines the saturation associated with each fractional-flow and total-mobility value for the coarse block. The two schemes considered are volume and outlet averaging, as described in Section 2.3. There are therefore four possible combinations in this assessment (two different boundary specifications, each with two averaging techniques).
Figure 2-5 shows the synthetic permeability (x-z) cross sections used to test the upscaling method. These fine-scale maps are 400 × 100 in dimension, and the physical dimensions in the horizontal (L_x) and vertical (L_z) directions are 1,200 and 1,300 ft, respectively. The fine-scale permeabilities (k_i) vary over six orders of magnitude (in md) and are anisotropic, with k_z = 0.1 k_x. The fields are characterized by two-point geostatistics (Deutsch and Journel, 1998), for which the mean (μ ln) and variance (σ^2 ln) of the natural logarithm of k_x are both 3. A spherical variogram model is used and the correlation structures of the fields are characterized by the dimensionless correlation length l, which for coordinate direction i, is defined as:

\[ l_i = \frac{\lambda_i}{L_i} \]  

(2.22)
Higher values of $l_i$ indicate higher degrees of layering. The fields shown in Figure 2-5 all have the same $l$ in the vertical direction ($l_z = 0.1$), but the horizontal value ($l_x$) ranges from 0.1 (Field A) to 5.0 (Field D).

We simulate a first-contact miscible process where solvent/gas is injected into a model containing oil and immobile water (connate water saturation, $S_{wc} = 0.3$). The wells are completed along the two lateral edges of the models. The oil and solvent viscosities are functions of pressure but are approximately 0.13 and 0.05 cp, respectively, over the pressure range simulated. The solvent injector is set at a constant bottomhole pressure (BHP) of 9,000 psi, while the producer has a constant gas rate of 100 MCF/day with a BHP constraint of 5,000 psi. The gas rate refers to the rate of produced gas at the separator. This consists of injected gas and gas associated with the oil.

We evaluate the effectiveness of using different combinations of boundary conditions and saturation-averaging schemes on the fields shown in Figure 2-5. Note that we always employ our ETLU formulation to generate the upscaled relative permeabilities ($k_{ij}^*$) for the $x$-direction and use the same $k_{ij}^*$ in the $z$-direction. For a few cases (see Appendix A.1), we also generated a set of $k_{ij}^*$ for the $z$-direction and did not observe much difference in the simulation results. We upscale the fine-grid model to a coarse grid of $20 \times 10$, so the model is coarsened by a factor of 200. The coarse grid is uniform and the absolute permeabilities $k^*$ are upscaled using a local procedure with standard (fixed pressure) boundary conditions.
CHAPTER 2. A NEW MISCIBLE UPSCALING TECHNIQUE

Field A ($l_x = 0.1$)

Field B ($l_x = 0.5$)
Figure 2-6 – Effects of using different boundary conditions and saturation-averaging schemes
We compare the oil-production rate versus pore volume injected (Figure 2-6) for the fine- and coarse-grid models. Since the gas-production rate is constant, the oil-production rate and oil-cut curves exhibit the same trends. We see that for $l_x \leq 1.0$ (Fields A through C), the proposed method (the use of EFBCs and outlet saturation-averaging scheme to compute $k^*_r$) clearly produces the coarse-scale solution with the best match to the fine-scale solution in terms of both breakthrough-time prediction and overall level of agreement. For Fields A through C, Figure 2-6 shows that the use of standard boundary conditions to compute $k^*_r$ (with an outlet saturation-averaging scheme) consistently leads to coarse models with early breakthrough-time predictions, as expected (see discussion in Section 2.3).

As discussed in Section 2.3, for layered systems in which high-permeability features are of length scales greater than the system size (i.e., $N_{mf} > 1$; see Eq. 2.21), the flux attenuation introduced by EFBCs is no longer appropriate. This is apparent in the results for $l_x = 5.0$ (Field D). Here, the system is almost completely layered and $N_{mf} \approx 5$. As a result, the use of EFBCs, leads to late breakthrough. In fact, for highly layered fields, the standard approach, which imposes fixed pressure-boundary conditions in the direction of flow, becomes applicable. This explains the closer match given by the standard approach for Field D. We could conceivably mitigate the oversuppression of flux by EFBCs in such fields by means of a generalization of the EFBC concept (Wallstrom et al., 2002b).

In Figure 2-6, we can see that, in general, the use of standard boundary conditions causes an earlier breakthrough as compared with the equivalent case employing EFBCs, as expected. The outlet averaging of saturations also leads to an earlier breakthrough as compared with the equivalent case employing volume averaging. This is because in the latter case, the solvent fractional-flow curve is shifted to the right, as each $k^*_r$ value is associated with a larger solvent saturation value (Figure 2-7). As suggested by Christie et
al. (1995), although the conventional use of a volume-averaging scheme for upscaling relative permeabilities compensates for numerical dispersion, it may not be appropriate in the context of a purely local upscaling approach, as is applied here. The compensation in this case may be excessive since we apply the boundary conditions on a small region of the reservoir (one coarse block) and these boundary conditions are not precisely those encountered in the global simulation. Using an extended local approach entailing buffer blocks around the region may lessen this effect, but this is not being investigated in this work. We also observe a “stair-step” behavior in the solutions using volume averaging (Figure 2-6). The reason is that the solvent saturation needs to build up to a critical level before any flow will occur and this critical saturation is different for each coarse block. When this occurs in a key region of the model, the sudden increase in oil flow brings about a plateau region on the curve.

![Graph](image)

Figure 2-7 – Typical EFBC $k_{ij}^*$ from different saturation-averaging schemes

We conclude that for partially layered fields ($l_x \leq 1.0$), the proposed method (ETLU formulation, EFBCs, and outlet saturation-averaging scheme) performs very well. In
addition to oil-production rates, we also compare the solvent-saturation maps for Field B, for which \( l_s = 0.5 \) (Figure 2-8). It is clear that the coarse-scale saturations generated with the proposed method display many of the essential features existing in the fine-scale solution. For example, the high-flow regions near the top of the fine-scale model that lead to solvent breakthrough are accurately captured by the third layer from the top of the coarse-scale model.

Figure 2-8 – Solvent saturation at breakthrough for Field B; coarse model from proposed upscaling technique

The speedups offered by our upscaling procedure are quite substantial. For the fairly typical case of Field B, the fine-scale simulation required approximately 20 hours of CPU time, while the coarse-scale model ran in only 27 seconds. The EFBC upscaling procedure required 22 minutes of CPU time. Thus, the overall coarse solution was accomplished in approximately 1/60\(^{th}\) of the CPU time of the fine-scale solution.
2.4.2 Evaluation for varying coarsening factor

In this section, we consider a wide range of coarsening factors and quantify our observations by computing the errors in oil cut and breakthrough time for the coarse-scale solutions. We compare the proposed method against other commonly used techniques (e.g., standard $k^*_p$, use of upcaled absolute permeabilities $k^*$ only, and nonuniform coarsening) using fields with different degrees of heterogeneity.

Although a visual inspection is useful as a gauge of the upscaling effectiveness, quantitative measures of the error are more instructive. Further, since many coarsening factors are being considered, it is difficult to compare all solutions on a single plot. To present the results concisely, we employ two parameters to quantify the mismatch in simulation results. The first parameter is the fractional $L_1$-norm of the error in oil cut, given by:

$$
\epsilon_{oc} = \frac{\int_{t_{D,fr}}^{t_{D,fc}} \left| f^c_o (t_D) - f^f_o (t_D) \right| dt_D}{\int_0^{t_{D,fc}} f^f_o (t_D) dt_D}, \quad (2.23)
$$

where $f_o$ is the dimensionless oil cut and $t_D$ is the pore volume injected. The error is thus the area of the absolute difference between the coarse (subscript $c$) and fine (subscript $f$) curves divided by the total area under the fine curve. The final time, $t_{D,fc}$, is chosen to be 1.4 PVI because the curves generally approach $f_o = 0$ at about this time (see Figure 2-6).

The other measure is the fractional error in breakthrough time ($\epsilon_{bt}$) defined as:

$$
\epsilon_{bt} = \frac{|t_{D,fr}^f - t_{D,fr}^c|}{t_{D,fr}^f}, \quad (2.24)
$$

The breakthrough time ($t_{D,fr}$) is defined as the dimensionless time (in PVI) at which the oil cut drops below 0.99.
In the results below, we consider a wide range of coarsening factors. We maintain the grid-aspect ratio, defined as the ratio of the number of blocks in $x$ to that in $y$, constant at 4. Similar results to those presented below were observed for a fixed aspect ratio of 2. The seven coarse-grid models used are $8\times2$, $16\times4$, $20\times5$, $40\times10$, $80\times20$, $100\times25$, and $200\times50$, hence the range of coarsening factors is quite large (4–2500). We simulate a first-contact miscible displacement with well conditions as described in Section 2.4.1. We consider two different fine-grid ($400\times100$) permeability fields: Field B from Figure 2-5 and a second system with much stronger heterogeneities, Field E (Figure 2-9). The permeability statistics for Field E are $\mu_{\ln} = 3$, $\sigma_{\ln}^2 = 5$, $l_x = 0.5$, and $l_z = 0.01$.

We compare the effectiveness of the proposed method against three other upscaling techniques. Thus, there are four methods being studied in this section:

1. Standard $k_{ij}^*$ (with outlet saturation averaging)
2. EFBC $k_{ij}^*$ (with outlet saturation averaging)
3. Upscaled permeabilities ($k^*$) only in a uniformly coarsened grid
4. Upscaled permeabilities ($k^*$) in a nonuniformly coarsened grid (Durlofsky et al., 1997)

Figure 2-9 – Synthetic Field E ($\mu_{\ln} = 3$, $\sigma_{\ln}^2 = 5$, $l_x = 0.5$, $l_z = 0.01$)
We are not fully exhaustive in our evaluation; i.e., although there are seven coarsening factors under consideration, we generate seven coarse models for each field for just two of the methods (EFBC $k_{ij}^*$ and $k^*$ only). We generate three coarse models for the two fields using the methods of standard $k_{ij}^*$ and nonuniform coarsening. Note that the aspect ratio will not be maintained at 4 for nonuniform coarsening because this would severely limit the degree of grid optimization possible.

### 2.4.2.1 Numerical results for Field B ($\sigma_{ln}^2 = 3, l_x = 0.5, l_z = 0.1$)

In Figure 2-10a, we show the curves of oil cut versus pore volume injected for the proposed method for Field B. The trends of the results and the level of agreement with the fine-scale solution are the same when plotted in a dimensionless or a dimensional form. Only three out of the seven coarse solutions (in the intermediate range of coarsening factors) are shown to keep the plots legible. We see that the coarse-scale solutions cluster around the fine-scale solution and that the matches are excellent. The proposed method is thus seen to be very robust with respect to coarsening factor. It is evident that the results from standard $k_{ij}^*$ (Figure 2-10b) and $k^*$ only (Figure 2-10c) are less accurate, being overly pessimistic (early breakthrough) and optimistic (late breakthrough) respectively. The nonuniformly coarsened models predict the breakthrough time accurately but are less accurate than the EFBC $k_{ij}^*$ results at later times (Figure 2-10d).
CHAPTER 2. A NEW MISCIBLE UPSCALING TECHNIQUE

(a) EFBC $k_{ij}^*$

(b) Standard $k_{ij}^*$
Figure 2-10 – Oil-cut curves from various methods (Field B)
Quantitatively, the fractional oil-cut errors from applying EFBC $k_r^*$ are all less than 0.1 and are clearly lower compared with errors from other methods (Figure 2-11). The trend of fractional breakthrough errors has a more erratic behavior, possibly because these are point values as opposed to integrated quantities like the oil-cut errors. One interesting feature is that the error (particularly the error in breakthrough time for the $k^*$ only simulations) does not necessarily decrease with increasing refinement. We will discuss this observation below in terms of the interplay of numerical dispersion and heterogeneity in Section 2.4.3.

For the nonuniformly coarsened models, the number of coarse layers in the $y$-direction more strongly impacts the accuracy; hence it is used as the abscissa in Figure 2-11. Due to this treatment, the comparison between the nonuniform coarsening and the other methods is not as straightforward. For instance, the $32 \times 15$ nonuniform model (480
blocks) is roughly comparable in size to the \(40 \times 10\) uniform model. We see that in terms of fractional oil-cut errors, the nonuniform coarsening method is inferior only to the proposed method and there is marked improvement with greater refinement in \(y\). Breakthrough time is predicted very accurately because the nonuniform model is generated to provide the highest resolution in important high-flow regions (Figure 2-12).

![Fine grid vs Coarse grid](image)

Figure 2-12 – The fine \((400 \times 100)\) and nonuniform coarse \((48 \times 40)\) models of Field B

![EFBC kij](image)

Figure 2-13 – Typical EFBC \(k_{ij}^*\) of one coarse block for two different fields; Field E is more heterogeneous than Field B
2.4.2.2 Numerical results for Field E ($\sigma_{ln}^2 = 5$, $l_x = 0.5$, $l_z = 0.01$)

Because Field E has a more extreme level of heterogeneity (i.e., higher $\sigma_{ln}^2$ and smaller $l_z$), it is more difficult to upscale. In general, the EFBC $k_{ij}$ for this field display a higher degree of nonmonotonicity (Figure 2-13). It is therefore not surprising that the match given by the proposed method (Figure 2-14a) is not as close as that observed in Field B (Figure 2-10a). Nevertheless, it is still clearly superior to the other methods (Figure 2-14b–d). Standard $k_{ij}$ and $k^*$ only give pessimistic and optimistic results respectively as before, but the mismatch is much more pronounced here. The nonuniform coarsening results are also consistent with previous observations: accurate breakthrough-time predictions but subsequent loss of accuracy.

In Figure 2-15, we can see that the error trends are different from those observed in Field B (Figure 2-11). Instead of staying roughly constant throughout the range of coarsening factors, the fractional oil-cut error for EFBC $k_{ij}$ is highest for the coarsest model and decreases with increasing refinement before becoming approximately constant for 40 blocks or more in $x$. This implies that a certain level of resolution is required in order to capture the essential features of such a heterogeneous field. We can discern a similar trend from the fractional breakthrough error plots. In contrast with the results for Field B, for which the breakthrough time predicted by the proposed method was somewhat erratic (Figure 2-11), here it generally decreases and thus moves closer to the fine-scale solution with increasing refinement. In fact, there appears to be general improvement for all methods with increasing refinement. Further, although the oil-cut errors for standard $k_{ij}$ are much higher compared with the proposed method, their breakthrough errors are similar or, in the $40 \times 10$ case, noticeably smaller. This is because the fine-scale model breaks
through quite quickly for this field and the pessimistic bias of the standard $k_{ij}^*$ is fortuitously beneficial.

(a) EFBC $k_{ij}^*$

(b) Standard $k_{ij}^*$
Figure 2-14 – Oil-cut curves from various methods (Field E)
One notable feature of the results presented in Section 2.4.2 is that the error does not necessarily decrease with increasing refinement. In the cases without relative permeability upscaling (e.g., $k^*$ only), we believe that this occurs as a result of the interplay of numerical dispersion, heterogeneity, and displacement physics. The first-order, finite-difference solution of the gas-saturation equation introduces truncation errors that smear sharp saturation fronts (Lantz, 1971; Fanchi, 1983). This numerical dispersion enhances spreading beyond what is physical and artificially disperses injected fluid longitudinally and transversely. Numerical dispersion is proportional to the grid spacing; thus for the coarser models with fewer blocks in the direction of predominant flow ($x$ in this case), the longitudinal numerical dispersion can become important, leading to an earlier break-
through. Since the breakthrough time tends to shift the entire solution, the overall match between the coarse- and fine-scale solutions will also be impacted. The effect of numerical dispersion on simulations of miscible displacements has been widely reported (e.g., Stalkup et al., 1990; Jessen et al., 2002).

Heterogeneity complicates the description given above. Heterogeneity is “lost” when important heterogeneous features like high permeability streaks that lead to early breakthrough are homogenized. Thus, an increasingly delayed breakthrough time would be predicted as the model is progressively coarsened (in the absence of numerical dispersion).

In Figure 2-16, we show the plots of breakthrough time (in days) as a function of number of coarse blocks in the x-direction from the use of $k^*$ upscaling only for Fields B and E. Since the grid-aspect ratio is kept at 4, as we decrease the number of blocks in x, the number of blocks in y also becomes lower. The lower resolution in x increases the numerical-dispersion effect, which leads to an earlier breakthrough. At the same time, the lower resolution in y increases the loss of heterogeneity, which tends to give later breakthrough. Given this competition, we expect to see nonmonotonicity in the breakthrough-time plot. For Field B, we can see that the breakthrough occurs later compared with the fine-scale solution as the model is coarsened from 200 blocks in x to 20 blocks in x. When there are fewer than 20 blocks in x, the effects of numerical dispersion start to dominate and the breakthrough of the coarse model occurs earlier and is closer to that of the fine-scale solution. In a more heterogeneous system like Field E (see Figure 2-16b), the loss of heterogeneity is more important. All coarse models predict a later breakthrough time than the fine-scale solution and numerical dispersion reverses the trend only in the coarsest model (8 blocks in x). This interplay can also be seen in the results from
previous work (Jerauld, 1998). A detailed assessment of this interplay was performed by Sablok and Aziz (2005).

![Graph (a) Field B](image)

![Graph (b) Field E](image)

Figure 2-16 – Breakthrough-time prediction using $k^*$ only
The above description is further complicated when the relative permeabilities are also upscaled. This is because the $k_{ij}^*$ account, to some extent, for the interaction between numerical dispersion and heterogeneity. Nonetheless, this interplay may in part explain the general trends in breakthrough-time error for the EFBC $k_{ij}^*$ results shown in Figures 2-11 and 2-15. We have observed a similar interplay in immiscible displacements.

2.5 Application to 3D systems

The extension of our miscible upscaling methodology to three dimensions is quite straightforward. See Hui et al. (2005a) for a brief description of the 3D formulation. Here, we will investigate the performance of the proposed technique for synthetic 3D fields and also evaluate its robustness with respect to changing well configurations (Section 2.5.1). Multiple permeability realizations will be considered in Section 2.5.2.

Figure 2-17 – Synthetic 3D Field $F$ with uniform fine grid ($50 \times 50 \times 25$)
2.5.1 Evaluation for varying production scenario

In this study, we consider Field F, the 3D fine-scale model shown in Figure 2-17, which consists of a uniform grid of $50 \times 50 \times 25$ blocks. This model is characterized by statistics similar to those of the 2D Field E (Figure 2-9): $\mu_{ln} = 3.0$, $\sigma_{ln}^2 = 5.0$, $l_x = l_y = 0.5$, $l_z = 0.01$. The physical dimensions in the horizontal ($L_x = L_y$) and vertical ($L_z$) dimensions are 500 and 125 ft respectively. The fine-scale permeability field is anisotropic, with $k_z = 0.1 k_x$ and $k_y = k_x$. The fine grid is uniformly upscaled by a factor of 5 in each coordinate direction, giving a $10 \times 10 \times 5$ coarse grid.

We simulate a first-contact miscible displacement in which the solvent is injected at a constant reservoir flow rate (in RB/day) and fluids are produced at the same balanced flow rate subject to a BHP constraint of 4,000 psi. The following production scenarios, involving fully-completed vertical wells (except for the corner-to-corner case), are evaluated:

1. Line-drive displacement: 5 injectors on one side and 5 producers on the opposite side, flow rate for each well specified as 30 RB/day
2. Edge-to-edge displacement: 1 injector at one edge and 1 producer at the diagonally opposite edge, flow rate of 150 RB/day
3. Corner-to-corner displacement: 1 injector completed only in the lower half of one edge of the reservoir while 1 producer completed only in the upper half of the diagonally opposite edge, flow rate of 150 RB/day
4. Five-spot pattern displacement: 1 injector surrounded by 4 producers, injection rate of 120 RB/day

Here we compare our miscible upscaling technique ($EFBC$ $k_r^*$) against the use of standard $k^*$ only, in terms of the match of the reference oil- and gas-production rates (Figures 2-18 through 2-21). For this evaluation, we do not consider the use of standard...
$k^*_r$, which consistently gave inaccurate coarse simulation results (see Section 2.4) for fields that are partially layered, as is Field F.

Figure 2-18 – Simulation flow results for 3D Field F; line-drive displacement

Figure 2-19 – Simulation flow results for 3D Field F; edge-to-edge displacement
We can see that except for the edge-to-edge configuration (Figure 2-19), the coarse simulation results provided by EFBC $k_{ij}^*$ are in reasonably close agreement with the fine-scale solutions. Even for the edge-to-edge case, the results from the use of EFBC $k_{ij}^*$ are clearly more accurate than those from $k^*$ only. The mismatch with EFBC $k_{ij}^*$ is possibly due to the extremely high degree of heterogeneity of Field F. In Section 2.5.2, we will again consider the edge-to-edge production scenario and show that an acceptable level of
accuracy is obtained from the application of EFBC \( k_{ij}^* \) to a less heterogeneous field (lower \( \sigma_{\text{in}}^2 \) and higher \( l_c \)).

Quantitatively speaking (see Figure 2-22, we will discuss the \( R_x' = 5.55 \) results below), it is apparent that the application of EFBC \( k_{ij}^* \) consistently reduces the oil-cut and breakthrough errors (see Eqs. 2.23 and 2.24). For example, the oil-cut error is decreased from 0.30–0.50 (\( k^* \) only) to 0.07–0.25 (EFBC \( k_{ij}^* \)). A greater degree of accuracy could possibly be achieved for these 3D cases by applying a more comprehensive upscaling methodology that involves a nonuniform coarsening procedure and a near-well treatment, as will be described in Chapter 3. The use of directional EFBC \( k_{ij}^* \) may also be beneficial here (see discussion in Appendix A.1).

![Figure 2-22 – Fractional oil-cut and breakthrough-time errors from EFBC \( k_{ij}^* \) and \( k^* \) only for different production scenarios (Field F)](image)

We note that the upscaling calculations, which took 25 CPU minutes, need only be performed once to compute the EFBC \( k_{ij}^* \) for each coarse block in the model. These sets of \( k_{ij}^* \) can then be employed in the simulations of the four different well configurations.
The four fine- and coarse-scale simulations required about 57 hours and 2 minutes of CPU time respectively. As such, for this study of a 3D field, our proposed miscible upscaling technique offers a very substantial overall speedup factor of 126. We thus conclude that reasonable accuracy and significant computational savings continue to be obtained from our miscible upscaling method in three dimensions.

It is evident from Figures 2-18 through 2-21 that the coarse models from the use of EFBC $k_{ij}$ tend to predict breakthrough times that are too late compared with the fine model. This suggests that the flux attenuation from EFBCs during our upscaling may be excessive for this field. For this anisotropic 3D system, for which $k_{Bx} \approx k_{By}$, $k_{Bx}/k_{Bz} \approx 67$ and $\bar{\lambda}_x \approx \bar{\lambda}_y \neq \bar{\lambda}_z$ (Eq. 2.17), the inclusions are spheroids and Wallstrom et al. (2002a) have shown that the asymptotic flux ratio in coordinate direction $i$, $R_i$, can be expressed as a function of the effective aspect ratio $A^i$ (defined in Eq. 2.15). Using their expressions, we find that $R_x = 5.55$ for Field F, as opposed to the value that we employed in our upscaling calculations ($R_x = 3$ for spherical inclusions). By recomputing the $k_{ij}^*$ with $R_x = 5.55$, we obtained simulation results that are closer to the fine-scale solutions compared with those using $R_x = 3$, as can be seen from the error plots in Figure 2-22. Although not a component of this work, a more comprehensive study of the effects of $R_i$ should be performed.

2.5.2 Evaluation for multiple realizations

We next evaluate the performance of our miscible upscaling technique for multiple (5) realizations of another synthetic 3D field, Field G in Figure 2-23. The fine-scale grid comprises $55 \times 55 \times 25$ blocks and has the following statistics: $\mu_{ln} = \sigma_{ln}^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$. The physical dimensions in the horizontal ($L_x = L_y$) and vertical ($L_z$) dimen-
sions are 550 and 125 ft respectively. The fine-scale permeability field is once again anisotropic, with $k_z = 0.1 k_x$ and $k_y = k_x$. The fine grid is uniformly upscaled by a factor of 5 in each coordinate direction, giving an $11 \times 11 \times 5$ coarse grid.

![Permeability field image](image)

$k: \begin{array}{c} 0.01 \\ 1000 \end{array}$

Figure 2-23 – One realization of synthetic 3D Field G with uniform fine grid

$(55 \times 55 \times 25)$

Here, we investigate the same edge-to-edge well configuration described in Section 2.5.1, in which solvent is injected from a fully-completed vertical well in one edge to a producer in the diagonally opposite edge. The balanced reservoir flow rate is 160 RB/day in this case. As in Section 2.5.1, our miscible upscaling technique (EFBC $k^*_i$) is compared against the use of standard $k^*$ only. We show the typical simulation curves for one permeability field realization in Figure 2-24. For both upscaling methods, the errors are smaller compared with those observed for Field F (Figure 2-19), presumably due to the lower degree of heterogeneity (lower $\sigma^2_{\ln s}$ and higher $l_z$) of Field G. Nevertheless, the
reduction in oil-cut and breakthrough errors from our method is still quite significant (Figure 2-25), e.g., the oil-cut error is decreased from 0.09–0.17 ($k^*$ only) to 0.01–0.09 (EFBC $k_{rj}^*$).

Figure 2-24 – Simulation results for a realization of 3D Field G; edge-to-edge displacement

Figure 2-25 – Fractional oil-cut and breakthrough-time errors from EFBC $k_{rj}^*$ and $k^*$ only for different realizations of Field G
2.6 Concluding remarks

A new miscible upscaling technique that enables accurate coarse-scale simulations of first-contact miscible displacements was presented. The method comprises two components: EFBCs and the ETLU formulation. The former leads to a more accurate breakthrough-time prediction by incorporating approximate global flow information into the local upscaling computations of upscaled relative permeabilities \( k^*_{ij} \), while the latter efficiently models immobile and unmixed oil \( S^*_{orb} \) to generate better-behaved \( k^*_{ij} \).

The miscible upscaling technique was evaluated on a sequence of synthetic 2D permeability fields with different correlation lengths \( l_x: 0.1–5.0 \). For partially layered systems \( (l_x \leq 1.0) \), the upscaling technique yielded a close match to the fine-scale simulation results for breakthrough time, oil-production curves, and saturation profiles. EFBCs become less applicable compared with standard boundary conditions as the permeability field becomes more layered, consistent with the theory underlying EFBCs. From this evaluation, we also conclude that it is more appropriate to employ outlet rather than volume averaging for saturation in the purely local computation of \( k^*_{ij} \).

By computing fractional oil-cut and breakthrough-time errors, we demonstrated that our proposed method consistently outperformed other techniques such as standard \( k^*_{ij} \), \( k^* \) only, and nonuniform coarsening over a wide range of coarsening factors (4–2500) for fields with different heterogeneity structures. An interesting interplay of numerical dispersion and heterogeneity effects was seen in the trends of breakthrough-time errors. The method was finally applied to synthetic 3D fields, and by investigating multiple production scenarios and permeability realizations, the use of EFBC \( k^*_{ij} \) was shown to consistently provide substantially more accurate results compared with the use of \( k^* \) only. For the synthetic 2D and 3D cases considered, significant speedup factors (60–126) were
provided by the proposed upscaling technique even though the upscaling computations have not been fully optimized.
Chapter 3
Field application of miscible upscaling technique

3.1 Introduction

In Chapter 2, we described a novel miscible upscaling technique, incorporating the use of effective flux boundary conditions and an extended Todd-Longstaff treatment of upscaled relative permeabilities ($r_j^k$). The technique was systematically evaluated using synthetic 2D and 3D models and was shown to be more accurate than other methods for various heterogeneity structures, correlation lengths, and coarsening factors as well as multiple production scenarios and permeability realizations.

In this chapter, we will present the results from the application of our miscible upscaling technique to the miscible gas injection study of a real field. Here, the overall upscaling procedure is extended to nonuniform 3D grids. A near-well upscaling procedure is also incorporated into the methodology. It is shown that the original fine-grid model must be refined areally to achieve numerical convergence. This refinement is easily (and efficiently) introduced into the local problems solved to determine the $r_j^k$. By considering
various realistic miscible displacement scenarios, we will demonstrate the practical applicability of our upscaling procedure for providing accurate simulation results and large speedup factors.

This chapter proceeds as follows. We will first give an overview of the miscible gas injection field problem and discuss our overall upscaling strategy in Section 3.2. In Section 3.3, we describe the observations from a grid refinement study for a cross section of the field. Next, we will present the results from the application of our upscaling technique to the field study. Here, miscible displacements similar to those planned in the field are simulated on 2D cross sections (Sections 3.4 and 3.5) as well as 3D sectors (Sections 3.6 and 3.7) extracted from the field model. The coarse-scale simulation predictions from various upscaled models will be compared with the converged, fine-scale reference solutions and the computational effort required by our upscaling technique will also be evaluated. Again, we used either a LINUX machine with a Pentium4 1.5 GHz CPU or an IBM machine with a Power4 1.3 GHz CPU for these simulations and upscaling computations.

Parts of the work presented in this chapter have been published (Hui et al., 2005a), though additional results and discussion are included here. We further note that in earlier work (Hui et al., 2004), we studied a related field problem involving a highly heterogeneous cross section. Results from this previous study are included in Appendix A.2.

### 3.2 Description of field problem

The reservoir under study is a large and deep isolated carbonate platform exhibiting a high degree of heterogeneity. The geocellular model for the 3D reservoir is of dimension $98 \times 104 \times 858$, giving rise to a model with almost 9 million blocks (Figure 3-1). The areal gridding is uniform and rather coarse (blocks of size $820 \text{ ft} \times 820 \text{ ft}$), while verti-
cally the gridding is nonuniform, with the layer thicknesses typically 1 foot or less. The reservoir oil and its associated gas are first-contact miscible at reservoir conditions. The focus of the field study is the optimization of the miscible gas injection for this field, which requires a robust upscaling method to enable the fast and accurate modeling of the miscible process.

The fine model is upscaled to a coarse model with $98 \times 104 \times 27$ blocks via the non-uniform coarsening approach (Durlofsky et al., 1997). The coarse-grid structure was determined by Chevron geologists and reservoir engineers. Note that the model is not upscaled areally. We use the extended local or border-region method (Wen et al., 2003) for the calculation of the upscaled absolute permeability $k^*$. By including a border region around each coarse block during permeability upscaling, the method in many cases provides $k^*$ that are more accurate than $k^*$ from a purely local approach.

![Geocellular model of the real field (98 × 104 × 858); vertically exaggerated by a factor of 5](image)

Within the context of this study, it is impractical to consider the field model in its entirety. We therefore extract smaller fine-scale models (2D cross sections as well as 3D...
sectors) for our scale-up validation studies. We evaluate the performance of the miscible upscaling technique by comparing the coarse-scale simulation results against the reference fine-scale solutions. In this assessment, various realistic miscible displacements are considered. However, the numerical values of the production parameters (rates, bottom-hole pressures) should not be viewed as exactly those actually employed in field operations. In some cases, we report normalized (rather than actual) results as some of the actual values are proprietary.

We will simulate the first-contact miscible displacements of various 2D cross sections as well as 3D sectors. The following upscaling methods are compared:

1. Permeability ($k^*$) upscaling only using the border-region method (Wen et al., 2003)
2. Our miscible upscaling technique (EFBC $k_{ij}^*$ and border-region $k^*$) in conjunction with a near-well treatment

Two different near-well treatments are investigated in this work. In Sections 3.4, 3.6, and 3.7, a near-well, single-phase (NW1P) upscaling procedure (Mascarenhas and Durlofsky, 2000; this technique will be discussed in detail in Chapter 4) is employed while a local grid refinement (LGR) around the well is introduced in Section 3.5. Before the results from the scale-up studies are presented, we will first describe a grid refinement study, which indicates that the fine-scale models need to be areally refined to yield numerically converged solutions.

3.3 Areal grid refinement study for a cross section

As the fine-scale field model (Figure 3-1) is rather coarse areally ($\Delta x = \Delta y = 820$ ft) but much finer vertically ($\Delta z \sim 1$ ft), there is a concern that the extreme grid-aspect ratio may introduce numerical effects that render the fine-scale simulation results inappropriate as
3.3. AREAL GRID REFINEMENT STUDY FOR A CROSS SECTION

the reference solutions. We therefore perform a grid refinement study to assess the convergence behavior of the fine-scale model with increasing areal refinement.

We consider the $10 \times 858$ cross-sectional model shown in Figure 3-2, which was extracted from the field model. We refine the model by a factor of 3, 5, and 9 in the $x$-direction, giving rise to refined fine models of $30 \times 858$, $50 \times 858$, and $90 \times 858$ respectively. Note that these models are identical from a heterogeneity standpoint because the permeabilities of the $10 \times 858$ model are simply projected onto the finer grids during refinement; i.e., no new permeability information is introduced.

Figure 3-2 – Fine-scale cross section ($10 \times 858$) extracted from the field model

We simulate a line-drive miscible displacement, with a vertical solvent injector on the right lateral edge of the cross section and a vertical producer on the left edge. We inject at a constant rate (5000 RB/day) and produce at the same balanced rate subject to a bottom-hole pressure (BHP) constraint. The solvent is first-contact miscible with the reservoir oil and the initial water saturation is at connate level ($S_{wc} = 0.1$).

The simulation results for a period of 70 years are shown in Figure 3-3. We use the dimensionless pore volume injected (PVI) as the abscissa. We can see that the fine-scale solutions rapidly converge beyond a refinement factor of 3 to 5. Although the oil-production rates and gas/oil ratios from the various fine models are close to one another, the average reservoir pressures, gas-injection rates, and well BHPs of the unrefined model are quite different from the converged solutions. These mismatches may be due to errors in the resolution of the near-well pressure profiles.
3.3. AREAL GRID REFINEMENT STUDY FOR A CROSS SECTION

Figure 3-3 – Areal grid refinement results for the $10 \times 858$ model shown in Figure 3-2
The well index $WI$ (given by Eq. 4.2 for a vertical well) couples the well to the well-block according to the relation described in Eq. 4.1. For a given specified flow rate $q$, the larger $\Delta x$ for the unrefined model therefore leads to a lower $WI$ and a higher $\Delta p$. At the start of the simulation, the injector wellblock pressure is the same for all models. For the unrefined model, given the higher $\Delta p$ needed to satisfy $q$, the injector BHP will be correspondingly higher. To obtain the match of the well variables (e.g., BHP), we have determined that it is necessary to either impose a local grid refinement near the wells or apply a near-well upscaling procedure (Mascarenhas and Durlofsky, 2000) on the wells. In subsequent sections, we will demonstrate that both approaches improve the prediction of well variables in our scale-up validation study results.

Another important complication is that fluid properties (viscosities, densities, and formation volume factors) are all functions of block pressures. Thus, once there are differences in wellblock pressures, the discrepancies in the flow responses of the models may grow with time. For example, the fact that the surface gas injection rates (in SCF/day) are so different (Figure 3-3), despite the identical reservoir rates (in RB/day) for all models, is due to the dependence of the solvent formation volume factor on pressure. In addition, we see that the average reservoir pressure, which is often considered to be reflective of the total mass of fluids within the system, differs substantially among the different models. This may be due, at least in part, to differences in reservoir fluid densities, which lead to different mass flowrates at the wells for the various models. Thus, at a particular time, the total mass of fluids within the various models can be quite different.

We observe a similar albeit smaller effect of areal grid refinement for an oil-water displacement in the same field (see Appendix B.1), indicating that the trends are also observed in immiscible cases but to a lesser degree. We also observe similar areal grid refinement trends in 3D cases, which will be described in Section 3.6. In essence, we
believe that the grid refinement behavior is attributed to a complicated interplay of numerical dispersion, near-well effects, miscibility, and pressure dependence of fluid properties. It is important to emphasize that the differences in simulation results among the fine models do not stem from heterogeneity differences but instead are numerical in nature. Based on these results, in our subsequent scale-up validation studies, we always refine the fine models areally by a factor of 3 to 5 (depending on the resultant computational requirements) to obtain the appropriate reference results. These refined models are also used for the calculation of $k^*_ij$.

### 3.4 Cross-sectional model (10 × 858)

For our scale-up validation studies, we first investigate the miscible displacement scenario described in Section 3.3 for the 10 × 858 field model shown in Figure 3-2. We upscale the 50 × 858 areally refined model (which yields essentially converged solutions) to the target 10 × 27 coarse model using the two methods indicated above. We will use the simulation results from the model with the largest refinement factor (90 × 858) from our refinement study in Section 3.3 as the reference solution.

In Figure 3-4, we compare the performance of the upscaling methods in terms of their match of the reference results. We also show the results for the unrefined 10 × 858 model to highlight the surprising observation that the original fine model provides a less accurate reproduction of the reference (refined) solutions than the considerably coarser upscaled model. We see that the combined application of the near-well and miscible upscaling techniques provides a closer match of the reference simulation curves, compared with the original fine model or the use of border-region $k^*$ only. It is noteworthy that our approach provides an almost perfect match of the well BHPs. However, as is evident
from the oil-production rate curves, our proposed approach leads to a slightly premature breakthrough compared with the reference model.
3.4. CROSS-SECTIONAL MODEL (10 × 858)

Figure 3-4 – Simulation results from different methods for the 10 × 858 model shown in Figure 3-2; line-drive displacement

The solvent-saturation maps shown in Figure 3-5 suggest that our miscible upscaling technique correctly predicts the regions where the solvent breakthrough first occurs (the top and near the bottom of the model). However, the solvent fronts move somewhat too
fast in the middle of the upscaled model, bringing about an early arrival in this portion of the reservoir, consistent with the trends of the oil-production rate curves in Figure 3-4.

Figure 3-5 – Solvent-saturation maps just after solvent breakthrough (the refined 90×858 model versus the 10 × 27 model from our miscible upscaling technique)

Besides the accuracy of the methods, it is also useful to compare the computational costs of the different methods. The reference (90 × 858) simulation run time exceeded 33 CPU hours while the miscible upscaling calculations and coarse-scale simulation required 26 minutes and 43 seconds respectively. The overall speedup factor (including upscaling time) is thus 75 in this case. Since the upscaled model can be used in many sensitivity runs (i.e., upscaling is only performed once), it is also instructive to consider the run-time speedup factor (without upscaling time). Here, the run-time speedup factor is nearly 2,800. If we compare against the 50 × 858 model, which was employed for upscaling purposes (simulation run time of 13 hours), the overall speedup factor is lower (30) but still significant. We reiterate that the miscible upscaling code has not been fully optimized, so greater speedups could likely be achieved.
3.5 Cross-sectional model (5 × 858)

The inter-well spacing applied for the 10 × 858 model in Section 3.4 is higher than the 3,300-foot spacing expected to be used in the field. To study a case with a smaller inter-well spacing, here we consider a 5 × 858 cross section (Figure 3-6), which is extracted from the full-field model (Figure 3-1). This model has only 5 blocks in the x-direction and the distance between the vertical injector and producer is 4,125 ft. Such a low areal grid resolution is in fact not uncommon in real field studies.

Figure 3-6 – Cross-sectional (5 × 858) model extracted from the field model in Figure 3-1

We first refine the original 5 × 858 fine model by a factor of 5 areally to a 25 × 858 model. The target coarse model consists of 5 × 27 gridblocks. Due to the coarseness of the model, we introduce narrow columns of blocks (Δx = 20 ft) on both edges of the cross-section for the wells. Permeability is set to that of the adjacent block. This is done to impose a more comparable set of well conditions for the different models. Specifically, we do not want the simulation results to be strongly dominated by the size of the “dead” zones, which are the regions between the wells and the lateral edges of the models that contain oil which is not available for miscible displacement. This treatment, which is essentially a form of local grid refinement (LGR), is important for a clear comparison of models with very few blocks between wells, especially when considering a first-contact
miscible displacement. In this case, in conjunction with our miscible upscaling technique, we employ this LGR treatment instead of the near-well upscaling procedure.
We simulate a miscible displacement with well conditions that are identical to those described in Section 3.3 but here the duration is shorter (45 years). Consistent with our observations for the 10 × 858 model (Figure 3-4), Figure 3-7 shows that the oil-
production rates and gas/oil ratios are not very sensitive to areal grid refinement: the results for the 5 × 858 and 25 × 858 models are quite similar. However, the well BHPs are again quite different. Here, our miscible upscaling technique again yields close matches to all reference simulation curves and provides better accuracy than both the original fine model and the use of border-region $k^*$ only. In this case, our method provides a very accurate reproduction of the well BHPs even without the application of the near-well upscaling procedure. Note that the LGR treatment is applied to all models, so the superior performance of our method is not entirely due to this. In addition, we see that the coarse model from our method captures the essential patterns in the reference saturation map, including the pathway through which injected solvent moves toward the producer (Figure 3-8).

![Solvent-saturation maps](image.png)

Figure 3-8 – Solvent-saturation maps just after solvent breakthrough (the refined 25×858 model versus the 5 × 27 model from our miscible upscaling technique)

Our miscible upscaling technique again offers significant computational savings. While the reference (25 × 858) simulation required 3 CPU hours, the upscaling step consumed 13 minutes and coarse-scale simulation run time was only 24 seconds. The
3.6. **SECTOR MODEL (15 × 15 × 466)**

We next investigate the effectiveness of our miscible upscaling technique in 3D cases. We extract a 15 × 15 × 466 sector model (104,850 blocks) from the full-field model (see Figure 3-9). We impose a five-spot pattern miscible gas injection scenario, with solvent injected at a constant rate while the producers operate at a maximum oil rate subject to a BHP constraint. We set a relatively high solvent-injection rate (100 MMSCF/day) because the model is physically large (12,375 ft × 12,375 ft × 409 ft) and we wish to observe solvent breakthrough during the simulation period (30 years). An areal refinement factor of 3 is applied to the fine-sector model shown in Figure 3-9 to obtain a refined 45 × 45 × 466 model (containing approximately 10^6 blocks).

As in Section 3.4, we compare the performance of our proposed upscaling approach (EFBC $k^*_r$, border-region $k^*$, and NW1P procedure) with those of the original fine model and the use of border-region $k^*$ only. We see from Figure 3-10 that, although the gas/oil ratio and gas-production-rate curves for the original (15 × 15 × 466) and refined (45 × 45 × 466) fine models are fairly similar, the normalized oil-production rates and average reservoir pressures are rather different. Hence, the areal grid refinement trends observed in the 2D cases in Section 3.3 appear to hold for 3D cases too. The use of border-region $k^*$ only produces a relatively poor match of the reference solutions. The use of our miscible upscaling technique in conjunction with the near-well upscaling procedure provides the best match of the reference solutions. We again observe that this solution outperforms overall and run-time speedup factors are therefore 13 and 432 respectively. These numbers are lower than those obtained for the 10 × 858 model in Section 3.4 because of the smaller grid sizes here.
the original \((15 \times 15 \times 466)\) fine model in terms of oil-production rate and average reservoir pressure. This superior performance is due in part to the areal grid refinement incorporated into our miscible upscaling calculations.

Figure 3-9 – A 3D \((15 \times 15 \times 466)\) sector model extracted from the full-field model in Figure 3-1

In terms of computational requirements, the reference \((45 \times 45 \times 466)\) simulation required 103 CPU hours, in addition to the effort invested in performing multiple restarts. The miscible upscaling calculations required 84 minutes, while the coarse-scale simulation consumed only 14 seconds, so the overall and run-time speedup factors are 74 and nearly 27,000 respectively.
3.6. SECTOR MODEL (15 × 15 × 466)

**Normalized oil-production rate**

- fine (45 x 45 x 466)
- fine (15 x 15 x 466)
- border k*
- EFBC krj* + NW1P

**Gas/oil ratio (MSCF/STB)**

- fine (45 x 45 x 466)
- fine (15 x 15 x 466)
- border k*
- EFBC krj* + NW1P
Figure 3-10 – Simulation results from different methods for the 15 × 15 × 466 sector model in Figure 3-9; five-spot pattern displacement
3.7 Sector model \((7 \times 7 \times 180)\)

We now consider a different \(7 \times 7 \times 180\) sector model (Figure 3-11) extracted from the full-field model shown in Figure 3-1. We impose a five-spot pattern with a solvent injector set at a constant BHP and 4 producers, each set at a maximum oil rate subject to a BHP constraint. Consistent with common field practice, we further constrain the producers to have a maximum gas/oil ratio, beyond which the wells will be shut in. The inter-well spacing in this case is 3,498 ft and we run the simulation for a duration of 20 years. We refine the original fine model by a factor of 3 in \(x\) and \(y\) to obtain the reference model \((21 \times 21 \times 180)\).

![Figure 3-11 – A 3D \((7 \times 7 \times 180)\) sector model extracted from the full-field model in Figure 3-1](image)

We upscale the \(21 \times 21 \times 180\) refined model to a \(7 \times 7 \times 7\) coarse model and compare the different upscaling methods (Figure 3-12). Consistent with the areal grid refinement trends observed in previous cases, we again see that the simulation results from the original \((7 \times 7 \times 180)\) fine model differ somewhat from the refined \((21 \times 21 \times 180)\) fine-scale solutions. The use of border-region \(k^*\) only produces a relatively poor match of the reference solution. This is in part because the time at which each producer is to be shut in (due
to excessive gas production) is not predicted correctly. We again obtain the closest match of the reference curves through the use of our miscible upscaling technique (EFBC \( k_{rj}^* \) and border-region \( k^* \)) in conjunction with the near-well upscaling procedure, though the accuracy is not quite as high in this case as in the previous example.
3.7. SECTOR MODEL (7 × 7 × 180)

Figure 3-12 – Simulation results for the 7 × 7 × 180 sector model in Figure 3-11; five-spot pattern displacement

Our proposed approach again offers significant speedup factors. The reference (21 × 21 × 180) simulation required a total of 30 CPU hours. In addition, a number of restarts were necessary to resolve numerical difficulties. The miscible upscaling calculations
required 83 minutes and the coarse-scale simulation run time was only 14 seconds. The overall and run-time speedup factors are thus 21 and about 7600 respectively.

3.8 Concluding remarks

In this chapter, the miscible upscaling procedure described in Chapter 2 was applied to a real miscible gas injection field study. The geocellular model of the field has a nonuniform, 3D grid that consists of close to 9 million gridblocks. Based on a grid refinement study, it was found that the original fine-grid model needed to be refined areally by a factor of 3 to 5 to achieve numerical convergence. This refinement was therefore introduced into our miscible upscaling procedure. In addition, it was determined that a near-well treatment, in the form of either a near-well, single-phase upscaling procedure or a local grid refinement around wells, was beneficial in improving the match in well variables such as BHP.

Various realistic miscible gas injection scenarios were simulated on 2D cross-sections and 3D sectors extracted from the full-field model. The performance of the proposed approach (miscible upscaling technique in conjunction with a near-well treatment) was shown to produce a very close match of the reference (refined) fine-scale solutions and outperformed the use of border-region $k^*$ only and even the original (unrefined) fine models. At the same time, our proposed approach provided significant computational savings, offering overall speedup factors of one to two orders of magnitude (13–75) for the cases considered, even though the upscaling calculations themselves have not been fully optimized for computational efficiency. We thus conclude that the miscible upscaling approach developed in this work is quite useful for practical field studies of miscible displacements.
Chapter 4
Near-well upscaling for high-mobility-ratio displacements

4.1 Introduction

In this chapter, a new two-phase upscaling approach for modeling well-driven, high-mobility-ratio displacements is developed and applied. For the near-well region, we extend the near-well, single-phase (NW1P) upscaling formulation proposed by Mascarenhas and Durlofsky (2000) to include a new near-well, two-phase (NW2P) upscaling procedure. By considering the well-driven flow problem of an extended local well model, the NW1P procedure computes the upscaled well indices ($W_i^*$) and transmissibilities ($T_{w_i}^*$) for all coarse wellblocks such that the steady-state, single-phase flow rates of the coarse- and fine-scale models are matched. Analogous in concept to the single-phase approach, the two-phase technique forces the agreement of oil and water flow rates between the coarse and fine grids for a well-driven flow problem by optimizing the upscaled relative permeabilities ($k_r^*$) for each coarse wellblock. These two procedures constitute the proposed near-well upscaling treatment, which is the focus of this chapter.
In some cases, the two-phase flow phenomena in regions away from wells must also be upscaled to achieve global accuracy. For this upscaling, the effective flux boundary conditions or EFBCs (Wallstrom et al., 2002a) are applied to compute the purely local $k_{ij}^*$ for coarse blocks in which no well is completed. The use of EFBCs in miscible displacements was described in Chapter 2 and further demonstrated in a field study in Chapter 3. We will show that these three upscaling components (NW1P, NW2P, and EFBC $k_{ij}^*$) form a sequence of methods that provide increasing levels of accuracy in the global simulation results.

Consistent with previous observations within the context of upscaled relative permeabilities (e.g., Hewett et al., 1998), we find that the optimized $k_{ij}^*$ from the NW2P procedure tend to exhibit a strongly nonmonotonic behavior. We formulate a semianalytical, subgrid well model based on the radial Buckley-Leverett solution (Buckley and Leverett, 1942) to diagnose the numerical reasons behind this behavior. We will show that by isolating the mobility effects of the near-well problem, which we capture by a mobility correction function $m \left( \overline{S_w} \right)$, the nonmonotonicity of the optimized well $k_{ij}^*$ is essentially eliminated.

This chapter proceeds as follows. In Section 4.2, we describe the NW1P procedure to determine the $W^*$ and $T^*$ procedures. We next present the NW2P procedure for the computation of upscaled well $k_{ij}^*$ in Section 4.3. We then briefly review the use of EFBCs to compute $k_{ij}^*$ in immiscible displacements in Section 4.4. In Section 4.5, we introduce our semianalytical well model and show that the model can isolate the effects of mobility and heterogeneity, which are accounted for by $m \left( \overline{S_w} \right)$ and $k_{ij}^*$ respectively. Finally in Section 4.6, we present the results from the application of our upscaling approach to a large number of examples. Here, we investigate multiple realizations of 3D synthetic fields with varying correlation structure and degree of spatial variability, as well as different fluid mobility...
4.2 Near-well, single-phase upscaling (NW1P)

We consider a fine-scale, structured, 3D reservoir model. The fine-scale permeability is highly variable and is considered to be a diagonal tensor \( \mathbf{k} \) with components \( k_x, k_y, \) and \( k_z \). The grid is assumed to be orthogonal and two-point flux approximations are applied on both the fine and coarse grids (i.e., off-diagonal terms in \( \mathbf{k}^* \) are neglected).

Gravitational effects are ignored in the local well model (LWM) calculations, though they are included in the global simulations. At the LWM level, viscous effects dominate, so the impact of gravity is expected to be small. We checked this by including gravity in some of the upscaling calculations and found the upscaled single- and two-phase parameters to be insensitive to gravitational effects. Note that the water-oil density contrast considered in this work is moderate (\( \rho_w/\rho_o = 1.35 \)).

In reservoir simulation, a well is coupled with the simulation gridblock in which it is completed via the well index (\( WI \)). The \( WI \) for a wellblock \( i,j,k \) relates the well rate \( q^w \) to the well bottomhole pressure \( p^w \) (evaluated at the block center) and gridblock pressure \( p_{i,j,k} \):

\[
q^w = WI_{i,j,k} \left( p_{i,j,k} - p^w \right). \tag{4.1}
\]

In other words, it is the flow conductivity parameter that relates the flow rate to a given pressure driving force. The value of \( WI_{i,j,k} \) is an input parameter to the flow simulator and
usually stays unchanged throughout the simulation. If the well rate $q^w$ is specified, then Eq. 4.1 can be used to determine the value of $p^w$ once $p_{i,j,k}$ is determined. Conversely, if $p^w$ is prescribed, then $q^w$ as a function of $p_{i,j,k}$ enters into the mass conservation equation.

We note that $p^w$ is constant and the same for all wellblocks because gravity effects are ignored. The value of $W_I$ is commonly computed using the equation proposed by Peaceman (1983). For a vertical well of radius $r_w$ completed in a gridblock, $W_I$ is given by:

$$W_I = \frac{2\pi \sqrt{k_x k_y \Delta z}}{\ln \left( \frac{r_0}{r_w} \right)},$$

(4.2)

$$r_0 = \frac{0.28 \left[ \left( \frac{k_y}{k_x} \right)^{1/2} \Delta x^2 + \left( \frac{k_x}{k_y} \right)^{1/2} \Delta y^2 \right]^{1/2}}{\left( \frac{k_y}{k_x} \right)^{1/4} + \left( \frac{k_x}{k_y} \right)^{1/4}},$$

(4.3)

where $\Delta x$, $\Delta y$, and $\Delta z$ are the gridblock dimensions.

Upon upscaling, a well that is completed in a particular location in the fine-scale model will be located in the corresponding gridblock in the coarse-scale model. A standard approach is to employ the dimensions and upscaled absolute permeability ($k^*$) of the coarse wellblock in the Peaceman expression (Eq. 4.2) to compute the coarse $W_I$. It has been amply demonstrated (Ding, 1995; Durlofsky et al., 2000) that this approach can lead to significant inaccuracies. One important problem with this approach is the assumption of a linear flow field in the calculation of $k^*$. Moreover, the Peaceman equation is derived based on a homogeneous wellblock and is therefore inappropriate for a coarse wellblock containing significant subgrid heterogeneity.

The near-well, single-phase upscaling (NW1P) procedure employed in this work is based on the work of Mascarenhas and Durlofsky (2000). We begin by considering the single-phase pressure equation for an incompressible fluid:

$$\nabla \cdot \left( \frac{k}{\mu} \nabla p \right) = q,$$

(4.4)
where $\mu$ is the (constant) fluid viscosity, $k$ is the spatially varying permeability tensor, $p$ is the pressure and $q$ is the source/sink term associated with wells (positive for producers).

For the near-well upscaling calculations, the size of the fine-scale computational domain around the well (extracted from the global simulation grid) and the boundary conditions to be applied on the domain must be prescribed. The size of the domain or local well model (LWM) can be quantified by the parameter $r^c$, which defines the local fine-scale region in terms of the number of rings of coarse blocks around the target well-block. Although usually a whole number, $r^c$ can be fractional, e.g., $r^c = 1.5$. When $r^c = 0$, the domain becomes purely local, i.e., only the coarse wellblock is considered. This is a common choice in standard upscaling procedures (Durlofsky, 1991), but may be inappropriate for near-well upscaling. The decomposition of the upscaling problem into a number of smaller sub-global computational domains offers computational efficiency. However, since no information about the flow away from the well is incorporated into the local flow computation, the simulated flow scenario is only an approximation of the actual flow in a global simulation. As $r^c$ increases, so does the domain size and the closer the local flow will be to the actual global flow (assuming other wells are also included in the calculations). At the upper limit, the entire global domain can be considered (Ding, 1995; Zhang et al., 2005), though the computational cost can become excessive. Previous investigations into the impact of domain size in the LWM (Mascarenhas and Durlofsky, 2000; Muggeridge et al., 2002) concluded that the use of $r^c = 1$ generally provides reasonable results. In our near-well upscaling methodology, we therefore employ a local well model with $r^c = 1$.

Unlike standard procedures, which employ linear boundary conditions (e.g., periodic or constant pressure-no flow), a well-driven flow problem is imposed on the LWM by
specifying the wellbore pressure \( p^w \) and a constant pressure at the domain boundaries \( p^b \) with \( p^w > p^b \). We note that although the well is assumed to be an injector, the use of a producer (applying \( p^w < p^b \)) leads to the identical set of upscaled near-well parameters because the flow is incompressible and single phase. In essence, the approach is to extract the extended local well model from the global grid and then impose a well-driven flow problem on the LWM to compute the upscaled near-well parameters, which will be used in the coarse-scale simulation of the global flow problem.

We illustrate our extended local approach in Figure 4-1, which can either represent the areal-section of a vertical well or the cross section of a horizontal well. We assume that the well is always fully completed at the center of the LWM and that the grid is uniform and Cartesian, thus complications like partial penetration, off-center completion and non-Cartesian or nonuniform grids are not addressed here. However, it is not difficult to incorporate these additional complexities into the procedure.

Figure 4-1 – Local well model showing the fine grid (green), coarse grid (blue), and coarse wellblock (orange)
In the NW1P procedure, the incompressible, single-phase, well-driven flow problem (as governed by Eq. 4.4) is first solved at the fine-scale level for the LWM, yielding the pressure at the center of each fine block. If \( i, j, \) and \( k \) are the fine-grid indices in the \( x-, y-, \) and \( z- \) directions of the LWM, then the fine-grid flow rate from block \( i,j,k \) to block \( i+1,j,k \), which we designate \( q_{i+1/2,j,k} \), is given by:

\[
q_{i+1/2,j,k} = T_{i+1/2,j,k} \left( p_{i+1,j,k} - p_{i,j,k} \right),
\]

where \( T_{i+1/2,j,k} \) is the fine-grid transmissibility in the \( x- \) direction. The expressions for flows in the \( y- \) and \( z- \) directions are analogous:

\[
q_{i,j+1/2,k} = T_{i,j+1/2,k} \left( p_{i,j+1,k} - p_{i,j,k} \right),
\]

\[
q_{i,j,k+1/2} = T_{i,j,k+1/2} \left( p_{i,j,k+1} - p_{i,j,k} \right).
\]

Designating \( I, J, \) and \( K \) as the coarse-grid indices in the \( x-, y-, \) and \( z- \) directions, in analogy with Eq. 4.5, we can compute the transmissibility between coarse wellblock \( I,J,K \) and neighboring block \( I+1,J,K \) as follows:

\[
T_{I+1/2,J,K}^* = \frac{\langle q \rangle_{I+1/2,J,K}}{\langle p \rangle_{I,J,K} - \langle p \rangle_{I+1,J,K}},
\]

where \( \langle p \rangle_{I,J,K} \) is the bulk-volume-weighted average of the fine-scale pressures over the region corresponding to coarse block \( I,J,K, \) and \( \langle q \rangle_{I+1/2,J,K} \) is the sum of the fine-scale flow rates through the areas corresponding to the interface between the coarse blocks \( I,J,K \) and \( I+1,J,K \). These quantities are computed via the following equations:

\[
\langle p \rangle_{I,J,K} = \frac{\sum_{k=1}^{n_z} \sum_{j=1}^{n_y} \sum_{i=1}^{n_x} V_{i,j,k} p_{i,j,k}}{\sum_{k=1}^{n_z} \sum_{j=1}^{n_y} \sum_{i=1}^{n_x} V_{i,j,k}},
\]

\[
\langle q \rangle_{I+1/2,J,K} = \sum_{k=1}^{n_z} \sum_{j=1}^{n_y} q_{n_z+1/2,j,k}.
\]

where \( n_x, n_y, \) and \( n_z \) are the number of fine gridblocks comprising the coarse wellblock in the \( x-, y-, \) and \( z- \) directions respectively and \( V_{i,j,k} \) is the volume of the fine block \( i,j,k \).
Therefore, $T_{I,J,K}^{w*}$ couples the coarse wellblock $I,J,K$ and its $x$-direction neighbor block $I+1,J,K$. The upscaled transmissibilities in the $y$- and $z$-directions are calculated in an analogous manner.

The number of neighbors that the coarse wellblock has, which determines the number of upscaled transmissibilities, depends on the well configuration. For example, in a 3D coarse model, a vertical well completed in a single block will have 6 neighbors while a horizontal well completed in multiple blocks will have 4 neighbors except at the well head and toe, where it has 5. The inter-block transmissibilities between adjacent coarse wellblocks and between non-well blocks are obtained in this work directly from the upscaled absolute permeabilities ($k^*$). For highly heterogeneous channelized systems (e.g., the system considered by Christie and Blunt, 2001), inter-block transmissibilities can be determined using the more sophisticated local-global transmissibility upscaling approach (Chen, 2005).

In analogy to Eq. 4.1, the upscaled well index, which couples the well to coarse well-block $I,J,K$, can be computed via:

$$W_{I,J,K}^* = \frac{\langle q_w^* \rangle}{\langle p \rangle_{I,J,K} - p_w^*}, \quad (4.11)$$

where $\langle q_w^* \rangle$ is the sum of the well flow rates associated with all fine wellblocks that are located within the coarse wellblock. For a coarse wellblock that consists of $n_{wb}$ fine wellblocks, this integrated quantity is given by:

$$\langle q_w^* \rangle = \sum_{i=1}^{n_{wb}} q_i^w. \quad (4.12)$$

The use of $T^{w*}$ and $W^*_{I,J,K}$ from these averaging procedures (Eqs. 4.8 and 4.11) in coarse-scale simulations has been shown to produce reasonable results in a variety of cases (Durlofsky et al., 2000; Muggeridge et al., 2002). However, these parameters may
not accurately reproduce the fine-grid solution if the averaged pressures used in Eqs. 4.8 and 4.11 differ considerably from the actual coarse-grid pressures computed during the coarse-scale simulation; i.e., if \( p_{l_{1},J,K}^{c} \neq p_{l_{1},J,K}^{c} \), where \( p_{l_{1},J,K}^{c} \) is the pressure of coarse block \( I,J,K \). Significant mismatches in these pressures are more likely to occur for highly heterogeneous permeability fields.

To improve the accuracy of the single-phase upscaling method, the optimization procedure proposed by Mascarenhas and Durlofsky (2000) is implemented to eliminate the mismatch between the fine- and coarse-grid LWM solutions of the single-phase, well-driven flow problem. The objective function, \( E \), of the optimization is defined as the sum of squared residuals or errors:

\[
E(\theta) = \frac{1}{2} \sum_{l=1}^{n_{wb}} \sum_{m=1}^{n_{l}} \left( q_{l_{m}}^{c} - \langle q \rangle_{l_{m}} \right)^{2} = \frac{1}{2} \sum_{k=1}^{N} r_{k}^{2} = \frac{1}{2} \| r(\theta) \|_{2}^{2},
\]

where \( n_{wb} \) is the number of coarse wellblocks, \( n_{l} \) is the number of relevant faces of the \( l^{th} \) coarse wellblock (\( n_{l} \) is typically 4 or 5), \( q_{l_{m}}^{c} \) and \( \langle q \rangle_{l_{m}} \) are the coarse-scale and integrated fine-scale flow rates across the \( m^{th} \) face of the \( l^{th} \) coarse wellblock, \( N = \sum_{l=1}^{n_{wb}} n_{l} \), \( r_{k} \) is the \( k^{th} \) residual and \( r \) is the residual vector. The parameter space, a vector of size \( \sum_{l=1}^{n_{wb}} n_{l} + n_{wb} \), is given by:

\[
\theta = \left[ T_{1,1}^{*} \ldots T_{1,n_{l}}^{*} \ldots T_{n_{wb,1}}^{*} \ldots T_{n_{wb,n_{l}}^{*}} \ldots WI_{1}^{*} \ldots WI_{n_{wb}}^{*} \right]^{T}.
\]

The aim is to find the set of parameters \( \theta^{opt} \) that minimizes \( E \), i.e., \( E(\theta^{opt}) = \min \{ E(\theta) \} \). We use the Levenberg-Marquardt algorithm (Gill et al., 1981; Bishop, 1995), which is a gradient-based method modified from the Gauss-Newton procedure and is specifically designed for least-square problems.

Consider a small change in \( \theta \) from iteration \( l \) to \( l+1 \) and perform a Taylor series expansion to obtain a quadratic approximation of the objective function as follows (Lendaris and Mathia, 1996):
\[ E(\theta_{i+1}) = E(\theta_i) + \Delta \theta^T \cdot g(\theta_i) + \frac{1}{2} \Delta \theta^T \cdot H(\theta_i) \cdot \Delta \theta , \]  \hspace{1cm} (4.15)

where \( g \) and \( H \) are the gradient vector and Hessian matrix evaluated at \( \theta_i \), respectively. These quantities can be expressed as:

\[ g_i = \frac{\partial E}{\partial \theta_i} \bigg|_{\theta_0} = \sum_{k=1}^{N} \left( r_k \frac{\partial r_{k}}{\partial \theta_i} \right) , \]  \hspace{1cm} (4.16)

\[ H_{ij} = \frac{\partial^2 E}{\partial \theta_i \partial \theta_j} \bigg|_{\theta_0} = \sum_{k=1}^{N} \left( r_k \frac{\partial r_{k}}{\partial \theta_i} \frac{\partial r_{k}}{\partial \theta_j} + r_k \frac{\partial^2 r_{k}}{\partial \theta_i \partial \theta_j} \right) . \]  \hspace{1cm} (4.17)

If we further define the Jacobian matrix \( J \) as:

\[ J_{ij} = \frac{\partial r_{i}}{\partial \theta_j} , \]  \hspace{1cm} (4.18)

then the gradient and Hessian can be written as:

\[ g = J^T \cdot r , \]  \hspace{1cm} (4.19)

\[ H = J^T \cdot J + Q , \]  \hspace{1cm} (4.20)

where \( J^T J \) and \( Q \) contain only the first and second derivatives of the Hessian matrix respectively. The Gauss-Newton procedure is based on the argument that the first derivatives tend to be much larger than the second derivatives, and that close to the minimum, the residual is small (Bishop, 1995), thus:

\[ Q_{ij} = \sum_{k=1}^{N} r_k \frac{\partial^2 r_{k}}{\partial \theta_i \partial \theta_j} = 0 . \]  \hspace{1cm} (4.21)

This linearization step gives us the final equation for the Gauss-Newton Hessian matrix:

\[ H_{GN} = J^T \cdot J . \]  \hspace{1cm} (4.22)

Differentiating Eq. 4.15 with respect to \( \Delta \theta \), the local approximation for the gradient is given by (Lendaris and Mathia, 1996):

\[ \nabla E = g + H \cdot \Delta \theta . \]  \hspace{1cm} (4.23)
In order to minimize \( E \), we set \( \nabla E \) to zero, yielding the step length or change in parameters over the Gauss-Newton iteration:

\[
\Delta \theta_{GN} = H^{-1}_{GN} \cdot (-g) = (J^T J)^{-1} \cdot (-J^T \cdot r).
\] (4.24)

The salient point of the Gauss-Newton formulation is that only first derivatives are used (Eq. 4.22), the consequence of which is that the Hessian (when not singular) will be always positive definite, guaranteeing the direction of descent. However, because of the linearization step in Eq. 4.21, the method is strictly applicable near the minimum and often results in too large a step length. To ameliorate this drawback, the Levenberg-Marquardt algorithm modifies Eq. 4.24 and finds the step length as follows:

\[
\Delta \theta_{LM} = (H_{GN} + \beta I)^{-1} \cdot (-g),
\] (4.25)

where \( I \) is the identity matrix. The parameter \( \beta \) represents the amount of “trust” placed in the linear Gauss-Newton approach. As \( \beta \) tends to a small value, the Levenberg-Marquardt method reduces to the Gauss-Newton method. When \( \beta \) is large, the Levenberg-Marquardt method becomes the steepest descent method with step length being inversely proportional to \( \beta \). At the beginning of the optimization, we set \( \beta = 0.1 \) and if the iteration brings us closer to the minimum, the value of \( \beta \) is decreased, otherwise it is increased.

The practical implementation of the algorithm requires the numerical evaluation of \( g \) and \( H_{GN} \) (Eqs. 4.19 and 4.22). We use forward difference approximations for these quantities. We evaluate \( J \), which contains first derivatives (Eq. 4.18), by perturbing each parameter in \( \theta \) (Eq. 4.14) and then performing a coarse-scale simulation to determine the change in residual vector, \( r \). In total, there are \( (\sum_{i=1}^{n_{wb}} n_{i} + n_{wb}) \) parameters to perturb. Since there are usually only a moderate number of coarse wellblocks, which typically have 4 faces each, the number of single-phase, coarse-scale simulations per iteration is not large.
Typically, with a reasonable initial guess for \( \theta \) (based on the averaged parameters from Eqs. 4.8 and 4.11), convergence is reached within 2–4 iterations, so the entire single-phase parametric optimization is achieved rapidly.

### 4.3 Near-well, two-phase upscaling (NW2P)

Although the near-well, single-phase upscaling technique described in Section 4.2 has been demonstrated to work well in a variety of scenarios (Mascarenhas and Durlofsky, 2000), we will show in Section 4.6 that two-phase upscaling is also needed when multi-phase effects are dominant, for instance in a high-mobility-ratio, oil-water or oil-gas displacement. In the near-well region, the displacement is expected to be viscous-dominated and the effects of gravity and capillarity tend to be less important. In this case, the flow of two incompressible phases (referred to here as oil and water) is governed by the following pressure and saturation equations:

\[
\nabla \cdot (u_t) = -q_t, \\
\phi \frac{\partial S_w}{\partial t} + \nabla \cdot (f_w u_t) = -q_w,
\]

where \( \phi \) is porosity, \( t \) is time, \( S_w \) is water saturation, \( q_t \) and \( q_w \) are the total and water source/sink terms, and \( u_t, f_w, \) and \( \lambda_t \) are the Darcy total velocity, fractional flow of water, and total mobility respectively, defined as:

\[
u_t = -k \lambda_t \nabla p, \\ f_w = \frac{k_{rw}/\mu_w}{k_{rw}/\mu_w + k_{ro}/\mu_o}, \\ \lambda_t = \frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o},
\]
where \( k_{rw} \) and \( k_{ro} \) are the water and oil relative permeabilities and \( \mu_w \) and \( \mu_o \) are the water and oil viscosities.

Prior to implementing the two-phase, near-well upscaling technique described in this chapter, two existing dynamic pseudo-relative-permeability upscaling methods were modified to account for well-driven flow and then evaluated. The first technique is the pore volume weighted method (Schlumberger, 2004a), which is based on the Kyte and Berry (1975) formulation. For an immiscible oil-water displacement, we compute the upscaled relative permeability for phase \( j \), designated \( k_{rj}^* \), for each coarse wellblock using the following expression:

\[
\begin{align*}
  k_{rj}^* &= \frac{\mu_j^* \sum_{m=1}^{n_l} \langle q_j \rangle_m \langle \Delta p \rangle_m}{\sum_{m=1}^{n_l} \langle T_m^w \Delta p \rangle_m}, \\
  \langle S_w \rangle &= \frac{\sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} \phi_{i,j,k} V_{i,j,k} S_{w,i,j,k}}{\sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} \phi_{i,j,k} V_{i,j,k}},
\end{align*}
\]

where \( \langle S_w \rangle \) is the pore-volume-weighted averaged water saturation for the wellblock; \( S_{w,i,j,k} \), \( V_{i,j,k} \), and \( \phi_{i,j,k} \) are the water saturation, volume, and porosity of the fine block \( i,j,k \) respectively; \( \mu_j^* \) is the viscosity of phase \( j \) evaluated at the bulk-volume-weighted averaged wellblock pressure \( \langle p \rangle_{i,j,k} \) (see Eq. 4.9); \( n_l \) is the number of wellblock faces; \( \langle q_j \rangle_m \) is the integrated fine-scale flow rate of phase \( j \) across the \( m^{th} \) face of the coarse wellblock; \( T_m^w \) and \( \langle \Delta p \rangle_m \) are the upscaled transmissibility and pressure driving force across the \( m^{th} \) face of the coarse wellblock respectively. For instance, for flow across the positive-\( x \) face, \( T_m^w = T_{i+1/2,j,k}^w \) (see Eq. 4.8) while \( \langle \Delta p \rangle_m = \langle p \rangle_{i,j,k} - \langle p \rangle_{i+1,j,k} \). Note that in the standard pseudo-relative-permeability upscaling approach (Kyte and Berry, 1975), instead of a summation over the coarse wellblock faces in Eq. 4.31, a linear pressure field is imposed and one set of \( k_{rj}^* \) is obtained for each coordinate direction.
The second method considered is the approach proposed by Stone (1991), which is based on a fractional flow/total mobility formulation and avoids the estimation of coarse-grid pressures. Again, we modified the formulation to handle well-driven flow problems. The upscaled relative permeabilities are computed as:

\[
k^*_j \left( \langle S_w \rangle \right) = f^*_j \lambda^*_j \mu^*_j,
\]

(4.33)

where \( f^*_j \) and \( \lambda^*_j \) are the averaged fractional flow and total mobility for the coarse well-block respectively, given by the following expressions:

\[
f^*_j = \frac{\sum_{m=1}^{n_l} \langle q_m \rangle_m}{\sum_{m=1}^{n_l} \langle q_m \rangle_m},
\]

(4.34)

\[
\lambda^*_j = \frac{\sum_{m=1}^{n_l} \langle T \lambda_j \rangle_m}{\sum_{m=1}^{n_l} \langle T \rangle_m},
\]

(4.35)

where \( \langle q_m \rangle_m \), \( \langle T \lambda_j \rangle_m \), and \( \langle T \rangle_m \) are the integrated fine-scale total flow rate, transmissibility-total mobility product, and transmissibility across the \( m^{th} \) face of the coarse wellblock. Other notation is as defined earlier.

We will show in Section 4.6 that these methods do not produce satisfactory results for the near-well, two-phase problems we have studied. As an alternative, we propose using an optimization procedure to obtain the upscaled relative permeabilities \( k^*_j \) that will reproduce the two-phase flow behavior of the fine-scale local well model (LWM). Our near-well, two-phase (NW2P) upscaling procedure is analogous to, and is performed in conjunction with, the NW1P procedure described in Section 4.2.

For our proposed NW2P upscaling technique, we first perform the NW1P procedure to obtain the set of optimized well indices \( W^* \) for the well and transmissibilities \( T^*_w \) for each face of the coarse wellblock (see Eq. 4.14). We next impose a well-driven two-phase flow problem on the fine-scale LWM (Figure 4-1), as governed by Eqs. 4.26 and
4.3. NEAR-WELL, TWO-PHASE UPSCALING (NW2P)

4.27. The following description is for an injection well. The domain is considered to be filled with oil initially and injected water will displace oil toward the boundaries (for an injector LWM with \( p^w > p^b \)). The fine-grid total and water flow rates through the areas that correspond to each face of the coarse wellblock are stored as a function of the average water saturation within the block (\( \langle S_w \rangle \)). The equivalent flow problem is next imposed on the coarse-scale LWM. The two-phase wellblock relative permeabilities (\( k^{*}_{ij} \)) of the coarse model are computed such that for each coarse block saturation \( S^c_w \), the two-phase flow rates of the coarse model match the reference, integrated fine-scale values at the corresponding average saturation \( \langle S_w \rangle \).

The objective function, \( E \), in the two-phase optimization procedure is defined as the following sum of squared residuals (notation similar to that in Eq. 4.13):

\[
E(\theta) = \frac{1}{2} \sum_{t=1}^{n_{r,t}} \left[ \alpha_t \left( \sum_{m=1}^{n_t} q^{t}_{i,m} - \sum_{m=1}^{n_t} \langle q_r \rangle_{im} \right)^2 + \alpha_w \left( \sum_{m=1}^{n_t} q^{t}_{w,m} - \sum_{m=1}^{n_t} \langle q_w \rangle_{im} \right)^2 \right],
\]

where \( q^{t}_{i,m}, \langle q_i \rangle_{im}, q^{c}_{w,m}, \) and \( \langle q_w \rangle_{im} \) are the coarse-scale total flow rate, integrated fine-scale total flow rate, coarse-scale water flow rate, and integrated fine-scale water flow rate across the \( m^{th} \) face of the \( l^{th} \) coarse wellblock respectively, while \( \alpha_t \) and \( \alpha_w \) are the optimization weights for the total and water flow rate residuals (we set both weights to be 0.5). For two-phase flow, the parameter space \( \theta \) is of size \( n^c_{wb} \times 2 \), and is given by:

\[
\theta = \left[ k^{*}_{ro,1} \ldots k^{*}_{ro,n_{wb}} k^{*}_{rw,1} \ldots k^{*}_{rw,n_{wb}} \right]^T.
\]

We again employ the Levenberg-Marquardt algorithm (see Eqs. 4.18 through 4.25) to find the set of parameters \( \theta^{opt} \) that minimizes \( E \). The input relative permeabilities employed in the fine model are used as initial estimates for the optimization. Unlike the single-phase procedure, for which there is a single set of optimized parameters, here we will determine the optimized \( k^{*}_{ij} \) as a function of \( S^c_w \). The fine-scale, two-phase simulation of the LWM provides the reference flow rates (\( \langle q_i \rangle_{im} \) and \( \langle q_w \rangle_{im} \)) and must be run out to a
relatively mature stage of flooding in order to cover a wide range of \( S_w \). During the coarse-scale simulation of the LWM, the optimization algorithm is applied at each time step, producing the set of optimized \( k_{ij}^r \) that force a match between the coarse- and fine-grid total and water flow rates for the entire range of \( S_w \) (\( \equiv \langle S_w \rangle \)). Although numerous coarse-scale simulations are performed during the optimization, the procedure is still not overly demanding because there are relatively few coarse wellblocks and only a few iterations are required at each time step. The fine-scale simulation of the LWM is the most time-consuming step of the overall procedure.

We note that the NW2P treatment described above is applicable to injectors only. We have found that for producers, adequate accuracy is generally achieved through the use of NW1P alone. However, the application of the NW2P procedure to a producer LWM is quite straightforward. The imposed boundary conditions will be different: the LWM is considered to be surrounded by water (\( S_w = 1.0 \) at the domain boundaries), and the water will displace oil toward the producer (\( p_w < p^b \)). Instead of Eq. 4.36, the objective function for the two-phase optimization here is given by:

\[
E(\theta) = \frac{1}{2} \sum_{l=1}^{n_w} \left[ \alpha_c \left( q_{t,l} - \langle q_t \rangle_l \right)^2 + \alpha_w \left( q_{w,l} - \langle q_w \rangle_l \right)^2 \right], \tag{4.38}
\]

where \( q_{t,l} \), \( \langle q_t \rangle_l \), \( q_{w,l} \), and \( \langle q_w \rangle_l \) are respectively the coarse-scale total flow rate, integrated fine-scale total flow rate, coarse-scale water flow rate, and integrated fine-scale water flow rate associated with the \( l^{th} \) coarse production wellblock (see Eq. 4.12). We will show in Section 4.6 that the use of NW2P upscaling of producers has a relatively small impact on the global accuracy for the cases considered.

The formulation of our procedure is quite general in the sense that it is applicable to any objective function as long as the problem is not overdetermined, i.e., the number of parameters should be at least equal to the number of variables we wish to match. For
instance, in Eqs. 4.36 and 4.37, there are \( n_{w}^c \times 2 \) flow rates to match and \( n_{w}^c \times 2 \) optimization parameters. If we seek to match the two-phase flow rates across each face of the wellblock (instead of the sums over the block), then the procedure can be modified through use of the following objective function:

\[
E(\theta) = \frac{1}{2} \sum_{l=1}^{n_{w}^c} \left[ \alpha_f \sum_{m=1}^{n_{f}} \left( q_{l,m}^e - \langle q_{l} \rangle_{lm} \right)^2 + \alpha_n \sum_{m=1}^{n_{n}} \left( q_{l,m}^n - \langle q_{l} \rangle_{lm} \right)^2 \right].
\] (4.39)

The parameter space will now contain one set of directional relative permeabilities for each relevant face of the coarse wellblock:

\[
\theta = \left[ k_{ro,1,x}^*, \ldots, k_{ro,1,z}^*, \ldots, k_{ro,n_{ro}^c,x}^*, \ldots, k_{ro,n_{ro}^c,z}^* \right].
\] (4.40)

The two-phase optimization procedure can then be used to determine the directional \( k_{ro,j}^* \) in Eq. 4.40.

### 4.4 Effective flux boundary conditions

The procedures described in Sections 4.2 and 4.3 represent our near-well upscaling treatment. To obtain global accuracy, two-phase upscaling may also be needed in regions away from wells. For this upscaling, we apply EFBCs to compute the purely local \( k_{ro,j}^* \) for each coarse block. The theory of EFBCs was developed by Wallstrom et al. (2002a) to improve the flux boundary conditions employed in the purely local computation of up-scaled relative permeabilities \( k_{ro,j}^* \). In Chapter 2, we described in detail the use of EFBCs for calculating \( k_{ro,j}^* \) for miscible displacements. Here, we will briefly review the use of EFBCs for immiscible displacements.

When performing purely local upscaling, only the fine-scale region that corresponds to the target coarse block is considered. In order to compute \( k_{ro,j}^* \) for each coarse block, we specify the effective flux boundary conditions at the inlet and outlet faces of the coarse
block, as well as the saturation at the inlet face \( (S_w = 1) \). The upscaled relative permeability for phase \( j (k^*_j) \) in each coordinate direction is computed as a function of \( \langle S_w \rangle \) using a Stone (1991) type fractional flow formulation (see Eq. 4.33). However, we do not employ Eqs. 4.34 and 4.35, which are derived assuming radial flow, to compute the averaged fractional flow and total mobility. Instead, \( f^*_j \) and \( \lambda^*_x \) (in the \( x \)-direction for a 3D problem) are determined using the following expressions:

\[
 f^*_j \left( \langle S_w \rangle \right) = \frac{\langle q_j \rangle_{I+1/2,J,K}}{\langle q_i \rangle_{I+1/2,J,K}}, \quad (4.41)
\]

\[
 \lambda^*_x \left( \langle S_w \rangle \right) = \frac{\langle T \lambda_x \rangle_{I+1/2,J,K}}{\langle T \rangle_{I+1/2,J,K}}, \quad (4.42)
\]

where \( \langle q_j \rangle_{I+1/2,J,K} \), \( \langle q_i \rangle_{I+1/2,J,K} \), \( \langle T \lambda_x \rangle_{I+1/2,J,K} \), and \( \langle T \rangle_{I+1/2,J,K} \) are respectively the integrated fine-scale phase \( j \) flow rate, total flow rate, transmissibility-total mobility product, and transmissibility in the \( x \)-direction across the outlet face of the coarse block (see Eq. 4.10). In this case, \( \langle S_w \rangle \) is computed by averaging the fine-scale saturations over the fine blocks lying on the outlet face of the target coarse block. In this manner, one set of \( k^*_j \) is determined for each coarse block in each coordinate direction.

### 4.5 Semianalytical subgrid radial well model

#### 4.5.1 Introduction

The optimized \( k^*_j \) obtained from the near-well, two-phase upscaling (NW2P) procedure (Section 4.3) often exhibit a nonmonotonic behavior, with relative permeability values larger than the end-point values (e.g., see Figure 4-7). This can pose numerical problems for some simulators. Nonmonotonicity was also observed by previous researchers who computed upscaled \( k^*_j \) (Zhang and Sorbie, 1995; Barker and Dupouy, 1999). In particu-
lar, Hewett et al. (1998) demonstrated this behavior in homogeneous, 1D linear cases and attributed it to coarse-grid mobility effects. They argued that apart from numerical dispersion (Lantz, 1971; Kyte and Berry, 1975), additional effects arise due to the numerical treatment of the fractional flow, saturation, and total mobility on the coarse grid. Specifically, in the commonly adopted finite difference scheme with single-point upstream weighting, the averaging region employed for the fractional flow (local interblock plane) is shifted relative to that for the saturation and total mobility (upstream block).

To understand this phenomenon within the context of near-well upscaling, we formulate a semianalytical subgrid well model that is based on the radial form of the Buckley-Leverett solution for immiscible, two-phase (oil-water) displacements in a homogeneous medium. Although similar to the approach taken by Hewett et al. (1998), our formulation is specifically tailored to address the radial flow pattern near a well. The application of this semianalytical model allows us to isolate (to some extent) the mobility and heterogeneity effects. These are accounted for in the upscaled model by a mobility correction function and the optimized $k_{ij}^*$ respectively. As a result, the optimized $k_{ij}^*$ become monotonic (or nearly so).

### 4.5.2 Theory

Welge et al. (1962) showed that the analytical Buckley-Leverett solution for 1D linear systems could be extended to represent radial flow by a coordinate transformation:

\[
\phi \frac{\partial S_w}{\partial t} + \frac{1}{r} \frac{\partial (ru_w)}{\partial r} = 0 ,
\]

where $t$ is time, $u_w$ is the radial component of the water Darcy velocity, and $r$ is the radial distance. For a vertical well (water injector) that is completed at the center of the radial system, $u_w$ is given by:
where \( q_t^w \) is the total injection flow rate and \( \Delta z \) is the thickness of the medium. Eq. 4.43 can now be written as:

\[
\frac{\phi \pi \Delta z}{q_t^w} \frac{\partial S_{w}}{\partial t} + \frac{1}{2r} \frac{df_w}{dS_w} \frac{\partial S_w}{\partial r} = 0.
\] (4.45)

By defining the following dimensionless variables:

\[
\xi_D = \frac{r^2 - r_w^2}{R^2 - r_w^2},
\]
(4.46)

\[
t_D = \frac{q_t^w t}{\phi \pi \Delta z (R^2 - r_w^2)},
\]
(4.47)

where \( R \) is the domain radius and \( r_w \) is the well radius, Eq. 4.45 can be written as follows:

\[
\frac{\partial S_{w}}{\partial t_D} + \frac{df_w}{dS_w} \frac{\partial S_w}{\partial \xi_D} = 0.
\] (4.48)

By defining another dimensionless variable:

\[
v_w = \frac{\xi_D}{t_D},
\]
(4.49)

we can show that Eq. 4.48 becomes:

\[
\frac{\partial S_{w}}{\partial v_w} \left( -\frac{v_w}{t_D} + \frac{1}{t_D} \frac{df_w}{dS_w} \right) = 0,
\]
(4.50)

the nontrivial solution of which is:

\[
v_w = \frac{df_w}{dS_w}.
\]
(4.51)

Hence the final form of Eq. 4.48 is as follows:

\[
\frac{\partial S_{w}}{\partial t_D} + v_w \frac{\partial S_w}{\partial \xi_D} = 0,
\]
(4.52)

where \( v_w \) is the dimensionless wave velocity:
4.5. SEMIANALYTICAL SUBGRID RADIAL WELL MODEL

\[ v_w = \frac{df_w}{dS_w} = \frac{\xi_D}{t_D} = \frac{(r^2 - r_w^2) \phi \pi \Delta z}{q^w t}. \] (4.53)

The solution to Eq. 4.52 is readily obtained using the method of characteristics (Buckley and Leverett, 1942). For a Riemann problem in which the injection is continuous and initial conditions are uniform (all oil), the solution is a function of the similarity variable \( v_w \) only and individual saturations travel along characteristic curves at a speed equal to \( v_w \).

We consider an injector in a homogeneous local well model (LWM). Our domain of interest is the injection coarse wellblock. We then cast the areally uniform \( (\Delta x^c = \Delta y^c) \) Cartesian grid into a volume-equivalent radial system (Figure 4-2) such that:

\[ \Delta x^c \Delta y^c \Delta z^c = \pi \left( R^2 - r_w^2 \right) \Delta z^c \text{ or } R = \sqrt{\frac{(\Delta x^c)^2}{\pi} + r_w^2}. \] (4.54)

In our investigation, relative permeability curves with a power-law dependence on water saturation are used:

\[ k_{rw} = k_{rw}^0 \left( \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}} \right)^{n_w} = k_{rw}^0 \left( S_{wn} \right)^{n_w}, \] (4.55)

\[ k_{ro} = k_{ro}^0 \left( \frac{1 - S_w - S_{or}}{1 - S_{wc} - S_{or}} \right)^{n_o} = k_{ro}^0 \left( 1 - S_{wn} \right)^{n_o}, \] (4.56)
where $S_{or}$ and $S_{wc}$ are the residual oil and connate water saturations, $k_{rw}^0$ and $k_{ro}^0$ are the water and oil relative permeability end-point values, and $S_{wn}$ is the normalized water saturation defined as follows: $S_{wn} = (S_w - S_{wc})/(1 - S_{wc} - S_{or})$. Since the relative permeabilities only depend on $S_{wn}$, the fractional flow of water (see Eq. 4.29) can be written as a unique function of $S_{wn}$:

$$f_w = \frac{M^0 (S_{wn})^{n_w}}{M^0 (S_{wn})^{n_w} + (1 - S_{wn})^{n_w}}, \quad (4.57)$$

where $M^0$ is the end-point mobility ratio defined as:

$$M^0 = \frac{k_{rw}^0 \mu_o}{\mu_w k_{ro}^0}. \quad (4.58)$$

By differentiating Eq. 4.57, $v_w$ from Eq. 4.53 can be expressed as a function of $S_{wn}$:

$$v_w = \frac{M^0 [n_w (S_{wn})^{n_w-1} (1 - S_{wn})^{n_w} - n_o (S_{wn})^{n_w} (1 - S_{wn})^{n_w-1}]}{(1 - S_w - S_{or}) \left[ M^0 (S_{wn})^{n_w} + (1 - S_{wn})^{n_w} \right]^2}. \quad (4.59)$$

This $v_w(S_{wn})$ relationship in combination with the definitions of $\lambda_t$ and $v_w$ (Eqs. 4.30 and 4.53) provides the $S_w(r, t)$ and $\lambda_t(r, t)$ relationships. In other words, from the Buckley-Leverett solution, the values of $S_w$ and $\lambda_t$ at every location $r$ and time $t$ are known.

We now illustrate the use of this semianalytical model to compute the mobility correction function for a coarse injection wellblock (Figure 4-2). We specify $S_{wc} = S_{or} = 0.0$, $k_{rw} = k_{ro} = 1.0$, $n_w = 1.0$, $n_o = 1.0$ (see Eqs. 4.55 and 4.56), resulting in the straight-line relative permeabilities shown in Figure 4-3. The choice of straight-line input relative permeabilities is made to keep the mathematics simple, as we will need to integrate $S_w$ and $\lambda_t$ with respect to $r$ to find averaged quantities for the coarse wellblock. We set $\mu_o = 0.29$ and $\mu_w = 5.0$, so $M^0 = 17.0$ (Eq. 4.58). The corresponding $f_w(S_w)$ curve is convex,
i.e., without an inflection point; hence no saturation shock occurs and the \( v_w(S_w) \) curve has no discontinuity (Figure 4-4).

![Figure 4-3 – Straight-line relative permeabilities](image)

![Figure 4-4 – Fractional flow and dimensionless wave velocity of water](image)
We next compute the averaged water saturation for the coarse wellblock (at time $t$) as a pore-volume-weighted average (note that porosity is constant within the homogeneous domain):

$$
\overline{S_w} = \frac{\int_0^{\Delta r} \int_0^{2\pi} \int_{r_e}^R S_w(r) \ r \ dr \ d\theta \ dz}{\int_0^{\Delta r} \int_0^{2\pi} \int_{r_e}^R r \ dr \ d\theta \ dz} = \frac{2\int_{r_e}^R S_w(r) \ r \ dr}{(R^2 - r_e^2)}.
$$ \hspace{1cm} (4.60)

In order to find the expression for the averaged total mobility for the coarse wellblock, we first consider the pressure equation in a radial system (Muskat, 1981):

$$
\frac{1}{r} \frac{\partial}{\partial r} \left( rk\lambda_i(r) \frac{\partial p}{\partial r} \right) = 0.
$$ \hspace{1cm} (4.61)

By setting pressure boundary conditions at radii $r_w$ and $R$, we can show that the solution to Eq. 4.61 is given by:

$$
p(r) = p(r_w) + \frac{p(R) - p(r_w)}{\int_{r_w}^R \frac{1}{k\lambda_i(r')} \ dr'},
$$ \hspace{1cm} (4.62)

where $r'$ is a dummy variable. The pressure gradient is given by:

$$
\frac{\partial p}{\partial r} = \frac{p(R) - p(r_w)}{\int_{r_w}^R \frac{1}{k\lambda_i(r')} \ dr'} \left( \frac{1}{rk\lambda_i(r)} \right).
$$ \hspace{1cm} (4.63)

The volumetric total flow rate in the radial direction $q_t$ is:

$$
q_t = -2\pi \Delta z' r \ k\lambda_i(r) \frac{\partial p}{\partial r}.
$$ \hspace{1cm} (4.64)

Substituting Eq. 4.63 into Eq. 4.64, and designating $\Delta p = p(r_w) - p(R)$, we obtain the final expression for $q_t$:

$$
q_t = 2\pi k \Delta z' \left[ \int_{r_e}^R \frac{1}{\lambda_i(r')} \ dr' \right]^{-1} \Delta p.
$$ \hspace{1cm} (4.65)
We now compute an averaged total mobility, $\overline{\lambda}$, for the coarse wellblock. This quantity is constant from $r = r_w$ to $r = R$ (at time $t$) and provides the same $q_t$ for the same $\Delta p$; i.e.,

$$q_t = \frac{2\pi k \Delta \varepsilon \overline{\lambda} \Delta p}{\ln \left( \frac{R}{r_w} \right)}.$$  \hspace{1cm} (4.66)

Comparing Eqs. 4.65 and 4.66, it is evident that $\overline{\lambda}$ is given by:

$$\overline{\lambda} = \frac{\ln \left( \frac{R}{r_w} \right)}{\int_{r_w}^{R} \frac{1}{\lambda_r(r)} \, dr}.$$ \hspace{1cm} (4.67)

If mathematically tractable, the integrals in Eqs. 4.60 and 4.67 are obtained analytically, otherwise they are computed numerically. As the displacement proceeds ($t$ increases), $\overline{\lambda}$ as a function of $S_w$ is obtained.

![Figure 4-5 – Comparison of original and averaged total-mobility curves](image)

The resulting averaged total mobility curve, $\overline{\lambda} \left( \overline{S_w} \right)$, is compared with the original curve, $\lambda_t \left( S_w \right)$ in Figure 4-5. The difference between the two curves represents the mobility correction that is required to account for the coarse-grid effect. We define the correction function, $m$, as follows:
The result for \( m \) is shown in Figure 4-6.

![Figure 4-6](image_url)

Figure 4-6 – Analytically computed mobility correction function \( m(S_w) \) for an injection coarse wellblock in a homogeneous permeability field

Our NW2P optimization procedure (Section 4.3) can now be modified to include the mobility correction function such that the well (injection) flow rate into coarse wellblock \( I,J,K \) and the total flow rate between coarse wellblock \( I,J,K \) and neighboring block \( I+1,J,K \) are respectively given by:

\[
q^w_{I,J,K} = WI_{I,J,K} m\left(S_w^c\right) I,J,K \hat{\lambda}^w I,J,K \left(p^w_{I,J,K} - p^w\right),
\]

\[
q^w_{I+1,J,K} = T_{I+1/2,J,K} m\left(S_w^c\right) I,J,K \hat{\lambda}^w I,J,K \left(p^w_{I,J,K} - p^w_{I+1,J,K}\right),
\]

where the \( WI^w \) and \( T^w \) are obtained from the NW1P procedure (see Eq. 4.14) while the upscaled total mobility \( \hat{\lambda}^w \) is determined from the \( k^w \) (see Eq. 4.30) that are computed to match the reference fine-scale, two-phase flow rates at the coarse block water saturation.
4.5. **SEMIANALYTICAL SUBGRID RADIAL WELL MODEL**

We note that the implicit assumption in using \( m(S^c_w) \) in Eq. 4.70 is that the analytically computed average saturation \( \bar{S}_w \) (Eq. 4.60) directly corresponds to the numerical coarse-scale saturation \( S^c_w \) encountered during the optimization.

In an independent work, Nakashima and Nomura (2004) proposed a related analytical computation of a discretized total mobility function for the well, as follows:

\[
\bar{k}_t \left( \bar{S}_w \right) = \frac{q^w}{WT \Delta p}, \tag{4.71}
\]

where \( \Delta p \) is the analytically integrated pressure driving force. This function was then applied with the upscaled relative permeabilities for the injection wellblock, which were separately obtained under the assumptions of a linear flow field and capillary limit conditions. Our approach differs from that of Nakashima and Nomura (2004) in the way the mobility correction function is determined and also in how the correction is employed in the computation of the final \( k^*_r \).

### 4.5.3 Illustrative examples

We next illustrate the use of this semianalytical subgrid radial formulation in several example problems. To demonstrate the coarse-grid mobility effect in the absence of heterogeneity, we first consider an injector in a homogeneous LWM \( (k = 50 \, \text{mD}) \). The fine- and coarse-scale models contain \( 21 \times 21 \times 1 \) and \( 3 \times 3 \times 1 \) blocks respectively, and the coarse-grid dimensions are \( \Delta x^c = \Delta y^c = 100 \, \text{ft}, \Delta z^c = 10 \, \text{ft} \), giving an equivalent radius \( R \approx 56.4 \, \text{ft} \) (Eq. 4.54). We employ the straight-line relative permeabilities (Figure 4-3) and fluid properties described above, so the \( m(\bar{S}_w) \) curve shown in Figure 4-6 is applicable. Water is injected at the center of the LWM, displacing oil outward at an unfavorable mobility ratio.
We first compute the optimized well $k_{ij}^*$ that match the fine-grid, two-phase flow rates for this homogeneous problem via our NW2P procedure. We find that the $k_{ij}^*$ indeed exhibit a highly nonmonotonic behavior, with values of $k_{ro}^*$ significantly exceeding unity (Figure 4-7). The NW2P procedure is next applied in conjunction with the use of the $m\left(S_w\right)$ (as defined in Eqs. 4.69 and 4.70). Figure 4-7 shows that the resulting optimized well $k_{ij}^*$ are very nearly monotonic and are quite close to the input curves, as would be expected for this homogeneous case. The optimized $k_{ij}^*$ do not exactly reproduce the input $k_{ij}$ due to numerical dispersion and because some inconsistency exists between the numerical fine-grid simulation (which is used as the reference for our optimization) and the analytical solution from which we derive our mobility correction function, i.e., $\bar{S}_w$ (Eq. 4.60) may be different from $\left\langle S_w \right\rangle$ (Eq. 4.32). Nonetheless, it is clear from the results of Figure 4-7 that the nonmonotonicity of $k_{ij}^*$ is largely due to the coarse-grid mobility effects, which are effectively captured through the use of $m\left(S_w\right)$.

![Figure 4-7](image)

Figure 4-7 – Effects of analytically computed $m\left(S_w\right)$ (Figure 4-6) on the optimized $k_{ij}^*$ from the NW2P procedure (homogeneous permeability field)
Figure 4-8 – LWM of a binary channel reservoir with water injector at center; $k_{\text{high}} = 50 \text{ mD}$ (yellow), $k_{\text{low}} = 5 \text{ mD}$ (green)

Figure 4-9– Effects of analytically computed $m\left(\overline{S_w}\right)$ (Figure 4-6) on the optimized $k_{ij}^*$ from the NW2P procedure (heterogeneous permeability field)

We next test our mobility correction approach in the presence of heterogeneity, by considering the injector LWM of a channel reservoir with a binary permeability field (Figure 4-8). This case is identical to the homogeneous system described above except for the permeability field. As such, the $m\left(\overline{S_w}\right)$ curve shown in Figure 4-6 is applicable.
We again apply the NW2P procedure and compare the optimized $k_{ij}^*$ obtained with and without the use of the mobility correction function $m\left(\overline{S_w}\right)$. From Figure 4-9, we see that the nonmonotonic behavior is again considerably mitigated through the use of $m\left(\overline{S_w}\right)$. At the same time, it appears that by employing $m\left(\overline{S_w}\right)$, the $k_{ij}^*$ no longer exceed unity. In this case, in addition to the effects discussed above, the optimized $k_{ij}^*$ also account for heterogeneity effects.

We conclude from these examples that our semianalytical, subgrid radial model offers a diagnosis of the numerical issues behind the nonmonotonic behavior of the optimized $k_{ij}^*$ from our NW2P procedure. In addition, the use of the semianalytical model allows us to isolate the coarse-grid mobility and heterogeneity effects, which are captured by $m\left(\overline{S_w}\right)$ and the optimized $k_{ij}^*$ respectively (to a first approximation).

![Figure 4-10](image-url)  

**Figure 4-10** – Numerically computed mobility correction function, $m\left(\overline{S_w}\right)$ for an injection coarse wellblock in a heterogeneous permeability field
Figure 4-11 – Effects of numerically computed $m\left(S^c_w\right)$ (Figure 4-10) on the optimized $k^*_j$ from the NW2P procedure (heterogeneous permeability field).

Although our test on a heterogeneous field produced promising results (Figure 4-9), it may be useful to extend the methodology to cases where the assumptions for the analytical Buckley-Leverett solution are grossly violated, e.g., a highly heterogeneous field or nonuniform initial conditions. In that case, we can compute the mobility correction via a purely numerical approach as follows:

$$m\left(S^c_w\right) = m\left(\langle S^c_w \rangle \right) = \frac{\langle q_w^c \rangle}{WT^* \lambda_1 \left( \langle S^c_w \rangle \right) \left( \langle p \rangle - p_w^c \right)},$$  \hspace{1cm} (4.72)

where $\langle S_w \rangle$, $\langle q_w^c \rangle$, and $\langle p \rangle$ are obtained from the numerical two-phase simulation of the fine-scale LWM (see notation in Eqs. 4.9 and 4.11). Using this approach for the heterogeneous case described above, we obtain the $m\left(S^c_w\right)$ curve shown in Figure 4-10, which is slightly different from the analytical curve in Figure 4-6. The corresponding optimized $k^*_j$ are shown in Figure 4-11; these display trends consistent with those seen in Figure 4-9 but the degree of nonmonotonicity is slightly lower here.
4.5.4 Summary

The two-phase near-well (NW2P) upscaling procedure described in Section 4.3 tends to produce optimized \( k_{ij}^* \) that exhibit a highly nonmonotonic behavior, with relative permeabilities significantly greater than unity. In this section, we presented a semianalytical approach based on the radial Buckley-Leverett solution to compute a mobility correction function, \( m\left(\bar{S}_w\right) \). Using \( m\left(\bar{S}_w\right) \) to account for high-mobility effects, we showed that the optimized \( k_{ij}^* \) from our NW2P procedure are very nearly monotonic.

We emphasize that these two approaches (NW2P with and without \( m\left(\bar{S}_w\right) \)) are equivalent and lead to identical simulation results assuming the simulator can handle nonmonotonic relative permeabilities. We further note that for the numerous cases considered later in this chapter (in Section 4.6), the nonmonotonic \( k_{ij}^* \) from our NW2P procedure did not pose any numerical difficulties for the simulator used here (Schlumberger, 2004b), though problems might be encountered with other simulators. In any event, the use of the semianalytical model may be more robust since the effects of high mobility and heterogeneity are accounted for separately via appropriate quantities, i.e., \( m\left(\bar{S}_w\right) \) and \( k_{ij}^* \) respectively. As such, for a sensitivity study of high-mobility-ratio displacements involving multiple permeability fields, the same analytically computed \( m\left(\bar{S}_w\right) \) can be used to provide the necessary mobility correction as long as the input relative permeabilities and fluid viscosities are unchanged.

4.6 Results and discussion

In this section, we evaluate the performance of our proposed upscaling approach on multiple realizations of 3D synthetic fields. We investigate the impact of varying the correlation structure and degree of spatial variability of the permeability fields as well as
changing the fluid mobility ratio. We consider two different production scenarios: a system of horizontal wells and a five-spot pattern of vertical wells. In Sections 4.6.2 and 4.6.3, we present the results from the application of our near-well upscaling procedures (NW1P and NW2P) to an injector and a producer local well model respectively. Next, in Sections 4.6.4 through 4.6.8, we compare the global simulation flow results obtained using a sequence of upscaling methods (standard $k^*$ only, NW1P, NW2P, NW2P plus EFBC $k^*_n$) with those from the reference, fine-grid model. The relative contribution of each upscaling component of our approach is assessed. For concise comparisons, the errors for the different coarse models are quantified using appropriate norms. Lastly, we present the typical speedup factors achieved by the various upscaling methods in Section 4.6.9.

The upscaling calculations (to compute $k^*$, $T^{w*}$, $W^{l*}$, and $k^*_n$) as well as flow simulations at the LWM level are performed using a research upscaling/simulation code that employs a fully-implicit formulation. We use the ECLIPSE simulator (Schlumberger, 2004b) to run the global (fine-scale and coarse-scale) flow simulations with a fully-implicit formulation. The simulator is able to handle the nonmonotonic relative permeability curves generated by our NW2P procedure (the near-well mobility correction discussed in Section 4.5 is not employed).

4.6.1 Description of flow problem studied

The 3D synthetic permeability fields we employ consist of $55 \times 55 \times 25$ blocks at the fine scale and are generated using unconditional, sequential Gaussian simulations (Deutsch, and Journel, 1998). The mean of the natural logarithm of $k_x$, designated $\mu_{ln}$, is fixed at 3.0 while the variance of the natural logarithm of $k_x$, $\sigma_{ln}^2$, ranges from 1.0 to 5.0. The physical
dimensions of the fields are as follows: \( L_x = L_y = 550 \, \text{ft}, L_z = 125 \, \text{ft} \). A spherical variogram is used. The dimensionless correlation length (see Eq. 2.22) in the vertical direction is constant \((l_z = 0.1)\) while that in the areal directions \((l_x = l_y)\) is varied from 0.2 to 0.7. In this manner, the effects of changing spatial variability and correlation structure are studied. The permeability fields are anisotropic at the fine scale, with \( k_z = 0.1k_x \) (and \( k_y = k_x \)). Unless otherwise specified, three realizations are generated for each set of permeability statistics. Realizations at each value of \( \sigma^2_{ln} \) are generated by rescaling a base set of realizations, so there is a high degree of correspondence between the models at different values of \( \sigma^2_{ln} \). We uniformly upscale the fine-scale models by a factor of 5 in each coordinate direction to obtain coarse-scale descriptions with 11 \( \times \) 11 \( \times \) 5 blocks.

For the oil and water relative permeabilities, we specify \( S_{wc} = 0.0, S_{or} = 0.15, k^0_{rw} = 0.43, k^0_{ro} = 1.0, n_w = 1.2, n_o = 1.5 \) (see Eqs. 4.55 and 4.56). The resulting power-law relative permeability curves (Figure 4-12) are used as input to the fine model. We also investigate the impact of changing the end-point mobility ratio, \( M^0 \) (defined in Eq. 4.58). We fix the water viscosity \( (\mu_w = 0.5 \, \text{cp}) \) and specify different oil viscosities \( (\mu_o = 11.63, 58.15 \text{ or } 116.3 \, \text{cp}) \) in order to obtain \( M^0 \) of 10, 50 or 100. A systematic study of the effects of changing mobility ratio will be presented in Section 4.6.6. The system (fluids and rock) are considered to be incompressible in this study. The two production scenarios, involving horizontal wells and a five-spot pattern element (with vertical wells), are illustrated in Figure 4-13. Note that in the five-spot pattern, there are only two coarse blocks on the diagonal between the injector and producers. This low degree of areal resolution is typical of that often encountered in practice. The injectors and producers are specified to flow at bottomhole pressures of 6,000 and 2,000 psi respectively in all cases.
4.6. RESULTS AND DISCUSSION

Figure 4-12 – Power-law fine-scale relative permeabilities

In our study, we consider the following upscaling methods in terms of their performance in matching reference, fine-scale flow results:

1. Standard upscaled absolute permeability ($k^*$ only)
2. NW1P procedure (see Section 4.2)
3. NW2P procedure (see Section 4.3)
4. NW2P and EFBC $k^r_{ij}$ (see Section 4.4)
5. Pore volume weighted well pseudo $k^r_{ij}$ (see Section 4.3)
6. Stone type well pseudo $k^r_{ij}$ (see Section 4.3)

We note that like the NW2P procedure, the pore volume weighted and Stone type well pseudo $k^r_{ij}$ techniques also include the NW1P procedure. Methods 5 and 6 are only considered in the study of the injector local well model in Section 4.6.2, where they serve to illustrate the inadequacies of such approaches. For our studies at the global simulation grid level (Sections 4.6.4 through 4.6.8), we only compare methods 1 through 4. This defines a sequence of upscaling approaches with increasing levels of sophistication. Method 4 is the full version of our proposed upscaling approach, comprising three com-
ponents (NW1P, NW2P, and EFBC $k_r^*$). We further note that although our standard NW2P treatment is only applied to injectors, we also present results from the additional application of NW2P to producers in Sections 4.6.3 and 4.6.4. There, we will demonstrate and explain why the two-phase upscaling of producers may not be worthwhile.

Figure 4-13 – Production scenarios: (a) horizontal wells, and (b) five-spot pattern. Coarse grid shown in (b) to emphasize low areal resolution
4.6. RESULTS AND DISCUSSION

4.6.2 Horizontal injector local well model ($M^0 = 100$)

We first illustrate the performance of the various upscaling methods at the local well model (LWM) level. We are particularly interested in validating the performance of our proposed near-well upscaling procedures (NW1P and NW2P), which are designed to produce accurate results at the LWM level. Although we have performed such studies on many different LWMs, here we will only show typical results for a permeability field with the following statistics: $\mu_{ln} = \sigma_{ln}^2 = 3.0$, $l_x = l_y = 0.5$, and $l_z = 0.1$. The LWM for the horizontal injector is first extracted from the global simulation grid (Figure 4-13a), as illustrated in Figure 4-14. The LWM consists of $45 \times 15 \times 15$ and $9 \times 3 \times 3$ blocks at the fine and coarse scale respectively, and the well is completed in 35 fine blocks and 7 coarse blocks. A well-driven flow problem, in which the injected water displaces oil toward the domain boundaries at an unfavorable mobility contrast ($M^0 = 100$), is simulated. No EFBC $k_{ij}^*$ are used in any of the LWM calculations. In coarse blocks not con-
taining a well, we apply purely local $k^*$ upscaling, which is generally adequate for the types of permeability fields considered here.

![Graphs showing flow results](image)

Figure 4-15 – Injector LWM flow results; $M^0 = 100$, $\mu_m = \sigma_{mn}^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$

As there is no producer in the LWM, the production of fluids is tracked at the domain boundaries. Figure 4-15 shows the plots of total flow rate ($q_t$) and oil flow rate ($q_o$) through the LWM boundaries versus pore volume injected ($t_D$), obtained using the differ-
ent models. Not surprisingly, the standard approach of $k^*$ only leads to a large disagreement with the reference, fine-scale solutions, resulting in 86% error in $q_t$ at $t_D = 1.2$ PVI. We also see that the use of the NW1P procedure alone does not produce satisfactory results for this case. This is because the total mobility effects are significant for such a high-mobility-ratio displacement, and these are not captured by single-phase upscaling. It is also notable that the reference total flow rate is not matched even at a late time (43% error in $q_t$ at $t_D = 1.2$ PVI) via the use of NW1P alone. This can be attributed to the high mobility ratio, which results in a very slow attainment of quasi-steady-state conditions (at which time essentially only one phase is flowing and the total flow rate from the NW1P model should be close to the fine-scale value).

In addition, Figure 4-15 clearly shows that the use of both pore volume weighted and Stone type well pseudo $k_{ij}^*$ lead to little or no improvement over NW1P alone. We observed this behavior for other cases as well and therefore will not consider these techniques further in subsequent investigations. In contrast, the application of our near-well upscaling technique (NW2P in conjunction with NW1P) provides a very close match of the reference results. It is worth reiterating that our near-well optimization procedures ensure the match of the fine-scale flow results at the coarse wellblock boundaries (see Eq. 4.36), whereas the comparison in Figure 4-15 is done at the domain boundaries. Thus the optimization does not assure exact agreement at the LWM level, though results of the accuracy illustrated in Figure 4-15 are typically obtained.

The optimized $k_{ij}^*$ from the NW2P procedure for one of the coarse wellblocks are shown in Figure 4-16. As explained in Section 4.5, the highly nonmonotonic behavior (with $k_{ij}^*$ larger than end-point values) is mainly due to high-mobility-ratio effects and can be mitigated via the use of our semianalytical well model to compute the mobility correction function $m(S_w)$. 
4.6.3 Horizontal producer local well model ($M^0 = 100$)

We now investigate the performance of a sequence of upscaling methods ($k^*$ only, NW1P, and NW2P) for a producer LWM. Again, although such studies have been performed on numerous producer LWMs, we will only show the results for a typical permeability field with the following statistics: $\mu_{in} = 3.0$, $\sigma_{in}^2 = 1.0$, $l_x = l_y = 0.5$, and $l_z = 0.1$.

The LWM for one of the horizontal producers shown in Figure 4-13a is first extracted from the global simulation grid. In this case, the fine- and coarse-scale LWMs comprise $35 \times 15 \times 15$ and $7 \times 3 \times 3$ blocks respectively, with the well completed in 25 fine blocks and 5 coarse blocks. A well-driven flow problem is simulated, with water at the domain boundaries encroaching as oil is produced at a high mobility ratio ($M^0 = 100$). We reiterate that we do not employ $k^*_{ij}$ in the LWM simulations and that purely local $k^*$ upscaling is applied to coarse blocks not containing a well.
4.6. RESULTS AND DISCUSSION

![Producer LWM flow results](image)

Figure 4-17 – Producer LWM flow results; \( M^0 = 100, \mu_m = 3.0, \sigma_m^2 = 1.0, l_z = l_y = 0.5, l_x = 0.1 \)

In Figure 4-17, we compare the different coarse models in terms of the total flow rate \( (q_t) \) and oil flow rate \( (q_o) \) at the producer. We see that the use of \( k^* \) only leads to significant errors (60% error in \( q_t \) at \( t_D = 1.0 \) PVI) whereas the use of NW1P and NW2P produce more accurate results (24% and 8% error in \( q_t \) respectively at \( t_D = 1.0 \) PVI). Com-
pared with Figure 4-15 for the injector LWM (practically no error in $q_w^*$ at all times), the error reduction from the application of NW2P is less significant here.

In Figure 4-18, we show the optimized $k^*_rj$ from the NW2P procedure for the coarse production wellblock with the largest $WI^*$. Compared with Figure 4-16 for the injector LWM, it is evident that these $k^*_rj$ are considerably less nonmonotonic and that $k^*_ro$ is closer to the input relative permeability curve. Accordingly, a smaller impact on the global simulation results is anticipated from the use of producer well $k^*_ro$, as will be confirmed in Section 4.6.4.

![Optimized $k^*_rj$ for a coarse production wellblock from the NW2P procedure compared with input relative permeabilities](image)

It may be surprising that $q^*_r$ and $q^*_o$ are not matched exactly in Figure 4-17 when NW2P is applied. This is likely because even though the well two-phase flow rates of the fine-scale producer LWM are matched (see Eq. 4.38), there may still be a mismatch between the coarse-scale and integrated fine-scale, two-phase flow rates across the coarse production wellblock boundaries, i.e., $q^*_r \neq \langle q_r \rangle$ and $q^*_o \neq \langle q_o \rangle$ (see notation in Eq. 4.36).
As a result, the pressures and saturations encountered by the coarse production wellblock at a particular time can be different from the conditions experienced by the fine blocks that comprise the coarse wellblock boundaries during the fine-scale, two-phase simulation of the LWM. This does not occur in the injector LWM since we directly match $\langle q_t \rangle$ and $\langle q_w \rangle$, and the conditions “upstream” of the injection wellblock (i.e., at the injector) are known and constant ($p = p^w$ and $S_w = 1$). It is not possible for our NW2P procedure (as currently implemented) to match $\langle q_t \rangle$ and $\langle q_w \rangle$ for a producer since in an upstream-weighted, finite-difference numerical scheme, the flow rates across the production wellblock boundaries are controlled by the mobilities of the (upstream) neighbor blocks, which do not depend on the well $k_{ij}^r$. Consistent with these observations, we will show in Section 4.6.4 that the application of the NW2P procedure to producers has limited efficacy in improving the accuracy of the global flow results for the cases considered.

4.6.4 Horizontal wells ($M^0 = 100$)

We now consider the full global problem for the production scenario involving horizontal wells (Figure 4-13a). Four upscaling methods are compared in terms of their ability to reproduce the reference, fine-scale simulation results. The use of $k^*$ only represents the base case, and each of the three proposed upscaling components (NW1P, NW2P, and EFBC $k_{ij}^r$) is added sequentially. Unless otherwise specified, the NW2P procedure is applied only to the injector. We investigate the effects of changing spatial variability ($\sigma_m^2$: 1.0 to 5.0; $l_x = l_y = 0.5$) and correlation structure ($l_x = l_y$: 0.2 to 0.7; $\sigma_m^2 = 3.0$) in the permeability field. Three realizations are considered for each set of statistics.
Figure 4-19 – Global simulation flow results; horizontal wells, $M^0 = 100$, $\mu_{in} = \sigma_{in}^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$
Figure 4-20 – Individual producer total flow rate; horizontal wells, $M^0 = 100$,

$$\mu = \sigma^2, l_x = l_y = 0.5, l_z = 0.1$$

For a representative field with $\mu = \sigma^2 = 3.0, l_x = l_y = 0.5$ and $l_z = 0.1$, we show the plots of field average pressure, field production rate, and field water cut in Figure 4-19, and the total flow rate of one of the producers in Figure 4-20. It can be seen that each additional upscaling component yields an incremental improvement in the accuracy of the upscaled model. The application of the full version of the proposed upscaling approach produces an excellent match of the reference results. Specifically, the field average pressure is matched almost exactly and the producer total flow rate shows an error of only 8% at 1 PVI.

It is clearly impractical to compare large numbers of cases by visual inspection of the simulation results alone. We therefore quantify the dimensionless upscaling errors using an $L^1$-norm of error, defined as:

$$\varepsilon_v = \frac{\int_0^{t_{p,f}} \left| v^f (t_D) - v^e (t_D) \right| dt_D}{\int_0^{t_{p,f}} v^f (t_D) dt_D},$$

(4.73)
where \( v \) is the result of interest (superscripts \( f \) and \( c \) indicate fine and coarse scale), \( t_D \) is the dimensionless time in pore volume injected, and \( t_{Df} \) is the time at which the fine-grid water cut reaches 0.95. For each set of field statistics, we compute the errors in the following variables: field average pressure (\( e_{fpr} \)), field production rate (\( e_{fqt} \)), field water cut (\( e_{fwc} \)), average of individual producer rates (\( e_{pqt} \)), and average of individual producer water cut (\( e_{pwc} \)). A composite error, \( \langle e \rangle \), which is the average of all these errors, is also computed:

\[
\langle e \rangle = \sum_{v=1}^{n_v} \alpha_v e_v,
\]

where \( n_v \) is the number of variables for which errors are computed using Eq. 4.73 and \( \alpha_v \) is the weight assigned to each error (we set \( \alpha_v = 1/n_v \)). It is reasonable to compute \( \langle e \rangle \) in this manner as all of the errors (\( e_v \)) are dimensionless quantities. Note that each error quantity is averaged over multiple (generally three) realizations, and thus contains a substantial amount of information.

Figure 4-21 – Composite error as a function of spatial variability; horizontal wells, \( M^0 = 100, l_x = l_y = 0.5 \)
Figure 4-21 presents the composite error $\langle \varepsilon \rangle$ for the various procedures as a function of $\sigma_m^2$. Because each point represents an average over 3 realizations, this figure displays results that involve 75 simulations. We see a trend of increasing $\langle \varepsilon \rangle$ with $\sigma_m^2$ for all upscaling methods. This is to be expected since a more spatially variable field is generally more difficult to upscale. It is also evident that each additional upscaling component leads to a reduction in upscaling error. For instance, for fields with $\sigma_m^2 = 5$, the use of $k^*$ only leads to $\langle \varepsilon \rangle \sim 0.16$, which is decreased to 0.11 via the application of NW1P. The use of both NW1P and NW2P procedures further reduces $\langle \varepsilon \rangle$ to 0.08, while the additional application of EFBC $k_{ij}^*$ gives an error of 0.05. The three upscaling components form a sequence of methods that offer increasing levels of accuracy (though at increasing computational cost).

It is also of interest to determine if the upscaling error trends described above hold on a realization by realization basis. To investigate this issue, we generate four additional realizations and rescale each to two different values of $\sigma_m^2$. Although these realizations are not rescaled to all five values of $\sigma_m^2$ considered above, for each realization we do consider $\sigma_m^2 = 5$. Using these additional realizations, the flow results for seven realizations (including the three realizations used to generate Figure 4-21) can be compared. For these seven realizations, we consistently observe the trend of increasing $\langle \varepsilon \rangle$ with increasing spatial variability for all four upscaling methods. The slopes of this increase do, however, vary appreciably from realization to realization. For six of the realizations, the relative performance of the different methods described earlier is observed. However, for one realization, the use of $k^*$ only is more accurate than the application of our near-well procedures (NW1P and NW2P), though the full method (NW1P, NW2P plus EFBC $k_{ij}^*$) still provides the lowest $\langle \varepsilon \rangle$. The unusually high degree of accuracy of $k^*$ only upscaling in this case is presumably due to statistical variation.
Figure 4-22 – Average errors in individual producer variables as a function of spatial variability; horizontal wells, \( M^0 = 100, l_x = l_y = 0.5 \)

For \( \sigma_n^2 = 5 \), the \( \langle \varepsilon \rangle \) values computed from the different methods (averaged over seven realizations) are almost identical to the values noted above, which are averaged over three realizations. This suggests that reasonably representative results for \( \langle \varepsilon \rangle \) may
be obtained using a small number of realizations, though verification of this speculation would require more extensive simulations. In any event, in all subsequent cases, we consider only three realizations.

![Figure 4-23](image)

**Figure 4-23** – Average error in individual producer water cuts (dimensional time) as a function of spatial variability; horizontal wells, $M^0 = 100$, $l_x = l_y = 0.5$

Although the composite error is a very compact metric, it is not directly physical because it combines information about different flow variables. As such, it is also instructive to examine upscaling errors in particular variables. We show plots of average errors in individual producer rates and water cuts in Figure 4-22. We see that upscaling errors increase with increasing spatial variability and that the sequential improvement evident in Figure 4-21 persists. Unlike the total field production rate, for which overestimation for one producer can be cancelled by underestimation for another producer, no cancellation of errors is introduced when computing the individual producer rates. As a result, the individual producer rates are relatively difficult to match and the average errors in these variables tend to be large: as high as 0.35 from the use of $k^*$ only (Figure 4-22a), which is
reduced to 0.04–0.11 (depending on $\sigma^2_{\text{ln}}$) via the application of our full approach (NW1P, NW2P, and EFBC $k_{ij}^*$).

By contrast, the average errors in individual producer water cut ($e_{\text{pwc}}$) from all upscaling methods are an order of magnitude smaller (Figure 4-22b)—less than 0.03 throughout. This suggests that the rapid breakthrough of the high-mobility injected fluid is insensitive to the coarse-scale representation. This finding is due in part to the use of dimensionless time ($t_D$) instead of dimensional time ($t$) as the abscissa in the simulation results, which means that time is rescaled and inaccuracies in total flow rate predictions from the coarse models ($q^c_t$) are masked. For all the cases considered in this study, we have also computed the errors using dimensional time and found that they are higher than the errors using dimensionless time. For example, from the plot of $e_{\text{pwc}}$ (computed using $t$ rather than $t_D$) as a function of $\sigma^2_{\text{ln}}$ in Figure 4-23, we can see that for $\sigma^2_{\text{ln}} = 5$, the use of $k^*$ only leads to $e_{\text{pwc}}$ of 0.11, while the application of NW1P, NW2P, and EFBC $k_{ij}^*$ reduces the error to 0.075, 0.045, and 0.04 respectively (compare these with values in Figure 4-22b, which are 0.03 or less). The use of $t$ rather than $t_D$ puts much more emphasis on the accuracy of $q^c_t$. Nevertheless, except for differences in magnitudes, the trends seen in the two types of error plots are generally consistent.
4.6. RESULTS AND DISCUSSION

Figure 4-24 – Errors as a function of areal correlation length; horizontal wells, $M^0 = 100$, $\sigma_{ln}^2 = 3.0$
Figure 4-25 – Errors as a function of spatial variability; horizontal wells, $M^0 = 100$, $l_x = l_y = 0.5$; with results from NW2P for injector and producers.
4.6. RESULTS AND DISCUSSION

We next present plots of upscaling errors for varying areal correlation length in Figure 4-24. The trends observed are consistent with those seen in Figures 4-21 and 4-22, in terms of the relative performance of the upscaling methods and the magnitudes of the errors. For instance, for $l_x = l_y = 0.7$, the use of $k^*$ only leads to $\langle \varepsilon \rangle \sim 0.16$. This error is reduced to 0.07 using the NW1P procedure, which is further decreased to 0.04 by additionally employing NW2P. The full upscaling procedure (NW1P, NW2P, and EFBC $k_{ij}^*$) is again very accurate, with $\langle \varepsilon \rangle \sim 0.03$. From Figure 4-24, it further appears that the upscaling errors are largely unaffected by the change in the correlation structure, for the range of $l_x$ and $l_y$ considered. However, as shown in Chapter 2 and in Hui et al. (2005b), we expect EFBCs to become less applicable for higher values of $l_x$ and $l_y$ ($> 1.0$). Consequently, the effectiveness of EFBC $k_{ij}^*$ may be diminished for highly layered fields.

As a fifth upscaling method, we now consider a variant of the NW2P method, in which we compute $k_{ij}^*$ for both the injector and the producers. In Section 4.6.3, we showed that, at the LWM level, the impact of the NW2P procedure for producers was not as dramatic as that for injectors. Here, we investigate the effect of the use of NW2P for producers on the global coarse-scale flow results. EFBC $k_{ij}^*$ are not applied. This upscaling is performed in addition to the application of NW2P to the injector. We consider the same cases studied earlier ($\sigma_{in}^2$: 1.0 to 5.0; $l_x = l_y = 0.5$), in which multiple realizations are considered for each set of statistics.

In Figure 4-25, we see that by additionally applying the NW2P procedure to the two horizontal producers, some reduction in $\langle \varepsilon \rangle$ and $\varepsilon_{pqt}$ is obtained for $\sigma_{in}^2 = 1.0$ and 3.0. However, for $\sigma_{in}^2 = 5.0$, the errors are either unchanged or higher. This could be due to the fact that the conditions actually encountered at the regions corresponding to the boundaries of the producer LWMs during the coarse-scale simulation differ more from those employed in the NW2P upscaling as the permeability field becomes more hetero-
geneous. This effect is greater for a producer because the LWM boundary conditions are more accurate in the case of an injector (see discussion in Section 4.6.3). Accordingly, with more accurate boundary conditions, perhaps by adopting an adaptive local-global upscaling framework (Chen, 2005), it is possible that the use of the NW2P procedure on producers will produce a greater impact. Nevertheless, in the current implementation, the application of NW2P upscaling to producers results in little, if any, improvement in the global simulation results. As such, from here on we will apply the NW2P treatment only to injectors. We note, however, that if a producer is adjacent to an aquifer and strong water-coning or edge-water effects develop around the well, then NW1P upscaling alone may be insufficient and the NW2P upscaling of the producer may also be needed.

4.6.5 Horizontal wells ($M^0 = 10$)

It is of interest to determine the impact of the fluid mobility contrast on the accuracy of the upscaling procedures. Here, we again consider the production scenario involving horizontal wells (Figure 4-13a) but specify a lower end-point mobility ratio ($M^0 = 10$). We investigate multiple realizations of fields with different spatial variability ($\sigma_{in}^2$: 1.0 to 5.0; $l_x = l_y = 0.5$).

Plots of field average pressure and field production rate for a representative realization are shown in Figure 4-26. The trends observed in these plots are different from those seen in Figure 4-19 ($M^0 = 100$). Here, while the use of $k^*$ only still results in large inaccuracies, the use of NW1P alone provides a reasonably close match of the fine-scale results. The additional application of NW2P and EFBC $k^*_j$ leads to only marginal improvement in accuracy. This observation implies that two-phase upscaling is much less crucial for this moderate mobility contrast case. This result is perhaps not surprising as it is clear that
the differences between the near-well $k^*_r$ and fine-scale relative permeabilities are
strongly affected by $M^0$. As $M^0 \rightarrow 1$, the curves will more closely resemble each other, so
the impact of $k^*_r$ on simulation results will lessen. The plot of composite error versus $\sigma^2_{ln}$
(Figure 4-27) shows that these conclusions hold for different field realizations and spatial
variabilities.

![Graph showing field average pressure and producer total rate over time for different conditions.](image)

Figure 4-26 – Global flow results; horizontal wells, $M^0 = 10$, $\mu_{ln} = \sigma^2_{ln} = 3.0$,
$l_x = l_y = 0.5$, $l_z = 0.1$
Figure 4-27 – Composite error as a function of spatial variability; horizontal wells, $M^0 = 10, l_x = l_y = 0.5$

4.6.6 Horizontal wells (effects of changing $M^0$)

We next systematically assess the impact of mobility ratio ($M^0 = 10, 50,$ and $100$). We employ multiple realizations for the permeability field with the following statistics: $\mu_{ln} = \sigma_{ln}^2 = 3.0, l_x = l_y = 0.5, l_z = 0.1,$ and consider the production scenario involving horizontal wells shown in Figure 4-13a.
4.6. RESULTS AND DISCUSSION

![Graph showing errors as a function of mobility ratio]

(a) Composite error of all variables, $\langle \varepsilon \rangle$

(b) Average error in individual producer rates, $\varepsilon_{pqt}$

Figure 4-28 – Errors as a function of mobility ratio; horizontal wells, $\mu_m = \sigma_m^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$
The plots of $\langle \varepsilon \rangle$ and $\varepsilon_{p,q,t}$ as a function of $M^0$ (Figure 4-28) display very similar trends, with the magnitudes of $\varepsilon_{p,q,t}$ being larger. The ranking of upscaling methods in order of increasing accuracy ($k^*$ only, NW1P, NW2P, and EFBC $k^*_{ij}$) is consistent with previous observations. It may be surprising that the errors from the use of $k^*$ only appear to decrease as $M^0$ becomes larger. This trend may be attributed to the interplay between loss of heterogeneity and numerical dispersion effects. A full elucidation of this aspect of the flow behavior will require a more in-depth investigation, which we will not perform in this work. The salient observation here is that the use of $k^*$ only leads to errors much larger than those from the other upscaling methods. Figure 4-28 shows that errors increase with increasing $M^0$ for both the NW1P and NW2P procedures, with the latter exhibiting more of an advantage as $M^0$ increases. The additional application of EFBC $k^*_{ij}$ consistently yields very accurate results, with $\langle \varepsilon \rangle \sim 0.03$ for the entire range of $M^0$ considered.

### 4.6.7 Five-spot pattern ($M^0 = 100$)

We now consider a different production scenario (the five-spot pattern shown in Figure 4-13b) but maintain all other settings described in Section 4.6.4; i.e., we again investigate a high-mobility-ratio, oil-water displacement ($M^0 = 100$) and employ the same permeability realizations. The important implication of using the same fields as those considered in Section 4.6.4 is that while the NW1P and NW2P upscaling calculations need to be performed again for this new well configuration, the EFBC $k^*_{ij}$ for each coarse block need not be recomputed.
Figure 4-29 – Global simulation flow results; five-spot pattern, $M^0 = 100$, $\mu_m = \sigma_m^2 = 3.0$, $l_x = l_y = 0.5$, $l_z = 0.1$
Figure 4-30 – Individual producer total flow rate; five-spot pattern, $M^0 = 100$,

\[ \mu_{in} = \sigma_{in}^2 = 3.0, \ l_x = l_y = 0.5, \ l_z = 0.1 \]

We again show the plots of field average pressure, field production rate, field water cut (Figure 4-29), and the total flow rate of one of the producers (Figure 4-30) for a representative model. These plots display differences when compared with the plots for the horizontal well configuration (Figures 4-19 and 4-20). The use of NW1P in this case results in very little improvement over $k^*$ only. Further, while the flow results from the application of NW2P are visibly closer to the reference curves, the most substantial improvement is provided by the additional use of EFBC $krj^*$. This suggests that flow effects away from the wells are important for this configuration. A reason for this may be that the small number of inter-well blocks and the predominantly lateral flow (recall that $k_z = 0.1k_x$) necessitate the use of upscaled relative permeabilities for the inter-well region. Nevertheless, consistent with Figures 4-19 and 4-20, Figures 4-29 and 4-30 indicate that the application of the full version of our upscaling approach (NW1P, NW2P, and EFBC $krj^*$) again produces a close match of the reference results.
Figure 4-31 – Errors as a function of spatial variability; five-spot pattern, $M^0 = 100$, $l_x = l_y = 0.5$
Figure 4-32 – Errors as a function of areal correlation length; five-spot pattern,

\[ M^0 = 100, \sigma_{\text{var}}^2 = 3.0 \]

The plots of composite errors and average errors in individual producer rates as a function of spatial variability (Figure 4-31) reaffirm our previous conclusions that upscaling errors are generally higher for more spatially variable fields and that each additional
upscaling component provides some amount of error reduction. For example, for fields with $\sigma^2_\tilde{m} = 5$ (Figure 4-31a), the use of $k^*_{ij}$ only leads to $\langle \varepsilon \rangle \approx 0.14$, which is reduced via the application of the NW1P and NW2P procedures to 0.13 and 0.12 respectively. The use of EFBC $k^*_{ij}$ has a more significant impact, further decreasing the error to only 0.06, again demonstrating that two-phase upscaling in the inter-well region is important here. The higher error values seen in Figure 4-31b once again illustrate the difficulty of matching individual producer rates in these high-mobility-ratio displacements.

We next consider the impact of varying permeability correlation structure for the five-spot pattern. Consistent with our conclusion from Figure 4-24 (for horizontal wells), Figure 4-32 indicates that the upscaling errors for this production scenario are largely independent of the permeability correlation structure, for the range of $l_x$ and $l_y$ considered. The relative performance of the different upscaling methods is very similar to that observed in Figure 4-31.

### 4.6.8 Five-spot pattern ($M^0 = 10$)

We now assess the five-spot pattern displacement with $M^0 = 10$. Multiple realizations of fields with varying spatial variability ($\sigma^2_\tilde{m}$: 1.0 to 5.0; $l_x = l_y = 0.5$) are investigated. The plots of field average pressure and field production rate for a representative field are shown in Figure 4-33.
Unlike the high-mobility-ratio case (Figure 4-29), here all upscaling methods lead to reasonable agreement with the fine-scale solution. The use of $k^*$ only still results in the largest inaccuracies while the use of NW1P provides some improvement in performance. The additional application of NW2P produces little impact, consistent with our observation in Section 4.6.5 that two-phase upscaling in the near-well region is not as important.
for moderate mobility contrasts. However, as highlighted in Section 4.6.7, the use of EFBC $k^*_ij$ results in improvement for this well configuration. This again demonstrates the need to capture inter-well, two-phase flow effects for this case. The plot of composite error versus $\sigma_{ln}^2$ (Figure 4-34) shows that these conclusions are applicable over multiple realizations and for a range of spatial variabilities. The application of our full upscaling approach (NW1P, NW2P, and EFBC $k^*_ij$) leads to very accurate results ($\langle \varepsilon \rangle$ of 0.03 to 0.05).

![Figure 4-34 – Composite error as a function of spatial variability; five-spot pattern, $M^0 = 10$, $l_x = l_y = 0.5$](image)

**4.6.9 Comparison of computational effort**

The goal of upscaling is to provide flow results of reasonable accuracy while gaining some computational savings. As such, it is useful to compare the computational effort required by the different upscaling procedures. We note that a PC with a Pentium4 3.0 GHz CPU was used to perform the various computations.
Here, we will compare the CPU times associated with the modeling of the two production scenarios on a representative field for $M^0 = 100$ (see Figures 4-19 and 4-29). The two fine-scale simulations required 11.6 hours of CPU time using a commercial simulator. By contrast, the most expensive coarse-scale simulations (using upscaled parameters from NW1P, NW2P, and EFBC $k^*_ij$) consumed one minute, so the run-time speedup is very substantial (a factor of $\sim 700$). The NW1P and NW2P upscaling calculations performed for the two well configurations required 1 minute and 25 minutes respectively, while the single upscaling run to compute the EFBC $k^*_ij$ for the entire field required 2.9 hours. Therefore, overall speedup factors (including upscaling times) from the use of the NW1P and NW2P procedures are 350 and 27 respectively. However, the full upscaling approach (NW1P, NW2P, and EFBC $k^*_ij$) provides an overall speedup factor of only 4, although more compelling speedups are possible if many well configurations are considered (recall that the $k^*_ij$ need not be recomputed). In addition, the upscaling calculations can be optimized for speed, e.g., by computing EFBC $k^*_ij$ only for coarse blocks in high-flow regions and through the use of the same set of EFBC $k^*_ij$ for the $x$- and $y$-directions.

We emphasize that our three upscaling components form a sequence of methods with increasing sophistication and it may not be necessary to employ all of them in some situations. For instance, NW1P alone produces reasonable results for horizontal wells with $M^0 = 10$ (Figure 4-27), while the additional use of NW2P is needed for horizontal wells with $M^0 = 100$ (Figure 4-21). On the other hand, the use of EFBC $k^*_ij$ significantly improves the accuracy for the five-spot (vertical wells) simulations with $M^0 = 100$ (Figure 4-31). Thus, depending on the specific flow problem, desired accuracy, and available computational resources, one or more upscaling components can be used.

Finally, we note that the fine grid considered in our study contains relatively few blocks (75,625) to allow for the simulation of large numbers of cases within a practical
4.7   CONCLUDING REMARKS

time frame. We anticipate more significant speedups for larger and more heterogeneous fields typically encountered in practice. For example, in Chapter 3, we applied EFBC $k_{ij}^*$ on a fine-scale model with $10^6$ blocks and obtained an overall speedup factor of 76. Thus, we expect the upscaling methodology presented in this chapter to provide both accuracy and speed for practical field studies of high-mobility-ratio displacements.

4.7 Concluding remarks

In this chapter, we presented a novel upscaling approach for modeling high-mobility-ratio, two-phase displacements where near-well effects are important. For the near-well region, we propose the application of two optimization-based upscaling procedures (NW1P and NW2P) on an extended local domain. For the inter-well region, we apply EFBCs for the purely local computation of $k_{ij}^*$. We also introduced a semianalytical subgrid radial well model to diagnose the nonmonotonic behavior of the optimized well $k_{ij}^*$ from our NW2P upscaling. We showed that this model can capture the high-mobility effects via a mobility correction function $m\left(S_w\right)$. Using this function, the well $k_{ij}^*$ are considerably less nonmonotonic and mainly account for heterogeneity (rather than mobility) effects.

We investigated the effectiveness of these approaches by considering a large number of oil-water displacements in synthetic 3D fields. Different permeability realizations, spatial variabilities, correlation structures, fluid mobility ratios, and production scenarios were investigated. For a horizontal injector local well model, we showed that the application of existing pseudoization methods did not produce satisfactory flow results for the near-well region (for $M^0 = 100$). On the other hand, the proposed near-well, two-phase (NW2P) treatment produced an almost perfect reproduction of the fine-scale, two-phase
flow rates over the LWM. For a horizontal producer LWM \((M^0 = 100)\), we showed that the improvement of flow results from the use of NW2P upscaling over the near-well, single-phase (NW1P) procedure was appreciable but less significant than that seen in the injector LWM.

By varying \(\sigma^2_{ln} (\text{variance of natural logarithm of permeability})\) from 1 to 5, we observed that upscaling errors generally increase with increasing \(\sigma^2_{ln}\), as expected. The errors are, however, largely insensitive to changes in the areal correlation structure over the range of \(l_x\) and \(l_y\) considered (0.2 to 0.7). Our upscaling components (NW1P, NW2P, and EFBC \(k^*_rj\)) constitute a sequence of methods, for which each additional component typically provides an increasing level of accuracy (relative to the global, fine-scale simulation results). These procedures consistently provided improvements over the standard approach of using only upscaled absolute permeabilities \(k^*_r\). By considering a range of fluid mobility ratios \((M^0 = 10, 50, \text{and } 100)\), we showed that upscaling errors generally increase with \(M^0\) and that our two-phase upscaling procedures (NW2P and EFBC \(k^*_rj\)) become more important as \(M^0\) increases. The additional application of the NW2P procedure on producers provided slightly better accuracy than the standard NW2P treatment (only applied to injectors) for moderate levels of heterogeneity \((\sigma^2_{ln} \leq 3)\) but was less accurate for highly heterogeneous fields \((\sigma^2_{ln} = 5)\).

For the production scenario involving horizontal wells and a high mobility ratio \((M^0 = 100)\), the application of our near-well procedures (NW1P in conjunction with NW2P) provided coarse-scale results with composite errors of 3–8%. The additional application of EFBC \(k^*_rj\) reduced the errors to 2–5%. For a moderate mobility contrast \((M^0 = 10)\), the use of NW1P upscaling alone led to very low composite errors (3–6%), obviating the need to employ either the NW2P procedure or EFBC \(k^*_rj\). For the five-spot pattern involving vertical wells and \(M^0 = 100\), the near-well procedures (NW1P and NW2P) led to
larger composite errors (6–12%), while the use of EFBC $k_{ij}^*$ decreased the errors to 3–6%. For this configuration, when $M^0 = 10$, the use of NW1P upscaling alone led to errors of 6–9%. The application of the near-well, two-phase procedure did not provide error reduction but errors of 3–5% were obtained via the additional use of EFBC $k_{ij}^*$.

For a representative case, our near-well procedures yielded a significant overall speedup (27) relative to the fine-scale simulations. Run-time speedup factors are very substantial (~700). However, the additional use of EFBC $k_{ij}^*$ reduced the overall speedup factor to a more modest value (4). Nevertheless, we expect more significant CPU savings for larger and more heterogeneous models and through the optimization of our upscaling calculations. From the results of the numerous cases considered in this work, we conclude that our proposed two-phase upscaling approach can provide coarse-scale models that are robust, fast, and accurate for the simulation of well-driven, high-mobility-ratio displacements.
Chapter 5

Conclusions and future work

In this dissertation, two new upscaling techniques were developed for the accurate and efficient modeling of two-phase displacements complicated by first-contact miscibility and near-well effects. The major conclusions from our investigations of these techniques and suggested future work are summarized below.

5.1 Miscible upscaling

- A new upscaling technique for coarse-scale simulations of first-contact miscible displacements was developed. The method incorporates two key components: effective flux boundary conditions (EFBCs) and the extended Todd-Longstaff with up-scaled relative permeabilities \( k^*_j \) or ETLU formulation. By incorporating approximate global information into the local upscaling calculations of \( k^*_j \), EFBCs provide upscaled models that lead to more accurate breakthrough-time predictions. The ETLU formulation efficiently models bypassed oil that is immobile and unmixed by solvent \( S^*_w \) within the limited-compositional framework.
• The upscaling technique was evaluated on synthetic 2D fields with varying correlation length \( (l_x: 0.1–5.0) \) and found to accurately reproduce fine-scale simulation results for partially layered fields \( (l_x \leq 1.0) \). Standard boundary conditions were more applicable than EFBCs for highly layered systems \( (l_x > 1.0) \). Outlet averaging was determined to be more appropriate than volume averaging for saturation.

• By quantifying the oil-cut and breakthrough-time errors, the technique was demonstrated to consistently outperform other techniques, such as standard \( k^* \), \( k^* \) only, and nonuniform coarsening, over a wide range of coarsening factors (4–2500) for fields with different heterogeneity structures. The breakthrough-time errors did not always decrease with increasing grid refinement, and this trend was analyzed and explained in terms of an interplay between effects from the loss of heterogeneity and numerical dispersion.

• By considering multiple production scenarios and permeability realizations for synthetic 3D fields, we showed that our upscaling technique again provided substantially more accurate results compared with the use of \( k^* \) only.

• Significant overall speedup factors of 60 and 126 were achieved for representative 2D and 3D cases respectively.

• The upscaling procedure was applied to a miscible gas injection field study involving a nonuniform, 3D, geocellular model. Our grid refinement study indicated that the original fine model needed to be refined areally by a factor of 3 to 5 to achieve numerical convergence. This refinement was introduced into the local problems solved to determine the \( k_{ij}^* \). In addition, a near-well treatment, in the form of either a near-well, single-phase upscaling procedure or a local grid refinement around wells, was found to be beneficial in improving the match in well variables.
The proposed approach (miscible upscaling technique in conjunction with a near-well treatment) was evaluated on 2D cross-sections as well as 3D sectors from the full-field model by considering realistic miscible gas injection scenarios. Our approach was seen to provide coarse-scale simulation results that closely matched the converged (refined) fine-scale solutions. The new upscaling procedure outperformed coarse models based on border-region $k^*$ only and even the original (unrefined) fine models. Substantial overall speedup factors of 13 to 75 were obtained. It was thus demonstrated that our miscible upscaling approach is quite useful for practical field studies of miscible displacements.

Issues that could be further investigated as extensions of this work are discussed below.

- Although only applicable to first-contact miscible displacements at present, it may be possible to extend our upscaling method to handle multicontact miscibility. Conceptually, it is straightforward to incorporate the two key ideas in our approach (the use of EFBCs and the $S_{wb}^*$ concept) into a fully-compositional miscible upscaling methodology (e.g., Jerauld, 1998). Instead of using the Todd-Longstaff formulation, fully-compositional simulations will need to be performed to compute the $k_{ij}'$. However, there are nontrivial challenges in the implementation of such an extension. The fully-compositional upscaling calculations may incur a prohibitive computational cost, although this may be mitigated by the use of more computationally-efficient techniques (e.g., Thiele et al., 1997; Nilsson et al., 2005). Another complication is the need to account for complex phase behavior and the development of miscibility.

- It may be of interest to study the adjustment of the flux attenuation behavior of EFBCs to handle systems with any degree of layering. In Chapter 2, a preliminary study of this issue was performed, in which the adjustment of the asymptotic flux ratio in accordance with the statistics of the 3D field was shown to result in a closer
match of the fine-scale solutions. With such a modification, it is hoped that the upscaling boundary conditions can transition smoothly from EFBCs to standard boundary conditions as the system of interest becomes more layered. A major challenge may be the accurate treatment of structures with different length scales, as the underlying EFBC theory assumes that all inclusions are of the same size.

\section*{5.2 Near-well upscaling}

- A novel upscaling approach for modeling well-driven, high-mobility-ratio displacements was presented. For the near-well region, we propose the application of single- and two-phase optimization-based upscaling procedures (NW1P and NW2P) on a local fine region around the well (the local well model or LWM) to provide upscaled well indices ($W^*$), wellblock transmissibilities ($T^*$), and relative permeabilities ($k^*$). For the inter-well region, we apply EFBCs for the purely local computation of $k^*$.

- Mobility effects in the near-well region can cause the $k^*$ from the NW2P procedure to be strongly nonmonotonic. A semianalytical subgrid well model based on the radial Buckley-Leverett solution was introduced to compute a mobility correction function, $m(S_w)$. By accounting for the mobility effects via $m(S_w)$, the optimized $k^*$ from our NW2P procedure become nearly monotonic and mainly account for heterogeneity effects.

- The performance of the overall upscaling approach was evaluated by considering oil-water displacements in synthetic 3D fields and investigating different permeability realizations, spatial variabilities, correlation structures, fluid mobility ratios, and production scenarios. For a horizontal injector LWM at high mobility ratio ($M^0 = 100$), the application of existing pseudoization methods did not yield accurate flow results.
In contrast, our NW2P upscaling reproduced the fine-scale, two-phase flow rates over the LWM almost perfectly. The improvement of flow results from the application of the NW2P procedure over NW1P upscaling was not as significant for a horizontal producer LWM ($M^0 = 100$).

- The spatial variability of the fields, characterized by the variance of the natural logarithm of permeability, $\sigma_{ln}^2$, was varied from 1 to 5. The upscaling errors from all methods were seen to increase with increasing $\sigma_{ln}^2$, as expected. However, no clear trend in the magnitude of the errors was observed as the areal dimensionless correlation lengths ($l_x = l_y$) were increased from 0.2 to 0.7.

- Our upscaling components (NW1P, NW2P, and EFBC $k^*_ij$) constitute a sequence of methods, for which each additional component typically provides an increasingly closer match of the global, fine-scale simulation results. Our procedures are consistently more accurate than the standard approach of using only upscaled absolute permeabilities $k^*$. As $M^0$ was increased from 10 to 100, upscaling errors generally increase and the two-phase upscaling components (NW2P and EFBC $k^*_ij$) become more important. The additional application of the NW2P procedure to producers led to some improvement in accuracy over the standard NW2P treatment (only applied to injectors) for moderately heterogeneous fields ($\sigma_{ln}^2 \leq 3$).

- For the production scenario involving horizontal wells ($M^0 = 100$), the application of our NW1P and NW2P procedures provided coarse-scale results with composite errors of 3–8%. The additional application of EFBC $k^*_ij$ reduced the errors to 2–5%. For $M^0 = 10$, neither NW2P nor EFBC $k^*_ij$ was required because the use of NW1P upscaling alone led to very low errors (3–6%).

- For the five-spot pattern involving vertical wells ($M^0 = 100$), the use of the near-well procedures alone led to larger composite errors (6–12%) while the additional use of
EFBC $k^*_i$ decreased the errors to 3–6%. For this configuration, when $M^0 = 10$, the use of NW1P upscaling alone led to errors of 6–9%. The application of the NW2P procedure did not provide error reduction for the $M^0 = 10$ case but errors of 3–5% were obtained via the additional use of EFBC $k^*_i$.

- For a representative case, our near-well procedures yielded a substantial overall speedup (27) relative to the fine-scale simulations. Run-time speedup factors are very substantial (~700). However, a more modest overall speedup (4) was obtained with the additional use of EFBC $k^*_i$. More significant computational savings could be achieved for larger and more heterogeneous fields (as considered in Chapters 2 and 3) and by optimizing the upscaling calculations.

The following related issues could be investigated in the future:

- For highly heterogeneous systems, we observed that the NW2P upscaling of producers did not provide significant benefit. We attribute this to the inaccuracy in the boundary conditions imposed on the LWM. One possible way to improve these boundary conditions is to apply an adaptive local-global (ALG) upscaling procedure (Chen, 2005), which may also enhance the accuracy of the NW2P procedure on injectors.

- We have not considered extremely challenging heterogeneity structures, such as the SPE 10 channelized reservoir (Christie and Blunt, 2001), which will likely require a more sophisticated single-phase upscaling technique such as ALG (Chen, 2005). For these systems, in which there are strongly preferential flow paths (along channel sands), the application of our NW2P procedure to compute directional $k^*_i$ to match the reference, two-phase flow rates across each face of the coarse wellblock (rather than the sum of the rates over all faces) may also be beneficial.
5.2. NEAR-WELL UPSCALING

- Our systematic quantification of upscaling errors—in which factors such as $\sigma_{ln}^2$, $l_s$, and $M^0$ were varied and multiple permeability realizations were considered—may allow this work to be combined with error modeling procedures (e.g., Lødøen et al., 2004). Linkages along these lines should be pursued.

- The near-well upscaling procedures developed here could be extended to handle three-phase flow. This would enable the proper calculation of the flow of free gas into production wells in coarse-scale simulation models.
Nomenclature

\( \langle \rangle \) averaged or integrated fine-scale quantity
BHP bottomhole pressure
\( E \) objective function for near-well optimization procedures
EFBCs effective flux boundary conditions
ETLU extended Todd-Longstaff with \( k^*_j \) formulation
\( f^*_j \) upscaled fractional flow of phase \( j \)
\( f_o \) oil cut
\( g \) gradient vector
\( J \) Jacobian matrix
\( H \) Hessian matrix
\( k \) absolute permeability tensor
\( k^* \) upscaled absolute permeability tensor
\( k^*_j \) upscaled relative permeability of phase \( j \)
\( k^0_j \) end-point relative permeability value of phase \( j \)
LC limited-compositional
\( l_i \) dimensionless correlation length in the \( i^{th} \) direction \((i = x, y, \text{ or } z)\)
\( L_i \) domain length in the \( i^{th} \) direction
LWM local well model
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>semianalytical mobility correction function</td>
</tr>
<tr>
<td>$M^0$</td>
<td>end-point mobility ratio</td>
</tr>
<tr>
<td>$N_{mf}$</td>
<td>parameter to gauge applicability of EFBCs</td>
</tr>
<tr>
<td>NW1P</td>
<td>near-well, single-phase upscaling</td>
</tr>
<tr>
<td>NW2P</td>
<td>near-well, two-phase upscaling</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
</tr>
<tr>
<td>PVI</td>
<td>pore volume injected</td>
</tr>
<tr>
<td>$q$</td>
<td>flow rate</td>
</tr>
<tr>
<td>$r$</td>
<td>radial distance or residual</td>
</tr>
<tr>
<td>$r$</td>
<td>residual vector</td>
</tr>
<tr>
<td>$r^c$</td>
<td>number of rings of coarse blocks around target upscaled block</td>
</tr>
<tr>
<td>$r_w$</td>
<td>well radius</td>
</tr>
<tr>
<td>$R'_i$</td>
<td>asymptotic flux ratio for the $i^{th}$ direction</td>
</tr>
<tr>
<td>$S^*_{orb}$</td>
<td>bypassed oil saturation of a coarse block in miscible flooding</td>
</tr>
<tr>
<td>$S_j$</td>
<td>saturation of phase $j$</td>
</tr>
<tr>
<td>$S_{jn}$</td>
<td>normalized saturation of phase $j$</td>
</tr>
<tr>
<td>$t$</td>
<td>dimensional time</td>
</tr>
<tr>
<td>$t_D$</td>
<td>dimensionless time (pore volume injected)</td>
</tr>
<tr>
<td>$T$</td>
<td>transmissibility</td>
</tr>
<tr>
<td>$T^*$</td>
<td>upscaled transmissibility</td>
</tr>
<tr>
<td>$T^{w*}$</td>
<td>upscaled wellblock transmissibility</td>
</tr>
<tr>
<td>$u$</td>
<td>Darcy velocity vector</td>
</tr>
<tr>
<td>$V$</td>
<td>volume</td>
</tr>
<tr>
<td>$WI$</td>
<td>well index</td>
</tr>
<tr>
<td>$WI^*$</td>
<td>upscaled well index</td>
</tr>
</tbody>
</table>
Greek

\( \beta \)  “trust” region parameter for Levenberg-Marquardt optimization

\( \Delta p \)  pressure difference

\( \Delta x, \Delta y, \Delta z \)  gridblock dimensions

\( \varepsilon \)  upscaling error

\( \langle \varepsilon \rangle \)  dimensionless composite error

\( \phi \)  porosity

\( \lambda \)  mobility or dimensional correlation length

\( \mu \)  viscosity

\( \nu_w \)  dimensionless wave velocity of water

\( \mu_{je} \)  effective viscosity of the phase or pseudophase \( j \)

\( \mu_{ln} \)  mean of the natural logarithm of permeability

\( \omega \)  Todd-Longstaff mixing parameter

\( \sigma_{ln}^2 \)  variance of the natural logarithm of permeability

\( \theta \)  parameter space/vector

Subscripts

\( B \)  background

\( bt \)  breakthrough-time

\( f \)  final

\( GN \)  Gauss-Newton

\( I \)  inclusion

\( i, j, k \)  coordinate indices

\( I, J, K \)  coarse-grid coordinate indices

\( inj \)  injector
NOMENCLATURE

$LM$ Levenberg-Marquardt

$m$ mixture

$o$ oil

$oc$ oil-cut

$pqt$ individual producer total flow rates

$pwc$ individual producer water cuts

$s$ solvent

$t$ total

$w$ water or well

$wb$ wellblock

$wc$ connate water

$x, y, z$ coordinate directions

**Superscripts**

$-$ analytically averaged quantity

* numerically averaged or upscaled quantity

$b$ boundary

$c$ coarse

$f$ fine

$w$ well
Bibliography


BIBLIOGRAPHY


Appendix A

First-contact miscible displacements: additional simulations

A.1 Directional upscaled relative permeabilities

None of the runs described in Section 2.4 employs directional upscaled relative permeabilities ($k_{ij}^*$). In other words, only one set of $k_{ij}^*$ (computed for the $x$-direction) is used for each coarse block. This is partly due to the limitation of simulator employed in this work (Chevron’s CHEARS simulator), which does not allow directional $k_{ij}^*$. In this section, we apply Schlumberger’s ECLIPSE simulator (Schlumberger, 2004b) to investigate the impact of using both the $x$- and $z$-direction $k_{ij}^*$ in a cross section (Field B of Figure 2-5).

We note that ECLIPSE was not used in Section 2.4 because it does not accept miscible $k_{ij}^*$ that exhibit a nonmonotonic behavior. Further, ECLIPSE does not model nonzero bypassed oil saturations in first-contact miscible displacements ($S_{orb}^*$). In these ECLIPSE runs, we did not include the $S_{orb}^*$ concept and we modified the $k_{ij}^*$ curves to force
monotonicity (these adjustments were relatively mild as Field B is only moderately heterogeneous).

Figure A-1 – Impact of using directional $k_{rj}^*$ in a corner-to-corner, cross-sectional problem (Field B of Figure 2-5)
We simulate a first-contact miscible displacement similar to that described in Section 2.4 but instead of a side-to-side linear flow pattern, we here consider a corner-to-corner flow pattern. The solvent injector is completed in the lower right hand corner (over 10 fine blocks) while the producer is completed in the upper left hand corner (over 10 fine blocks) of the cross section. There will therefore be significant flow in the vertical direction and we should expect the $z$-direction $k_{z_j}^*$ to impact the results. We consider coarse models of dimensions $20 \times 10$ and $80 \times 20$.

We show the plot of oil cut versus PVI for the case in which the $x$-direction $k_{ri}^*$ are used for both ($x$-$z$) directions in Figure A-1a. From the plot, we can see that although the overall match is good, both coarse models predict a breakthrough time that is too early. This is because in this case, the use of $x$-direction $k_{ri}^*$ in the vertical direction causes the injected fluid to move upward too rapidly. When directional $k_{ri}^*$ are employed (Figure A-1b), the overall match is slightly better while the breakthrough-time prediction is noticeably improved.

![Figure A-2 – Error comparison to show the effects of employing directional $k_{ri}^*$](image-url)
The plots of upscaling errors as a function of number of coarse blocks in the \( x \)-direction are shown in Figure A-2. We see that the quantification of upscaling errors confirms our observations above. When we employ only the \( \kappa_{ij}^* \) computed for the \( x \)-direction, the oil-cut error (see Eq. 2.23) is low (\( \varepsilon_{oc} \sim 0.06 \) to 0.08) while the breakthrough-time error (see Eq. 2.24) is substantial (\( \varepsilon_{bt} \sim 0.23 \) to 0.33). With the use of directional \( \kappa_{ij}^* \), the oil cut error is slightly smaller (\( \varepsilon_{oc} \sim 0.04 \) to 0.06) but the breakthrough-time prediction is significantly more accurate (\( \varepsilon_{bt} \sim 0.09 \) to 0.13).

These limited tests suggest that for a problem where there is flow in both the transverse and longitudinal directions, directional \( \kappa_{ij}^* \) can improve the performance of the coarse model. Consistent with this speculation, for the linear displacement problem considered in Section 2.4, we find that the results are essentially the same regardless of whether we use directional \( \kappa_{ij}^* \) or not. This trend may also be due to permeability anisotropy (\( k_z = 0.1k_x \)), which reduces the importance of vertical flow. While this \( x-z \) anisotropy is fairly typical for a cross-sectional model, directional \( \kappa_{ij}^* \) may become necessary for models in which the flow is different in characteristics but equally important in both the \( x \)- and \( y \)-directions. We reiterate that the use of only one set of \( \kappa_{ij}^* \) for each coarse block is purely due to a limitation of the simulator used (CHEARS). Whenever possible, it is probably more accurate to employ directional \( \kappa_{ij}^* \), albeit at a greater computational cost.
A.2 Miscible upscaling of a field cross section

In this section, we again apply the miscible upscaling method described in Chapter 2 to a miscible gas injection field problem similar to the one studied in Chapter 3. However, here we consider a different field cross section. Note that the results presented in this section have been published (Hui et al., 2004). This case is of interest because the permeability field has an extremely high degree of heterogeneity. Further, unlike the cases considered in Chapter 3, here we also investigate the use of standard $k_{ij}^*$ and a range of coarsening factors.

The geocellular model is 6,720 ft $\times$ 1,382 ft in size (240 $\times$ 432 blocks) and the permeability field is shown in Figure A-3. The system has the following statistics: $\mu_{ln} = -0.76$, $\sigma_{ln}^2 = 12.7$, $l_x = 0.22$, $l_z = 0.07$. The $N_{mf}$ value (Eq. 2.21) for this field is 1.49, slightly beyond the range for which the proposed method is theoretically applicable (maximum of 1.0). This field is also extremely heterogeneous ($\sigma_{ln}^2$ is very high), but we will show that the results are nevertheless quite encouraging.

![Figure A-3 – Cross section of a real field (Field X)](image)

The production scenario is similar to that used in previous test cases (Sections 2.4.1 and 2.4.2): the injector is set at a constant BHP of 11,000 psi while the producer has a
constant oil rate of 80 STB/day subject to a BHP constraint of 3,700 psi. We uniformly upscale the fine-scale model to three coarse models: $20 \times 36$, $15 \times 27$, and $10 \times 24$, which give coarsening factors of 114, 256, and 432 respectively. We again compare the oil cut versus PVI curves given by the coarse- and fine-scale simulations.

![Figure A-4 – Oil-cut curves from EFBC $k_{ij}^*$ (Field X)](image)

![Figure A-5 – Oil-cut curves from standard $k_{ij}^*$ (Field X)](image)
Figure A-6 – Oil-cut curves from $k^*$ only (Field X)

From Figure A-4, we can see that the oil-cut results given by EFBC $k^*_{ij}$ are in close agreement with the reference solutions. Using standard $k^*_{ij}$ (Figure A-5), we once again observe early breakthrough and low predictions for oil cut. The use of $k^*$ only yields surprisingly good results (Figure A-6), though the error results in Figure A-7 indicate that the proposed method still generally provides the most accurate coarse models. We note that an even coarser model ($10 \times 18$) did not provide accurate results using the proposed method, and a late breakthrough time was predicted. This may be related to the extremely high degree of heterogeneity of the field, which emphasizes the effect of loss heterogeneity as the model is vertically coarsened (see Section 2.4.3). A combination of nonuniform coarsening and EFBC $k^*_{ij}$ may be required for higher accuracy in this case.

Apart from plots of oil cut, we also compare the oil-production rates, gas-production rates, gas-injection rates, and average field pressures versus actual dimensional time (not PVI) in Figure A-8. The overall matches are quite good until the producer switches from rate to BHP control at ~4,600 days. In general, the coarse models predict the oil-
production rates and average field pressure with reasonable accuracy even after the switch in well control, though the gas-production and gas-injection rates display some error. This suggests that perhaps the well injectivities and productivities are not accurate; i.e., the well treatments in the coarse and fine-scale models may not be exactly equivalent. The use of a near-well, single-phase upscaling procedure (Mascarenhas and Durlofsky, 2000) may resolve this issue. At the time of the switch (~4,600 days), the oil fractional flow is much less than 0.1, so the mismatch at late time is not of much practical concern. We note that for the test cases shown in Chapter 2, the EFBC $k_r^*$ results are equally good whether we use dimensional or dimensionless plots.

![Figure A-7](image)

Figure A-7 – Fractional oil-cut and breakthrough errors from various methods (Field X)

In Figure A-9, we show the improvement in oil-production rate prediction (using dimensional time) given by the proposed method over that using $k^*$ only for the $15 \times 27$ model. We also see from Figure A-10 that the coarse-scale saturation map given by the proposed method captures the essential features of the fine-scale map very well, including the layer that leads to breakthrough. Consistent with our conclusions in Chapter 3,
these results indicate that the proposed method is able to provide useful predictions for a practical miscible displacement study.

For this case, the fine-scale simulation required about 68 hours of CPU time while the coarse-scale simulation ran in only 57 seconds. The EFBC upscaling procedure took 64 minutes, giving an overall speedup factor of 63. Optimizing the simulation parameters
(e.g., time step, material balance tolerance) in the fine run reduced the run time by about a factor of 6; in this case, the speedup factor is only about 10. It will, however, be difficult in general to determine these optimal parameters without performing extensive simulations.

![Figure A-9 – Oil-production curves for Field X (EFBC $k_r^*$ and $k^*$ only)](image)

![Figure A-10 – Solvent-saturation maps at breakthrough for Field X (15 x 27 model obtained using EFBC $k_r^*$)](image)
Appendix B

Immiscible displacements: additional simulations

B.1 Areal grid refinement of field cross section

In Section 3.3, we discussed the sensitivity of the miscible displacement simulation results to the areal resolution of the grid. In an effort to eliminate the effects of miscibility and reduce the fluid-property dependence on pressure, here we consider an immiscible oil-water displacement. We refine the original 10 × 858 fine model from Figure 3-2 by a factor of 3 to obtain a 30 × 858 refined model. We inject water at a constant rate of 1,000 RB/day on the right lateral edge of the model and produce at the same balanced rate with a BHP constraint of 4,000 psia on the left edge. The simulation results for a period of 100 years are shown in Figure B-1, with the dimensionless pore volume injected (PVI) as the abscissa.

Like Figure 3-3 for the miscible case, we find that the oil-production-rate and fractional-flow (water-cut) curves of the unrefined model are quite close to the refined model, consistent with the fact that the models have identical heterogeneity patterns. However, if we compare the 10 × 858 and 30 × 858 models at PVI = 0.6, the fractional difference in
average reservoir pressures is only about 0.02 here compared with 0.11 in Figure 3-3. Even more striking is the observation that the fractional difference in water-injection rates here (at PVI = 0.6) is merely 0.02 (Figure B-1), compared with the 0.33 fractional difference in gas-injection rates (Figure 3-3). This indicates that miscibility and the pressure dependence of fluid properties (e.g., gas formation volume factors) plays a significant role in the magnitude of the discrepancies of results between the unrefined and refined models in Section 3.3.

Figure B-1 – Results of areal grid refinement for an oil-water displacement in the 10×858 cross-sectional model shown in Figure 3-2