EFFECTIVE MODELS OF FRACTURED SYSTEMS

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Abstract

The geological complexity of fractured reservoirs necessitates the use of simplified models for flow simulation. This is often addressed in practice by using flow modeling procedures based on the dual-porosity/dual-permeability concept. However, there is often not a systematic and quantitative link between the underlying discrete fracture model (DFM) and the parameters appearing in the flow model.

In this work, a systematic upscaling methodology is presented to construct a generalized dual-porosity/dual-permeability model from detailed discrete fracture characterizations. The technique, referred to as a multiple subregion (MSR) method, introduces local subregions (or subgrids) to resolve dynamics within the matrix and provides appropriate coarse-scale parameters describing fracture-fracture, matrix-fracture and matrix-matrix flow. The geometry of the local subregions, as well as the required parameters for the coarse-scale model, are determined efficiently from local single-phase flow solutions using the underlying discrete fracture model. The subregions thus account for the fracture distribution and can represent accurately the matrix-matrix and matrix-fracture transfer. Three variants of the method are developed and tested. The first procedure provides a generalized dual-porosity model and is appropriate for systems with weak or nonexistent gravitational effects. The second procedure, a dual-porosity/dual-permeability approach, introduces connections between matrix regions in vertically adjacent blocks to capture phase segregation due to gravity. The third approach is a full dual-porosity/dual-permeability representation and includes connections between matrix regions in vertically and horizontally
adjacent blocks. All connections in the coarse-scale model are characterized in terms of upscaled transmissibilities, and the resulting coarse model can be used with any connection list-based reservoir simulator.

The methods are applied to simulate single-phase, two-phase and three-phase flow in 2D and 3D fractured reservoir models. Viscous, gravitational and capillary pressure effects are considered. The MSR models are shown to provide results in close agreement with the underlying DFM at computational speedups of 80-180. The MSR model is also applied to the simulation of compositional systems, specifically six-component miscible gas injection. In these simulations, the full dual-porosity/dual-permeability formulation is applied, and global single-phase flow information is introduced in the computation of the upscaled interblock transmissibilities. In a series of 3D simulations involving both connected and disconnected fracture systems, it is shown that the MSR method provides results of reasonable accuracy. Computational speedups of order 1,000 are achieved for the compositional models.

A hybrid method that allows some regions of the model to be treated using the DFM and others with the MSR method is also presented. This treatment may be appropriate for systems containing both connected and disconnected fractures or when enhanced accuracy is required in some reservoir regions (e.g., in the vicinity of wells). The hybrid representation can be implemented naturally into flow simulators using the connection list approach. Though it is more expensive computationally than the global MSR method, the hybrid approach does show enhanced accuracy.
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To my parents
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Chapter 1

Introduction and Literature Review

Naturally fractured reservoirs contain a significant portion of global hydrocarbon reserves. These reservoirs are characterized by a system of fractures existing within a background rock matrix. Nelson (2001) identified four types of naturally fractured reservoirs. In Type-I reservoirs, fractures provide the essential reservoir storage capacity (porosity) and permeability. In Type-II systems, fractures provide the essential permeability, but the matrix provides the essential porosity. In Type-III reservoirs, the matrix permeability is relatively high, with the fractures acting to further increase flow capacity. In Type-IV fractured reservoirs, the fractures are filled with minerals and provide no additional porosity or permeability. In this case the fractures create significant reservoir anisotropy and tend to form barriers to fluid flow and partition formations into relatively small blocks. In this thesis, we develop efficient techniques for simulating flow in Type-II and Type-III reservoirs.

Geological characterization is required in order to construct reliable flow models for reservoir simulation or hydrogeological modeling. The development of fractured reservoir characterization has lagged behind simpler matrix dominated systems (referred to as “single-porosity” systems) due to the difficulty of quantifying fracture parameters such as fracture spacing, length, orientation, porosity, connectivity, aperture and permeability. Outcrops provide a direct way of determining fracture
length and connectivity, and also give valuable information on fracture spacing and direction. However, as Aydin et al. (1996) demonstrated, weathering and stress effects may affect parameters in the outcrops and render them different than in-situ conditions. Cored wells, borehole image logs, seismic data, and production history are other valuable sources that can be used to infer fractured reservoir parameters. When applied in conjunction with geostatistical methods and/or geomechanical modeling, they can be used to generate detailed geological realizations of fractured systems.

Even given a detailed geological model, there are still a number of challenges associated with predicting flow through fractured systems. In general, there are two approaches for simulating flow in such systems. The discrete fracture model (DFM), or discrete fracture network (DFN) model, is one such approach. With this approach, flow is explicitly modeled in each fracture and in the matrix using Darcy’s law. As it represents a direct simulation of the fine-scale geological model, DFM is accurate but extremely expensive (for example, as discussed in Chapter 4 of this thesis, a DFM simulation performed for a 3D model containing 131,817 cells required three weeks of simulation time).

The second approach is to represent the system with an equivalent continuum model. With this approach, equivalent continuum properties assigned to model cells represent the combined effects of fractures and matrix. This approach is typically formulated using a dual-porosity concept to idealize the reservoir on the global scale. In this model, most fluid storage is in the matrix, with the large-scale flow occurring through the fractures (Type-II and III fractured reservoirs as classified by Nelson, 2001). The flow between matrix and fracture (and matrix to matrix in the dual-porosity/dual-permeability model) is represented by a “transfer function.” Traditional dual-porosity
approaches are more approximate but computationally much less expensive than DFM. For this reason, the dual-porosity approach is applied for most fractured reservoir simulations in industry. We now describe the two approaches in more detail.

### 1.1 Discrete Fracture Model (DFM)

Discrete fracture modeling represents each fracture as a geometrically well-defined entity. In DFM, each fracture is modeled explicitly and individually using highly resolved Cartesian or unstructured grids. Figure 1.1 illustrates a portion of an outcrop of a fractured formation and the corresponding unstructured grid.

![Figure 1.1: Fractures in the Sisquoc Formation at Halama Beach, CA. Image on the left shows a detail of the formation outcrop (photo by Atilla Aydin). Image on the right shows a triangulated mesh for a large portion of this outcrop which can be used for DFM simulation.](image)

Since its first introduction in the late 1970s, discrete fracture modeling has been studied by many researchers within both finite-element and finite-difference frameworks. Within the finite-element context, Baca et al. (1984) proposed a 2D model for single-phase flow with heat and solute transport in fractured formations. Juanes et al. (2002) presented a general finite-element formulation for 2D and 3D...
single-phase flow in fractured porous media. In a more recent paper, Matthai et al. (2005) applied a control-volume finite-element (CVFE) approach and presented a set of two-phase flow simulations with fractured rocks represented by unstructured 3D hybrid meshes. Within a finite-difference context, Sarda et al. (2002) presented a systematic procedure for discretization of fracture networks by manipulating the nodes of every fracture element and their relative positions. Combined with local grid refinement (LGR) in both fractures and matrix, this method shows flexibility in handling a wide range of reservoirs in terms of the fracture distribution and connectivity.

A simplified discrete fracture model suitable for use with Stanford’s General Purpose Research Simulator (GPRS) was recently presented by Karimi-Fard et al. (2004). This model is formulated as a finite-volume procedure and applies a connection list to represent the unstructured grid. This method is applicable for 2D and 3D systems with multiphase flow and will be used extensively in our work.

Recently, Vitel and Souche (2007) presented a “pipe network” approach to construct fine-scale and upscaled fracture models. A discrete fracture network and a corner-point grid are jointly discretized using a dual approach (pipe network). Nodes of the pipe network represent either discrete fractures or matrix blocks, and matrix-matrix, fracture-fracture and matrix-fracture connections are represented by pipes. This method has the advantage that the underlying system need not be gridded using an unstructured mesh.

Previously, DFM was limited by our ability to describe the fractured reservoir accurately and in terms of the computational effort required to account for every
fracture. Advanced characterization methodology and software are now able to provide realistic realizations of fracture networks. This, combined with increased computing capabilities, enable DFM simulations in some cases. However, the use of DFMs for flow modeling at the field scale is still too computationally demanding, especially when the displacement mechanism is complex and various flow scenarios must be considered.

1.2 Dual-Porosity Model

The most commonly used flow model for practical simulations of fractured systems is the dual-porosity model. Here the basic idea is to dissociate the flow inside the fracture network and the matrix and to model the exchange between these two media using a transfer function. This concept was first introduced by Barenblatt and Zheltov (1960). In the original model a complete set of equations for slightly compressible single-phase flow was written for both the fractures and the matrix, and transfer between them was assumed to occur at pseudo-steady state. Warren and Root (1963) presented a practical model for fractured systems. They considered an idealized case comprised of a set of identical rectangular parallelepipeds, representing the matrix blocks, which are separated by fractures. A simplified dual-porosity version of the Barenblatt and Zheltov (1960) flow model was used, in which the block to block flow takes place only through the fracture network, with the matrix feeding the fractures through a transfer function.

The model proposed by Warren and Root (1963) has been a framework for many applications and a number of subsequent investigations focused on the evaluation of the transfer function, also referred to as the shape-factor. This parameter depends on
the shape of the matrix block and the flow mechanisms. Kazemi et al. (1976) presented an extension of the dual-porosity model of Warren and Root (1963) to two-phase flow which could account for relative fluid mobilities, gravitational effects, imbibition, and variation in formation properties. Thomas et al. (1983) developed a three-dimensional, three-phase model for simulating the flow of water, oil, and gas in fractured systems.

Recently, Donato and Blunt (2003) presented a model combining a streamline simulation technique with a dual-porosity model. This approach is appealing as it applies streamline techniques for transport (flow in the fractures), while modeling the exchange of fluid between streamlines and the matrix by a transfer function. In contrast to standard streamline techniques, in which capillarity may pose difficulties, the capillary pressure effects are in this case modeled accurately through the transfer function.

Although originally developed based on physical considerations, the dual-porosity model has since been derived rigorously using two-scale homogenization procedures. Specifically, in a series of papers, Douglas and Arbogast (1990) and Arbogast (1993) considered single and two-phase flow in uniformly fractured systems and showed that the dual-porosity description is recovered via homogenization. These developments provide the equations governing flow in both the matrix and fractures and demonstrate the local character of matrix-fracture transfer and the global character of flow through the fractures. We use the results of these homogenization procedures to motivate the form of the (dual-porosity) coarse-scale model used in this work.
Existing dual-porosity representations can be used to model large-scale flow through (connected) naturally fractured systems and have proved useful in many settings. However, there are a number of approximations commonly used in these models that are not always appropriate. For example, a clear link between a particular discrete fracture characterization and the corresponding dual-porosity representation is not always established (meaning that systematic procedures for determining dual-porosity parameters from discrete fracture models are lacking). In addition, many dual-porosity implementations neglect spatial variation within local matrix regions; i.e., they model pressure and saturation as constant within the matrix (we refer to formulations of this type as “standard dual-porosity models”). These assumptions are justified when spatial variations in pressure and saturation in the matrix are small, but in other cases this will lead to inaccuracy.

To improve the representation of matrix-fracture transfer for complex fracture characterizations, Bourbiaux et al. (1998) developed a technique to evaluate the size of the matrix block using an optimization process. Their intent was to find the optimal equivalent block size to provide the same imbibition behavior as the underlying fractured media. Along similar lines, Sarda et al. (2002) applied an unstructured approach where a matrix volume was defined around each fracture using a distance criterion. In this case the fracture network was represented explicitly. Though useful, these approaches are based on some simplified assumptions, e.g., the fractures in 3D are purely vertical, the matrix medium is assumed to have constant petrophysical properties, and the invasion of displacing fluid into the matrix is piston-like.

The need for improved transfer functions may arise because the local matrix properties (such as pressure and saturation) cannot be assumed to be uniform; i.e., spatial
variability must be modeled. The underlying matrix flow dynamics can be approximately captured via time or saturation dependent transfer functions. Procedures along these lines were introduced by, among others, Dykhuizen (1990), Zimmerman et al. (1993), Penuela et al. (2002) and Sarma and Aziz (2004). Variable transfer functions are adequate for modeling imbibition processes, though gravity effects require additional consideration due to the directionality of the flow. Pseudo-capillary pressure has been used to account for gravity effects without including the gravity term explicitly in the flow equations (Coats et al., 1971; Rossen and Shen, 1987; Dean and Lo, 1986). This can account for gravitational effects to some extent but it is not always accurate. In addition, this treatment can impact the accuracy of the models for other processes such as imbibition or drainage.

The approaches described thus far are efficient as they maintain the same number of unknowns as the standard dual-porosity model. With proper definition of the transfer and pseudo functions, they can provide accurate flow models. However, these approaches lack full generality as different transfer functions or pseudo-capillary pressures need to be defined for different fracture distributions and flow types.

A more general (but computationally more expensive) approach to capture spatial variability in the matrix is to resolve the matrix using a subgrid. Different types of subgridding have been developed for different types of flow. Pruess and Narasimhan (1985) and Wu and Pruess (1988) proposed a nested grid (multiple interacting continua or MINC model) to simulate fluid and heat flow. This type of grid is appropriate for flow processes such as imbibition and heat transfer but is not ideal for capturing flow with specific directionality, such as gravity-driven flow. To account for phase segregation, other subgridding methods (e.g., stacked blocks) have been
proposed (Gilman, 1986; Gilman and Kazemi, 1988). Different types of subgrids can be used in the same model. For example, Beckner et al. (1991) combined nested and stacked grids to accurately model imbibition and gravity-driven flow.

1.3 Dual-Porosity/Dual-Permeability Model

The dual-porosity models discussed above do not represent interblock matrix-matrix flow. This approximation is reasonable when large-scale flow is solely through the fractures. When matrix-matrix interblock flow is significant (Type-III fractured reservoirs as classified by Nelson, 2001), and must therefore be included in the model, we require a dual-porosity/dual-permeability representation. Models of this type were first introduced by Blaskovich et al. (1983) and Hill and Thomas (1985). By adding the matrix to matrix connections, the matrix blocks are no longer isolated, and contribute to the overall fluid flow. Being more general than the dual-porosity model, which is limited to strongly connected fractured reservoirs, the dual-porosity/dual-permeability model is capable of simulating a wide variety of problems ranging from slightly fractured to highly fractured systems.

Because this model accounts for matrix-matrix connections, flow between matrix blocks due to phase segregation can also be modeled, as described by Blaskovich et al. (1983) and Hill and Thomas (1985). Gilman and Kazemi (1988) presented a modified form of the dual-porosity/dual-permeability approach with fine gridding and matrix connections only in the vertical direction to account for gravity effects.
1.4 Motivation for this Work

Flow simulators of the dual-porosity or dual-porosity/dual-permeability type have been widely developed and are commonly used in industry. These approaches are, however, usually based on idealized fracture distributions. The actual geological description of the fracture network is seldom considered in full detail in the evaluation of the dual-porosity parameters. This issue has been addressed to varying degrees by previous investigators (e.g., Cacas et al., 1990; Bourbiaux et al., 1998; Bourbiaux et al., 2002). In these papers, methods to create the link between the geological model and continuum flow models were proposed. Cacas et al. (1990) demonstrated an approach for estimating the large-scale permeability and dual-porosity parameters of fractured rock using statistical data on network geometry in combination with a model calibration using small-scale hydraulic and tracer tests. These approaches maintain the link to geology to some extent but they lead to large models due to the complexity of the fracture network. In addition they do not usually account for the spatial variability of flow in the matrix. A number of other investigators (e.g., Snow, 1969; Long et al., 1985; Lee et al., 2001; Bogdanov et al., 2003) computed effective permeabilities for fractured systems. In these studies, however, the parameters required for dual-porosity representations were not determined.

A more accurate but also more expensive procedure is to combine a discrete fracture network model with the dual-porosity/dual-permeability concept. This idea has been explored by Dershowitz et al. (2000) and Sarda et al. (2002) for the case of single-phase flow. They combined a discrete fracture network model (explicit representation of the fractures) with a dual-porosity concept to account for the matrix contribution. In a recent paper by Uba et al. (2007), an application of a dual-porosity/dual-permeability
1.4 Motivation for this Work

representation of large-scale fractures to the simulation of a giant carbonate reservoir was discussed. The main idea in their approach is to compute equivalent dual-porosity/dual-permeability parameters of the large-scale fractures from explicit fracture descriptions using the DFM simulator of Sarda et al. (2002). However, only a very limited number of fractures were considered in this work.

As is clear from the discussion above, the geological complexity of fractured reservoirs requires the use of simplified models for practical flow simulation. This is often addressed in practice by using flow modeling procedures based on the dual-porosity/dual-permeability concept. However, in most existing approaches, there is not a systematic and quantitative link between the underlying discrete fracture models and the parameters appearing in the flow model. In fact, in his recent review of flow and transport in fractured systems, Berkowitz (2002) identified the issue of appropriate integration of continuum and discrete fracture models (DFMs) as an important open question.

The goal of this work is to develop and apply a systematic procedure for upscaling a fine-scale fracture description to a continuum flow simulation model. Approaches such as this are required in order to take full advantage, for purposes of flow simulation, of the detailed fracture models that recent measurement and modeling techniques are able to generate. The methods developed here provide a general framework for constructing effective fractured reservoir models that are capable of handing viscous, capillary and gravitational flow effects for any given fracture description.
We refer to our upscaling methodology as the “multiple subregion” (MSR) method. This method incorporates a systematic approach for developing coarse-scale continuum models from detailed fracture descriptions. The coarse models are in the form of a generalized dual-porosity or dual-porosity/dual-permeability representation. Local subgrids are introduced to resolve dynamics within the matrix. The method provides appropriate coarse-scale parameters describing fracture-fracture, matrix-fracture and matrix-matrix flow. The geometry of the local subgrids, as well as the required parameters (transmissibilities) for the coarse-scale model, are determined from local flow solutions using the underlying discrete fracture model.

The MSR model is simulated using Stanford’s General Purpose Research Simulator (GPRS), with the connections defined using a connectivity list. To account for gravity effects, the local problems are solved using different boundary conditions and the matrix-matrix connections between vertically neighboring coarse blocks are included. This dual-porosity/dual-permeability type of connection list allows us to model fluid flow with combined viscous and gravitational forces.

The MSR method is applied to simulate 2D and 3D fracture models, with viscous, gravitational and capillary pressure effects. It is also applied to 3D models for compositional simulation of a first-contact miscible gas injection process with six components. The MSR is shown to provide accurate coarse-scale models in close agreement with the underlying discrete fracture model at much less computational cost. Speedup factors of about 100 are typically observed for oil-water simulations and about 1000 for compositional simulations.
We also introduce a “hybrid” treatment that combines DFM with MSR. This treatment is appropriate for systems containing both connected and disconnected fractures. Similar formulations have been proposed previously, though the method presented here appears to be the most comprehensive to date. For example, Lee et al. (2001) proposed a hierarchical approach for modeling fluid flow in a naturally fractured reservoir with multiple length-scale fractures. In their approach, long fractures were modeled explicitly and the permeability contribution (single-porosity) from short fractures was determined analytically. Voelker et al. (2003) presented an application of multiple point geostatistical modeling combined with DFM for a fractured reservoir field study. Training images of facies were developed based on conceptual depositional models of the reservoir, and simulation of flow in the discrete fractures was performed for well blocks. These hybrid models aim to capture geological elements with different length scales in a hierarchical way, thus maintaining both efficiency and accuracy. In our work, we use DFM to model regions that require higher resolution (well blocks and/or disconnected fractures) and MSR to model regions where fractures dominate the fluid flow (i.e., strongly connected fractures). The DFM/MSR hybrid approach shows a significant advantage in modeling systems containing both highly connected and isolated fractures.

1.5 Dissertation Outline

This dissertation proceeds as follows. Chapter 2 presents the basic multiple subregion (MSR) method for the generation of coarse-scale continuum flow models from detailed fracture characterizations. The method developed in this chapter is appropriate for systems with weak or zero gravitational effects. The procedure
involves two steps. First, the geometry of the subregions and the transmissibilities linking them are determined by solving local flow problems (using the DFM) within each coarse block. The second step in the procedure accounts for the flow between coarse blocks and entails the determination of the upscaled interblock transmissibility. All connections in the coarse-scale model are characterized in terms of upscaled transmissibilities, and the resulting coarse model can be used with any connection list-based reservoir simulator. The method is applied to 2D and 3D single-phase, two-phase and three-phase flow problems and the accuracy of the coarse models is assessed relative to fully resolved discrete fracture simulations. For the cases considered, it is shown that the technique is capable of generating highly accurate coarse models with many fewer unknowns than the detailed characterizations. As indicated above, speedups of about a factor of 100 are achieved. The work presented in Chapter 2 was recently published in *Water Resources Research* (Karimi-Fard et al., 2006).

In Chapter 3, the MSR method described in Chapter 2 is extended to construct a generalized dual-porosity/dual-permeability model. This entails re-defining the local flow problems and modifying the connection list. With this extension, the matrix subregions are connected to matrix in vertically adjacent blocks (as in a dual-permeability model) to capture phase segregation due to gravity. Such linkages are not included in the MSR procedure in Chapter 2. Two-block problems are again solved to provide fracture-fracture flow effects. Several two-phase flow examples involving strong gravity effects are considered and the approach is again shown to provide a high degree of accuracy and efficiency. This work will be published in *SPE Journal* (Gong et al., 2007).
In Chapter 4, compositional simulations for a six-component miscible gas injection process are performed for 3D connected and disconnected fracture systems. This work was performed with Dr. Mun-Hong (Robin) Hui from Chevron Energy Technology Company. For this application, the MSR method is further extended to a full dual-porosity/dual-permeability formulation and global single-phase flow information is used to compute the upscaled interblock transmissibilities. The resulting MSR models are applied for compositional simulations using two of the 3D fracture systems considered in Chapters 2 and 3. The MSR models are shown to provide results in reasonable agreement with the DFM but with very substantial computational speedups (typically of order 1,000). This suggests that the general MSR procedure can be applied to practical cases involving complex displacement processes. This work will be presented at the 2007 SPE Annual Technical Conference and Exhibition (Hui et al., 2007).

Chapter 5 introduces a hybrid method that allows some regions of the model to be treated using DFM and others with the MSR method. As an example, we use the MSR method to model portions of the reservoir where fractures are well connected and the DFM for regions where fractures are sparse and not strongly connected. This hybrid approach can be implemented naturally into our flow simulator using the connection list approach. Simulation results demonstrate the capabilities and accuracy of the hybrid formulation for oil-water and compositional models.

In Chapter 6, we draw conclusions and present our recommendations for future research in fractured reservoir modeling and upscaling.

Appendix A provides a brief overview of the software developed in this work.
Chapter 2

Generation of Coarse-Scale Continuum Flow Models from Detailed Fracture Characterizations

2.1 Introduction

The determination of accurate and efficient flow models from detailed characterizations of fractured reservoirs represents a key challenge. Existing flow models include continuum representations (such as the dual-porosity model) and fully-resolved discrete fracture models. The dual-porosity model is prevalent in industry because of its simplicity, though its link to the underlying discrete fracture characterization is often tenuous. DFM is the most accurate approach for fractured reservoir modeling, though it suffers from excessive computational demands.

In this chapter, a systematic methodology for constructing an upscaled model from a detailed, geometrically complex fracture characterization is developed and applied. This approach, referred to as the “multiple subregion” (MSR) method, establishes an explicit link between discrete fracture characterizations and the dual-porosity concept. By introducing local subregions, the upscaled coarse model is in the form of a
“generalized” dual-porosity representation, in which matrix rock and fractures exchange fluid locally while large-scale flow occurs through the fracture network. This methodology yields a connection list that includes all of the internal and interblock transmissibilities, which can then be input directly to a simulator such as GPRS.

This chapter proceeds as follows. First, relevant homogenization results and the dual-porosity formulation are discussed. Our overall procedure is outlined and the difference between the MSR approach and the standard dual-porosity model is addressed. Then the governing equations and the discrete fracture model used for the local simulations are presented. Next, the upscaling procedure is derived in detail. Finally, the MSR upscaling formulation is applied to several flow models, including a 2D compressible single-phase flow case, two-phase flow examples in 2D and 3D, and a 3D three-phase flow example.

2.2 Homogenization Results and Dual-Porosity Representation of the Coarse Model

The approach described in this work is best viewed as an upscaling procedure in which we coarsen the fine-scale (discrete fracture) model into a coarse-scale (continuum) model. There are several issues to consider in any upscaling procedure. These include (1) the form of the coarse-scale equations, which dictates the upscaled parameters that must be computed (in general, the form of the coarse-scale equations may differ from that of the fine-scale equations), (2) the domain to be used for the determination of the upscaled parameters (e.g., local or global) and (3) the boundary
2.2 Homogenization Results and Dual-Porosity Representation of the Coarse Model

conditions and post-processing to be applied in the computation of the upscaled parameters. Upscaling procedures for porous media flow problems are related to finite element- and finite volume-based multi-scale methods but differ in some important aspects. For a discussion of the similarities and differences between upscaling and multiscale procedures, see Gerritsen and Durlofsky (2005). Upscaling techniques for non-fractured systems are discussed in detail in recent reviews (e.g., Farmer, 2002; Gerritsen and Durlofsky, 2005).

![Matrix and Fracture]

**Figure 2.1**: Illustration of an idealized periodic fractured system

Homogenization procedures have been applied for flow modeling of both non-fractured and fractured systems, and are very useful for providing the form of the upscaled model. We focus here on fractured systems. In a series of investigations, Douglas and Arbogast (1990) and Arbogast (1993) developed homogenized models for single- and two-phase flow by considering individual matrix-fracture blocks to be periodically replicated in space (the assumption of periodicity is common in homogenization procedures) and to be of a size $\varepsilon$ relative to the global domain (of size...
1), with $\varepsilon \ll 1$, indicating that there are many repetitions of the basic matrix-fracture unit. A schematic illustration of such a system is shown in Figure 2.1. Application of homogenization theory was shown by Douglas and Arbogast (1990) to provide equations of the same form as the standard dual-porosity model, usually derived through application of physical arguments. Specifically, for the case of compressible single-phase flow, the governing fine-scale equation is:

$$
\frac{\phi}{c} \frac{dt}{dp} - \nabla \cdot \left( \frac{k \rho}{\mu} \nabla p \right) = q
$$

(2.1)

where $\rho$ is density, $p$ is pressure, $\phi$ is porosity, $k$ is permeability, $\mu$ is viscosity and $q$ is the external (e.g., well) source term. Note that $\frac{dp}{c \rho dp}$, where $c$ is compressibility. Equation 2.1 applies for flow in both the fractures and the matrix; i.e., the flow in the fractures is assumed to be governed by Darcy’s law. This assumption is commonly used, though it is not always appropriate (Berkowitz, 2002).

Homogenized models of equation 2.1 proceed by introducing expansions for the dependent variables $p$ and $\rho$ in terms of the small parameter $\varepsilon$, for both the matrix and the fractures. The fine scale (with variations on the scale of $\varepsilon$) is designated $y$ and the coarse scale (global scale) is designated $x$. Denoting pressure in the fractures and matrix as $p_f$ and $p_m$ (and similarly for other variables), we write $p_f = p_f^0 + \varepsilon p_f^1 + \cdots$, etc. The homogenized model for this system is (Douglas and Arbogast, 1990; Arbogast, 1993):
2.2 Homogenization Results and Dual-Porosity Representation of the Coarse Model

\[
\phi_m \frac{\partial P_m^0}{\partial t} - \nabla_y \cdot \left( \frac{k_m P_m^0}{\mu} \nabla_y P_m^0 \right) = 0 \quad \text{(matrix)} \tag{2.2}
\]

\[
\phi_f \frac{\partial P_f^0}{\partial t} - \nabla_x \cdot \left( \frac{k_f P_f^0}{\mu} \nabla_x P_f^0 \right) = q + \tau \quad \text{(fractures)} \tag{2.3}
\]

\[
\tau = -\frac{1}{V} \int_{V_m} \phi_m \frac{\partial P_m^0}{\partial t} \, dy \quad \text{(transfer term)} \tag{2.4}
\]

where \( \nabla_y \) and \( \nabla_x \) are fine- and coarse-scale operators, respectively, the superscript 0 designates the leading \( O(1) \) term (which is the term of interest here), \( V \) designates the volume of the unit cell (and \( V_m \) the matrix volume) and \( k_m \) and \( k_f \) represent effective (upscaled) matrix and fracture permeabilities. At the matrix-fracture interface, \( P_m^0 = P_f^0 \). These equations differ from the fine-scale equations, with the key difference being that the matrix equations are solved on the local \( y \) scale and the fracture equations on the coarse global scale \( x \). In other words, a dual-porosity model results from the application of homogenization theory, in which the large-scale flow (on the scale of \( x \)) occurs only through the fractures. The matrix and fractures interact through the integral term in equation 2.4, governed by the solution of the local matrix flow equation.

Analogous results were obtained by Douglas and Arbogast (1990) and Arbogast (1993) for two-phase flow systems. Specifically, again in direct analogy to dual-porosity models, they showed that the homogenized model contains fracture and matrix equations (for both phases), with the matrix equations defined over the local matrix region and the fracture equations acting globally. In cases where matrix regions are very small (densely fractured systems), they showed that the matrix equations can be
approximated via the assumption of constant pressure and saturation in the local region, but in more general cases (larger matrix regions) the two-phase matrix flow equations must be solved numerically.

These homogenization results are for an idealized case, but they are useful as they provide the form for the coarse-scale (or effective) model. Our coarse-scale model is in fact of the form of equations 2.2, 2.3 and 2.4. Specifically, we construct a model in which the matrix and fractures exchange fluid locally while large-scale flow occurs through the fractures. As our procedure starts with a detailed discrete fracture model, which is much more complex geometrically than the idealized system illustrated in Figure 2.1, there are a number of numerical issues that must be addressed. As is the case with coarse-scale models for other porous media flows, homogenization theory provides the general form of the model, but issues pertaining to the numerical representation of the various terms are outside the scope of homogenization theory itself.

Our overall procedure can be outlined as follows. Starting with a general discrete fracture model that we wish to upscale (i.e., model via a continuum description on a coarse scale), the first step is to form the coarse grid. Ideally, this would be done in such a manner that the matrix and fractures contained within each grid block form a closed system; i.e., the fractures in (and bordering) the block drain only the matrix rock contained within the block. In this case, there would be no flow from the matrix within this block to any other block and the model would conform to the assumptions used in the homogenization procedure. Given a general fracture characterization, it may be possible to generate a grid that approximately satisfies this condition, but the grid would be unstructured with very general shaped cells, which would in turn lead to
2.2 Homogenization Results and Dual-Porosity Representation of the Coarse Model

a number of numerical discretization issues. Rather than proceeding in this way, we impose a structured Cartesian grid on the system and then, in the determination of the matrix-fracture and matrix-matrix interactions (described in detail in the next section), specify boundary conditions that restrict these flows to occur only within the target grid block. Large-scale flow occurs from grid block to grid block and is modeled via an upscaled transmissibility which captures the effective fracture permeability.

Before describing the specific numerical procedures, it is worthwhile highlighting how our approach differs from standard dual-porosity modeling. In the simplest approaches, the matrix is represented as being of uniform pressure and saturation. In this case, each matrix region is essentially a tank and the transfer term $\tau$ can be approximated via:

$$\tau = \frac{k_m p_m \sigma}{\mu} \left( p_f - p_m \right)$$

where $\sigma$ is the so-called shape factor which depends only on fracture geometry. For two-phase flow, we have a $\tau$ for each phase; e.g., for water (Kazemi et al., 1976):

$$\tau_w = \frac{k_m k_{rw} (S_m) p_m^w \sigma}{\mu_w} \left( p_f^w - p_m^w \right)$$

where $k_{rw}(S_m)$ is the relative permeability to water (which is a function of water saturation in the matrix, $S_m$, assumed constant) and $p_f^w$ and $p_m^w$ are the water pressures in the fracture and matrix. An analogous transfer function, $\tau_o$, is defined for the oil phase.
Models of this type are well-suited for some purposes but are limited in their ability to resolve transient and multiphase flow phenomena, as spatial variation within the matrix is not modeled. These effects can be approximated by introducing additional time or saturation dependencies into the models for $\tau_w$ and $\tau_o$. Such approaches have been successfully applied in a number of cases (Dykhuizen, 1990; Zimmerman et al., 1993; Penuela et al., 2002; Sarma and Aziz, 2004), but they require that new transfer functions be determined when new physics is introduced into the problem. This may pose challenges for complicated systems involving, for example, compositional effects or many different types of fractures. The approach applied here is quite general in that we actually solve the equations governing matrix flow; i.e., the multiphase analogs of equations 2.2 and 2.3. Our method can thus be applied with any level of physics, though higher resolution may be required for the solution of the matrix flow problem in complex settings. The disadvantage of our approach is that more unknowns will appear than would be required if we had access to a “perfect” transfer function. In any event, this approach represents an alternative to the use of complex transfer functions and as such may be useful in a variety of applications.

As indicated in Chapera 1, previous investigators (e.g., Pruess and Narasimhan, 1985; Gilman, 1986; Wu and Pruess, 1988) also introduced spatial discretization into the matrix flow problem. Our approach differs from these earlier efforts in that we start with a specific discrete fracture model rather than an idealized representation and we determine the subregion geometries (i.e., the grids for matrix flow) and the properties of the model through solution of appropriate local flow problems using the discrete fracture representation.
2.3 Governing Equations and Discrete Fracture Model

The equations describing compressible two-phase flow in porous media are obtained by writing an equation of the form of equation 2.1 for each phase (designated $o$ and $w$):

$$\frac{\partial (\phi p_o S_o)}{\partial t} = \nabla \cdot \left( \frac{\rho_o k_{ro}}{\mu_o} \nabla p_o \right) + q_o$$  \hspace{1cm} (2.7)

$$\frac{\partial (\phi p_w S_w)}{\partial t} = \nabla \cdot \left( \frac{\rho_w k_{rw}}{\mu_w} \nabla p_w \right) + q_w$$  \hspace{1cm} (2.8)

where all variables are as defined previously. The full description also requires the saturation constraint $S_o + S_w = 1$, and the capillary pressure relation, in which the pressure difference between the phases is defined as a function of saturation, $p_o - p_w = p_c(S)$. For simplicity, in our fine-scale computations permeability is taken to be locally isotropic and equal to either $k_f$ or $k_m$ (both constants), though this is not a requirement of the method.

This set of equations applies as written for the fully resolved model (again we assume flow in the fractures can be modeled via a Darcy’s law description). We also solve these equations locally for the determination of the upscaled model parameters. In our upscaled model, by contrast, we solve similar equations but in a dual-porosity formulation in which the matrix flow is localized (internal to the coarse block) and large-scale flow occurs only through fractures.
For simulations of the fully resolved (fine-scale) model and the local upscaling calculations (i.e., solutions of equations 2.7 and 2.8), any discrete fracture simulation procedure (e.g., Bogdanov et al., 2003; Monteagudo and Firoozabadi, 2004; Matthai et al., 2005; Hoteit and Firoozabadi, 2005) could be applied. In this work we use a recently developed finite-volume-based discrete fracture model, presented by Karimi-Fard et al. (2004) and also described (for a different application involving flow in systems characterized by thin but extensive low-permeability compaction bands, which act as “anti-fractures”) in Sternlof et al. (2006). The technique can handle unstructured 2D and 3D grids and can thus capture accurately the geometry of the fracture network. An advantage of this approach is that it can model the fractures using control volumes that are of the same thickness as the fracture; i.e., the fracture aperture need not be resolved by the grid (similar ideas have been used within finite-element or control volume finite-element contexts; see e.g., Bogdanov et al., 2003; Monteagudo and Firoozabadi, 2004; Hoteit and Firoozabadi, 2005). This reduces the overall number of cells and simplifies considerably the gridding procedure, especially in three dimensions. In addition, the very small control volumes that appear at fracture intersections are eliminated using a “star-delta” connectivity transformation as described in Karimi-Fard et al. (2004). This improves substantially the numerical stability and time step size for IMPES (implicit pressure, explicit saturation) techniques.

In finite-volume procedures, each control volume is characterized by its bulk volume and porosity. The discretized flow terms can be represented in terms of a list of connected control volumes. Connections are quantified by the cell to cell transmissibility, which relates the flow rate to the difference in cell pressures:
2.4 Upscaling Technique

\[ Q_{l,ij} = T_{ij} \rho_l \lambda_i \left( p_{l,i} - p_{l,j} \right) \]  

Here \( Q_{l,ij} \) is the mass flow rate of phase \( l \) (\( l = o, w \)) from cell \( i \) to cell \( j \), \( p_{l,i} \) is the pressure of phase \( l \) in cell \( i \), \( T_{ij} \) is the rock and geometric part of the transmissibility (commonly referred to simply as transmissibility), \( \rho_l \) is phase density and \( \lambda_i = k_{ri} / \mu_i \) represents the phase mobility (based on upstream information). Note that, in the case of multiphase flow, although \( \lambda_i \) is different for each phase, \( T_{ij} \) is the same for each phase and is provided by the discretization technique. Integrating equations 2.7 and 2.8 over each control volume and expressing flow rates using equation 2.9 provides the discrete form of the flow equations, which are solved to obtain the pressure and saturation. A general purpose research simulator (GPRS) originally developed by Cao (2002) is used to perform the flow simulations.

2.4 Upscaling Technique

For now we consider the fine model to be a fracture network that is well connected over the entire domain (or over significant portions of the domain) and the associated matrix rock. Fracture permeability is taken to be large compared to typical matrix permeability. As discussed in Section 2.4.3, limited regions of disconnected fractures can be handled within the general procedure, though the treatment in such regions will be more approximate. The objective of the upscaling procedure is to construct a coarse model that provides approximately the same flow behavior (e.g., approximately the same flow rates and phase fractions for wells operating at prescribed pressure or flow rate) as the original DFM. As motivated by homogenization results for fractured
systems (discussed in detail in Section 2.2), the coarse model here is a dual-porosity description with flow in the matrix resolved spatially. The coarse model is therefore described by equations of the general form of equations 2.7 and 2.8, though the connectivity of the coarse-grid blocks is modified to represent the dual-porosity character of the coarse system.

**Figure 2.2**: Schematic of the connectivity list for a 2D Cartesian model showing a cell (block 1) and surrounding neighbors. Heavier lines represent block to block fracture connections, while lighter lines depict internal or subgrid (fracture-matrix and matrix-matrix) connections.

Before describing the determination of the coarse-model parameters, it is instructive to consider the implications of the dual-porosity description on the connectivity of the discrete model. This is illustrated schematically in Figure 2.2 for a single-phase flow problem solved on a structured 2D grid (for which a single-porosity description leads to the usual five-point finite difference stencil). The heavier lines represent fracture-
fracture (F-F) connections and the finer lines fracture-matrix (F-M) and matrix-matrix (M-M) connections. In the figure, the superscript on M denotes the coarse block to which the subregion corresponds while the subscript designates the subregion number, with the first subregion corresponding to the fractures. Due to the dual-porosity nature of the model, the fracture-matrix and matrix-matrix connections are internal to each coarse block. The fracture-fracture connections, by contrast, link adjacent grid blocks and thus enable large-scale flow to occur. The goal of our upscaling procedure is to determine the parameters required for a coarse model in the form illustrated in Figure 2.2. Because flow is still simulated using a finite-volume scheme, the required parameters are the coarse-scale transmissibilities (quantifying fracture-fracture, fracture-matrix and matrix-matrix flow), geometrical quantities (e.g., bulk and pore volumes) and the topology (connectivity) of the coarse grid (described in terms of the connection list).

Prior to the upscaling calculations, an unstructured grid, based on triangular elements in two dimensions, and tetrahedral elements in three dimensions, is generated to resolve the discrete fracture description. This is accomplished using standard constrained Delaunay grid generation procedures (Shewchuk, 1996; Si, 2005). A coarse grid is then defined over the detailed model. As mentioned before, in this chapter we will only consider Cartesian coarse grids, though this is not required by the methodology. The upscaling technique presented here has two distinct steps. In the first step, the fractures and matrix internal to each coarse block are considered in isolation to determine the local flow exchange between the fracture network and the matrix. In the second step, the connections between coarse blocks are computed. We reiterate that only local simulations are required for the upscaling. We now describe the detailed procedures.
2.4.1 Internal Fracture-Matrix and Matrix-Matrix Connections

A given coarse block typically contains a highly permeable network of fractures embedded in a matrix (which can be homogeneous or heterogeneous). To capture accurately the exchange between the fracture and the matrix and the transient effects that occur internal to the coarse block, we resolve flow internal to the block using a flow-based subgridding technique. This grid comprises a number of subregions, which are linked together in sequence as shown in Figure 2.2. As a result of this construction, the internal connections are always logically one-dimensional regardless of the dimensionality of the original problem (by logically one-dimensional we mean that the internal connections can be represented via a linear sequence or linear connectivity list).

As this local solution provides the connections (transmissibilities) involving the fracture network and matrix internal to the coarse block, and because this involves transient effects, we determine these connections from the solution of a compressible single-phase flow equation:

$$\frac{\phi c}{\partial t} = \nabla \cdot \left( \frac{k}{\mu} \nabla p \right)$$

(2.10)

where $c$ is constant. This solution is performed using the discrete fracture model described previously. The upscaling calculations will be illustrated using the example fracture distribution shown in Figure 2.3. Equation 2.10 is solved inside the coarse-block region with impermeable (zero flux) boundaries surrounding the block and a constant injection rate at a point inside the fracture network. The no-flux boundary
condition is consistent with the fact that we view the coarse block in isolation at this point; i.e., there is no exchange between coarse blocks through the matrix (see discussion in Section 2.2). As the system is isolated, the overall pressure of the block will increase with time and, after a transient period, will reach a pseudo-steady state profile. For this problem (fixed injection rate, no-flux boundary conditions), at pseudo-steady state (also referred to as semi-steady state or depletion state) \( \frac{\partial p}{\partial t} \) will reach a constant value that depends only on the injection rate, compressibility and pore volume (Dake, 1978; Joshi, 1991). As shown in Figure 2.3, due to the high permeability of the fracture network, the pressure everywhere inside the fractures is approximately the same and the pressure variation inside the matrix behaves like a diffusion process. The shapes of the iso-pressure curves at pseudo-steady state depend only on the fracture geometry and permeability variation within the coarse block and are independent of the injection rate and fluid properties.
Figure 2.3: Determination of subregions using iso-pressures. Equation 2.10 is solved locally and the iso-pressure curves are constructed from the pressure solution. The resulting subregions and their connections are illustrated in the right column. The first subregion is always associated with the fractures (top right)
We use this pressure solution for the construction of the multiple subregion model. This entails both the determination of the subregion geometry and the transmissibilities describing flow from one subregion to another. The subregion geometry is formed from the iso-pressure contours from the local fine-grid solution of equation 2.10. These curves are grouped into \( n \) non-overlapping subregions. The use of these contours provides a natural one-dimensional grid for this problem as there is no flow along the iso-pressure curves defining the grid block boundaries. This enables the accurate and efficient reproduction of the local solution.

Figure 2.3 (right column) depicts five subregions, with the fracture subregion in the uppermost figure and the other figures representing four matrix subregions. Note that the subregions are in general disconnected, as they are formed from iso-pressure curves, which are themselves disconnected. This does not represent any problem for the method and is due to the complexity of the underlying fracture description. If we take the coarse block to be bordered by fractures, with no internal fractures present (as illustrated in Figure 2.1, assuming here that the coarse block is of size \( \varepsilon \)), the subregion geometries will be essentially concentric (rounded) “squares.”

Our specific approach for forming the subregions is as follows. The coarse block \( \Omega^k \) is subdivided into \( n \) non-overlapping subregions designated \( \Omega^k_i \),

\[
\Omega^k = \bigcup_{i=1}^{n} \Omega^k_i \tag{2.11}
\]

where the superscript \( k \) designates the coarse block and the subscript \( i \) refers to the subregion. This subdivision is performed in a straightforward manner by sorting all of
the fine-grid cells in block $k$ according to their pressure values from $p_{\text{max}}^k$ to $p_{\text{min}}^k$ (the maximum and the minimum value of the pressure in coarse block $\Omega^k$). The first subregion $\Omega_1^k$ is constructed from the fine-grid fracture cells. As the solution is obtained by injecting fluid inside the fracture network, the subregion $\Omega_1^k$ has the highest average pressure. The remaining matrix cells are subdivided into $(n-1)$ groups defining $(n-1)$ additional subregions. The iso-pressures defining the borders of each subregion are obtained by applying a simple optimization procedure that minimizes the pressure variance inside each subregion. This will provide a reasonable (optimal or near optimal) set of subregions. Higher accuracy can be achieved by increasing the number of subregions.

The general approach is adaptive, meaning that the number of subregions can vary from block to block. This allows for the use of more resolution where required and leads to enhanced computational efficiency. In the results presented below, such adaptivity is applied in some of the coarse models. In these models, the number of subregions used for each block is determined from the variation in pressure observed for the block. Those blocks displaying the maximum pressure variation are modeled with the maximum number of subregions while those with the minimum pressure variation are modeled with the minimum number. A linear interpolation is used to determine the number of subregions for blocks displaying intermediate degrees of pressure variation.

The bulk volume $V_i^k$ of each subregion as well as the average porosity can be computed once the $\Omega_i^k$ are determined. These are given by:
2.4 Upscaling Technique

\[ V_i^k = \sum_{j \in \Omega_i^k} v_j \]  
\[ \overline{\phi}_i^k = \frac{\sum_{j \in \Omega_i^k} v_j \phi_j}{\sum_{j \in \Omega_i^k} v_j} \]  

where \( j \) designates the fine-scale cells associated with subregion \( \Omega_i^k \). For the determination of the subregion transmissibilities, other variables such as the average pressure \( \overline{p}_i^k \) and density \( \overline{\rho}_i^k \) must also be computed (\( \mu \) is assumed constant). For pressure, this is accomplished using an equation of the form of equation 2.13 with \( p_j \) replacing \( \phi_j \). For density, we apply a pore-volume (rather than bulk-volume) weighted average:

\[ \overline{\rho}_i^k = \frac{\sum_{j \in \Omega_i^k} v_j \phi_j \rho_j}{\sum_{j \in \Omega_i^k} v_j \phi_j} \]  

For the subregion transmissibility calculation we also need to determine the mass accumulation within each subregion at pseudo-steady state. This quantity, designated \( A \), is computed via:

\[ A_i^t = \sum_{j \in \Omega_i^t} v_j \phi_j \frac{\partial \rho_j}{\partial t} \]  

At pseudo-steady state, \( \partial \rho / \partial t \) is constant and the accumulation term is proportional to the pore volume of the subregion.
Given the mass accumulation term \( A^k_i \) in each subregion and the one-dimensional character of the subregion connectivity, we can evaluate the mass flow rate between two consecutive subregions \( (Q^k_{i,i+1}) \) and can thus compute the transmissibility linking them. It is most natural to start with the last matrix subregion \( n \), as this subregion is connected only to subregion \( (n-1) \). The flow rate between these two subregions is therefore equal to the mass accumulation in subregion \( n \):

\[
Q^k_{n-1,n} = A^k_n
\]  

(2.16)

In the case of \( n = 2 \) (standard dual-porosity model) this is the only flow rate required. For \( n > 2 \), the other inter-subregion flow rates are computed using:

\[
Q^k_{i,i+1} = A^k_i - Q^k_{i+1,i+2} \quad i = 1,2,\cdots,n-2
\]  

(2.17)

which simply states that the net mass flow into (or out of) subregion \( \Omega^k_i \) is balanced by the accumulation term.

As is apparent from our previous discussion and from equation 2.9, transmissibility expresses the block to block mass flow rate in terms of the difference in pressure between the two blocks. From the local solution we have the subregion pressures \( (\bar{P}^k_i \) and \( \bar{P}^k_{i+1} \)) and the flow rates between them \( (Q^k_{i,i+1}) \). We can thus compute upscaled transmissibility using an expression of the form of equation 2.9:
2.4 Upscaling Technique

\[ T_{i,j}^{k} = \frac{Q_{i,j}^{k} \mu}{\overline{\rho}_{i}^{k} (\overline{p}_{i}^{k} - \overline{p}_{j}^{k})} \quad i = 1, 2, \cdots, n - 1 \]  

These transmissibilities (along with the connection list defining the linkages), combined with the associated subregion volumes, \( V_{i}^{k} \), and porosities, \( \overline{\phi}_{i}^{k} \), fully define the local matrix-fracture and matrix-matrix flow model inside each coarse block.

2.4.2 Connections between Coarse Blocks

The global connectivity of the fracture network is captured by connections between the coarse blocks (as illustrated in Figure 2.2). In this work we apply a straightforward two-point transmissibility upscaling which provides accurate results for our problems. If the orientation of the fracture network is systematically skewed to the grid, or if the shapes of coarse blocks are irregular, a more advanced technique (i.e., one compatible with a multipoint flux approximation) may be required. Permeability upscaling and subsequent calculation of upscaled transmissibility (via weighted harmonic averaging of upscaled permeability) could also be applied, though direct transmissibility upscaling has been shown to provide better accuracy for highly heterogeneous systems (Romeu and Noetinger, 1995; Chen et al., 2003).

Figure 2.4 depicts the problem setup for the transmissibility upscaling procedure. The fine-scale region (DFM) associated with two adjacent coarse blocks \( \Omega^{l} \) and \( \Omega^{l'} \) is considered. A steady-state flow problem is solved with a pressure difference imposed between the two boundaries. The average pressure and fluid properties inside each block, as well as the flow rate \( Q^{k,l} \) through the interface between the blocks, are
computed from the local fine-grid solution. The transmissibility can then be determined via:

$$T^{k,l} = \frac{Q^{k,l} \mu}{\rho (\bar{p}^k - \bar{p}^l)}$$  \hspace{1cm} (2.19)

The mass flow rate $Q^{k,l}$ is computed over the entire interface, though it will be dominated by flow through the fractures when the fracture network is connected (as it generally will be for models in which a dual-porosity formulation is applicable). For those blocks in which the fractures are disconnected, this treatment will provide a reasonable approximation for $T^{k,l}$ even in cases when the interblock flow is from the matrix in block $k$ to the matrix in block $l$.

Figure 2.4: Schematic illustrating transmissibility upscaling to account for block to block flow
2.4 Upscaling Technique

These transmissibilities, in addition to the parameters determined in the first step of the upscaling procedure, fully define the coarse continuum flow model. This model is applicable for single-phase flow as well as multiphase flow with capillary pressure effects, as we will see in Section 2.5. The relative permeability and the capillary pressure of the fine-scale problem are used directly (without upscaling) on the coarse scale. In addition, there is no iteration required in the determination of the coarse-grid properties, though comparisons with the fully resolved fine-scale model can be used (when practical) to assess the accuracy of the coarse model.

2.4.3 Additional Numerical Implementation Considerations

The preceding sections described the determination of the coarse-model structure and parameters. Here we briefly consider some aspects of the numerical implementation and address the treatment of coarse blocks with disconnected fractures.

As our emphasis is on densely fractured models, most of the coarse blocks will contain connected fractures. In such cases, at pseudo-steady state the pressure in the fractures is essentially constant in space and the $\frac{\partial p}{\partial t}$ term becomes constant (independent of position) throughout the entire coarse-block region (Dake, 1978; Joshi, 1991). In this case equation 2.10 can be approximated directly via the following Poisson equation:

$$\nabla \cdot \left( k \frac{1}{\mu} \nabla p \right) = F \quad (2.20)$$

where $F$ represents a forcing term that is constant everywhere in the coarse block (without loss of generality, we set $F = 1$ in our solutions). No-flux boundary conditions are again specified on the block boundaries and the pressure in the fractures
is fixed. Solution of equation 2.20 rather than equation 2.10 results in efficiency gains as time-stepping is avoided.

Figure 2.5: Comparison between pressure contours determined from equations 2.10 and 2.20 for one coarse block

To illustrate this point, we now compare the subregions resulting from solution of equations 2.10 and 2.20. This comparison is shown in Figure 2.5. It is clear that the subregions determined from the two approaches are essentially identical, as are the resulting transmissibilities. Thus, in the results presented below, we solve equation 2.20 for the determination of the fracture-matrix and matrix-matrix parameters.

Even in the connected fracture systems for which our approach (and the dual-porosity framework in general) applies, because our input is a discrete fracture characterization,
there are likely to be some coarse blocks with disconnected fractures or no fractures at all. Although our methodology is not specifically designed for models with disconnected fractures, flow in these blocks can be modeled approximately within the general procedure. For blocks in which the fracture network is disconnected, the main effect of the fractures is to enhance overall block permeability, and thus block to block flow. This effect is essentially captured in the upscaled interblock transmissibility $T^{k,j}$ computed via equation 2.19.

Disconnected fractures also interact with the matrix, and our treatment of this interaction, which we approximate via the same dual-porosity representation used for connected fractures, will incur some error. In the results presented in this chapter, we did not introduce any special treatment for these blocks (i.e., we applied the specified number of subregions to these blocks as well as to blocks with connected fractures). However, we did perform tests in which we varied the number of subregions ($n$) for blocks with disconnected fractures (fixing $n$ for blocks with connected fractures) and found that the results were very insensitive to $n$ for blocks with disconnected fractures. This insensitivity may result because blocks with disconnected fractures often have low fracture density, which means that the computed $T^{k,j}$ will be small. Thus, errors in this term may not have much effect on the global results. We note that a much more general (hybrid) treatment for disconnected fractures is presented in Chapter 5.

In the numerical results below, the following notation is used to define the grid for the coarse model. In 2D cases, $n_x \times n_y \times (n)$ represents an $n_x$ by $n_y$ Cartesian grid with $n$ subregions in each block. Analogously, in 3D cases, $n_x \times n_y \times n_z \times (n)$ represents an $n_x$ by $n_y$ by $n_z$ Cartesian grid with $n$ subregions per block. When representing an adaptive
coarse model, the number of subregions will be defined by two integers \((n_{\text{min}} - n_{\text{max}})\) which specify the minimum and the maximum number of subregions. The actual number of subregions for a given block depends on the magnitude of the pressure variation within the block (as computed during the first step of the upscaling) compared to the blocks with the minimum and maximum pressure variations.

### 2.5 Applications

We now demonstrate the accuracy of the upscaling technique via 2D and 3D simulations. The technique is first applied to compressible single-phase and two-phase flows. In all simulations, water is incompressible and the oil phase compressibility \((c_o)\) is \(1.379 \times 10^{-5}\) psi\(^{-1}\). The reference densities of water and oil are \(\rho_w = 62.4\) lb/ft\(^3\) and \(\rho_o = 49.9\) lb/ft\(^3\), and the viscosities are \(\mu_w = 0.55\) cp and \(\mu_o = 1.2\) cp. The matrix rock is characterized by a permeability of \(k_m = 1.6 \times 10^4\) md and a porosity of \(\phi_m = 25\%\). All of the fractures are considered to be identical, though our methodology allows for general property variation. The fracture aperture \(e\) and permeability \(k_f\) are \(e = 0.344\) and \(k_f = 1.6 \times 10^9\) md. These permeability values are much larger than would typically be observed for naturally fractured reservoirs. These large magnitudes are due to a unit conversion error in GPRS which was discovered after all of the simulations were performed (the values input to the simulator were physically realistic, but the unit conversion error resulted in the simulations being performed with the values given here). In any event, the key quantity for our purposes is the ratio \(k_f/k_m (=10^5)\), and this value is physically reasonable. The fractures are considered to be fully open with a porosity of \(\phi_f = 100\%\). Wells in all cases intersect fractures. For two-phase flow
simulations, relative permeability and capillary pressure data are also required. The data for the matrix are summarized in Table 2.1. Two sets of capillary pressure (designated \( p_c^1 \) and \( p_c^2 \)) are considered in order to explore the accuracy of the upscaling technique in different parameter ranges. In all simulations the capillary pressure inside the fractures is neglected. Straight-line relative permeabilities are used in the fractures. Other data can be used for the fractures if available.

**Table 2.1:** Relative permeability and capillary pressure data for the matrix

<table>
<thead>
<tr>
<th>( S_w )</th>
<th>( k_{rw} )</th>
<th>( k_{rm} )</th>
<th>( p_c^1 ) (psi)</th>
<th>( p_c^2 ) (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
<td>200</td>
<td>2.0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0</td>
<td>0.875</td>
<td>50</td>
<td>0.5</td>
</tr>
<tr>
<td>0.4</td>
<td>0.04</td>
<td>0.43</td>
<td>9</td>
<td>0.1</td>
</tr>
<tr>
<td>0.6</td>
<td>0.125</td>
<td>0.1</td>
<td>2</td>
<td>0.04</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3</td>
<td>0.0</td>
<td>0.5</td>
<td>0.02</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**2.5.1 2D Case: Single-Phase and Two-Phase Flow**

Figure 2.6 depicts a 2D (1000×1000 ft\(^2\)) fractured system representing a portion of a model introduced in Lee et al. (2001). It contains 70 fractures and the fine-grid DFM includes 12,280 cells. The cells are composed of 1,976 fracture segments and 10,304 triangular matrix elements. This model is upcaled to a 6×6 coarse grid with different numbers of subregions. Note that some coarse blocks contain isolated or disconnected fractures. A constant number of subregions per block (from 1 to 5), as well as an adaptive number of subregions based on local pressure variation, are considered. The same upscaled model is used for compressible single-phase and two-phase flow.
For the case of compressible single-phase flow, a formation at an initial pressure of 4,000 psi is considered. Oil is produced from a single well at the center of the formation at constant pressure (1,000 psi). The evolution of cumulative oil recovery with time is presented in Figure 2.7. Several coarse models with different numbers of subregions are considered. A single-region model (SR = 1) corresponds to a standard (single-porosity) upscaling technique which accounts only for the connectivity between coarse blocks. A two-subregion treatment (SR = 2) corresponds to a standard dual-porosity model with the transfer function determined from the underlying discrete fracture model.
It is evident that a higher number of subregions acts to more accurately capture the transient effects inside the matrix. As is evident from Figure 2.7, the results obtained by the one-subregion model differ substantially from the reference solution. This is because matrix storage is completely missing from this model. Significant improvement is obtained by the SR = 2 model, illustrating that it is important to model matrix storage effects. By increasing the number of subregions to 5, we obtain very close agreement with the DFM. This illustrates the systematic improvement attainable using our general framework. We note that the importance of transient effects in single-phase flow problems will depend on the complexity of the fracture network, the compressibility of the fluid, and the time scale of observation.
Figure 2.8 displays pressure distributions for the fine and coarse (SR = 5) models after about 0.5 days of production. These distributions correspond to the volume-averaged pressure of the coarse solution (\( \bar{\mathbf{p}}^{k,c} \)) and the volume-averaged pressure of the fine solution (\( \bar{\mathbf{p}}^{k,f} \)), where both averages are computed over regions corresponding to coarse blocks (designated \( k \)) and are given by:

\[
\bar{\mathbf{p}}^{k,c} = \frac{\sum_{j=1}^{n} v_j^k p_j^c}{\sum_{j=1}^{n} v_j^k}, \quad \bar{\mathbf{p}}^{k,f} = \frac{\sum_{j=1}^{N} v_j^k p_j^f}{\sum_{j=1}^{N} v_j^k} \tag{2.21}
\]

where \( n \) is the number of subregions in block \( k \) (in the coarse model), \( N \) is the number of fine-scale control volumes over the region corresponding to coarse block \( k \), and \( p_j^c \) and \( p_j^f \) represent the coarse- and fine-scale pressures respectively. The block edges appear jagged because the blocks are defined in terms of the underlying triangulation, which does not conform exactly to a rectangular grid. There is clearly close agreement between the fine and the coarse results for pressure. The relative pressure error for block \( k \) is given by \( \left| \bar{\mathbf{p}}^{k,f} - \bar{\mathbf{p}}^{k,c} \right| / \bar{\mathbf{p}}^{k,f} \). Computing the \( L_2 \) norm of this error for the results presented in Figure 2.8, we obtain an average coarse-scale relative error of 1.37%, again indicating a high degree of accuracy in the coarse solution.
2.5 Applications

We now consider two-phase flow with capillary pressure. Capillary pressure effects are often very important for flow in fractured porous media as the recovery of oil is driven by imbibition of water into the matrix. The same 2D model considered above is applied here. In this case the domain is initially saturated with oil. Water is injected (100 bbl/day) at the lower left corner and liquid is produced from the upper right corner of the model. Both sets of capillary pressure data from Table 2.1 are used.

**Figure 2.8:** Pressure distributions for the fine model (left) and coarse model with SR=5

![Average pressure](image)
Figure 2.9: Oil recovery for fine-grid solution and several coarse models including the adaptive model for strong capillary pressure ($p_c^1$) case.
Figures 2.9 and 2.10 display cumulative recovery as a function of injected water for each set of capillary pressure data. Similar to the single-phase compressible flow case, we observe that the single-porosity ($SR = 1$) model does not capture the physics of the flow and provides inaccurate results. In this case even a dual-porosity model is not very accurate. The systematic improvement in accuracy offered by increasing the number of subregions is clearly apparent in both figures. This demonstrates the general efficacy of our approach and the benefit of multiple subregions in coarse-grid models. We also observe that the adaptive approach (in this case a $6 \times 6 \times (2-5)$ model, with a total of 224 unknowns) is more accurate than the $6 \times 6 \times (4)$ model (not shown) containing 288 unknowns (recall that there are two unknowns per cell for this two-
phase case). Thus, there is some benefit in adaptively introducing subregions on the basis of the local pseudo-steady state solutions. Note that in Figure 2.9 the coarse model converges to the fine model from below, while in Figure 2.10 convergence is from above. These different behaviors are due to differences in the form and magnitude of the capillary flux for the two cases.

**Figure 2.11:** Average water saturation distributions at 0.5 PV of water injection for fine model (left) and upscaled model with SR = 5 (right). Results for strong capillary pressure ($p_c^1$) are shown in the upper figures and those for weak capillary pressure ($p_c^2$) in the lower figures.
Figure 2.11 depicts the water saturation (averaged over regions corresponding to coarse blocks as defined in equation 2.21 except here using pore-volume weighting) observed after 0.5 pore volume of water injection. We observe close agreement between the fine and upscaled (SR = 5) models for both sets of capillary pressure data. The relative pressure and saturation errors measured in the $L_2$ norm are presented in Table 2.2. The pressure error is computed as described above (using equation 2.21). Saturation errors are computed analogously but using pore-volume weighted averages. The errors are quite reasonable for both sets of $p_c$ data, again indicating the general accuracy of the method.

![Figure 2.12](image_url)

**Figure 2.12**: Recovery curves demonstrating the advantage of using more subregions instead of more coarse blocks
Table 2.2: Average relative errors at 0.5 PV water injection for pressure and saturation in 2D cases with strong ($p_c^1$) and weak ($p_c^2$) capillary pressure

<table>
<thead>
<tr>
<th>$L_2$ pressure error</th>
<th>$L_2$ saturation error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_c^1$</td>
<td>3.66%</td>
</tr>
<tr>
<td></td>
<td>5.37%</td>
</tr>
<tr>
<td>$p_c^2$</td>
<td>3.87%</td>
</tr>
<tr>
<td></td>
<td>5.89%</td>
</tr>
</tbody>
</table>

The accuracy of the upscaled model can be improved by increasing the number of coarse blocks and/or the number of subregions. An example is considered to investigate the relative impact of these two treatments. For this purpose the same number of cells (128) is distributed differently in two coarse models. In one model the dimensions are $8 \times 8 \times (2)$ and in the other model $4 \times 4 \times (8)$. Figure 2.12 shows the recovery curves for these models compared to the fine model. The $4 \times 4 \times (8)$ model is clearly more accurate, demonstrating that, at least for this case, the use of more subregions has the greater impact on global accuracy. More study of this issue will be required in order to better characterize the convergence properties of the MSR approach.

We also simulated these 2D cases using more realistic values of $k_m$ and $k_f$, namely $k_m = 100 \text{ md}$ and $k_f = 10^7 \text{ md}$ (these simulations were performed after the correction of the unit conversion error in GPRS). Note that the ratio $k_f/k_m$ is still $10^5$, as in the previous examples. Simulation results for the DFM and MSR (SR = 5) models, using the $p_c^1$ and $p_c^2$ data, are shown in Figures 2.13 and 2.14 respectively. The curves are shifted somewhat compared with the earlier results shown in Figures 2.9 and 2.10, though the general level of agreement between the DFM and MSR models is quite comparable to
that observed earlier. This suggests that the MSR modeling procedure can be expected to perform well over a range of $k_m$ and $k_f$ values.

![Graph showing oil recovery vs. water injected for DFM and MSR (SR = 5) solutions with $k_m = 100$ md and $k_f = 10^7$ md for strong capillary pressure ($p_c^1$) case](image)

**Figure 2.13**: Oil recovery for DFM and MSR (SR = 5) solutions with $k_m = 100$ md and $k_f = 10^7$ md for strong capillary pressure ($p_c^1$) case
The upscaling technique developed here is not limited to 2D models and can be applied directly to 3D systems, as we now illustrate. Figure 2.15 represents a 1000×1000×200 ft³ model containing 28 intersecting fractures. The fractures are near-vertical though they do have slight inclination. This model is discretized using 52,059 cells (5,247 triangles for the fractures and 46,812 tetrahedra for the matrix). The system is initially saturated with oil and water is injected at constant rate (1,000 bbl/day) at one edge and liquid is produced from the opposite edge as shown in Figure 2.15. The upscaled model has a coarse grid of 9×9×3 and several subregion configurations are considered. The simulations are performed for the weak capillary pressure case ($p_c^2$). The recovery results are plotted in Figure 2.16. These results are
consistent with those for the 2D case (Figure 2.10) and clearly demonstrate the applicability of the method to 3D systems.

Figure 2.15: Synthetic 3D model with 28 intersecting fractures (discretized model contains 52,059 cells)
Figure 2.16: Oil recovery for fine-grid solution and several coarse models for the 3D two-phase flow case

We now consider a three-phase flow example. The fracture distribution and reservoir geometry are the same as in Figure 2.15. The initial reservoir pressure is 4,783 psi and the system is initially saturated with oil and connate water ($S_{wi} = S_{wc} = 0.12$). Water is injected at constant rate (1,000 bbl/day) at one edge and fluid is produced from the opposite edge using a constant BHP (3,500 psi) control as shown in Figure 2.15. The properties for oil, water and gas are given in Tables 2.3-2.5. The bubble point for reservoir fluid is 4,014.7 psi. The oil-water and gas-oil relative permeabilities are given in Tables 2.6 and 2.7.
Table 2.3: Oil phase property data for the 3D three-phase case

<table>
<thead>
<tr>
<th>$p$ (psi)</th>
<th>$B_o$ (RB/STB)</th>
<th>$\mu_o$ (cp)</th>
<th>$R_{go}$ (SCF/STB)</th>
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</thead>
<tbody>
<tr>
<td>14.7</td>
<td>1.062</td>
<td>1.04</td>
<td>1</td>
</tr>
<tr>
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<td>0.975</td>
<td>90.5</td>
</tr>
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<td>514.7</td>
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<td>0.91</td>
<td>180</td>
</tr>
<tr>
<td>1014.7</td>
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<td>0.83</td>
<td>371</td>
</tr>
<tr>
<td>2014.7</td>
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<td>0.695</td>
<td>636</td>
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<td>775</td>
</tr>
<tr>
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<td>1.671</td>
<td>0.549</td>
<td>1270</td>
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</table>

Table 2.4: Water phase property data for the 3D three-phase case

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<th>$\mu_w$ (cp)</th>
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<tr>
<td>4014.7</td>
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<tr>
<td>9014.7</td>
<td>1.013</td>
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<td>0</td>
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</table>

Table 2.5: Gas phase property data for the 3D three-phase case

<table>
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<tr>
<th>$p$ (psi)</th>
<th>$B_g$ (RB/bbl)</th>
<th>$\mu_g$ (cp)</th>
<th>$R_{gg}$ (SCF/STB)</th>
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<td>0.047</td>
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</table>
Table 2.6: Oil-water relative permeabilities for the 3D three-phase case

<table>
<thead>
<tr>
<th>$S_w$</th>
<th>$k_{rw}$</th>
<th>$k_{ro}$</th>
<th>$p_{cow}$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0</td>
<td>1</td>
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</tr>
<tr>
<td>0.121</td>
<td>1.67E-12</td>
<td>1</td>
<td>1.75</td>
</tr>
<tr>
<td>0.14</td>
<td>2.67E-07</td>
<td>0.997</td>
<td>1.41</td>
</tr>
<tr>
<td>0.17</td>
<td>1.04E-05</td>
<td>0.98</td>
<td>1.05</td>
</tr>
<tr>
<td>0.24</td>
<td>0.000346</td>
<td>0.7</td>
<td>0.89</td>
</tr>
<tr>
<td>0.32</td>
<td>0.002668</td>
<td>0.35</td>
<td>0.72</td>
</tr>
<tr>
<td>0.37</td>
<td>0.006514</td>
<td>0.2</td>
<td>0.58</td>
</tr>
<tr>
<td>0.42</td>
<td>0.013507</td>
<td>0.09</td>
<td>0.43</td>
</tr>
<tr>
<td>0.52</td>
<td>0.042688</td>
<td>0.021</td>
<td>0.26</td>
</tr>
<tr>
<td>0.57</td>
<td>0.068379</td>
<td>0.01</td>
<td>0.2</td>
</tr>
<tr>
<td>0.62</td>
<td>0.10422</td>
<td>0.001</td>
<td>0.15</td>
</tr>
<tr>
<td>0.72</td>
<td>0.21611</td>
<td>0.0001</td>
<td>0.09</td>
</tr>
<tr>
<td>0.82</td>
<td>0.40037</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.7: Gas-oil relative permeabilities for the 3D three-phase case

<table>
<thead>
<tr>
<th>$S_g$</th>
<th>$k_{rg}$</th>
<th>$k_{ro}$</th>
<th>$p_{cgo}$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2.0</td>
</tr>
<tr>
<td>0.001</td>
<td>0</td>
<td>1</td>
<td>1.95</td>
</tr>
<tr>
<td>0.02</td>
<td>0</td>
<td>0.997</td>
<td>1.56</td>
</tr>
<tr>
<td>0.05</td>
<td>0.005</td>
<td>0.98</td>
<td>1.35</td>
</tr>
<tr>
<td>0.12</td>
<td>0.025</td>
<td>0.7</td>
<td>1.15</td>
</tr>
<tr>
<td>0.2</td>
<td>0.075</td>
<td>0.35</td>
<td>0.93</td>
</tr>
<tr>
<td>0.25</td>
<td>0.125</td>
<td>0.2</td>
<td>0.78</td>
</tr>
<tr>
<td>0.3</td>
<td>0.19</td>
<td>0.09</td>
<td>0.63</td>
</tr>
<tr>
<td>0.4</td>
<td>0.41</td>
<td>0.021</td>
<td>0.46</td>
</tr>
<tr>
<td>0.45</td>
<td>0.6</td>
<td>0.01</td>
<td>0.33</td>
</tr>
<tr>
<td>0.5</td>
<td>0.72</td>
<td>0.001</td>
<td>0.25</td>
</tr>
<tr>
<td>0.6</td>
<td>0.87</td>
<td>0.0001</td>
<td>0.12</td>
</tr>
<tr>
<td>0.7</td>
<td>0.94</td>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td>0.85</td>
<td>0.98</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The upscaled model has a coarse grid of $9 \times 9 \times 3$ with each coarse grid containing 5 subregions.
The simulation results are plotted in Figures 2.17-2.20. The comparisons for gas and oil production rates and for pressure at the injector between the fine-grid and MSR solutions demonstrate a high degree of accuracy for this 3D three-phase system. This is encouraging as it demonstrates that the same MSR model can be used (at least in this case) for different flow and recovery processes.

**Figure 2.17:** Gas production rate for fine-grid and MSR solutions for the 3D three-phase flow case
Figure 2.18: Oil production rate for fine-grid and MSR solutions for the 3D three-phase flow case

Figure 2.19: Water production rate for fine-grid and MSR solutions for the 3D three-phase flow case
2.6 Concluding Remarks

The upscaling procedure results in significant computational savings. Specifically, running on a P4 2.4GHz CPU processor, the time required to simulate the fully resolved 3D fine-scale two-phase model was 2,700 seconds. The upscaling calculations plus the coarse-scale simulation required only 25 seconds. For the 3D three-phase model, these timings were 198 hours for fine-scale simulation and 2.1 hours for the upscaling process plus coarse-scale simulation. These timings indicate overall speedup factors of about 100. Runtime speedup is much more substantial and will be achieved each time the model is simulated (as the upscaling calculations need only be performed once). This level of speedup was also observed for 2D simulations.

Figure 2.20: Pressure at injector for fine-grid and MSR solutions for the 3D three-phase flow case

2.6 Concluding Remarks

In this chapter a systematic methodology for constructing an upscaled model from a
detailed, geometrically complex fracture characterization was developed and applied. The technique is applicable for 2D and 3D systems. The method was successfully applied to single-phase, two-phase and three-phase flow problems, including the effects of compressibility and capillary pressure. In the approach presented here gravity effects are neglected inside each coarse block and are captured only through coarse-block connections, as in the standard dual-porosity model. In the next chapter, we extend the approach to treat gravitational effects.

The upscaling procedure as presented here is most suitable for systems with strong fracture connectivity. In real formations the fractures are not always well connected and a hybrid simulation technique, in which key fractures are modeled discretely while the bulk of the domain is upscaled using the procedure developed here, may be required. Such an approach is developed in Chapter 5.
Chapter 3

Upscaling Discrete Fracture Characterizations to Dual-Porosity/Dual-Permeability Models for Efficient Simulation of Flow with Strong Gravitational Effects

3.1 Introduction

In Chapter 2, we developed an upscaling technique to construct dual-porosity models from detailed fracture characterizations. This method, referred to as a multiple subregion (MSR) technique, entails the determination of a flow-based subgrid (and appropriate transmissibilities) for each coarse block based on the fracture geometry and matrix properties inside the block. The subregions and transmissibilities are determined through solution of a local (discrete fracture) problem. Block to block (fracture-fracture) linkages are determined through a transmissibility upscaling procedure. This approach was successfully applied to single-, two-, and three-phase flow with capillary pressure effects for 2D and 3D configurations. However, due to the way in which we construct the subgrid and to the lack of connectivity between coarse...
matrix blocks, systems with strong gravity effects cannot be modeled accurately using this procedure.

In this chapter, we extend the MSR upscaling technique developed in Chapter 2 to account for strong gravitational effects. This entails modification of the local problems used to determine the subregions as well as adjustments to account for vertical connections between matrix blocks. This introduces a dual-permeability aspect to the method, as in some of the formulations discussed in Chapter 1. The method is applied to simulate flow in 2D and 3D fractured reservoir models. Two-phase flow, including compressibility, capillary pressure and gravity, is considered. We show that the technique is capable of generating accurate coarse models with many fewer unknowns relative to fully-resolved discrete fracture models. An overall speedup of about a factor of 80 is achieved for a 3D example.

This chapter proceeds as follows. First, the governing equations and the upscaling procedure are presented. Next, the MSR upscaling formulation is applied to several two-phase flow models in 2D and 3D.

### 3.2 Upscaling Technique

The governing equations describing compressible oil-water flow with gravity are similar to equations 2.7 and 2.8, except that we now include gravitational effects:

\[
\frac{\partial (\phi \rho_o S_o)}{\partial t} = \nabla \cdot \left( \frac{\rho_o k_{ro}}{\mu_o} (\nabla P_o - \rho_o g) \right) + q_o
\]  

(3.1)
3.2 Upscaling Technique

\[
\frac{\partial(\phi p_w S_w)}{\partial t} = \nabla \cdot \left( \frac{\rho_w k_{rw}}{\mu_w} (\nabla p_w - \rho_w g) \right) + q_w
\]  

(3.2)

where \( g \) is the gravity vector.

The discretized versions of equations 3.1 and 3.2 for the discrete fracture model can be represented in terms of a connection list and simulated in GPRS, as described in Section 2.3. This provides the reference solutions used to validate the upscaled model. The DFM is also solved locally in the upscaling computations to provide the subregion geometry and the transmissibilities appearing in the coarse model.

As in the formulation in Chapter 2, the upscaling consists of two separate steps; namely (1) a local solution within each coarse block to determine the subregion geometry and transmissibilities characterizing flow between the fracture network and the matrix (i.e., fracture-matrix and matrix-matrix connections) followed by (2) solution of a two-block problem to compute the transmissibility linking adjacent blocks. In step (2), in the absence of gravity (dual-porosity model as in Chapter 2), the linkages involve only fracture-fracture connections, while in the presence of strong gravity effects (dual-porosity/dual-permeability model) they involve both fracture-fracture and matrix-matrix connections.

We first review the local solutions in the dual-porosity (\( g = 0 \)) case and then describe the dual-porosity/dual-permeability computations.
3.2.1 Local Solution in the Absence of Gravity

As presented in Chapter 2, the local solutions provide the subgrid geometry and the fracture-matrix and matrix-matrix transmissibilities internal to each coarse block. The solutions are computed over the local fine-grid (discrete fracture) region corresponding to each coarse block. Because these fracture-matrix and matrix-matrix flows involve transient effects, we can determine the relevant quantities by solving a compressible single-phase flow equation:

$$\nabla \cdot \left( \frac{k}{\mu} \nabla p \right) = \phi c \frac{\partial p}{\partial t} + q_f$$  \hspace{1cm} (3.3)

where \( q_f \) designates injection into the fractures (note that \( q_f \) here and in subsequent discussion is based on volume, while \( q \) in equations 3.1 and 3.2 is based on mass). No-flow boundary conditions are applied along all of the faces of the coarse block region. These boundary conditions are appropriate because the matrix in the dual-porosity formulation is assumed to interact only with the fracture network within the target block, so there is no flow out of the block through the matrix. After an initial transient, pseudo-steady state (i.e., \( \partial p/\partial t = \text{constant} \)) is achieved. In fact, in Chapter 2 we solve a Poisson equation, in which \( \partial p/\partial t \) is specified to be a constant, rather than equation 3.3. This avoids the need to integrate in time and leads to computational savings. From the pseudo-steady state solution of equation 3.3, we construct iso-pressure contours, which define the multiple subregions (i.e., subgrid), and then compute the fracture-matrix and matrix-matrix transmissibilities (see Section 2.4).

As shown in Chapter 2, this approach works well for cases in which gravity is not important. However, as demonstrated below, it does not perform adequately when
gravity is important, so a modified procedure is required for such cases. We now describe our dual-porosity/dual-permeability approach.

### 3.2.2 Local Solution and MSR Determination with Gravitational Effects

When gravitational effects are important, the subregion geometry as determined from solution of equation 3.3 and the connection list shown in Figure 2.2 are not fully adequate. This is because the logically one-dimensional connections evident in Figure 2.2 are not in general ordered by decreasing depth (which limits the ability of the model to capture local gravity segregation) and because vertical interblock matrix-matrix transfer, which is not included in the model, may be important. We now describe procedures that address both of these issues.

Rather than solve equation 3.3 subject to no-flow boundary conditions (with flow driven only by $q_f$) as above, we now solve equation 3.3 with flow prescribed to occur both by injection into the fractures and by flow driven in the vertical direction. This essentially superimposes a vertical flow and a constant injection flow. The vertical flow component is intended to mimic gravitational effects and, as we will see, leads to vertically ordered subregions. Because we now have a competition between these two effects, a new parameter $\alpha$ is introduced to quantify their relative magnitudes.

The problem set up is as follows. We specify a total volumetric flow rate $Q_T$ at the top of the local region, the same total flow rate $Q_B$ at the bottom of the local region ($Q_B = Q_T$), and a total fracture injection rate $Q_F$. We define $\alpha = Q_b/Q_F$. No-flow boundary conditions are specified at the left and right boundaries (and at front and back
boundaries in 3D problems). The system is illustrated schematically in Figure 3.1. Note that $\alpha = 0$ results in the same local solution as in our earlier procedure.

**Figure 3.1**: Boundary conditions applied for the local one-block problem with gravity

The total flow rates are divided among the boundary and fracture cells as follows:

$$\frac{\partial p_i}{\partial n} = 0 \text{ on left and right boundary cells,} \quad (3.4a)$$

$$q_i = \frac{Q_s}{\sum_{j \in \text{bdy}} k_j} \times k_i \text{ on top and bottom boundary cells,} \quad (3.4b)$$

$$q_i = \frac{Q_f}{\sum_{j \in \text{frac}} k_j} \times k_i \text{ for fracture cells not on top and bottom boundaries,} \quad (3.4c)$$
3.2 Upscaling Technique

\[ q_i = 0 \] for matrix cells not on top and bottom boundaries. \hspace{1cm} (3.4d)

This specification results in \( Q_B \) and \( Q_T \) partitioned to boundary cells in proportion to the cell permeability \( k \), which should approximate how the flux will distribute itself in a large-scale system. Similarly, in the fractures, \( Q_F \) is apportioned to each fracture volume proportional to its permeability. In the internal matrix, \( q \) is zero. In the case of a connected fracture network within the local region, the solution will be insensitive to the precise way in which \( Q_F \) is apportioned. For disconnected systems, we would expect to observe some sensitivity, and other approaches for apportioning \( Q_F \) could be investigated. The current approach assures that all fractures receive some portion of \( Q_F \), which seems reasonable because fractures that are disconnected within the block may connect with fractures in other blocks. Thus these fractures may not be truly isolated, so it seems appropriate to inject fluid into them in the local solutions.

The solution of equation 3.3 subject to equations 3.4a-d again leads to a pseudo-steady state after an initial pressure transient. The shapes of the iso-pressure contours will depend on the fracture geometry and fracture and matrix permeabilities in addition to the parameter \( \alpha \). Iso-pressure curves at pseudo-steady state (used to construct the subgrid) for the system shown in Figure 3.1 for various values of \( \alpha \) are shown in Figure 3.2.
We note that a steady state solution is not achieved for the specified conditions except in the case $\alpha = \infty$. This is because fluid is continuously injected into the system (due to $Q_F$), so pressure continues to increase with time. Thus we use a pseudo-steady state solution rather than a steady state solution for the determination of the subregion geometry in the general case.

The contours in the left figure ($\alpha = 0$) are those that result from our earlier procedure. These contours are appropriate when gravitational effects are not important (note that the disconnectedness of the subregions resulting from these contours is not a problem for the method – see Section 2.4.3 for discussion). If gravity is significant, it is clear that this set of subregions will not be optimal. This is the case because (1) the subregions are not ordered in decreasing depth and (2) the linkage between the uppermost (or lowermost) matrix subregion to the adjacent matrix subregion in the block above (or below) is not readily accomplished.

Both of these limitations are addressed by the subregion model shown in the middle image ($\alpha = \infty$) in Figure 3.2. Here, flow is driven only by $Q_T$ and $Q_B$, which results in
an essentially layered subregion geometry. This means that gravity segregation within the block can be captured and that interblock (vertical) matrix-matrix transfer can be readily represented. This subregion geometry may not be optimal, however, in cases when both gravitational and viscous effects are important. In such cases an intermediate value of $\alpha$ may be most appropriate. The subregions obtained using an intermediate value ($\alpha = 1$) are shown on the right in Figure 3.2. This geometry appears to be a reasonable compromise between the two limits. Note that, for $\alpha \neq 0$, the fractures are in general linked to multiple subregions (Figure 3.2, center and right), in contrast to the case of $\alpha = 0$ in which the fractures are linked only to the first matrix subregion. This will impact the connectivity structure of the numerical model as shown below.

For any particular flow problem involving viscous and gravitational effects, there will be an optimum value of $\alpha$ (we note that it would be possible to vary $\alpha$ over the global domain but this is not considered here). It will be inconvenient in practice, however, to explore a wide variety of $\alpha$ values, as this will require the full upscaling procedure to be run multiple times. In the results below, we present simulations demonstrating the sensitivity of the results to $\alpha$. Although an optimal value clearly exists, we observe relatively little sensitivity in the results to $\alpha$ over a fairly wide parameter range. This is encouraging and suggests that, while it is important to introduce some degree of layering in the subregion model, as illustrated in Figure 3.2 (center and right), the results are not overly sensitive to the details of the subregion geometry.

As described in Chapter 2, an adaptive approach can be used to determine the number of subregions in each block. This is accomplished by prescribing the maximum and minimum number of subregions (say 5 and 2). Then each coarse block is assigned a
number of subregions according to the pressure variation over that block relative to that observed on average for all of the blocks. Once the number of subregions is determined, cells in the block are grouped into subregions by minimizing the pressure variance within each subregion. After constructing the subregions, we can calculate the required transmissibilities, as described in the following section.

### 3.2.3 Transmissibility Calculations

To obtain the intrablock fracture-matrix and matrix-matrix transmissibilities, we first compute the pseudo-steady state solution of the local problem. From this solution, the subregions are formed from the iso-pressure contours. Quantities such as bulk volume, porosity, depth and pressure for each subregion are then computed as a bulk-volume weighted average (see equation 2.13 in Section 2.4.1). The determination of the subregion transmissibilities is slightly different than the approach described in equations 2.14-2.18 (Section 2.4.1). In Chapter 2, we used the accumulation term in the computations of fracture-matrix and matrix-matrix flux. This was appropriate because the problem was solved subject to no-flow boundary conditions. For the modified local problem in Figure 3.1, we instead apply a direct flow integration approach to calculate the fracture-matrix and matrix-matrix flux. Specifically, we first calculate the total volumetric flow rate $Q^k_{n1,n2}$ between two subregions (here designated $n1$ and $n2$) by summing the flows between any connected fine cells in the two subregions via:

$$Q^k_{n1,n2} = \sum_{lca1, mca2} T^k_{l,m} \frac{(p^k_l - p^k_m)}{\mu}$$  \hspace{1cm} (3.5)
where $k$ designates the $k^{th}$ coarse block, $(l,m)$ designates any connected fine cell pairs from subregions $n1$ and $n2$ respectively, and $p$ designates the pressure from the local solution of equation 3.3. The transmissibility linking subregions $n1$ and $n2$ can now be computed via:

$$T_{n1,n2}^{k} = \frac{Q_{n1,n2}^{k} \mu}{(\bar{p}_{n1} - \bar{p}_{n2})}$$  \hspace{1cm} (3.6)$$

where overbar denotes average. These transmissibilities, combined with the associated subregion volumes, porosities, and depth, fully define the local matrix-fracture and matrix-matrix flow model inside each coarse block. We note that, in this model, the fracture subregion can interact with multiple matrix subregions. This is evident from Figure 3.2 (for $\alpha = \infty$ and $\alpha = 1$). These linkages (transmissibilities) are determined from equations 3.5 and 3.6 where the volumetric flow rate $Q_{n1,n2}^{k}$ is between the fracture subregion ($n1$) and each of the matrix subregions ($n2$) to which it connects.

As in Chapter 2, we use a standard two-point transmissibility upscaling procedure to compute the fracture-fracture transmissibility between coarse blocks. The main difference between the approach in Chapter 2 and that applied in this chapter is in our computation of the vertical interblock matrix-matrix transmissibility. Figure 2.4 in Chapter 2 shows a schematic of the two-block problem (blocks are designated $k$ and $l$). The steady-state single-phase pressure equation is solved with fixed pressure and no-flow boundary conditions as indicated. The average pressure and fluid properties inside each block are computed from the local pressure solution. For horizontal ($x$ and
y) connections, we simply compute the total flow rate $Q^{k,j}$ through the interface between the blocks. Transmissibility is then determined by:

$$T^{k,j} = \frac{Q^{k,j} \mu}{(p^k - p^l)}$$  \hspace{1cm} (3.7)

As discussed in Chapter 2, for blocks with disconnected fractures, this treatment will provide a reasonable approximation for $T^{k,j}$ even in cases when the interblock flow is from the matrix in block $k$ to the matrix in block $l$.

In the vertical direction, we also need to compute interblock matrix-matrix transmissibility. This is accomplished by computing the flow across the interface occurring through the fractures and matrix separately. We then apply equation 3.7 for the determination of each transmissibility individually.

The coarse continuum flow model is now fully defined. It differs from the model developed in Chapter 2 both in terms of the subregion geometry and the connectivity structure of the model. The connection list for the new model is illustrated schematically in Figure 3.3. It is noticeably more complex than that shown in Figure 2.2. The additional connections appearing here are (1) those between the fracture subregion and multiple (or all) matrix subregions (which can occur whenever $a_i \neq 0$; see Figure 3.2 – center and right) and (2) those between, for example, the uppermost matrix subregion in block 1 and the lowermost matrix subregion in block 5 (designated by dashed lines in Figure 3.3). Note that analogous connections will appear for all neighboring matrix subregions in vertically adjacent blocks. Simulation
using this connectivity structure is, however, still straightforward if a connection-list based simulator such as GPRS is used. Indeed, in all of the results below, we apply GPRS using the connection list generated by the upscaling procedure.

![Diagram of MSR connection list for 2D problems with gravity](image)

**Figure 3.3**: Schematic of the MSR connection list for 2D problems with gravity

### 3.3 Applications

To demonstrate the capabilities of the MSR upscaling procedure, several examples are considered. We first demonstrate the inaccuracy that results from directly applying the method with \( \alpha = 0 \) and no interblock matrix-matrix connections (this corresponds to our earlier procedure, which is suitable for viscous-dominated cases) to a 2D system undergoing gravitational settling and then show that the modified method is able to provide sensible results. Next we consider 2D examples in which viscous, capillary
and gravitational effects are important. We also investigate the sensitivity of the results for such cases to the specific value of $\alpha$. Finally, we apply the procedure to a 3D fractured reservoir.

In all simulations, we consider two-phase oil-water flow. Water is incompressible and the oil compressibility ($c_o$) is $1.38 \times 10^{-5}$ psi$^{-1}$. The reference densities of water and oil are $\rho_w = 62.4$ lb/ft$^3$ and $\rho_o = 49.9$ lb/ft$^3$; the viscosities are $\mu_w = 0.55$ cp and $\mu_o = 1.2$ cp. The matrix rock is characterized by a porosity of $\phi_m = 0.25$. The fractures are considered to be fully open with a porosity of $\phi_f = 1$. Wells in all cases intersect fractures. Capillary pressure in the matrix is included in the model (unless otherwise indicated) but it is neglected in the fractures. Straight-line relative permeabilities are used in the fractures. Relative permeability and capillary pressure data for the matrix are summarized in Table 3.1. Note that the capillary data labeled $p_c^1$ is applied in most cases. In the example in Section 3.4.2 with “weak” and “strong” capillary pressure, weak corresponds to $p_c^1$ and strong to $p_c^2$.

**Table 3.1**: Relative permeability and capillary pressure data for the matrix

<table>
<thead>
<tr>
<th>$S_w$</th>
<th>$k_{rw}$</th>
<th>$k_{rn}$</th>
<th>$p_c^1$ (psi)</th>
<th>$p_c^2$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
<td>2.0</td>
<td>200</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0</td>
<td>0.875</td>
<td>0.5</td>
<td>50</td>
</tr>
<tr>
<td>0.4</td>
<td>0.04</td>
<td>0.43</td>
<td>0.1</td>
<td>9</td>
</tr>
<tr>
<td>0.6</td>
<td>0.125</td>
<td>0.1</td>
<td>0.04</td>
<td>2</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3</td>
<td>0.0</td>
<td>0.02</td>
<td>0.5</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

In the 2D examples, the matrix rock is characterized by an isotropic permeability of $k_m = 1.6 \times 10^4$ md. All of the fractures are considered to be identical, though this is not required by the method. The fracture aperture $e$ and permeability $k_f$ are $e = 0.344$ mm and $k_f = 1.6 \times 10^9$ md.
3.3 Applications

3.3.1 Gravity Segregation Example

Figure 3.4: Fracture model and coarse grid for gravity segregation example

In this 2D case, there are 29 fractures, shown as black lines in Figure 3.4. The fine grid contains 4,659 cells and is of physical dimensions 100 ft × 100 ft. We upscale the model to 6×6 coarse blocks, with each coarse block containing 5 subregions (for a total of 180 coarse cells). In this model there are no wells and the boundary conditions are no-flow. Capillary pressure effects in both the fractures and matrix are neglected, so flow is driven only by gravity. The initial condition is set as $S_w = S_o = 0.5$ everywhere in the reservoir. As the simulation proceeds, water moves down and oil moves up due to the density difference, and the system eventually reaches a final state with water filling the lower half and oil filling the upper half. There is a sharp transition between the two regions due to the neglect of capillary pressure.

Simulation results for water saturation are shown in Figure 3.5. The three plots in the top row depict fine-grid (discrete fracture) solutions at 10 days, 100 days and 1000
days. The fluids clearly segregate with time as expected. The three plots in the middle row display results for the MSR model using the method in Chapter 2, in which we use $\alpha = 0$ for the local solutions and do not link matrix regions in vertically adjacent blocks. The MSR model here does not capture the physics of the segregation process and results in isolated patches of oil and water. This is because the subregions are not ordered (even approximately) in the gravity direction and also because there is no matrix-matrix interblock flow. Both of these effects need to be included in order to obtain results in correspondence with the fine-scale simulations.

We note that the qualitative behavior of the MSR ($\alpha = 0$) solution observed here (isolated regions of oil and water) was also observed in MSR ($\alpha = 0$) models that used two subregions in all coarse blocks and in idealized dual-porosity simulations using an industry simulator. This suggests that this problem is not caused by the use of multiple subregions but is rather due to the limited vertical communication in the standard dual-porosity model, as indicated above.
3.3 Applications

Figure 3.5: Water saturation profiles for gravity segregation example (no capillarity)

As discussed in detail in Section 3.2, the modified procedure with $\alpha \neq 0$ addresses the deficiencies of the earlier method. We now run the model with very large $\alpha$ ($\alpha = 10^4$) to provide an MSR model suitable for the limit of purely gravitationally-driven flow. These results are shown in the bottom row of Figure 3.5. The simulations are now in accord with the fine-scale results and demonstrate the accuracy of the modified method for this case.
Neglecting capillary pressure results in a sharp transition between the oil and water regions at late time, as is apparent in Figure 3.5. In order to assess the performance of the MSR model with gravity and strong capillary pressure effects, we now simulate this case using the $p^2_c$ data given in Table 3.1. Simulation results for water saturation are shown in Figure 3.6. The plots in the upper and lower row present DFM and MSR ($\alpha = 10^4$) solutions at 10 days, 100 days and 1000 days. The MSR results display close agreement with the DFM simulations for this case. Both models depict a significant transition zone, as would be expected with strong capillary effects.

Figure 3.6: Water saturation profiles for gravity segregation example (strong capillarity)
3.3.2. Combined Viscous and Gravitational Effects (2D)

Figure 3.7 depicts a 2D (1,000 ft × 1,000 ft) fractured system representing a portion of a model introduced by Lee et al. (2001). This system, considered in Chapter 2, contains 70 fractures. The fine-grid DFM includes 12,280 cells (1,976 fracture segments and 10,304 triangular matrix elements) and is upscaled to a 6×6 coarse grid. In these simulations, we consider both fixed numbers of subregions (1, 2, 3 and 5 in each block), and variable (adaptive) numbers of subregions. For all of the MSR models we used $\alpha = 1$. The injection well is specified to inject at 100 STB/d and the producer operates at a fixed bottomhole pressure of 1,000 psi. This is a low injection rate and is used in order to emphasize the effects of gravity (at high rates viscous effects are dominant and our treatment of gravitational effects would not have much impact on flow results). We observed that the pressure contours were approximately layered, which provides verification that we are in a gravity dominated flow regime.

![Fracture model and coarse grid for second example](image)
The upper plot in Figure 3.8 displays oil recovery (presented in terms of pore volume recovered) as a function of time. The heavy solid curve is the fine-scale result and the other five curves represent varying numbers of subregions, as indicated. The single subregion model, which corresponds to a single-porosity model, shows substantial error. The MSR results systematically improve in accuracy with increasing numbers of subregions, and the five-subregion model shows only 2.3% error at the end of the simulation. This model contains only 180 cells, which is about a factor of 70 reduction relative to the fine model.
Figure 3.8: Oil recovery curves (top) and water saturation profiles (bottom) for case 2 (blue is water and red is oil)
The lower plots in Figure 3.8 depict the average water saturation observed at 0.5 pore volume of water injection. The saturation results are here averaged onto the coarse rectangular grid for easier comparison. We observe close agreement between the fine (lower left plot) and upscaled (SR = 5, lower right plot) models. The relative pressure and saturation errors in the upscaled model (measured in the $L_2$ norm, for details see Section 2.5.1) are 3.48% and 4.97% respectively.

For the 2D model shown in Figure 3.7, we evaluated the sensitivity of the simulation results to the particular value of $\alpha$. Two different matrix capillary pressure curves, designated weak and strong capillary pressure ($p_c^1$ and $p_c^2$ in Table 3.1), were considered. Oil recovery results for these cases are shown in Figure 3.9. In both cases, the most accurate results are achieved with $\alpha = 1$. It is evident, however, that there is relatively little sensitivity in the oil recovery results to the exact value of $\alpha$ over a wide range. This is encouraging, as it suggests that it is not necessary to perform extensive numerical experimentation in order to determine a reasonable value for $\alpha$. 
Figure 3.9: Oil recovery results with varying $\alpha$
### 3.3.3 3D Case with Gravity

We now consider application of the MSR upscaling procedure to 3D systems. Figure 3.10 represents a 5,000 ft × 5,000 ft × 300 ft model containing 30 essentially vertical fractures, some of which are disconnected. The permeability for fracture and matrix are $k_f = 1.6 \times 10^8$ md and $k_m = 160$ md. This model was provided by Chevron ETC and is derived from a portion of a real fractured reservoir. The model is discretized using GOCAD (2006) into 110,877 cells (9,568 triangular cells for the fractures and 101,309 tetrahedral cells for the matrix). The system is initially saturated with oil and water is injected at a constant rate of 1,000 STB/day, with the wells as shown in Figure 3.10. The coarse-grid model is 10×10×3 and we vary the number of subregions as in the previous example. In all cases we set $\alpha = 1$.
Oil recovery results are shown in the upper plot in Figure 3.11. As was observed in the 2D results (Figure 3.8 upper plot), the accuracy of the model systematically improves as the number of subregions is increased. Using 5 subregions, the error in oil recovery is 1.8% at the end of the simulation. For this case the relative pressure and saturation errors (in $L_2$) for the upscaled model are 3.71% and 5.22% respectively. Water saturation results are shown on the lower right images in Figure 3.11. These results are averaged onto the coarse blocks and are then averaged vertically. The lower left plot corresponds to the fine model and the lower right plot to the upscaled model (SR = 5); agreement between the two is clearly very close. The MSR model (with SR = 5) contains a total of 1,500 cells, which represents about a factor of 70 reduction relative to the fine (discrete fracture) model.
Figure 3.11: Oil recovery curves (top) and water saturation profiles (bottom) for 3D example (blue is water and red is oil)
The computational requirements for the MSR model include both the model generation step (which entails the solution of a sequence of local problems) and the global simulations. For this 3D example, using a Pentium IV 3.0GHz processor, the fine model required 5,320 seconds while the MSR model required 67 seconds (which includes model generation and global simulation) of CPU time. The speedup in this case is therefore about 80, which is consistent with the speedups of roughly 100 observed in Chapter 2. We note that these speedups will be significantly higher if the MSR model is simulated multiple times (e.g., with different well rates or well locations) as the model generation step need be performed only once.

3.5 Concluding Remarks

In this chapter we generalized the multiple subregion (MSR) model for the coarse-scale simulation of fractured reservoirs. The following conclusions can be drawn from this work:

- The MSR procedure of Chapter 2 was successfully extended to treat models in which gravitational effects are important. In contrast to the previous approach, which was applicable for viscous-dominated systems, the new approach accounts for matrix-matrix interblock transfer and thus provides a dual-porosity/dual-permeability model. A direct linkage between the underlying discrete fracture characterization and the coarse-scale dual-porosity/dual-permeability model is maintained.

- A new parameter ($\alpha$) was introduced to control the subregion geometry. This parameter interpolates between our previous approach for generating multiple
subregions (which corresponds to $\alpha = 0$) and a purely layered subgrid ($\alpha = \infty$). It was shown that large-scale simulation results are not overly sensitive to $\alpha$ over a reasonable range though, when gravity is important, $\alpha$ should be large enough such that the subregion displays a degree of layering.

- Simulation results for several 2D and 3D problems, including viscous, capillary and gravitational effects, demonstrated the accuracy of the method and the systematic improvement in the coarse model with increasing numbers of subregions. For the 3D case, an overall speedup of a factor of 80 was observed.
Chapter 4

First-Contact Miscible Gas Injection Simulations Using a Fully-Compositional Model

4.1 Introduction

Field-scale simulations of naturally fractured reservoirs using discrete fracture models (DFMs) are not commonly performed. When such simulations are conducted, they are generally for oil-water systems (e.g., Guaquirian et al., 2007; Matthai et al., 2005). In this chapter we perform both DFM and MSR simulations for the 3D fracture models introduced in Chapters 2 and 3. The displacement process is complex – specifically a six-component, first-contact miscible gas injection. The reasonable level of agreement achieved between the MSR and DFM procedures indicates the potential applicability of the MSR approach for flow problems involving complex recovery mechanisms.

This chapter proceeds as follows. First a simple case using the relevant six-component fluid characterization is simulated using GPRS and Chevron’s proprietary simulator CHEARS to ensure that the two simulators provide consistent results for this compositional simulation. Then a full dual-porosity/dual-permeability implementation
of the MSR upscaling technique is presented. The use of global, single-phase flow information for the MSR transmissibility upscaling procedure is also introduced. These new MSR upscaling procedures are then applied for simulating the miscible displacement via a fully-compositional formulation in which the miscible fluids are represented by six hydrocarbon pseudo-components. Two different fracture systems are studied – one in which there is connectivity between the injector and producer through the fractures, and one in which this connectivity is absent. The effect of varying areal grid resolution on upscaling accuracy is also evaluated. The upscaled model shows a reasonable degree of accuracy and reduces the simulation time by a factor of over 1000.

4.2 Comparisons between CHEARS and GPRS

Before simulating the DFM sector model in GPRS, we designed a simple 3D compositional case in order to verify the GPRS simulation results against Chevron’s proprietary simulator CHEARS. The model is a homogeneous reservoir with a 15×15×2 Cartesian grid (Figure 4.1). The permeability and porosity are $k_x = k_y = k_z = 0.1$ md and $\phi = 1\%$. The initial reservoir pressure is 11,950 psi. A gas injector and an oil producer are located at the two opposite corners. We apply constant BHP controls for the injector (13,000 psi) and producer (4,500 psi). This is a fully compositional simulation model using the six-component fluid characterization given in Table 4.1.
4.2 Comparisons between CHEARS and GPRS

Table 4.1: Compositions for Reservoir Oil and Injected Gas

<table>
<thead>
<tr>
<th>Component</th>
<th>Reservoir Oil</th>
<th>Injected Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂S</td>
<td>0.1252</td>
<td>0.1623</td>
</tr>
<tr>
<td>Methane</td>
<td>0.4510</td>
<td>0.6067</td>
</tr>
<tr>
<td>C₂-3</td>
<td>0.1675</td>
<td>0.1957</td>
</tr>
<tr>
<td>C₄-7</td>
<td>0.1133</td>
<td>0.0344</td>
</tr>
<tr>
<td>C₈-26</td>
<td>0.1376</td>
<td>0.0009</td>
</tr>
<tr>
<td>C₃₉</td>
<td>0.0054</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 4.1: Homogeneous model for GPRS and CHEARS comparison - the colors correspond to pressure

Figures 4.2 and 4.3 display the simulation results for molar rate of injected and produced components for GPRS and CHEARS. The agreement is very close for all quantities. This indicates that GPRS and CHEARS are equivalently formulated in terms of compositional simulation.
Figure 4.2: Molar rates of injected components from GPRS and CHEARS simulations

Figure 4.3: Molar rates of produced components from GPRS and CHEARS simulations
4.3 Multiple Subregion Modeling Techniques

4.3.1 Full Dual-Porosity/Dual-Permeability MSR Formulation

Our initial implementation of the MSR procedure was a dual-porosity formulation in which there are linkages between fractures and matrix within a coarse block and between the fracture subregions in adjacent blocks but not between matrix subregions in adjacent blocks (Section 2.4 in Chapter 2). In Chapter 3, we developed a dual-porosity/dual-permeability MSR model that additionally included linkages between matrix subregions in vertically adjacent blocks. This treatment was shown to be beneficial for capturing gravitational effects in the MSR simulations. In the MSR model used in this chapter, in an attempt to achieve higher accuracy, we retain these connections and introduce additional connections linking matrix subregions in horizontally adjacent blocks. Specifically, we apply equations 3.3 and 3.4a-d to compute the intrablock MSR geometries as well as transmissibilities. In addition, we apply equations 3.5-3.6 to calculate the interblock transmissibilities not only for fracture-fracture connections, but also for interblock matrix-matrix connections. As a result, this approach is essentially a full dual-porosity/dual-permeability MSR model.

4.3.2 Use of Global Solutions to Determine Interblock Transmissibilities

In geological models characterized by complex permeability structures, it is known that local permeability or transmissibility upscaling procedures (as applied in Chapters 2 and 3 and illustrated in Figures 2.3, 2.4 and 3.1) can lead to errors in large-scale flow. This is due to inaccuracy in the assumed boundary conditions used in local upscaling
methods. In such cases, the use of global information has been shown to provide improved coarse-scale models (e.g., Chen and Durlofsky, 2006). In an attempt to enhance some of our MSR models, particularly in cases with disconnected fracture networks, in which the large-scale connectivity of the model can be impacted significantly by the local boundary conditions, a global upscaling technique was developed. This method is along the lines of that described by Mallison et al. (2006). In this approach, the global steady-state single-phase pressure equation is solved using the DFM. Flow is driven by the actual wells and global boundary conditions. Although this requires a global fine-scale solution, it is not a significant computational expense as it entails only a single solution of the pressure equation.

Following this solution, all of the interblock subregion-to-subregion flow rates and the average subregion pressures are computed via a volumetric average approach. Given these quantities, we then determine the fracture-fracture and matrix-matrix transmissibilities using equation 3.7. We do not use this global steady-state solution to determine the intrablock matrix-fracture and matrix-matrix transmissibilities or the subregion geometries. Instead, these are still determined from the single-block problems as described in Section 3.2.

### 4.4 Model Upscaling Using MSR Method

We apply our proposed dual-porosity/dual-permeability MSR procedure on two different 3D models. The first model, shown in Figure 4.4, is a synthetic system in which there is direct connectivity between the injector-producer pair through the fracture network (this model was also shown earlier in Figure 2.13). The second model,
shown in Figure 4.8, is extracted from a DFM generated using real field data from a fractured carbonate reservoir (this model was shown earlier in Figure 3.9). In contrast with the first model, there is no direct flow path between the injector and producer; hence any injected fluid must traverse through some matrix regions to arrive at the producer. For brevity, we will refer to these two models as “connected” and “disconnected” respectively.

For both models, we simulate a miscible gas injection process using a fully-compositional formulation with six pseudo-components (see Table 4.1 for fluid compositions). The system is in all cases initially saturated with oil and the wells always intersect fractures. The fracture and matrix properties (indicated below for completeness) are the same as described in Section 2.5.2 for the connected model and Section 3.3.3 for the disconnected model respectively.

In the results presented below we investigate the effect of varying the areal grid resolution as well as the impact of global upscaling. We set the number of subregions to be five (uniformly) when fractures are present within a coarse block and use no subregions for coarse blocks that contain only matrix.

### 4.4.1 Connected Fracture Model

Here we employ the same model considered in Section 2.5.2. The model contains 28 intersecting fractures of near-vertical orientation (see Figure 4.4). The fractures are of permeability $k_f = 1.6 \times 10^7$ md and the matrix is homogeneous and of permeability $k_m = 160$ md. The DFM generated for this model comprises 52,059 cells. The physical dimensions here are $100 \times 100 \times 20$ ft$^3$. As indicated in Figure 4.4, we complete the
injector at one edge of the model and the producer at the opposite edge. For the upscaled model, we use two grids of different areal resolution: 10×10×1 and 20×20×1, which contain 484 and 1,428 cells respectively. We specify a constant gas injection rate of 300 MMSCF/day (the injection gas composition is given in Table 4.1) and a constant oil production rate of 35 STB/day.

![Model with a network of 28 fractures; direct connectivity exists between the injector and producer](image)

**Figure 4.4:** Model with a network of 28 fractures; direct connectivity exists between the injector and producer

A key advantage of the MSR modeling approach is that the same model can be used for different production scenarios. There are therefore no additional coarse-scale parameters (e.g., upscaled transmissibilities or transfer functions) that need to be computed for these compositional simulations. There is, however, no guarantee that the same level of accuracy will be achieved for all types of simulations, and it is possible that simulations involving more complex physical mechanisms may require higher levels of grid resolution.
Comparisons between simulation results using the DFM and MSR procedures are presented in Figures 4.5-4.7. We show plots of cumulative gas production (gas fraction is determined from a flash of produced fluids at standard conditions), methane concentration at the producer, and injector BHP as a function of PVI. Plots of the other component concentrations are comparable to that shown here for methane. In Figures 4.5 and 4.6, we observe that all the MSR models yield reasonably good results relative to DFM. The use of global single-phase flow information in the finer (20×20×1) MSR model yields the most accurate results, especially for methane concentration at the producer and injector BHP.

![Figure 4.5: Cumulative gas production for DFM and MSR simulations (connected model)](image-url)
**Figure 4.6:** Methane concentration at producer for DFM and MSR simulations (connected model)

**Figure 4.7:** Injector BHP for DFM and MSR simulations (connected model)
From these plots, we also observe that areal grid refinement alone fails to provide any improvement in accuracy for this compositional simulation. In addition, even for the 20×20×1 grid with global upscaling, there is still noticeable error in injector BHP. Nevertheless, it bears reiterating that a reasonably close reproduction of the DFM results for a rather challenging first-contact-miscible displacement is obtained without any special accounting of the fully-compositional aspects of the process (e.g., through use of pseudo-functions).

There are dramatic differences in the computational requirements for the DFM and MSR models. Using an AMD Athlon 64 processor, the most accurate (and most computationally expensive) MSR model in this case (20×20×1 with global flow information) required only 78 seconds for upscaling and 125 seconds for coarse-scale simulation. The DFM simulation, which is very demanding computationally due to the complex physics, high permeability contrasts and the large number of cells, required 57 CPU hours. Thus the speedup factor for this model is about 1000. Compared with typical speedup factors obtained for oil-water cases presented in previous chapters (order 100), the considerably larger speedup achieved here reflects the added difficulty of performing compositional simulations using DFM. The speedup is clearly very substantial and demonstrates the potential impact of our MSR procedure for modeling displacements involving complex physical mechanisms.

4.4.2 Disconnected Fracture Model

We now apply the MSR upscaling procedure to a model in which the fractures were generated based on actual field data for a fractured carbonate reservoir (Figure 4.8). This reservoir description, which contains 30 essentially vertical fractures, was studied
previously in Section 3.3.3. For the modeling in this chapter, we generated a new (finer) DFM grid that consists of 131,817 cells. The physical dimensions are also different (600×600×36 ft³) and we do not include gravity or capillarity in these simulations. We set \( k_f = 1.6 \times 10^8 \) md and \( k_m = 160 \) md. We complete the injector at one edge of the model and the producer at the opposite edge. In contrast to the “connected” model considered in the previous section, we can see in Figure 4.8 that injected fluids must flow through matrix in order to reach the portion of the fracture network with which the producer intersects. We specify a constant gas injection rate of 3,000 MMSCF/day and a constant producer BHP of 9,000 psi.

Figure 4.8: Model with a network of 30 vertical fractures; no direct connectivity exists between the injector and producer
4.4 Model Upscaling Using MSR Method

We again generate two grids of different areal resolution for the upscaled model: 10×10×3 and 20×20×3, which contain 844 and 2,408 cells respectively. By incorporating global flow information into the upscaling of these two MSR models, we obtain two additional coarse models (for a total of four models, as in the connected case).

Comparisons between simulation results using the DFM and MSR procedures are presented in Figures 4.10-4.14. From the plots of cumulative oil production, cumulative gas production, and methane concentration at the producer as a function of PVI (Figures 4.10-4.12), we observe improvement in accuracy from both areal refinement and incorporation of global flow information. We also see that the use of global flow information in the finer (20×20×3) MSR grid yields results of relatively high accuracy for all curves except injector BHP. The highest accuracy in BHP is achieved with the 10×10×3 model with global upscaling.
Figure 4.10: Cumulative oil production for DFM and MSR simulations (disconnected model)

Figure 4.11: Cumulative gas production for DFM and MSR simulations (disconnected model)
4.4 Model Upscaling Using MSR Method

Figure 4.12: Methane concentration at producer for DFM and MSR simulations (disconnected model)

Figure 4.13: Injector BHP for DFM and MSR simulations (disconnected model)
Figure 4.14: Methane concentration maps at 0.25 pore volume of gas injected from the DFM and the 20×20×3 MSR with global flow information (disconnected model)
4.6 Concluding Remarks

We also compare the average methane concentration map at 0.25 pore volume of gas injected (Figure 4.14) from the 20×20×3 MSR model (with global flow information) to that from the DFM. All concentrations are averaged onto a coarse grid of 20×20×1 for ease of comparison (this averaging is as described in Section 2.5.1). Relatively close agreement between the fine and upscaled models is observed, and the $L_2$ norm error in methane concentration is only 3.24%.

The fully-compositional simulation of the DFM in this case is extremely time consuming, requiring over 550 CPU hours. In contrast, the most accurate (and most computationally expensive) MSR model (20×20×3 with global flow information) required 1,500 seconds for upscaling and only 32 seconds for the coarse-scale simulation. We thus obtain an overall speedup factor of 1,240 using the MSR procedure compared to the DFM approach while maintaining a reasonable level of accuracy.

4.6 Concluding Remarks

In this chapter we presented applications for first-contact miscible gas injection simulations using a fully-compositional model. The MSR upscaling technique presented in Chapter 3 was extended to a full dual-porosity/dual-permeability implementation. In addition, we introduced the use of global, single-phase flow information into our transmissibility upscaling procedure. The resulting models were used for simulations of six pseudo-component compositional systems on two fracture systems – one in which there was connectivity between the injector and producer through the fractures, and one in which this connectivity was absent.
Results of a reasonable level of accuracy relative to the reference discrete fracture models were achieved for both the connected and disconnected fracture systems. The MSR results for bottomhole pressure, however, did show some inaccuracy. The use of global information in the upscaling acted to improve the accuracy of some, but not all, of the MSR models. In the other cases it had very little effect on the results, though no degradation in accuracy was observed. The MSR model was shown to provide a speedup factor of over 1000. These results are very encouraging and suggest that the MSR procedure can be applied to model complex displacements, though more cases will need to be tested.

Future work should address the generation of 3D unstructured grids for complex discrete fracture characterizations involving many fractures. These meshes are required as the first step in the MSR procedure, though their generation is still quite difficult for complex fracture models. In addition, the use of global flow information for the determination of upscaled interblock transmissibilities should be further investigated. It may also be useful to incorporate near-well upscaling into the MSR modeling procedure. This may act to improve the MSR results for bottomhole pressure.
Chapter 5

Development of a Hybrid Discrete Fracture / Multiple Subregion Model

5.1 Introduction

Fractures in the subsurface are often distributed in a complex manner and commonly display significant variation in connectivity and size over the formation. For example, large and strongly connected fractures are typically located near bedding planes and fault zones, while small and disconnected fractures are usually located away from those regions. In addition, as discussed by Aydin et al. (1996), the dimensions and spatial frequency of fractures are impacted by the thickness of the confining stratigraphy (megasequence, sequence, parasequence set, parasequence, bedset, and bed).

Because of these variations in fracture properties, it is reasonable to model different zones of the reservoir using different treatments. As discussed in Chapter 1, such approaches have been proposed previously by various researchers. Our goal in this chapter is to proceed in this direction by linking the MSR and DFM treatments. This
will provide a hybrid approach that can treat accurately both connected and disconnected fractures within one model.

In previous chapters, we have performed global flow simulations using both the DFM (to obtain the fine-scale reference solutions) and MSR procedures. Because the MSR model is a generalized dual-porosity/dual-permeability approach, it is most appropriate when all of the fractures within the coarse-grid block are connected (or nearly so). When fractures are disconnected, by contrast, we may need to apply a different approach in order to represent their effects accurately. We note that, as discussed in Chapter 2, if there are relatively few blocks in the model with disconnected fractures, the MSR approach can still be applied globally without significant loss of accuracy (recall that the single-porosity upscaled permeability is captured by the MSR method). However, if a substantial portion of the reservoir contains disconnected fractures, or the disconnected fractures are in key regions of the reservoir (such as the near-well region), then the global use of the MSR treatment may lose accuracy. The hybrid approach presented here will, however, be applicable for such cases.

This chapter proceeds as follows. First, the hybrid methodology is described and the computations for the internal and interblock connections are explained. Next the hybrid approach is applied to several cases (2D and 3D) for two-phase, three-phase and compositional flow examples. These results demonstrate the improvement in accuracy attainable from the hybrid procedure. We also discuss computational demands for this approach, which are important to consider because the hybrid method is more expensive than the global MSR procedure.
We note that the examples considered in this chapter involve synthetic fracture systems. In models containing both connected and disconnected fractures, the system is constructed such that there is a clear delineation between these two types of regions. For practical problems this distinction may not be immediately evident, and criteria will need to be established to determine the appropriate treatment (MSR or DFM) for a particular reservoir region. Such a determination is, however, beyond the scope of this work.

5.2 Methodology for Hybrid DFM/MSR Procedure

The hybrid method models some coarse blocks using the DFM representation and others using the MSR approach. Figure 5.1 illustrates two regions (each corresponding to one coarse block) that would be modeled differently (the left block using MSR and the right block using DFM). Consistent with our MSR framework, the hybrid method again involves the determination of two types of transmissibilities, corresponding to internal connections and interblock connections.

![Figure 5.1](image.png)

**Figure 5.1**: Motivation for hybrid method: coarse blocks to be modeled using MSR (left) and DFM (right)
The subregion geometry and internal transmissibilities for MSR coarse blocks are determined by solving one-block problems exactly as described previously (Section 2.4.1 for zero or weak gravity effects and Section 3.2.2 for strong gravity effects). The internal connections for DFM blocks (or groups of blocks) remain in the fine-scale form; i.e., the transmissibilities are determined directly from the discretization on the underlying unstructured fine grid. Thus, no new computational procedures need to be introduced for these transmissibilities. Similarly, if an MSR block borders another MSR block, we use the previously described computations for interblock transmissibility.

![MSR Block and DFM Block](image)

**Figure 5.2**: Illustration of matrix subregions for MSR block (left) and fine cells for DFM block (right)

The treatment of the interblock transmissibility is, however, quite different when adjacent blocks are modeled using different procedures. Consider two adjacent blocks as shown in Figure 5.2, one modeled using MSR (left) and the other using DFM (right). The MSR block contains four matrix subregions, each depicted by a different color. The geometries of these subregions are determined by solving the local flow
problems described in Chapters 2 and 3. The DFM block, by contrast, is fully unstructured and contains 707 triangular cells.

![Figure 5.3: Interblock connection determination for neighboring MSR and DFM blocks](image)

Figure 5.3 illustrates how the MSR block $k$ is linked to the DFM block $l$. Specifically, the 4th subregion (in yellow) from the MSR block is linked to all of the fine cells in the DFM block with which it shares a physical interface. Linkages between other subregions in block $k$ and fine cells in block $l$ are defined in a similar manner.

The hybrid model thus involves connections between particular subregions (in MSR blocks) and multiple fine cells (in DFM blocks). To compute the actual transmissibility values, a steady-state single-phase flow problem is solved with a pressure difference imposed between the two boundaries (see Figure 5.3). This solution is performed on the underlying discrete fracture representation. We designate the superscript $k(i)$ to denote subregion $i$ in the MSR block $k$ and $l(j)$ to denote fine cell $j$ in DFM block $l$. The average pressure and fluid properties inside each subregion, as
well as the flow rate $Q^{k(i)l(j)}$ through the interface between the blocks, are computed from the local fine-grid solution. The transmissibility can then be determined via:

$$T^{k(i)l(j)} = \frac{Q^{k(i)l(j)} \mu}{\rho(\bar{p}^k - \bar{p}^l)} 
(5.1)$$

where all quantities are as previously defined. The mass flow rate $Q^{k(i)l(j)}$ is computed over the shared interface of subregion $i$ in the MSR block $k$ and fine cell $j$ in the DFM block $l$. We then apply equation 5.1 for the determination of each of the required transmissibilities.

Figure 5.4: Sketch of connection list between one MSR block and one DFM block (setting corresponds to Figure 5.3)
The connection list is more complicated in this case than it is for the MSR procedures described in Chapters 2 and 3. A portion of the connection list for the case considered above is shown in Figure 5.4. Green and red lines represent internal and interblock connections respectively. This figure illustrates that the 3rd, 4th, and 5th subregions of MSR block \( k \) are connected to 3, 8, and 5 fine cells respectively of DFM block \( l \). This connection list is input directly into GPRS for flow simulation. We now present results for several representative cases.

5.3 Applications

The hybrid method is applied to two 2D cases and a 3D case. In the first example, fractures are connected throughout the reservoir. We model the injection and production well blocks using DFM to better resolve the interactions between the wells and fractures; the other coarse blocks in the model are represented using MSR. In the second case, fractures are disconnected in one portion of the reservoir and we use DFM to model these isolated fractures. In the 3D example, the near-well fractures are disconnected and we use DFM to model them.

5.3.1 2D Model with Connected Fractures

As shown in Figure 5.5, a simple synthetic 2D model containing 32 connected fractures is considered. An injector and producer are located in the lower left and upper right corners of the reservoir, and they both intersect fractures. Two simulations are performed, one for an oil-water system and one for miscible gas injection. For the oil-water flow simulation, we use constant injection rate (500 STB/day) for the
injector and constant BHP (4,000 psi) for the producer. For the miscible gas injection simulation, we use the same fluid characterization for reservoir oil and injected gas as in Chapter 4, and apply constant BHP controls for both the injector (12,500 psi) and producer (11,500 psi). Our coarse model is upscaled to 4×4 coarse blocks (MSR blocks contain 5 subregions). This represents a high degree of coarsening and some inaccuracy would be expected in the global MSR model.

Figures 5.6 and 5.7 display the simulation results for these cases. We present results for the DFM (reference fine-scale results), global MSR, and hybrid approaches. For both the oil-water (case 1) and miscible gas injection (case 2) simulations, we observe that, although the MSR approach yields a model of reasonable accuracy, the hybrid treatment clearly provides improved accuracy. In fact, the hybrid results are nearly indistinguishable from the reference DFM results.
Figure 5.5: Synthetic 2D model with 32 fractures
Figure 5.6: Oil recovery results for DFM, global MSR and hybrid solutions for synthetic 2D oil-water flow (case 1)

Figure 5.7: Oil recovery results for DFM, global MSR and hybrid solutions for synthetic 2D miscible gas injection (case 2)
5.3.2 2D Oil-Water Model with Disconnected Fractures

Figure 5.8 displays a synthetic model with 35 disconnected fractures in the upper right portion of the model and 36 connected fractures elsewhere in the reservoir. This case is somewhat artificial but it would be expected to be a good candidate for the hybrid treatment. We use DFM to model the four blocks in the upper right corner and MSR (with 5 subregions) for the other twelve blocks. For the wells, we use constant injection rate (500 STB/day) for the injector and BHP control (400 psi) for the producer.

Figure 5.8: Synthetic 2D model with 36 connected fractures and 35 disconnected fractures
Figure 5.9 displays simulation results for injector BHP. Due to the disconnected fractures in the well region, the MSR solution shows significant error relative to the reference DFM simulation. The hybrid method, by contrast, provides results in close agreement with DFM solution. The hybrid model also gives more accurate results for oil production rate, as shown in Figure 5.10. This example clearly illustrates the potential for loss of accuracy when the MSR procedure is applied in regions of disconnected fractures and the improved results that can be achieved using the hybrid procedure.

![Figure 5.9: Producer BHP results for DFM, global MSR and hybrid solutions for synthetic 2D oil-water flow with disconnected fractures (case 3)](image-url)
5.3 Applications

Figure 5.10: Oil production rate results for DFM, global MSR and hybrid solutions for synthetic 2D oil-water flow with disconnected fractures (case 3)

5.3.3 3D Three-Phase Model

The hybrid method is now applied to a 3D three-phase example. The same DFM, as well fluid properties and well configurations, are considered as in Section 2.5.2 (see Figure 2.15 and Tables 2.3-2.7). In Chapter 2, we demonstrated that the MSR formulation using a $9 \times 9 \times 3$ coarse grid gives a highly accurate coarse model (Figure 2.16). We now apply a $4 \times 4 \times 1$ coarse grid (much coarser than $9 \times 9 \times 3$) for MSR upscaling. This coarser MSR model is expected to yield less accurate simulation results. We then apply the hybrid approach to model the injection and production well blocks using DFM. The other coarse blocks are treated using MSR with 5 subregions.
Figures 5.11-5.13 display the simulation results (gas and oil production rates and pressure at the injector) for the 3D three-phase case. Again, we present results for the DFM, global MSR, and hybrid approaches. We observe that the MSR approach using the 4×4×1 coarse grid shows substantial error, especially at early time. By resolving the injection and production coarse blocks explicitly using DFM, the hybrid treatment clearly provides significantly improved accuracy.

Figure 5.11: Gas production rate results for DFM, global MSR and hybrid solutions for synthetic 3D three-phase flow (case 4)
5.3 Applications

Figure 5.12: Oil production rate results for DFM, global MSR and hybrid solutions for synthetic 3D three-phase flow (case 4)

Figure 5.13: Producer BHP results for DFM, global MSR and hybrid solutions for synthetic 3D three-phase flow (case 4)
5.4 Discussion

The results shown in the previous sections demonstrate the enhanced accuracy of the hybrid approach. This accuracy results from the use of finer resolution in some regions of the model and therefore leads to increased computational cost, which we now quantify. Compared to the MSR approach, the increased numerical burden will depend on the number of DFM coarse blocks. Tables 5.1-5.4 below summarize the problem size and simulation times for the DFM, global MSR and hybrid approaches for the four cases studied in this chapter.

Table 5.1: Comparison of problem size and simulation times for DFM, global MSR, and hybrid approaches for case 1

<table>
<thead>
<tr>
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<th>Number of Cells</th>
<th>Number of Connections</th>
<th>Simulation Time (s)</th>
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</thead>
<tbody>
<tr>
<td>DFM</td>
<td>4659</td>
<td>7619</td>
<td>92</td>
</tr>
<tr>
<td>MSR</td>
<td>80</td>
<td>88</td>
<td>2.2</td>
</tr>
<tr>
<td>Hybrid</td>
<td>647</td>
<td>1203</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 5.2: Comparison of problem size and simulation times for DFM, global MSR, and hybrid approaches for case 2

<table>
<thead>
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<tbody>
<tr>
<td>DFM</td>
<td>4659</td>
<td>7619</td>
<td>245</td>
</tr>
<tr>
<td>MSR</td>
<td>80</td>
<td>88</td>
<td>3.6</td>
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<tr>
<td>Hybrid</td>
<td>647</td>
<td>1203</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 5.3: Comparison of problem size and simulation times for DFM, global MSR, and hybrid approaches for case 3

<table>
<thead>
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<td>DFM</td>
<td>11677</td>
<td>18460</td>
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<tr>
<td>MSR</td>
<td>80</td>
<td>88</td>
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<tr>
<td>Hybrid</td>
<td>2909</td>
<td>4610</td>
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</tr>
</tbody>
</table>
Table 5.4: Comparison of problem size and simulation times for DFM, global MSR, and hybrid approaches for case 4

<table>
<thead>
<tr>
<th></th>
<th>Number of Cells</th>
<th>Number of Connections</th>
<th>Simulation Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFM</td>
<td>52,059</td>
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<td>198</td>
</tr>
<tr>
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<td>80</td>
<td>88</td>
<td>2.1</td>
</tr>
<tr>
<td>Hybrid</td>
<td>8,019</td>
<td>16,568</td>
<td>25.6</td>
</tr>
</tbody>
</table>

It is clear from Tables 5.1-5.4 that the hybrid treatment leads to many more cells and connections. For these examples, the global MSR offers about a two-order of magnitude speed up relative to the reference DFM, while the hybrid procedure provides only about a one-order of magnitude speed up relative to DFM. The computational requirements for the hybrid method will of course depend directly on the number of fine cells included. The timings in Tables 5.1-5.4 suggest that it will be important to limit the number of regions that are fully resolved, or to model these regions using a coarse DFM.

5.5 Concluding Remarks

In this chapter we presented and tested a hybrid procedure that represents some reservoir zones using the multiple subregion model and other zones using the DFM approach. The hybrid method naturally generalizes the MSR representation, which may encounter problems when applied to reservoir regions with disconnected fractures. In addition, the hybrid method enables enhanced accuracy in key reservoir zones such as the near-well region. The examples clearly demonstrate the improved accuracy of the hybrid approach, though the additional computational costs (relative to global MSR) of using this method are significant.
It will therefore be important to develop procedures for determining appropriate treatments (MSR or DFM) for reservoir zones in large models. This determination, which was not considered here, might involve first computing various fracture statistics (such as length and connectivity measures). These could then be used to prescribe the appropriate modeling technique for the various reservoir zones. It may also be possible to incorporate some type of iteration procedure in order to assure self-consistency in the model. In analogy to local-global upscaling procedures (Chen et al., 2003), this could entail performing a global solution for a simplified (e.g., single-phase flow) problem with the initial hybrid model and then using this simulation result to determine the proper treatment for each reservoir zone.
Chapter 6

Conclusions and Future Work

This thesis presented upscaling procedures for constructing generalized dual-porosity and dual-porosity/dual-permeability simulation models from detailed discrete fracture reservoir characterizations. This work was motivated by the need for accurate representations of fracture systems that are appropriate for flow modeling but maintain the effects of fine-scale heterogeneity. We now summarize this work and offer recommendations for future research directions.

6.1 Summary and Conclusions

The following conclusions can be drawn from this study:

- A systematic methodology for constructing an upscaled model from a detailed, geometrically complex fracture characterization was developed and applied. The technique, which we call the multiple subregion or MSR method, is applicable for 2D and 3D systems. The initial implementation was a dual-porosity type formulation, which is most suitable for systems with weak or nonexistent gravitational effects. The MSR approach was applied successfully to single-
phase, two-phase and three-phase flow problems, including the effects of compressibility and capillary pressure. Systematic improvement in the coarse model with increasing numbers of subregions was observed. Computational speedups relative to the underlying (reference) discrete fracture model (DFM) of about 100 were achieved.

- This dual-porosity type MSR method was then extended to treat models in which gravitational effects are important. In contrast to the initial MSR approach, which was applicable for viscous (and capillary) dominated systems, the extended model accounts for matrix-matrix interblock transfer. It can thus be viewed as a generalized dual-porosity/dual-permeability model. Simulation results for 2D and 3D problems, including viscous, capillary and gravitational effects, demonstrated the accuracy of the method. Overall speedups of about a factor of 100 were typically observed.

- The MSR approach was further extended to a full dual-porosity/dual-permeability representation that includes connections between matrix regions in vertically and horizontally adjacent blocks. We also introduced the use of global single-phase flow information to compute the upscaled interblock transmissibilities. This extended MSR representation was applied to 3D models for a six-component, first-contact miscible displacement. The MSR models provided results in reasonable agreement with the reference DFM simulations at computational speedups of about 1,000. This suggests that the general MSR procedure is applicable even for complex displacement processes.
6.2 Recommendations for Future Work

- A hybrid procedure, which represents some reservoir zones using the MSR model and other regions using the DFM approach, was also developed. This enables higher resolution and accuracy in selected regions and was shown to lead to more accurate simulation results. Computational costs with the hybrid approach are, however, significantly higher than with the global MSR model (though still much less than with the global DFM).

6.2 Recommendations for Future Work

- As indicated in Chapter 4, the use of global flow information for the determination of upscaled interblock transmissibilities should be further investigated. This may lead to improved accuracy in BHP and other quantities in MSR models.

- For the hybrid method presented in Chapter 5, it will be useful to develop procedures for determining appropriate treatments (MSR or DFM) for various reservoir zones. This might be accomplished using an iterative approach. In addition, given that the computational costs of the hybrid method are significantly higher than those of the global MSR method, it may be worthwhile to investigate the use of coarser DFM representations for resolved blocks.

- Our current treatment of wells in the upscaled models is most appropriate for wells that intersect the fracture network. For wells completed in the matrix or in isolated fractures, an enhanced approach may be required. This problem can be
viewed within the context of near-well upscaling and existing techniques (e.g., Hui and Durlofsky, 2005) can likely be adapted to address this issue.

- The procedures presented in this work used an underlying triangular grid in 2D and a tetrahedral grid in 3D for the fine-scale calculations. Structured Cartesian or stratigraphic grids defined the coarse grid. The use of general polyhedral grids for both the fine and coarse-scale models should be explored. This may provide improved accuracy and flexibility for complex geological characterizations.

- In our experience, the generation of high quality fine-scale grids that resolve complex fracture descriptions is very challenging. Thus, new gridding procedures should be explored. Problems related to gridding can be circumvented to some extent by modifying the current workflow to avoid generating a global grid. Instead, the grid could be constructed over one or two coarse blocks at a time, as required for the local upscaling calculations. With this approach it will not be possible to simulate the global DFM, but such simulations may not be necessary in many cases.

- The current method employs two-point flux approximation (TPFA) for all fine-scale transmissibility calculations. It will therefore be useful to incorporate the grid optimization techniques under development by Mohammad Karimi-Fard, or multipoint flux approximation (MPFA), to reduce errors due to grid nonorthogonality or general anisotropy.
6.2 Recommendations for Future Work

- In order to refine the approach and workflow, the MSR formulation should be tested on field cases and then generalized and enhanced as required.

- The MSR method can be considered for related problems (e.g., geothermal simulations) involving flow and heat transfer in fractured systems.
Appendix A

Workflow for 3D DFM Simulations and MSR Code Description

A.1 3D DFM Simulations

Figure A.1: Workflow to generate discrete fracture models
A schematic of the workflow used to construct the DFM and perform flow simulations is presented in Figure A.1. This workflow was used for constructing the disconnected fracture model (see Figures 4.8 and 4.9) studied in Section 4.4.2. It can also be applied for other realistic 3D models. First, we input the fracture characterization (provided by the fracture modeling software FRED or another fracture modeling tool) into GOCAD to build the detailed geological model (DFM). In GOCAD, the reservoir matrix is modeled as an “SGrid” object and the discrete fractures are modeled as “surface” objects. GOCAD is also used to identify all of the intersections between fracture-fracture and fracture-matrix surfaces. TetGen (Si, 2004) is then used to construct a tetrahedral mesh for the geological model defined by GOCAD. Next, the Grid2Trans program (written by Mohammad Karimi-Fard, 2006) is applied to compute all cell volumes, cell to cell transmissibilities (using the procedures described in Karimi-Fard et al., 2004), and the connection list. This information is input to GPRS and simulations are performed. The simulation results can then be read back into GOCAD for visualization.

We now provide more specifics on the software tools and procedure. The four tools applied are:

GOCAD, which is used to:
- load the fracture characterization
- visualize the geological model
- identify the intersections between fractures
- visualize the simulation results

TetGen, used to:
- perform unstructured gridding honoring the surface constraints defined by the fractures

Grid2Trans, used to:
- calculate the cell volumes and center point coordinates for each cell
- compute the transmissibility between each pair of cells

GPRS, used to:
- run reservoir simulations using the connection list

The step-by-step procedure is as follows:

1. Load the fracture sets as surfaces and the matrix (constructed after SGrid) as surfaces.
2. Combine all of the fracture surfaces into one, and similarly for the matrix surfaces.
3. Beautify the surfaces by choosing from the GOCAD menu “Edit → Optimize” or “Edit → Beautify.” This acts to improve the quality of the gridding of the fracture surfaces.
4. Identify the intersections between the fractures and between fracture and matrix surfaces. This can be achieved by one of the following approaches (or various combinations):
   - In “Surface” mode, choose from the menu “Model3d → New from surfaces,” pick the fracture and matrix surfaces, then a model3d will be constructed with all of the surface intersections identified.
   - In “Surface” mode, choose from the menu “Edit → Cut → Cut by surfaces,” pick the fracture and matrix surfaces, then the two surfaces will be updated with all the surface intersections identified.
In “Surface” mode, choose from the menu “Edit → Cut → Cut by each other,” pick the fracture and matrix surfaces, then the two surfaces will be updated with all of the surface intersections identified.

5. Save the surfaces as “fracture.ts” and “matrix.ts.”

6. Run “gocad2tet” to generate the input file for TetGen (i.e., to write “dfm.poly”).

7. Run “tetgen dfm.poly” to generate unstructured meshes. Some of the command line switches we use include:
   - tetgen –q dfm.poly: to improve the mesh quality
   - tetgen –a5000 dfm.poly: to force the largest volume of generated meshes to be less than 5000
   - tetgen –T1e-9 dfm.poly: to set the precision to be 1e-9

   Refer to TetGen manual for examples of different command line switches.

8. Run “tet2grid” to generate the input file (i.e., “*.grid”) for “grid2trans” by processing the TetGen output files.

9. Run “grid2trans *.grid *.trans” to calculate the inter-cell transmissibilities and cell volumes.

10. Run “trans2gprs” to generate the input files (“conn.in,” “volume.in,” “depth.in,” etc.) for GPRS.

11. Run GPRS for reservoir simulation.

12. Run “gprs2gocad” to create point sets (x, y, z coordinates, pressure, phase saturation, composition, etc. for each cell) at different time steps and the corresponding solid at the last time step.

13. Load the point sets and solid generated from step 12 to GOCAD for visualization.
A.2 MSR Code Description

The code is written in C++. It includes the main function and three subroutines. Figure A.2 illustrates the workflow, which we now describe.

1. Read in information from the DFM including $x$, $y$, $z$ coordinates and volume for each fine cell, as well as the connection list.
2. Initialize the problem by identifying the coarse grid ID for each fine cell and building a map containing all fine-cell properties. The boundary cells are identified through look ups from the DFM connection list. The $\alpha$ factor is also specified.
3. Perform the one-block calculations using the governing equation and boundary conditions described in Section 3.2.2. This computation provides the subregion geometry and intrablock fracture-matrix and matrix-matrix transmissibilities.
4. Perform the two-block calculations using the governing equation and boundary conditions described in Section 2.4.2. As an option, global pressure information can be used for the transmissibility calculations. Either a dual-porosity (DP) or dual-porosity/dual-permeability (DPDK) formulation can be used in computing the block to block linkages.
5. Output the MSR model parameters such as volume, porosity and depth for each subregion, as well as the MSR connection list.
Appendix A

Start

Initialization

Build a map containing all fine cells with coordinates and coarse grid
Identify boundary cells for each coarse grid block
Specify α factor

Intra-block calculation

Subregion geometry
Fracture-matrix and matrix-matrix transmissibilities

Global pressure information

Inter-block calculation

DP formulation
DPDK formulation

Fracture-fracture and matrix-matrix transmissibilities

End

Figure A.2: Workflow for MSR code
Nomenclature

\( A \)  
Accumulation term

\( c \)  
Compressibility

\( D \)  
Discrete fine cell

\( F \)  
Fracture region

\( g \)  
Gravity vector

\( k \)  
Absolute permeability (tensor)

\( k \)  
Absolute permeability (scalar)

\( k_r \)  
Relative permeability

\( M \)  
Matrix region

\( p \)  
Pressure

\( p_c \)  
Capillary pressure

\( Q \)  
Total flow rate (volume or mass)

\( q \)  
Flow rate (volume of mass)

\( S \)  
Saturation

\( T \)  
Transmissibility

\( t \)  
Time

\( V \)  
Volume

Greek

\( \alpha \)  
Parameter for MSR geometry

\( \varepsilon \)  
Micro-scale length

\( \lambda \)  
Mobility

\( \mu \)  
Viscosity

\( \Omega \)  
Coarse block
Nomenclature

\( \phi \) Porosity
\( \rho \) Density
\( \sigma \) Shape factor
\( \tau \) Transfer function

Subscripts

\( B \) Bottom
\( f, F \) Fracture
\( g \) Gas
\( i, j \) Fine or coarse cell index
\( l \) Phase
\( l, m \) Connected fine cell pair index
\( m \) Matrix
\( n, n1, n2 \) Subregion index
\( o \) Oil
\( T \) Top
\( w \) Water

Superscripts

\( \_ \) Averaged
\( 0 \) First order
\( c \) Coarse scale
\( f \) Fine scale
\( l,k \) Block index
\( w \) Water
\( x, y, z \) Coordinate direction

Acronyms

DFM Discrete fracture model
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>MSR</td>
<td>Multiple subregion</td>
</tr>
<tr>
<td>SR</td>
<td>Subregion</td>
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Bibliography


GOCAD version 2.1.4, Earth Decision Sciences, Houston, Texas.


