REAL-TIME CALIBRATION OF DRIFT-FLUX FLOW MODELS

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By
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I certify that I have read this report and that in my opinion it is fully adequate, in scope and in quality, as partial fulfillment of the degree of Master of Science in Petroleum Engineering.

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Abstract

One of the key near-term challenges of the energy industry is the need to exploit new and mature fields more efficiently and at lower costs than ever before. This report describes an approach for addressing this challenge in the specific area of modeling of multiphase flows in oil production systems (specifically wells and pipelines) and continuous calibration of models for the purpose of better multiphase flow predictions. The overall objective is to make flow-related decisions more rapidly, i.e., in real-time. To enable this objective, this work proposes two novelties: a new drift-flux flow model for upward bubbly flows in small-to-large diameter wells and pipes, and a real-time calibration algorithm of well and pipe flow models with field sensor measurements.

In formulating the new drift-flux model, the restrictions of existing drift-flux models were analyzed and flow phenomena that were deemed important were explicitly related to the two parameters of the drift-flux model, i.e. the distribution parameter and the drift velocity. These flow phenomena were bubble coalescence/breakup, evolution of radial bubble distributions and asymmetric radial bubble distributions in deviated well and pipe flows. The new drift-flux model was tested with an existing model at the department, MSWell, using large (7.25 and 6 inches) and small (1.79 inches) diameter data from the experiments described in Hill (1992), Oddie et al. (2003), and Spedding and Nguyen (1976), respectively. With these experiments, the predictions of void fraction using the new model was shown to be satisfactory for gas-water two-phase flows, better for oil-water-gas three-phase flows and best for oil-water two-phase flows. Identification of the average bubble sizes and regimes matched fairly well with visual observations of the Oddie et al. (2003) dataset.

The role and use of flow model calibration in a real-time surveillance strategy was analyzed in the context of calibration as a tool for integration of real-time sensor
measurements, field data and flow models. In formulating the new calibration algorithm, the flow of sensors data from an automation network on a facility to a computer model on a remote desktop was analyzed, and a proposed method was used in calculations using these remote sensors data. Additionally, in the calibration algorithm itself, a direct three-phase holdup objective function is derived and used in calibrations of the drift-flux model’s parameters in alignment with an existing three-phase holdup calculation procedure in the department. The calibration algorithm was implemented in a simple oil well case with simulated real-time sensor measurements, for illustrating how minimal field operating data could be used to achieve calibration of the drift-flux model to pressure sensors data. For this example, the calibration algorithm has been proven to work under certain conditions. Restricting conditions, in which the algorithm stalls (or becomes unstable), are shown to be the large amount of memory requirements, and the difficulty of using un-preprocessed real-time data in a gradient-based, non-linear least squares optimization routine. The latter causes stalling whenever there is a series of unfeasible calibrated drift-flux parameters present, whereas the former causes stalling due to low system resources.

Overall, despite the complex physics and computational requirements, the results of the computer model of this research, called SURF, are both qualitatively and quantitatively encouraging. It is hoped that continued work in this area of research will eventually lead to the ability to make faster, better-informed flow-related decisions that minimize long term production losses.
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Chapter 1

1. Introduction

1.1. Statement of the Problem

For decades now, two key things have been happening in the oil and gas industry:

1) The average size of new oil and gas discoveries has been dropping.
2) The complexity of reservoirs has been fueling a ballooning volume of data.

Finding new giant reserves is a solution for 1) above. There is also the need to exploit new and mature fields more efficiently and at lower costs than ever before, i.e., a solution for 2) above. Both 1) and 2) are key near-term challenges that need to be met for providing hydrocarbon (or natural resources) to meet the rapidly increasing global demand for energy. This report describes an approach for addressing 2) above in the specific area of modeling of multiphase flows in smart oil production systems (specifically wells and pipelines) and continuous calibration of models for the purpose of better multiphase flow predictions. The overall objective is to make flow-related decisions more rapidly, i.e., in real-time.

In the oil and gas industry, data are acquired, stored and transmitted using various information and computing systems at the upstream appraisal stage; and data exists in distributed control hierarchies in instrumented facilities at the upstream development stage, i.e. during production. The data explosion that currently results from the range of control systems being used to monitor and interact with oil platforms demands a need for integrated modeling with real-time sensor data. The problem remains however, that the oil and gas industry continues to struggle with adapting to and implementing digital-age technology of the type as noted above. The challenge appears to be the integration of digital-age technology across the different work flows of subsurface evaluation and surface facility operations. The current approach of using individual applications based on discipline-centric work flows is often unsuccessful. The ability to integrate
data flows from the subsurface, through wells and network production monitoring, and into surface control systems still proves a challenge both in theory and in practice. Hence, this research puts forward two novelties (specific objectives) in response to the challenge described above. They are:

1) A drift-flux flow model for upward bubbly flows in small-to-large diameter wells and pipes that is continuous and differentiable.
2) Real-time calibration of well and pipe flow models with field sensors data.

The specific technical objectives of this research are to provide solutions to the following questions:

1) What multiphase flow model is most appropriate for the small-to-large diameter piping components of an oil production facility (i.e., well tubing-strings and pipelines)?

2) How to automatically optimize flow model parameters within a real-time time-frame, i.e., of the order of seconds, whilst retaining physically meaningful constraints (box and/or linear constraints) on parameters? How to update the flow model with both external (periodic updates from file) and internal (continuous inputs from results of calibration) changes?

3) How to securely access streaming real-time sensor data over the internet for use in a client-side computer model?

The specific impacts of this research are in:

1) Timely well intervention and pipeline-related decisions. Traditionally, reservoir, production and facility engineers have utilized aggregated daily or monthly production values. With real-time surveillance, they can access data at any desired time interval, on demand. Comparing predicted with actual rates they can identify problem wells earlier.

2) Retention of future production in worst-case scenarios, i.e., making faster, better-informed decisions that minimize long term production losses.
1.2. Multiphase Flow in Pipes

1.2.1. Terminology

Fluid particles, as used in this report, is a generic term for the dispersed phase particles in a multiphase mixture (denoted as $p$), namely gas bubbles (in gas-liquid systems), liquid globules (or emulsions, in liquid-liquid systems) or liquid droplets (in liquid-gas systems). This research focuses on the specific case of the dispersed phase being gas bubbles or liquid globules in a carrier liquid.

The *facility* model for this research is restricted to:

1) The production of gas, oil and water in *wells*.
2) The transportation of gas, oil and water in *pipelines* on the surface.
3) The transportation of gas, oil and water in platform-to-platform *flow-lines*.

For the scope of this research, ultra large diameter platform-to-plant *trunk-lines* are not considered.

The *multiphase flow* in production facilities piping is defined as a mixture of the phases of oil, water and gas. A phase is *continuous* if it occupies continuously connected regions of space. The continuous (or carrier) phase denoted by subscript $c$ can either be a liquid or a gas, but in this report, will always refer to a liquid. A phase is *dispersed* if it occupies disconnected regions of space. The dispersed phase denoted by subscript $d$, can either be a liquid or a gas. Fluid particles formed by a gaseous phase are denoted as *bubbles*, while fluid particles formed by a liquid phase are called *globules* or *droplets*, depending on the prevailing flow pattern and system. An arbitrary phase is denoted as $k$.

1.2.2. Flow Models

The analysis of black oil production facilities involving the multiphase pipelining of untreated or unseparated reservoir fluids requires a good understanding of the flow phenomena. In these systems, the hydrodynamics of the flow is of prime interest to the facility engineer. In
accordance with the objectives of this research, the usual approach of the hydrodynamic modeling of facility flows is also adopted.

The difficulties of the modeling and the subsequent numerical computation of multiphase flows mainly arise from the existence of moving interfaces separating the phases in the mixture. In general, the geometry of the interface and spatial distribution of the phases are not known a priori but are part of the solution. **Modeling** difficulties concern the physical transfer processes taking place across the interface such as momentum, heat and mass transfer, and phase change. **Numerical** difficulties arise from the fact that: 1) the interface is moving, and 2) some properties of the phases are discontinuous across the interface, e.g. density, viscosity. Depending on the strength of the coupling between the phases, different modeling approaches are suggested. In general, they can be classified into homogeneous (no slip) flow models, mixture models (with slip) and full multi-fluid models. Combinations of these are also possible. In most models, each phase is treated as an interpenetrating continuum with an in-situ volume fraction parameter (a.k.a. holdup).

Multiphase flow simulation utilizing a **full** multiphase flow model (i.e., one that considers the mass, momentum and energy conservation equations for each phase) is impractical for flows with wide distributions in the fluid particle size or density. Various approximations are therefore made to simplify the computational task. In the simplest approach, the multiphase mixture is represented by a homogeneous single phase system and the influence of the fluid particles is taken into account in the values of the physical properties. In this formulation, the hydrodynamics of a mixture refer only to the motion of the **centre of mass** of the system. However, in surface facility wells and pipes, there are often bubbly flows in which the dispersed phase particles can vary in their concentration in both the axial and radial directions of flow. Thus flows with these characteristics of distributed concentrations of bubbles in various parts of the pipe, can alter the hydrodynamic behavior of the mixture, and hence a homogeneous single phase approach may not be applicable. This **homogeneous** flow model is applicable only in drag dominated flows in which the phases are strongly coupled and their velocities equalize over short spatial length scales. All phases are assumed to move at the same velocity. The velocity of the
mixture is solved for from a single momentum equation. For each phase, an individual continuity equation is solved to obtain its volume fraction.

In the case of oil production facilities, bubbly/globular dispersed phase flow is often present in the main piping components. In these flows, buoyancy, inertia, viscous and surface tension forces tend to cause in-situ velocity differences (i.e., *slip*) among the phases, which lead to a low degree of dispersed phase clustering. This implies that the spatial length-scales related to fluid particle acceleration are often short, and hence particles reach their terminal velocities in short time-scales. Additionally, low dispersed phase clustering diminishes the total effective drag force, which implies that the terminal velocity of the dispersed phase particles can be assumed equivalent to that derived from a local force equilibrium between the buoyancy and drag forces on a single dispersed phase particle. The flow model described above is commonly referred to as the *mixture* model, of which there can be several different groups depending on the exact formulation of the *slip* velocity between the phases of the system.

**1.2.3. Phase and Velocity Averaging**

In principle, a multiphase flow model could be formulated in terms of the *local* instant variables pertaining to each phase and matching boundary conditions at all phase interfaces. Obtaining a solution from this formulation would be impossible in practice (Worner, 2003). However, it can be used as a starting point for derivation of *macroscopic* equations which replace the *local* instant description of each phase by some collective or *continuum* description of the phases. In this research, the formulation of the flow model is based on a *macroscopic* description of all the phases in the mixture, using spatial averaging across the direction of flow. In this case, the principles of continuum mechanics for a single phase are generalized to several inter-penetrable continua. The basic assumption is that, at any instant of time, all phases are present at every material point. The formulation of the flow model of Chapter 2 uses three types of averaging:

1) Averaging across the direction of flow (quantities with this averaging will be referred to as *mean* quantities).
2) Integrals over the flow field, i.e. the cross-section of the pipe (quantities with this averaging will be referred to as cross-section averaged quantities). For example, quantity $f$ in this type of averaging is denoted as: $\langle f \rangle = \frac{1}{A} \int_A f \, dA$

3) Weighted, cross-section averaging (Wallis, 1969), i.e. quantities averaged by 2), and also weighted by the cross-section averaged dispersed phase volume fraction. For example, quantity $f$ in this type of averaging is denoted as: $\langle \{ f \} \rangle = \frac{\langle \alpha_d f \rangle}{\langle \alpha_d \rangle}$

### 1.2.4. Key Multiphase Flow Forces

For this research, the multiphase forces that are considered to be most significant to the modeling of flow in wells and pipes on oil producing facilities are (Soo, 1990):

1) Buoyancy force – a volume force that accounts for the net action of gravity in systems with non-uniform density.

2) Inertia force – a volume force that acts to retain the actual direction and magnitude of motion of phases unchanged.

3) Viscous force – a surface force that acts to diminish the velocity differences between phases, i.e., making the flow-field uniform.

4) Surface Tension force – a line force that acts to minimize the surface area of the phase interface.

The three dimensionless numbers that are defined as ratios of the above forces, on a per unit volume basis, are given below. In each definition, the characteristic fluid particle velocity, $\bar{u}$, is represented as the particle terminal velocity, $u_{Cd,\infty}$, and the characteristic fluid particle size, $\bar{d}$, is represented as the volume-equivalent particle diameter, $d_{p,eq} = \left( \frac{6V_p}{\pi} \right)^{\frac{1}{3}}$. They are (Worner, 2003):
\[ \frac{Ea_p}{Eotvos \, No.} = \frac{F_{\text{Buoyancy}}}{F_{\text{Surface Tension}}} = \frac{g(\rho_c - \rho_d)}{\frac{\sigma_{dc}}{d^2}} = \frac{d_{p,eq}^2 g(\rho_c - \rho_d)}{\sigma_{dc}} \]  

(1)

\[ \frac{Re_p}{Reynolds \, No.} = \frac{F_{\text{Inertia}}}{F_{\text{Viscous}}} = \frac{\rho_c |\mathbf{u}|^2}{\frac{d}{\mu_c |\mathbf{u}|}} = \frac{\rho_c d_{p,eq} |\mathbf{u}|_{Cd,\infty}}{\mu_c} \]  

(2)

\[ \frac{Mo}{Morton \, No.} = \frac{F_{\text{Buoyancy}}}{(F_{\text{Inertia}})^2 (F_{\text{Surface Tension}})^3} = \frac{g(\rho_c - \rho_d)\left(\frac{\mu_c |\mathbf{u}|}{d}\right)^4}{\left(\frac{\rho_c |\mathbf{u}|}{d}\right)^2 \left(\frac{\sigma_{dc}}{d^2}\right)^3} = \frac{g(\rho_c - \rho_d)\mu_c^4}{\rho_c^3 \sigma_{dc}^3} \]  

(3)

### 1.2.5. Flow Patterns

This drift-flux flow modeling, as described in this report, applies to upward bubbly/globular flows in oil production systems as denoted in the flow patterns of Figure 1 below. The top row of Figure 1 below depicts the gas-liquid flow patterns that are prevalent in upward bubbly flows from 0 to 45 degrees to the vertical, and the bottom row depicts the liquid-liquid flow patterns in this range of inclination. The testing of the drift-flux flow model developed in this report, as outlined in Chapter 4, is restricted to this range of inclination angles from the vertical. For clarity, in the case of gas-liquid upward bubbly flows, the bubble flow pattern is observed only in vertical and off-vertical flows in relatively large diameter pipes, while dispersed bubble flow is found over a wider range of pipe inclinations (Barnea, 1986). The elongated bubble flow, as depicted below, occurs at pipe inclinations where there is no longer axial symmetry in the dispersed bubble flow case, and where larger bubbles of irregular shape (not Taylor bubbles) migrate to the upper side of the pipe (particularly in larger diameter pipe). The differences between the stable and unstable emulsions (globular flows) is due to the differences in uniformity in the axial (\(\theta \geq 0^\circ\)) and radial (\(\theta > 0^\circ\)) directions.
The flow-field scale descriptions (a.k.a. flow patterns) shown below are not predicted by the new drift-flux flow model in this report, since the results of the new model only provide particulate-scale descriptions of the dispersed phase particles. The relationship between flow pattern identification/transition criteria and fluid particle size is not trivial and warrants separate research in this area. The flow patterns of Figure 1 are shown only to depict the range of applicability (i.e., flow patterns and pipe inclinations) of the drift-flux model of this report in upward bubbly flows in oil production systems.

Figure 1: Upward bubbly flow patterns in oil production systems (flow regime figures taken from Worner (2003), Soo (1990) and Guet et al. (2003)).
1.2.6. Pertinent Literature Review

The following review is restricted to past and recent developments in one-dimensional two-phase (gas-in-liquid, liquid-in-liquid) and three-phase (gas-in-liquid-in-liquid) mixture flow models, and particularly drift-flux models. In general, the mixture models of interest to this research are those which can be described as homogeneous mixtures or mixtures of pseudo-homogeneous fluids, i.e., the fully-developed laminar or turbulent Newtonian flow of multi-component fluids in thermodynamic equilibrium in which one of more phases is dispersed in a carrier phase, and where phases migrate relative to the bulk motion (i.e., drift) or relative to each other (i.e., slip). Mixture model equations of this type largely resemble those for a single-phase flow but are represented in terms of a mixture density, viscosity and velocity. The volume fraction for each dispersed phase is solved from a phase continuity equation. Depending on the exact formulation of the equations used to determine the slip velocity between the phases of a system, applicability and on the personal preference of the researchers, mixture models in literature have been referred to in several ways as (Manninen et al., 1996):

- the Drift-flux model (Zuber et al., 1965),
- the Mixture model (Ishii, 1975),
- the Algebraic-slip model (Pericleous et al., 1986),
- the Suspension model (Verloop, 1995),
- the Diffusion model (Ungarish 1993, Ishii, 1975), and
- the Local-equilibrium model (Johansen et al. 1990).

The mixture flow models above are all generalizations of the homogeneous flow model, which in turn is one of three commonly used flows models in the context of petroleum engineering, i.e., empirical correlations, homogeneous models with slip and mechanistic models. The applicability and drawbacks of these flow models are noted in Shi et al. (2005).

In petroleum engineering applications, such as counter-current flow of heavier and lighter phases as can be found in well shut-in cases (Hasan and Kabir, 1999), or in reservoir simulator linkage and modeling of wells, the drift-flux mixture flow model is particularly suited because it is simple, continuous and differentiable (Holmes, 2001). The origins of the one-dimensional drift-
flux flow model can be traced back to a paper by Zuber and Findlay (1965), in which a general method was proposed that could be used for predicting void fraction while accounting for non-uniform flow and concentration distributions (i.e., distribution parameter) as well as the slip velocity between two phases. Since then many researchers have refined this model for various applications such as in:

- two-phase flow regimes (Ishii, 1977, 1984; Kataoka et al., 1987; Wallis 1969; Richardson et al., 1954; Chen, 2001),
- transient conditions (Nassos et al., 1967; Alkaya, 2002),
- vertical and deviated wells (Hasan and Kabir, 1988, 1999),
- multi-segmented wells (Holmes, 2001; Shi et al., 2003),
- three-phase flows (Shi et al., 2004),
- mechanistic models (Petelas and Aziz, 1997, 1998; Ansari et al., 1994), and
- parametric optimization studies (Diaz, 2004; Shi et al., 2005).

Existing drift-flux flow models in literature, to this author’s knowledge, share the following restrictions:

1) They are applicable in one-dimensional flows.

2) There are often different drift-flux models for different flow regimes. A notable exception is the MSWell model of Shi et al., (2003), originally developed by Holmes (1977).

3) Transverse (or lateral) forces, also known as non-drag forces, are not explicitly considered. This implies that no bubble coalescence/breakup model is incorporated in existing drift-flux models (i.e., interactions between dispersed phase particles is not explicitly considered), which in turn affects flow pattern prediction with these models.

4) A symmetric radial void fraction distribution is assumed, even in deviated pipe flows where density differences cause phase migration and hence an asymmetric radial void fraction profile. Effects of pipe deviation are usually only accounted for as a multiplier of the drift velocity in existing drift-flux models.

5) Pipe wall peaking flows and the peaking transition phenomena (i.e., radial void fraction evolution from central to wall peaking profiles and vice versa) in larger diameter pipes, as
observed by several researchers (Tomiyama et al., 1998, Krepper et al., 2000; Guet et al., 2003) are not explicitly accounted for.

6) A specific bubble size is usually assumed (also usually considered as spherical, rigid bubbles moving through a stagnant carrier phase) and taken as constant for all bubbly flow systems.

The drift-flux flow model of this research attempts to address the restrictions noted in 2) – 6) above, by providing explicit ways in which these phenomena can be related to the two drift-flux parameters, \( C_0 \) and \( V_d \), for the specific case of upwards bubbly flows. The basic assumption is that these phenomena can be accounted for by these two parameters. The model will then be tested with an existing drift-flux model, i.e., the MSWell model at the department (which is essentially the same as that used in Eclipse (Geoquest, 2001)).

### 1.3. Applicability of Drift-flux Flow Model for Real-time Calculations

The drift-flux flow model is a particular type of mixture model (see Sections 1.2.2 and 1.2.6) in which the slip velocity between the phases of a system is not a function of space, but only of time (Manninen et al., 1996). Hence the same slip velocity is used within the modeling domain. Simplifications made in calculating the slip velocity from the terminal velocity of the dispersed phase particles, e.g., as approximated by an algebraic expression, restrict the applicability of the drift-flux flow model to cases in which the fluid particles reach their terminal velocity in a short time period compared to the characteristic time scale of the flow of the mixture, i.e. fully developed flow. This then explains why the drift-flux model is so useful since it provides a simple way to obtain the slip velocity between the phases from the terminal velocity of the dispersed phase particles, which is constant within the modeling domain. Hence only the terminal velocity is solved, thus reducing the computational effort considerably, especially in cases where several dispersed phases need to be defined due to varying densities.

The most important advantage of the drift-flux flow model, in the context of above, is thus the considerably smaller number of variables to be solved when compared to the full multi-fluid models. Hence the reduction in the need for computing resources combined with the simplicity
of computation makes the drift-flux flow model applicable for real-time calculations in fully-developed flows, where the speed of simulation becomes the decisive factor. Moreover, drift-flux model calculations require less computer memory and disk space than full multi-fluid models, which usually in turn affects the computing time.

1.4. Integration of Real-time Sensor Measurements, Field Data and Flow Models

Effective calibration of the parameters of a pipe or well flow model, such as the drift-flux model, with real-time field sensors measurements requires an understanding of:

1) the role of real-time calibration in instrumented or “smart” oil production facilities,
2) the use of calibration results in the real-time surveillance strategy of a facility management team, and how it links theory to real-time practice to facilities management, and
3) the kinds of data input the calibration routine requires.

Figure 2 depicts the role of real-time calibration in instrumented or smart oil production facilities, commonly referred to in industry as Normally Unmanned Installations (NUI). It is useful to note at this point that the term “smart” as used above really stems from way the various process instruments are connected. Conventional process technology connects multiple process instruments via individual cables to a central control system (Koninckx, 1998). Fieldbus technology (in NUI’s), connect multiple “smart” instruments (self-diagnostic) to a common network cable permitting them to communicate with each other and with host control systems.

As is seen in this Figure 2, remote platforms operated by platform operators constitute the server side of the automation network on the installation. Real-time sensor data such as pressures, temperatures, flow rates and valve statuses from the SCADA systems on the facility are stored in a data manager or Raw Process Historian (Ngai, 1999), which serves as a “data warehouse” for continuous feeds of operational data. The ability to view and use any combination of this operational data on a well-by-well basis, via a web-browser or database-reporter in real-time, is made possible by a Well Test Manager which also resides on the server side of the automation network. Both the Well Test Manager and the Raw Process Historian enable real-time
monitoring of the facility for those on the client side of the automation network, i.e., land-based production operations engineers and technicians.

![Diagram](image)

**Figure 2: Role of real-time calibration in smart production facilities (picture on top-left is a rig over BP’s Kapok Platform; process data-flow from Ngai, 1999).**

Figure 3 shows how calibration results are used in the real-time surveillance strategy of a facility management team, and how it links theory to real-time practice to facilities management. The theoretical foundation any kind of real-time optimization, of which calibration is a particular form, is the Boyd cycle or the OODA loop, i.e., observing, orienting, deciding and acting. An application of the Boyd cycle (Saputelli *et al.*, 2004) to an automation network is shown in the leftmost part of Figure 2. In practice real-time monitoring, control and diagnostics, shown in the middle part of Figure 2, are performed by various production operations teams. Finally, calibration results are used to enable better flow prediction, as can be seen in the rightmost “circle”, i.e., the real-time surveillance model.
Figure 3: Use of calibration results in a real-time surveillance model, and its linkage of theory to real-time practice to facilities management (figures in middle taken from PlantScape’s automation software suite).

Figure 4 shows the kinds of data input a real-time calibration routine requires if using field operating data. Basically, separator sampling and lab PVT data provide periodic, post-quality-controlled inputs that the calibration routine uses to periodically update fluid property parameters. Real-time sensor measurements and proprietary correlations provide continuous automatic inputs for automatic flow model parameter correction, i.e. calibration. The real-time sensor measurements used as input are: TSP, FTP and FBHP pressure data; TST, FTT and FBHT temperature data; and surface oil, water and gas flow rates.
Figure 4: Data input into calibration routine.
Chapter 2

2. New Drift-Flux Flow Model Formulation

2.1. Drift-flux Flow Model as a Subset of a Class of Mixture Models

In any kind of buoyancy-driven bubbly flow where there is one or more phases (in the form of gas-bubbles or liquid-globules) dispersed in another carrier phase, we can define four relative velocities that exist. They are:

\[
\begin{align*}
\text{Fluctuating velocity: } & \quad u_{Fd} = u_{Ld} - u_d \\
\text{Diffusion velocity: } & \quad u_{Md} = u_d - u_m \\
\text{Drift velocity: } & \quad u_{Vd} = u_d - j_m \\
\text{Slip velocity: } & \quad u_{Cd} = u_d - u_c
\end{align*}
\]

The fluctuating velocity in Eqn. (4) above is used in a small class of mixture models whose field equations consist of (Soo, 1990):

1) a term in the dispersed phase continuity equation (or equations) describing the turbulent diffusion due to dispersed phase concentration gradients, i.e., a term containing a turbulent diffusion coefficient (a function of fluctuating velocity), and
2) a term in the phase momentum equation (or equations) containing a turbulent stress tensor (also a function of fluctuating velocity).

In mixture models such as above, the mean in-situ velocity of a phase \( k \), \( u_k \), is defined as \( \overline{u}_{Lk} \), where the overbar indicates an average inside some averaging domain, e.g., volume, time-step, a set of experiments, or a group of particles. The fluctuating and local velocities, \( u_{Fk} \) and \( u_{Lk} \) respectively, can be eliminated from the dispersed phase continuity equation in redefining \( u_k \) by weighting it with the local density, \( \rho_{Lk} \):
\( u_k = \frac{\rho_{lk} u_{lk}}{\bar{\rho}_{lk}} = \frac{\rho_{lk} u_{lk} A_k}{\rho_k A_k} = \frac{\bar{m}_{lk}}{\rho_k A_k} = \frac{\bar{m}_k}{\rho_k A_k} \quad (8) \)

The averaging above is referred to as \textit{Favre}-averaging (Ishii, 1975), and allows for a phase continuity (mass conservation) equation to be defined as (using same notation as Ishii, 1975):

\[
\frac{\partial}{\partial t} \sum_{k=1}^{n} (\alpha_k \rho_k) + \nabla \cdot \sum_{k=1}^{n} (\alpha_k \rho_k u_k) = \sum_{k=1}^{n} \Gamma_k \quad (9)
\]

Throughout this report, \( u_k \) denotes the \textit{Favre}-averaged velocity. The averaging in Eqn. (8) together with the phase continuity equation in Eqn. (9) is the basis for the majority of mixture model classes in the literature, in which flows are usually steady-state, phase densities are considered constants over a differential length and the inter-phase mass transfer is neglected. These assumptions lead to a simplified form of the phase continuity equation in Eqn. (9) as:

\[
\nabla \cdot \sum_{k=1}^{n} (\alpha_k \rho_k u_k) = 0 \quad (10)
\]

Application of the diffusion velocity in Eqn. (5) to Eqn. (10) leads to a diffusion-velocity equation. Accordingly, this mixture model is referred to as the \textit{Diffusion model} (Ungarish, 1993). Application of the drift velocity in Eqn. (6) to Eqn. (10) leads to a drift-velocity equation. Since velocity is related to flux, this mixture model is preferentially referred to as the \textit{Drift-flux model}. Applying the averaging described in Section 1.2.3 to the drift velocity definition in Eqn. (6), we can arrive at a definition of weighted, cross-section averaged drift velocity, \( \langle \langle u_{id} \rangle \rangle \), i.e., the cross-section averaged drift velocity \( \langle u_{id} \rangle \), weighted by the cross-section averaged dispersed phase volume fraction, \( \langle \alpha_d \rangle \). Thus:

\[
\langle \langle u_{id} \rangle \rangle = \frac{\langle \alpha_d u_{id} \rangle}{\langle \alpha_d \rangle} = \frac{\langle \alpha_d u_d \rangle}{\langle \alpha_d \rangle} - \frac{\langle \alpha_d j_m \rangle}{\langle \alpha_d \rangle} \quad (11)
\]

Where \( u_d \) and \( j_m \) are the in-situ dispersed phase velocity and mixture volume flux, respectively.
Rearranging Eqn. (11) and multiplying the numerator and denominator of last term on the right-hand side by \( \langle j_m \rangle \), gives (from the drift-flux model formulation of Takamasa et al., 2004):

\[
\langle \langle u_d \rangle \rangle = \frac{\langle \alpha_d j_m \rangle}{\langle \alpha_d \rangle \langle j_m \rangle} \langle j_m \rangle + \langle \langle u_{rd} \rangle \rangle
\]  

(12)

The square-bracketed term in Eqn. (12) is referred to as the distribution parameter, \( C_0 \), and can clearly be seen as a term accounting for combined variations of mixture volume flux and void fraction (or concentration) profiles. The nature of these variations, however, is not obvious from Eqn. (12). The last term in Eqn. (12) is the drift-velocity which is related to the slip velocity in Eqn. (7). Eqn. (12) is the drift-flux equation of Zuber and Findlay (1965), and can be re-written in the case of gas-liquid flows, for example, in a form with “traditional” parameters as:

\[
V_g = C_0V_m + V_d
\]  

(13)

Substituting the distribution and drift-velocity parameters of Eqn. (13) above in Eqn. (12), we see that (Wallis, 1969):

\[
\langle \langle u_d \rangle \rangle = \frac{\langle j_d \rangle}{\langle \alpha_d \rangle} = C_0\langle j_m \rangle + V_d
\]  

(14)

The void fraction can thus be found from Eqn. (14) as:

\[
\langle \alpha_d \rangle = \frac{\langle j_d \rangle}{C_0\langle j_m \rangle + V_d}
\]  

(15)

Hence, the determination of void fraction in Eqn. (15) above depends on arriving at appropriate constitutive equations for the parameters \( C_0 \) and \( V_d \). The formulation of these constitutive equations is the greatest difficulty when developing a Drift-flux flow model for a practical application (Drew et al., 1979). The problem is that the constitutive equations applied include considerable uncertainties. Empirical information thus forms an essential part of any Drift-flux model.
2.2. Constitutive Equation for the Distribution Parameter

From Eqn. (12) above, we see that \( C_0 \) is always a positive scalar quantity, and is logically expected to take values surrounding 1. We further note the relation below:

\[
\langle \alpha_d \rangle C_0 = \left[ \frac{\langle \alpha_d j_m \rangle}{\langle \alpha_d \rangle \langle j_m \rangle} \right] \langle \alpha_d \rangle = \frac{\langle \alpha_d j_m \rangle \text{ always}}{\langle j_m \rangle} \leq 1
\]

(16)

Eqn. (16) above shows two constraints:

1) The first constraint is clear, i.e., there is a maximum value for \( \langle \alpha_d \rangle C_0 \) of unity, and as can be seen, as \( \langle \alpha_d \rangle \) approaches unity, \( C_0 \) will also approach a value of 1.

2) A second constraint can be inferred by noting that as \( \langle j_m \rangle \) takes on higher values (i.e., at higher mixture volume fluxes), the influence of \( \langle \alpha_d \rangle \) on the value of \( C_0 \) diminishes (flatter profile as turbulence increases), and \( C_0 \) approaches unity in this case.

Hence, since \( C_0 \) must obey these constraints above, we can arrive at constitutive equations for \( C_0 \) in gas-liquid and liquid-liquid flows by assuming a form for the relation between \( C_0 \) and \( \langle \alpha_d \rangle \) in these cases, and constraining the relations using the two constraints above. Mathematically, we require:

\[
C_0 = f(\langle \alpha_d \rangle, \langle j_m \rangle)
\]

(17)

Subject to:

\[
As \; \langle \alpha_d \rangle \to 1, \; C_0 \to 1
\]

(18)

\[
As \; |j_m| \uparrow, \; C_0 \to 1
\]

(19)

The Drift-flux model of Shi et al., 2003 (originally developed by Holmes, 1977) is of the form of Eqns. (17) – (19) for gas-liquid and liquid-liquid flows, shown schematically as the left-most and right-most diagrams of Figure 5, respectively. In Figure 5, \( C_0 \) for gas-liquid flows is given as:
\[ C_{0,\text{Gas-Liquid}} = \frac{A}{1 + (A-1)\left(\frac{\beta - B}{1-B}\right)^2} \] (20)

Where,

\[ \beta = \max\left(\alpha_g, F_v \frac{\alpha_g |\mathbf{j}_m|}{|\mathbf{j}_{g,f}|}\right) \] (21)

And,

\[ |\mathbf{j}_{g,f}| = Ku(D_d) \sqrt{\frac{\rho_l}{\rho_g}} \left(\frac{\sigma_{dg}(\rho_l - \rho_g)}{\rho_l^2}\right)^{\frac{1}{4}} \] (22)

Similarly, in Figure 5, \( C_0 \) for liquid-liquid flows is given as:

\[ C_{0,\text{Liquid-Liquid}} = \begin{cases} 
A' & \text{if } \langle \alpha_o \rangle \leq B'_1 \\
1 & \text{if } \langle \alpha_o \rangle > B'_2 \\
A' - (A' - 1) \left(\frac{\langle \alpha_o \rangle - B'_1}{B'_2 - B'_1}\right) & \text{otherwise}
\end{cases} \] (23)

The details of the derivation and parameters of Eqns. (20) – (23) can be found in Shi et al., 2003 and Diaz, 2004. Note that unlike \( C_0 \) for gas-liquid flows where the upper limit of the mixture volume flux, \( \mathbf{j}_m \), is taken as the superficial gas flooding velocity, \( \mathbf{j}_{g,f} \), there is no similar upper limit of the mixture volume flux in the liquid-liquid Eqn. (23).

\[ \text{Figure 5: Form of } C_0 \text{ in gas-liquid and liquid-liquid flows (Shi et al., 2003).} \]
The third constraint can be inferred from the first constraint, shown as the relation in Eqn. (18), i.e., \( C_0 \) will approach unity as the density of the carrier phase approaches the density of the dispersed phase, regardless of the actual value of \( \langle \alpha_d \rangle \). This dependency of \( C_0 \) on the ratio of phase densities has also been noted in by several other researchers (Ishii, 1977; Sun et al., 2004). For example, the Ishii (1977) equation for \( C_0 \) is simply:

\[
C_0 = 1.2 - 0.2 \frac{\rho_d}{\rho_c}
\]  

(24)

Hence the \( C_0 \) relation of Eqn. (17) can be improved if we extend \( C_0 \) to include this third constraint. So we now have:

\[
C_0 = f \left( \langle \alpha_d \rangle, \langle j_m \rangle, \sqrt{\frac{\rho_d}{\rho_c}} \right)
\]  

(25)

As noted in the last part of Section 1.2.6, there are several key buoyancy-driven bubbly flow phenomena that are not included in existing drift-flux models to-date, and which have been shown to be relevant especially in larger diameter pipes (Shoukri et al., 2003). The interplay between buoyancy and void fraction (or bubble) distribution results in a complex flow that is not well understood (Mudde, 2005). The transversal lift force in bubble/fluid swarms and the structure of the turbulence in the bubbly mixture are important examples of inadequately understood physical phenomena, providing many challenges for fundamental and applied research on bubbly flows. The flow phenomena noted in Section 1.2.6 all relate to the understanding of the radial void fraction distribution, which is necessary for the calculation of \( C_0 \) as defined in Eqn (12).

Figure 6 below, taken from gas-liquid regimes in Mudde (2005), shows how buoyancy-induced flow can increase the inhomogeneity of the bubble distribution, leading to a bubble-induced turbulent flow. In this flow, specific scales are identified (Chen et al., 1992):

1) On the intermediate scale there are vortical structures, or eddies of liquid, with a size on the order of the diameter of the column, that stir the liquid and radially transport the bubbles.
2) On the small scale there is the local stirring of the bubbles. At low flow rates the bubbles move upward through a basically stagnant liquid. At higher gas flow rates, such as found in most industrial applications (e.g., gas-liquid flows in the wells and piping of oil production facilities) the bubbles tend to form clusters, i.e., they coalesce.

![Figure 6: Transition of buoyancy-driven bubbly flow with increasing gas fraction. Two-phase gas-liquid regimes are taken from Mudde (2005).](image)

There is also a large-scale circulatory flow (not shown in Figure 6 and not considered in this research), in which the liquid (or carrier phase) flows upward in the center of the pipe and downward close to the wall, which usually leads to a central-peaking void fraction profile (De Nevers, 1968). As Figure 6 shows, coalescence eventually sets in and there is a transition toward another flow regime. However, in larger pipe diameters, say 4” and above, the gravity-induced liquid flow is usually turbulent. The liquid turbulence causes breakup of the larger bubbles and a dynamic equilibrium sets in. Hence, in general, bubble coalescence/breakup is determined by:

1) Local parameters of turbulence. In this case, there can be radial void fraction profile smoothing due to bubble-induced turbulence, or, due to large bubble fluctuation.

2) Local bubble-size distribution. In the case, void fraction (or bubble) distribution is governed by the transversal shear-induced lift force (which acts toward the pipe wall), and the transversal lubrication lift force (which acts away from the pipe wall).
Pipe wall peaking flows and the peaking transition phenomena (i.e., radial void fraction evolution from central to wall peaking profiles and vice versa) in larger diameter pipes such as in production facility wells and pipes, is another flow phenomena that is quite important but not incorporated into existing drift-flux models. The experiments of Tomiyama et al., (1998) on the lift force in bubbly/fluid swarm flow, as well as of Krepper et al., (2000) have shown that there is a critical bubble Eotvos number above which bubbles move from the wall to the center region of the pipe. They noted a tendency of small bubble fractions to move towards the wall. This is explained by the fact that increase of void fraction near the wall leads to an intensification of coalescence, which leads to a generation of bubbles with \( d_b > 5.5 \) mm in this region. These bubbles experience an inverse lift force, pushing them back towards the centre of the pipe. Hence, two separate cases of dispersed bubbly flows exist and need to be accounted for:

1) bubbly flows exhibiting both central peaking and wall peaking void fraction distributions (laminar flow of small fluid particles, typically \( d_b < 5.5 \) mm (= \( L_1 \)), and

2) bubbly flows exhibiting central peaking void fraction distributions (turbulent flow of larger fluid particles).

The other key phenomenon that influences the radial void fraction distribution is the asymmetry of the void fraction profile in deviated flows. In existing drift-flux models, a symmetric radial void fraction distribution is assumed, even in deviated pipe flows where density differences cause transversal phase migration and hence an asymmetric radial void fraction profile. Effects of pipe deviation are usually only accounted for as a multiplier of the drift velocity in existing drift-flux models.

The above discussion of the key flow phenomena that relates to the understanding of the radial void fraction distribution is summarized in Figure 7 below. Our next steps will be to propose mathematical models for each of these phenomena to arrive at a new constitutive equation for \( C_0 \), as defined in Eqn. (12).
To arrive at a bubble coalescence model, we look for an empirical relation that describes the transition of bubbles of different sizes and configurations over a wide range of the dimensionless numbers of Section 1.2.4. These dimensionless numbers are chosen because they relate to the multiphase flow forces that are of interest to our problem. A bubble coalescence/breakup model exists that satisfy the requirement above – it is the Grace (1973) relation that provides a mapping of $\text{Re}_p$ versus $\text{Eo}_p$ for various system Mo numbers for any generalized two-phase system, i.e., a dispersed phase (bubbles or drops) in unhindered gravitational motion through a carrier phase. This relation, which was arrived at after considering a broad range of fluid properties and particle sizes (Clift et al., 1978) is shown in Figure 8 below. This relation is used later in Section 2.3 to calculate the volume equivalent particle diameter, $d_{p,eq}$ (or bubble size).
Next, we would like to arrive at a peaking transition model that describes the radial void fraction evolution from central to wall peaking profiles and vice versa. We require a relation that will explicitly describe the variation of the radial void fraction distribution from a starting central peaking profile to a wall peaking profile and back to a central peaking profile in accordance to the observations of Tomiyama et al., (1998) and Krepper et al., (2000). Additionally, this closure relation would need to have a mathematical form of being able to describe a symmetric radial void fraction distribution for vertical flows, and an asymmetric distribution for deviated flows.

The approach taken to satisfy the above specific requirements of the varying radial void fraction distribution is to find a continuous, differentiable mathematical equation with coefficients that
can be changed in a systematic manner to match an estimated behavior of the void fraction evolution, in accordance with the experimental observations noted previously. Such an equation would need to possess three traits:

1) Transform from a symmetric to asymmetric form.
2) Transform from a smooth to peaking form.
3) Have a steady concentration transition at the pipe centerline for any given radial void fraction profile.

An example of the expected radial void fraction evolution can be seen in the Figure 9 below:

![Figure 9: Example of evolution of radial void fraction profiles capturing the peaking transition phenomenon.](image)

The advantage of having one equation with variable coefficients that describes Figure 9 above, that relates to bubble profile (peaking) and pipe deviation, is that any regular or non-regular profile can be incorporated in the calculation of \( C_0 \), i.e., a general \( C_0 \) for all flows. An equation that satisfies the special form requirements as described above is a modified form of the First-Order, Kinetic Enzyme-Reaction equation (Systat, 2005):
\[
\frac{\alpha_d}{\alpha_{d0}} = 1 - e^{-n_1 \left( \frac{r}{R} \right)} - \frac{n_2}{n_2 + n_3} \left( 1 + \frac{n_2 e^{-\left( n_2 + n_3 \right) \frac{r}{R}}}{n_2 + n_3 - n_1} \right) \tag{26}
\]

If Eqn. (26) is combined with a power-law velocity profile as:

\[
\frac{j_m}{j_{m0}} = \left( \frac{r}{R} \right)^{\frac{1}{m}}
\]

Then applying Eqns. (26) and (27) into the definition of C0 in Eqn. (12), we get:

\[
\frac{\langle \alpha_d j_m \rangle}{\langle \alpha_d \rangle \langle j_m \rangle} = \frac{\frac{1}{\pi R^2} \int_0^R \left[ (\alpha_d j_m) 2\pi (R - r) \right] dr}{\frac{1}{\pi R^2} \int_0^R \left[ \alpha_d 2\pi (R - r) \right] dr \left( \frac{1}{\pi R^2} \int_0^R \left[ j_m 2\pi (R - r) \right] dr \right)} \tag{28}
\]

Which, after integration, leads to:

\[
C_0 = \frac{\sum_{k=1}^{5} C_k}{2 \left( \sum_{k=1}^{5} N_k \right) \left( \frac{m^2}{1 + m} \right) \left( 1 + 2m \right)} = f(n_1, n_2, n_3, m) \tag{29}
\]

The coefficients of Eqn. (29) are in Appendix B. Next, we require a modification to the radial void fraction profile to account for cases where there is a transversal phase migration leading to asymmetric profiles. An example of this modification is shown in Figure 10 below.
Figure 10: Example of asymmetry in radial void fraction distribution in deviated flows. Leftmost figure shows a radial profile for vertical flow. Rightmost figure shows a radial profile for the same flow at 5° to vertical.

The best way to combine all three phenomena of Figures 8, 9 and 10 is to define one of the coefficients of on the right-hand side of Eqn. (29) in terms of \( d_{p,eq} \), \( L_1 \) and \( \theta \), and relate the remaining coefficients to that chosen coefficient. In the proposed model of research, \( n_1 \), \( n_2 \) and \( n_3 \) are defined in Figure 11 below in accordance to the assumed radial void fraction evolution model of Figure 9 and the profile modification model of Figure 10. Any void fraction evolution model or profile modification model can be assumed, thereby requiring different definitions of \( n_1 \), \( n_2 \) and \( n_3 \). The definitions of the coefficients below were formulated to match one assumption of the radial void fraction evolution and profile modification, i.e., exactly as that shown in Figures 9 and 10. There can be many more assumptions depending on a prior understanding of the expected flow in the wells or pipes of the system.

Assuming a power-law velocity profile is applicable, \( C_0 \) is not sensitive to the value of the coefficient, \( m \), as defined in Eqn. (27) (Govier and Aziz, 1972). In this report, \( m = 7 \).
Figure 11: Definitions of the coefficients of Eqn. (29) as a function of \( d_{\text{p,eq}}, L_1 \text{ and } \theta \).

Hence, we note that:

\[
C_{0,dL0} = f(n_1, d_{\text{p,eq}}, L_1, \theta, n_2(n_1, \theta), n_3(n_1), m)
\]  

(30)

We rename \( C_0 \) in Eqn. (29) as \( C_{0,dL0} \) in Eqn. (30). The next step is to combine \( C_{0,dL0} \) with the three constraints described previously to arrive at a new constitutive equation for \( C_0 \) of the form:

\[
C_0 = f\left(\langle a_d \rangle, \langle j_m \rangle, \sqrt{\frac{P_d}{P_c}}, d_{\text{p,eq}}, L_1, \theta\right)
\]  

(31)

This new equation is:

\[
C_0 = \frac{C_{0,dL0} - (C_{0,dL0} - 1)A_3}{1 + (C_{0,dL0} - 1)(1 - A_3)\left(\frac{A_{1,2} - B}{1 - B}\right)^2}
\]  

(32)
Where:

\[
C_{0,dt,\theta} = \frac{\sum_{k=1}^{s} C_k}{2 \left( \sum_{k=1}^{s} N_k \right) \left( m^2 \left( 1 + m \right) \left( 1 + 2m \right) \right)}
\]  

(33)

And,

\[
A_{1,2} = \begin{cases} 
\max \left( \frac{\langle \alpha_d \rangle \left| j_m \right|}{j_{g,f}} \right), & \text{Gas} - \text{Liquid}, \\
\max \left( \frac{\langle \alpha_d \rangle \left| j_m \right|}{\left| j_{o,max} \right|} \right), & \text{Oil} - \text{Water} 
\end{cases}
\]  

(34)

And,

\[
A_3 = \sqrt{\frac{\rho_d}{\rho_c}}
\]  

(35)

The parameter B in Eqn. (32) is a tunable parameter (defaulted to 0.4) and defined in Shi et al., 2003. In Eqn. (34), there is an upper limit of the mixture volume flux in gas-liquid flows, \( j_m \), as the superficial gas flooding velocity, \( j_{g,f} \), i.e., at the onset of annular gas-liquid flow. A new parameter, \( j_{o,\text{max}} \), is introduced to provide a similar upper limit of the mixture volume flux in oil-water flows (the detailed derivation of this new parameter is shown in Appendix C), as (Banwart, 1998):

\[
\left| j_{o,\text{max}} \right| = \langle \alpha'_o \rangle \left| j_m \right| + \left( \nu_{reg} k \langle \alpha'_o \rangle^2 \left( 1 - \langle \alpha'_o \rangle \right)^w \right)
\]  

(35b)
2.3. Constitutive Equation for the Drift Velocity

The distribution parameter in Eqn. (32) requires the value of \( d_{p,eq} \), i.e., the volume equivalent dispersed phase particle diameter. However, the determination of a generalized expression for bubble/droplet size in bubbly flows is a significant problem. This important variable is used in determining the bubble terminal (or rise) velocity, which is used in calculating the drift velocity. Another term commonly used instead of bubble terminal velocity is the slip velocity, which is the same as described in Eqn. (7) of Section 2.1. At the bubble scale, if a bubble is considered as a rigid sphere, a local force balance between the drag and buoyancy forces the bubble is subjected to can be shown as (Worner, 2003):

\[
\left( \frac{\pi d_{p,eq}^3}{6} g \Delta \rho \right)_{\text{buoyancy force}} = \left( \frac{\pi d_{p,eq}^2}{4} C_{D,\text{Total}} \rho_c \frac{u}{2} | u |_{\text{drag force}} \right)
\]  

(36)

Rearranging Eqn. (36) yields a definition for the slip velocity of an isolated, smooth rigid sphere in an infinite carrier phase as:

\[
u_{Cd,\infty} = \sqrt{4 \frac{d_{p,eq} g \Delta \rho}{3 C_{D,\text{Total}} \rho_c}}
\]

(37)

Hence, if the drift velocity can be calculated from the slip velocity in the general form as:

\[
\left\langle \langle u_{vd} \rangle \right\rangle = V_d = \nu_{Cd,\infty} \left( d_{p,eq} C_{D,\text{Total}} \right) \cdot f \left( \langle \alpha_d \rangle \right)
\]

(38)

Then it seems that the two key unknowns, other than void fraction, are the bubble size and the total drag coefficient. However, if the total drag coefficient is set constant, say 0.44, using a drag correlation for isolated rigid spheres, such as the Schiller and Naumann (1933) correlation, then the only unknown remains as the bubble size, \( d_{p,eq} \). By setting the drag coefficient as a constant, a high Re\( p \) flow is assumed. If a further assumption is made such as the bubble shape and size is largely determined by the Eo\( p \) number, then, assuming that fluid particles with an Eo\( p \) number < 13 is spheroidal in shape, the bubble size can be eliminated from Eqn. (37) by multiplying Eqn. (37) by a factor as:
\[ u_{CD,\infty} = \left(0.88 \text{Eo}^{-\frac{1}{4}} \right) \left( \frac{4}{3} \right)^{\frac{1}{4}} \left( \frac{d_{p,eq} g \Delta \rho}{C_{D,\text{Total}} \rho_c} \right) \]

\[ = \left(0.88 \sqrt{\frac{4}{3}} \sqrt{0.44} \right) \left( \frac{\sigma_{dc}}{d_{p,eq}^2 g \Delta \rho} \right)^{\frac{1}{4}} \left( \frac{d_{p,eq} g \Delta \rho}{\rho_c} \right) \]

\[ = 1.53 \left( \frac{\sigma_{dc} g \Delta \rho}{\rho_c^2} \right)^{\frac{1}{4}} \]  \hspace{1cm} \text{(39)}

Eqn. (39) above was derived by Harmathy (1960) and is a common expression for slip (or rise) velocity in existing drift-flux models in the literature. Similar forms of slip velocity equations as Eqn. (39) also exist where the only difference is in the constant of 1.53. For example, if the bubble size is assumed to lie in the corresponding Eo\(p\) range of 13 < Eo\(p\) < 40, then the constant of 0.88 in Eqn. (39) is replaced by 0.81, which in turn replaces 1.53 with 1.41.

Drift-flux models that use slip velocities of the form of Eqn. (39) assume a specific bubble size, in which bubbles are usually considered as rigid isolated spheres moving through a stagnant infinite carrier phase, and this bubble size is taken as constant for all bubbly flow systems in which the models are applied. Hence, these drift-flux models do not represent the whole range of bubbly flows which occur in engineering practice. For small enough Morton numbers, i.e. if Mo < 10E-2 (corresponding to very viscous carrier phases), there are no range of diameters for which the slip velocity is independent of bubble size (Comolet, 1979). There is therefore a need to predict the bubble size. Small bubbles have a lower rise velocity in a carrier phase than large ones. Hence, the average void fraction in a pipe is for small bubbles higher than for large bubbles. This means that the average density of the mixture is lower and the carrier phase rate is larger. Bubble size also has a drastic influence on the evolution of the radial void fraction distribution (Serizawa et al., 1975). Small bubbles in an upward flow move toward the pipe wall, whereas large bubbles move to the center of the pipe.
The gas-liquid drift-flux model of MSWell uses the slip velocity of Eqn. (39) combined with other equations relating to the flooding velocity (Wallis and Makkenchery, 1974), to get (details of other equations in Shi et al., 2003):

\[
V_d = \frac{(1 - \alpha_g C_o) C_o K(\alpha_g, D_D) \left( \frac{\sigma_l g \Delta \rho}{\rho_l^2} \right)^{1/4}}{\alpha_g C_o \sqrt{\frac{\rho_g}{\rho_l}} + 1 - \alpha_g C_o}
\]

(40)

Where:

\[
K(\alpha_g, D_D) = \begin{cases} 
1.53 \frac{C_o}{\alpha_g}, & \forall \alpha_g \leq a_1 \\
\left(K_u(\alpha_g, D_D) - 1.53 \frac{\alpha - a_1}{C_o} \right) \left(\frac{\alpha - a_1}{\alpha - a_2}\right) + 1.53 \frac{C_o}{\alpha_g}, & \forall a_1 \leq \alpha_g \leq a_2 \\
K_u(\alpha_g, D_D), & \forall \alpha_g \geq a_2
\end{cases}
\]

(41)

The oil-water drift-flux model of MSWell (Geoquest, 2001) uses the slip velocity of Eqn. (39) in a relation of the type of Eqn. (38) as:

\[
V_d = 1.53 \left( \frac{\sigma_{on} g \Delta \rho}{\rho_w^2} \right)^{1/4} \left(1 - \alpha_o \right)^{n'}
\]

(42)

Where \(n' = 2\) is a default value for Eqn. (42). The details of the derivation of Eqns. (40) – (42) is found in Shi et al., (2003), and Diaz (2004). The drift velocities above are multiplied by an empirical deviation factor to account for flows in deviated pipes as (Hasan and Kabir, 1999):

\[
\bar{m}(\theta) = \begin{cases} 
m(\theta)_{\text{Gas-Liquid}} = \left(m_0 (\cos \theta)^{n_1} (1 + \sin \theta)^{n_2}\right) & \forall \theta \leq 70^\circ \\
m'(\theta)_{\text{Liquid-Liquid}} = \left(n_1' \cos \theta + n_2' \sin 2 \theta + n_3' \sin 3 \theta\right)_{\text{Shi et al., 2004}} & \forall \theta \leq 88^\circ
\end{cases}
\]

(43)

The default and optimized parameters of Eqn. (43) for small and large diameter pipes are shown in Figure 12 below.
The assumptions of the slip velocity in Eqn. (39) restrict the applicability of drift-flux model of Eqns. (40) – (42). Hence we require a drift velocity of the form of Eqn. (38) that does depend on bubble size and total drag. Furthermore, we would like to use the bubble coalescence/breakup model in Figure 8 in solving the bubble size, since the dimensionless numbers that the relation in Figure 8 are based on, relate to the multiphase flow forces that are of interest to our problem.

The Richarson and Zaki (1955) drift velocity equation for the rise of dispersed phase particles in swarms is given below. This is the constitutive drift velocity equation used in the new drift-flux model of this research:

$$\langle \langle u_{vd} \rangle \rangle = V_d = \left( u_{cd,\infty} \langle d_{p,eq} C_{D,Total} \rangle \left( 1 - \langle \alpha_d \rangle \right)^{n-1} \right) \bar{m}(\theta)$$  \hspace{1cm} (44)$$

Where:
The exponent, n, as defined in Eqn. (45) is for gas-liquid flows. For liquid-liquid flows, n was initially set to a value of 3, in order that Eqn. (44) would equate to Eqn. (42) with the slip velocity of Eqn. (42) replaced by Eqn. (37) and with n-1 = n’ = 2. However, after testing Eqn. (44) with the large diameter SCR oil-water experiments, n having a value of 2 gave the best oil holdup predictions. Hence in the new drift-flux model n is set to a value of 2 for liquid-liquid flows. With n = 2, the exponent of \((1 - \langle \alpha_p \rangle)\) in Eqn. (44) for oil-water flows is actually n-1 = 1. Interestingly, the exponent of \((1 - \langle \alpha_p \rangle)\) as 1 was noted by Shi et al. (2005) in their parametric optimization studies as the optimum value of this parameter (same as n’) in liquid-liquid flows.

Now, as is observed in Eqns. (44) and (45), the key unknown parameters are \(d_{p,eq}\) and \(C_{D,Total}\). Since both \(Re_p\) and \(Eo_p\) depends on \(d_{p,eq}\) (see Eqns. (1) and (2)), it is logical to start our procedure of determining \(d_{p,eq}\) from the total drag coefficient. Most correlations used for the drag coefficient (Clift et al., 1978) are based on isolated or suspension rigid sphere experiments where \(C_{D,Total}\) is typically of the form:

\[
C_{D,Total} = \begin{cases} 
  f\left(Re_p\right) & \forall\ Re_p < Re_{p,Limit} \\
  Constant & \forall\ Re_p \geq Re_{p,Limit} 
\end{cases}
\]

The problem with Eqn. (46) is not only that \(Re_p\) is an unknown, since it depends on \(d_{p,eq}\), but that the experiments used to arrive at drag correlations of the form of Eqn. (46) are based on rigid sphere flows in a carrier phase. We require drag correlations based on the swarm flow of fluid spheres in a carrier phase. Furthermore we require the calculation of a limit of \(C_{D,Total}\) which
when exceeded, signifies that $C_{D,\text{Total}}$ is no longer a function of $Re_p$. Hence we seek a drag correlation of the following form:

$$C_{D,\text{Total}} = \begin{cases} 
    f(Re_p) & \forall C_{D,\text{Total}} < C_{D,\text{Limit}} \\
    \text{unknown constant} & \forall C_{D,\text{Total}} \geq C_{D,\text{Limit}} 
\end{cases}$$

(47)

An appropriate $C_{D,\text{Limit}}$ is found in Ishii & Mishima’s (1984) drag correlation for swarm flow of fluid spheres for high-$Re_p$ flows ($Re_p > 1000$):

$$C_{D,\text{Limit}} = 0.45 \left( 1 + 17.67 \left[ \frac{\mu_c}{\mu_{\text{app}}} \sqrt{1 - \langle \alpha_d \rangle} \right]^6 \right) \left( \frac{18.67}{\mu_c} \frac{\mu_c}{\mu_{\text{app}}} \sqrt{1 - \langle \alpha_d \rangle} \right)^{-\frac{2.5}{\omega}}$$

(48)

Where:

$$\mu_{\text{app}} = \mu_c \left( 1 - \frac{\langle \alpha_d \rangle}{\alpha_{pm}} \right)^{-2.5 \omega^*}$$

(49)

And,

$$\mu^* = \frac{\mu_d + 0.4 \mu_c}{\mu_d + \mu_c}, \quad \alpha_{pm} = \begin{cases} 
0.62, & \text{Liquid - Liquid} \\
0.775, & \text{Gas - Liquid} 
\end{cases}$$

(50)

However, Eqns. (48) and (49) for high $Re_p$ flows contains the dispersed phase holdup. Since this is what we eventually want to solve for, i.e., $\langle \alpha_d \rangle$, we replace $\langle \alpha_d \rangle$ in Eqns. (48) and (49), with the input (no-slip) dispersed phase cut, $C_{\text{sd}}$, with the assumption that slip would be considerably smaller (i.e., negligible) at higher $Re_p$ flows. Hence the final $C_{D,\text{Limit}}$ equation is Eqns (48) and (49) with $C_{sd}$ in place of $\langle \alpha_d \rangle$. $\mu^*$ in Eqn. (51) remains the same as defined in Eqn. (50):
\[
C_{D,\text{Limit}} = 0.45 \left( 1 + \frac{17.67}{18.67} \left( \frac{\mu_c}{\mu_{app}} \sqrt{1 - C_{sd}} \right)^6 \right), \quad \text{where} \quad \mu_{app} = \mu_c \left( 1 - \frac{C_{sd}}{\alpha_{pm}} \right)^{-2.5a_{pm}^{1/3}} \tag{51}
\]

The next step is to find simple power law-type drag correlations as functions of \( \text{Re}_p \), based on experiments of swarm flow of fluid spheres in a carrier phase, for a wide range of \( \text{Re}_p \) and flow systems. These experiments were found in the data of Abdel-Alim and Hamielic (1975) shown in Figure 14 below. The reason for the specific requirement of simple power law-type drag correlations, is that the form of \( C_{D,\text{Total}} \) as a power law function of \( \text{Re}_p \), facilitates a derivation of an equation of \( \text{Re}_p \) as a function of \( \text{Eo}_p \) and \( \text{Mo} \) only. This equation can then be combined with the Grace (1973) relation in Figure 8, approximated in the form of another equation of \( \text{Re}_p \) as a function of \( \text{Eo}_p \) and \( \text{Mo} \). Using these two equations containing the two unknowns \( \text{Re}_p \) and \( \text{Eo}_p \), \( \text{Re}_p \) can then be eliminated in order to arrive at one equation with one unknown, \( \text{Eo}_p \). \( d_{p,\text{eq}} \) can thus be determined by rearranging Eqn. (1).
Figure 13: $C_{D,Total}$ experimental data for swarm flow of fluid-spheres in a carrier fluid, with power law-type fits. Data from Abdel-Alim and Hamielic (1975).

In Figure 14 above, $C_{D,Total}$ values are shown for the flow systems of gas bubbles in liquid, liquid (water) droplets in gas (air), with corresponding results for rigid spheres. As seen in Figure 14, the viscous liquid sphere has essentially about the same total drag as a rigid sphere. Power law fits to the data above are:

For gas bubbles in liquid ($\kappa = 0$):

$$C_{D,Total} = 15.1Re_p^{-0.77}$$  \hspace{1cm} (52)

Alternatively, the Haas et al., (1972) relation can be used for this flow system as:

$$C_{D,Total} = 14.9Re_p^{-0.78}$$  \hspace{1cm} (53)

For liquid globules in liquid based on a flow system with $\kappa = 0.3$: 

38
\[ C_{D,\text{Total}} = 16.4 \Re_p^{0.75} \]  \hspace{1cm} (54)

For liquid droplets in gas based on flow systems with \( \kappa = 55 \) or \( \kappa = \infty \):

\[ C_{D,\text{Total}} = 17.4 \Re_p^{0.6} \]  \hspace{1cm} (55)

From the above Eqns. (52) – (55) above, we can arrive at a general definition of \( C_{D,\text{Total}} \) as:

\[ C_{D,\text{Total}} = \alpha \Re_p^\beta \]  \hspace{1cm} (56)

Where:

\[ \alpha = \begin{cases} 15.1 + 4.333 \kappa & \forall \kappa \leq 0.3 \\ 16.395 + 0.018 \kappa & \forall \kappa > 0.3 \end{cases} \quad \text{and} \quad \beta = \begin{cases} -0.77 + 0.67 \kappa & \forall \kappa \leq 0.3 \\ -0.75 + 0.003 \kappa & \forall \kappa > 0.3 \end{cases} \]  \hspace{1cm} (57)

Now we can substitute \( d_{p,eq} \) defined in Eqn. (1) and the slip velocity derived in Eqn. (37) into Eqn. (2) to arrive at:

\[ \Re_p = \frac{\rho_c}{\mu_c} \sqrt{\frac{Eo_p \sigma_{dc}}{g(\rho_c - \rho_d)}} \frac{4 \left( \sqrt{\rho_c} - \sqrt{\rho_d} \right) \left( \rho_c \Re_{p,\text{Total}} \right)}{3 \left( g(\rho_c - \rho_d) \right)^{\frac{1}{3}} \mu_c (\sigma_{dc})^{\frac{1}{3}}} \]  \hspace{1cm} (58)

Eqn. (58) above can be re-arranged as:

\[ \Re_p = \frac{1}{\sqrt{C_{D,\text{Total}}}} \left( Eo_p \right)^{\frac{3}{2}} \left[ \frac{4 \left( \sqrt{\rho_c} - \sqrt{\rho_d} \right)}{3 \left( g(\rho_c - \rho_d) \right)^{\frac{1}{3}} \mu_c (\sigma_{dc})^{\frac{1}{3}}} \right] \]  \hspace{1cm} (59)

The square term Eqn. (59) can be simplified using Eqn. (3) as:

\[ \left( \frac{16}{9} M_{\text{system}} \right)^{\frac{1}{4}} = \sqrt[4]{\frac{4 \left( \rho_c^2 \sigma_{dc}^3 \right)}{3 \left( g(\rho_c - \rho_d) \mu_c^4 \right)}} = \sqrt[3]{\frac{4 \sqrt{\rho_c}}{3 \left( g(\rho_c - \rho_d) \mu_c^4 \right)}} \frac{1}{\mu_c (\sigma_{dc})^{\frac{1}{3}}} \]  \hspace{1cm} (60)

Hence substituting Eqn. (60) in Eqn. (59), \( \Re_p \) can be expressed as:
\[ Re_p = \frac{1}{\sqrt{C_{D,\text{Total}}} (Eo_p)^{\frac{1}{4}} \left( \frac{16}{9Mo_{\text{system}}} \right)^{\frac{1}{4}}} \]  \hspace{1cm} (61)

Now, if \( C_{D,\text{Total}} \) is a function of \( Re_p \), Eqn. (56) can be substituted in Eqn. (61) to get:

\[ Re_p = \frac{1}{\sqrt{\alpha Re_p} (Eo_p)^{\frac{1}{4}} \left( \frac{16}{9Mo_{\text{system}}} \right)^{\frac{1}{4}}} \]  \hspace{1cm} (62)

If we take the logarithm of both sides of Eqn. (62) and define \( y = \log_{10}(Re_p) \) and \( x = \log_{10}(Eo_p) \), we will arrive at:

\[ y = \sigma + \delta x \]  \hspace{1cm} (63)

Where:

\[ \sigma = -\frac{1}{2} \log_{10}(\alpha) + \frac{1}{4} \log_{10}\left( \frac{16}{9Mo_{\text{system}}} \right) \]  \hspace{1cm} (64)

And,

\[ \delta = \frac{3}{4 \left( 1 + \frac{\beta}{2} \right)} \]  \hspace{1cm} (65)

Hence, if we can get another equation involving \( y = f(x) \), then \( y \) can be eliminated to form an equation involving only \( x \). This other equation of \( y = f(x) \) can be found in by approximating the relation in Figure 8 as shown in Figure 15 below.
Figure 14: Fourth-degree polynomial fit of the coefficients a, b and c of a quadratic approximation of the Grace (1973) relation.

From Figure 15 above, quadratic coefficients a, b and c is stated as functions of the Mo number of the system as:

\[ a, b, c = f(M_{\text{system}}) = q_0^{a,b,c} + q_1^{a,b,c} M' + q_2^{a,b,c} (M')^2 + q_3^{a,b,c} (M')^3 + q_4^{a,b,c} (M')^4 \]  \hspace{1cm} (66)

Where:

\[ M' = \log_{10}(M_{\text{system}}) \]  \hspace{1cm} (67)
The values of \( q_0 \) to \( q_5 \) in Eqn. (66) for the coefficients \( a \), \( b \) and \( c \), are shown in Table 1 below:

<table>
<thead>
<tr>
<th></th>
<th>( q_0 )</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_3 )</th>
<th>( q_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>0.000070</td>
<td>0.000500</td>
<td>-0.020500</td>
<td>-0.564800</td>
<td>-1.266000</td>
</tr>
<tr>
<td>( b )</td>
<td>-0.000090</td>
<td>-0.001000</td>
<td>0.005100</td>
<td>0.139200</td>
<td>1.601700</td>
</tr>
<tr>
<td>( c )</td>
<td>0.000020</td>
<td>0.000200</td>
<td>-0.000400</td>
<td>-0.009700</td>
<td>-0.163300</td>
</tr>
</tbody>
</table>

**Table 1: Polynomial fit coefficients for \( a \), \( b \) and \( c \).**

Hence, we can combine Eqn. (63) with the quadratic equation \( y = a + bx + cx^2 \) as shown in Figure 15, to arrive at:

\[
x = \log_{10}(Eo_p) = \frac{1}{2} \left( -B \pm \sqrt{B^2 - 4C} \right) = f(\text{Mo}_\text{system}, \alpha(\kappa), \beta(\kappa), a, b, c) \quad (68)
\]

Where:

\[
B = \frac{b - \delta}{c}, \quad \text{and} \quad C = \frac{a - \sigma}{c} \quad (69)
\]

\( \sigma \) and \( \delta \) in Eqn. (69) are the same as in Eqns. (64) and (65) respectively. So our final solution procedure for determining the bubble size is to firstly assume that \( C_{D,\text{Total}} \) can be expressed as a function of \( \text{Re}_p \) and check if Eqn. (68) has real solutions, such as shown in the leftmost figure of Figure 16. In this case, solutions are chosen such that the solved \( \text{Eo}_p \) lies within the range Figure 8. Once a solution to Eqn. (68) is found, Eqn. (1) is used to determine \( \text{d}_{p,\text{eq}} \).

In the case that Eqn. (68) does not have a real solution, this implies that either:

1) the assumption of the total drag coefficient being a function of \( \text{Re}_p \) is not valid, i.e., the total drag coefficient is an unknown value (a constant) which is not related to \( \text{Re}_p \), or,

2) some of the assumptions made in arriving at Eqn. (68) are not valid.

Indeed, it is not clear whether 1) or 2) is in fact the case. To justify which approach to proceed with we note where exactly are the assumptions made in arriving at Eqn. (68). In arriving at Eqn. (68), two sets of empirical data were used (\( C_{D,\text{Total}} \) experiments and Grace (1973)’s relation) and
one other equation was derived (Eqn. (37)) from a force balance considering only the buoyancy and drag forces on a particle. Since the derivation of Eqn. (37) is widely used and accepted as a valid basis for the derivations of many existing slip velocity equations (Manninen et al., 1996), and since both empirical relations were developed using wide ranges of fluid properties (Clift et al., 1978), then it is a reasonable assumption to proceed from 1) above.

So in the case of no real solution of Eqn. (68), we assume the total drag coefficient is a constant which is not related to Re_p, and which remains to be found. In this case, as shown in the bottom right figure of Figure 16, the drag coefficient is too low to enable a solution of Eqn. (68). Hence the required higher drag coefficient is found by restating Eqn. (62) as:

\[
Re_p = \frac{1}{\sqrt{C_{D,Total}}} \left(E_0 p \right)^{\frac{1}{4}} \left(\frac{16}{9M_{o,system}}\right)^{\frac{1}{4}}
\]  

(70)

And hence, Eqns. (64) and (65) are reformulated as:

\[
\sigma = \frac{1}{4} \log_{10} \left(\frac{16}{9M_{o,system} \left(C_{D,Total}^k\right)^2}\right), \text{ and } \delta = \frac{3}{4}
\]  

(71)

Thus we can find a real solution to Eqn. (68), reformulated with \(\sigma\) and \(\delta\) defined in Eqn. (71), by setting the total drag coefficient at \(C_{D,Limit}\) and incrementally increasing it until a real solution to the reformulated Eqn. (68) is found. This is shown in the top right figure of Figure 16 below.
Finally, once a solution to Eqn. (68) is found, Eqn. (1) is used to determine $d_{p,eq}$. This bubble size is then used in Eqn. (37), which is substituted in the drift velocity equation, Eqn. 44.

The above procedure can be summarized in the steps below:

**Step 1:** Assume that $C_{D,\text{Total}}$ can be expressed as a function of $Re_p$ and check if Eqn. (68) has real solutions, such as shown in the leftmost figure of Figure 16 above.

**Step 2:** If no solutions are found, such as shown in the lower rightmost figure of Figure 16 above, go to **Step 3**. If one solution is found, go to **Step 5**. If two solutions are found, chose a valid solution such that the solved $Eo_p$ lies within the range of Figure 8, then go to **Step 5**.

**Step 3:** Assume that $C_{D,\text{Total}}$ is an unknown constant which is not related to $Re_p$, and reformulate Eqn. (68) with $\sigma$ and $\delta$ defined in Eqn. (71).

**Step 4:** Set $C_{D,\text{Total}}$ to $C_{D,\text{Limit}}$ and incrementally increase it until a real solution to the reformulated Eqn. (68) of **Step 3** is found (upper rightmost figure of Figure 16).

**Step 5:** Use Eqn. (1) to determine $d_{p,eq}$. Substitute this $d_{p,eq}$ in Eqn. (37), then substitute Eqn. (37) in Eqn. (44) to determine the drift velocity.
Chapter 3

3. New Real-time Calibration Algorithm

3.1. Real-time Calibration of the Drift-flux Model

In terms of shorthand notation, for this chapter:

- $\alpha_k$ refers to $\langle \alpha_k \rangle$
- $V_{sg}$ refers to $\langle j_g \rangle$ and $V_{so}$ refers to $\langle j_o \rangle$
- $V_m$ refers to $\langle j_m \rangle$
- $C_0$ and $V_d$ are defined as in Eqns. (20) and (23), and Eqns. (40) and (42) respectively.

A generic model calibration algorithm has the following features (Yip et al., 2002):

1) A broad range of information is used to constrain the problem without adding features to the model to attain model fit if they contradict other information about the system.
2) The problem is well-posed.
3) Many kinds of data is included as measured variables in the calibration (provides more information about the system).
4) Weights are used to reflect measurement errors. For example, in a production facility, particular sensors may be less frequently calibrated than others and can be expected to produce a greater number of faulty measurements.

Incorporating the general calibration guidelines above into the real-time calibration of the Drift-flux flow model with continuously streaming sensors data, requires the use of non-linear least-squares minimization (or parameter estimation) at two levels. These levels and details of the proposed real-time calibration algorithm are depicted in Figure 17 below. The drift-flux flow model chosen for use in the calibration algorithm of this research is the same as that of MSWell. The MSWell model was a considerably simpler numerical model than the new drift flux model
for coding within the calibration routine, shown in Figure 17, given the time that was available for this research. Any other drift-flux model can be substituted depending on its application. Also, the calibration algorithm below is applied to a three-phase flow system since this algorithm will be later applied to a simulated three-phase flowing oil well case in Section 4.6. Any two-phase flow system can be substituted and used in the calibration algorithm.

![Figure 16: Real-time Calibration of the drift-flux model, as used in SURF.](image)

The calibration algorithm can be described in the following eleven steps in accordance with Figure 17 above:

**Step 1:** At iteration level $k = 0$ (at start), the drift-flux parameter vector is initialized to some vector of default values. Otherwise (at $k \neq 0$), this parameter vector is the calibrated parameter vector as a result of the drift-flux model update after **Step 7**. This drift-
flux parameter vector is always used as the initial parameter vector (starting points) in the Levenburg-Marquardt non-linear least squares minimization of Step 7.

**Step 2:** The distribution and drift velocity parameters of the drift-flux model, i.e., $C_0$ and $V_d$, are calculated using the drift-flux parameter vector of Step 1 and their constitutive equations, i.e., For $C_0$: Eqns. (20) and (23), and for $V_d$: Eqns. (40) and (42).

**Step 3:** $C_0$ and $V_d$ calculated in Step 2 are used to calculate the three-phase gas, oil and water phase holdups, using the two-stage iterative procedure of Shi et al., (2004) as shown in Figure 18 of Section 3.2.

**Step 4:** The three-phase holdups calculated in Step 3 are used as starting points for the FTHP calibration, posed as a non-linear, weighted least squares minimization of measured FTHP sensor data and calculated FTHP. The calculation of FTHP in the minimization is performed using the pressure drop calculation of Section 4.6 (replacing $\alpha_j$ with $p_j$, for $j = 0,1,2$) with the FLMV pressure data serving as the upstream pressure. The minimization solves for the optimum (or matched) phase holdups, i.e., the parameters, and is performed using a Levenburg-Marquardt non-linear, weighted least-squares minimization with a combination of: 1) a linear penalty function enforcing the summation of the phase holdups equaling 1, and 2) box constraints for each of the three holdups lying between 0 and 1. Sixty FTHP sensor measurements are used in the minimization. Mathematically, the parameter estimation problem of this step is posed as:

$$
\text{min} \sum_{i=1}^{N_{\text{meas}}=60} w_i^{\text{press}} \left( P_{\text{meas}}^{\text{FTH}_i} - P_{\text{calc}}^{\text{FTH}_i} \right)^2
$$

s.t. \[ \begin{cases} 0 \leq p_j \leq 1 & \forall \ j = 0(\alpha_g),1(\alpha_o),2(\alpha_w) \\ p_0 = 1 - p_1 - p_2 \end{cases} \]  

However, in the objective function above, the linear constraint requirement of the parameters is not exactly honored from the results of the minimization. This is
remedied if the objective function is re-posed as below, subject to the same constraints in Eqn. (74):

\[
\min \sum_{i=1}^{N_{\text{pars}}+60} w_i^{\text{press}} \left( P_{\text{FTH}}^{\text{meas}} - P_{\text{FTH}}^{\text{calc}} + (1 - p_0 - p_1 - p_2) \right)^2
\]  

(74)

**Step 5:** \(N_{\text{pars}}+1\) matched holdup vectors are generated using the minimization of **Step 4**, corresponding to FTHP measurement-sets \([1\) to \(60], [2\) to \(61], [3\) to \(62], \ldots, [N_{\text{pars}}+1\) to \(60+N_{\text{pars}}+1]\). Each of the holdup vectors are results of the minimization in **Step 4** from \(N_{\text{pars}}+1\) different independent FTHP measurement-sets (each measurement-set has one new point as we progress in the simulation).

**Step 6:** The parameter values of the drift-flux parameter vector in **Step 1** are now used as starting points for the drift-flux parameters calibration, posed as a non-linear least squares minimization of matched phase holdups and calculated phase holdups. The calculation of three-phase holdups in the minimization is performed using the two-stage iterative procedure of Shi et al., (2004) with the constitutive equations for \(C_0\) (Eqns. (20) and (23)) and \(V_d\) (Eqns. (40) and (42)). The minimization of this step solves for the optimum drift-flux parameters, and is performed using a Levenburg-Marquardt non-linear least-squares minimization with box constraints on each parameter. \(N_{\text{pars}}+1\) holdup vectors generated in **Step 5**, representing the holdup “data” to be matched, are used in the minimization. Mathematically, the parameter estimation problem of this step is posed as (same equation as Eqn. (88) in Section 3.2):

\[
\min \left\{ \sum_{i=1}^{N_{\text{pars}}+1} \left( \alpha_{\text{matched}}^{g_i} - \alpha_{\text{calc}}^{g_i} \right)^2 + \sum_{i=1}^{N_{\text{pars}}+1} \left( \alpha_{\text{ol}}^{\text{matched}} - \alpha_{\text{ol}}^{\text{calc}} \right)^2 \right\}
\]

(75)

s.t. \(LB_j \leq p_j \leq UB_j \quad \forall \ j = 0, \ldots, N_{\text{pars}}\)  

(76)

**Step 7:** The calibrated drift-flux parameter vector resulting from **Step 6** represents the start of iteration level \(k+1\), and the calibration algorithm returns to **Step 1** where the drift-flux parameter vector from iteration level \(k\) is replaced by the calibrated drift-flux
parameter vector of this step. This is the drift-flux model updating step. As the global iterations continue, this updating of the drift-flux parameters should drive the calculated FTHP towards the measured FTHP sensor data, i.e., calibrate the model.

**Step 8:** $C_0$ and $V_d$ are calculated using the calibrated drift-flux parameter vector of Step 7 and their constitutive equations, i.e., For $C_0$: Eqns. (20) and (23), and for $V_d$: Eqns. (40) and (42). $C_0$ and $V_d$ in this step are graphed in real-time against $C_0$ and $V_d$ in Step 2.

**Step 9:** $C_0$ and $V_d$ calculated in Step 8 are used to calculate the three-phase gas, oil and water phase holdups, using the two-stage iterative procedure of Shi et al., (2004). The dispersed phase holdups, i.e., gas and oil, are graphed in real-time against the dispersed phase holdups in Step 3.

**Step 10:** The calibrated FTHP is calculated using the pressure drop calculation of Section 4.6, the three-phase holdups in Step 9 and the FLMV sensor data. This calibrated FTHP is graphed in real-time against the FTHP sensor measurements.

**Step 11:** The streaming real-time FTHP sensor measurements are fed into the algorithm from a web-browser interface. The details of this procedure are given in Section 4.1 of Chapter 4.

3.2. **Method of Determination of Drift-flux Model Parameters**

In determining the drift-flux parameters using a non-linear, gradient-based, least squares minimization routine, such as the Levenburg-Marquardt routine, the goal is to predict the phase holdups as accurately as possible. Hence the objective function can be posed in a way to directly minimize the error between the measured holdup and the calculated holdup. In the case of two phase gas-liquid flows, this minimization can be posed as:
\[
\min \left\{ \sum_{i=1}^{N} (\alpha_{i,j}^{\text{meas}} - \alpha_{i,j}^{\text{calc}})^2 \right\} \\
\text{s.t. } LB_j \leq p_j \leq UB_j \ \forall \ j = 0, ..., N_{\text{pars}}^{\text{Gas-Liquid}}
\]  
(77)

Where:

\[
\alpha_{g}^{\text{calc}} = \frac{V_{sg}}{C_{0,\text{Gas-Liquid}}(\alpha_{g}^{\text{calc}}, V_m) \cdot V_m + V_{d,\text{Gas-Liquid}}(\alpha_{g}^{\text{calc}}, V_m)}
\]  
(79)

Similarly, in the case of two-phase liquid-liquid flows, e.g. oil-water, the minimization problem can be posed as:

\[
\min \left\{ \sum_{i=1}^{N} (\alpha_{o,i}^{\text{meas}} - \alpha_{o,i}^{\text{calc}})^2 \right\} \\
\text{s.t. } LB_j \leq p_j \leq UB_j \ \forall \ j = 0, ..., N_{\text{pars}}^{\text{Liquid-Liquid}}
\]  
(80)

Where:

\[
\alpha_{o}^{\text{calc}} = \frac{V_{so}}{C_{0,\text{Liquid-Liquid}}(\alpha_{o}^{\text{calc}}, V_m) \cdot V_m + V_{d,\text{Liquid-Liquid}}(\alpha_{o}^{\text{calc}}, V_m)}
\]  
(82)

In the case of three-phase gas-liquid-liquid flows, such oil-water-gas flows, a direct objective function such as in Eqns. (78) and (81) is required. To arrive at this three-phase objective function for holdup, the formation of the objective function should be in close alignment with the two-stage calculation procedure for three-phase hold-ups. Details of this two-stage procedure are given in Shi et al., (2004), but for illustration, a simple schematic of the procedure is shown in Figure 18 below:
Figure 17: Two-stage iterative procedure for three-phase holdup calculation from volumetric flow rates.

In Figure 18 above, the light-green boxes indicate calculations in the first-stage, whereas the yellow boxes indicate calculations in the second (iterative) stage of the three-phase holdup calculation procedure. The procedure is described in the following way:

**Step 1:** A no-slip assumption is used for $\alpha_{oi}^{k-1}$, from which pseudo-properties are formed.

**Step 2:** These fluid properties are used to calculate $\alpha_{g,\text{Gas-Liquid}}^k$.

**Step 3:** A drift-flux model to solve for the oil holdup in the oil-water combined liquid is used to determine $\alpha_{oi,\text{Oil-Water}}^k$. 

\[
\begin{align*}
Q_{so} &= V_o A, \quad Q_{sw} = V_w A \\
\alpha_{oi}^{k-1} &= \frac{Q_o}{Q_{io} + Q_{iw}} \quad \alpha_{ow}^{k-1} = \frac{Q_w}{Q_{so} + Q_{sv}} \\
\alpha_{oi}^{k+1} &= \frac{\alpha_{oi}^{k-1} \sigma_{og} + \alpha_{ow}^{k-1} \sigma_{ug}}{\sigma_{og} + \sigma_{ug}} \\
\rho_i^{k+1} &= \frac{\alpha_{oi}^{k-1} \rho_o + \alpha_{ow}^{k-1} \rho_w}{\rho_o + \rho_w} \\
\end{align*}
\]
**Step 4:** The oil holdup in the oil-water-gas mixture, $\alpha_{o}^{k+1}$, can now be formed and used to re-compute $\alpha_{g,Gas-Liquid}^{k}$ using the updated values of $\alpha_{ol,Oil-Water}^{k}$ and $\alpha_{ow,Oil-Water}^{k}$.

**Step 5:** Steps 2 to 5 are continued until the holdups no longer change.

Hence, for gas hold-up in the first gas-in-combined-liquid stage, $\alpha_{g,Gas-Liquid}$, we can use an objective function like Eqn. (78), since the gas holdup is calculated using a gas-liquid drift-flux model. For oil holdup, $\alpha_{o}$, in the second oil-water combined liquid stage however, we cannot use a least squares minimization on the oil holdup, since this holdup is actually the oil holdup in the oil-water-gas mixture, i.e., $\alpha_{o} = \alpha_{o,Three-phase}$. We require an optimization on the oil-in-combined-liquid holdup, $\alpha_{ol}$, in order to match the calculation procedure of the second stage of the three-phase procedure. Therefore, our objective function for the oil holdup in the combined oil-water liquid can be posed as:

$$\sum_{i=1}^{N} (\alpha_{ol_i}^{\text{meas}} - \alpha_{ol_i}^{\text{calc}})^2$$

(83)

subject to $LB_{j} \leq p_{j} \leq UB_{j} \forall \ j = 0,\ldots,N_{\text{pars}}^{\text{Oil-Water}}$ (84)

Where:

$$\alpha_{ol}^{\text{meas}} = \frac{\alpha_{o}^{\text{meas}}}{\alpha_{o}^{\text{meas}} + \alpha_{w}^{\text{meas}}}$$

(85)

And,

$$\alpha_{ol}^{\text{calc}} = \frac{V_{m}^{sp}}{C_{0,Liquid-Liquid}(\alpha_{ol}^{\text{calc}}, V_{m}) \cdot V_{m} + V_{d,Liquid-Liquid}(\alpha_{ol}^{\text{calc}}, V_{m})}$$

(86)

Hence, the direct objective function for non-linear least squares minimization of three-phase holdups can be posed as:

$$\min \left\{ \sum_{i=1}^{N} (\alpha_{g_i}^{\text{meas}} - \alpha_{g_i}^{\text{calc}})^2 + \sum_{i=1}^{N} (\alpha_{o_i}^{\text{meas}} - \alpha_{o_i}^{\text{calc}})^2 \right\}$$

(87)

subject to $LB_{j} \leq p_{j} \leq UB_{j} \forall \ j = 0,\ldots,N_{\text{pars}}^{\text{Oil-Water-Gas}}$ (88)
Finally, there can be multiple local minima for the objective-functions of Eqns. (78), (81) and (88), and optimized parameter values are non-unique. Box constraints as shown in Eqns. (79), (82) and (89) should be considered within the context of their physical meanings.
Chapter 4

4. Code Validation and Discussion of Results

4.1. SURF Program Features

4.1.1. Integration of Real-time Sensor Data and SURF Program

Any real-time programming involves the acquisition of real-time data at a reception rate appropriate for the phenomena under study, in our case, multiphase flows in the wells and pipes of surface production facilities. In this case, real-time data from pressure and flow sensors along the facility are fed into a common server-side process database (previously discussed in Section 1.4 of Chapter 1) at reception rates ~O(seconds). A useful feature with regards to production operations monitoring is having this data to be securely accessed over the internet from anywhere in the world. The proposed steps needed to access streaming real-time data over the internet from a remote server for use in the client-side computer program is shown in Figure 19 below:

Figure 18: Proposed steps for accessing streaming real-time data from a remote internet server for use in the client-side program.
The procedure of integrating real-time sensor data from a surface facility’s automation network suite into a client-side computer model starts with the surface facility automation network. Raw sensor-data from an automation software vendor suite is transmitted to the process history database (discussed in Section 1.4) at unknown real-time sampling rates native to individual sensors. If the sampling rate of a sensor is greater than that of the sampling rate of chosen for the calibration, $n$, then the PolyInterpret program performs a polynomial interpolation of the incoming data to filter it to a $n$-second reception rate. This is performed for each sensor. Output XML data files for each sensor are then posted on an internet server for direct access by a client-side computer program.

In the PolyInterpret program, no smoothing of data is performed, only polynomial interpolation. The program filters the unknown reception rates native to the various sensors into regular $n$-second increments, but does not pre-process the incoming data. In practice, any real-time application must be data-tolerant. Raw sensor data are generally useless to an end-user unless there is a system in place to validate and screen it. The extent to which a real-time application makes it easy to “scrub” incoming data and to supply reasonable substitutes for missing data, will have a powerful influence on the success of the simulation effort. Though the importance of pre-processing incoming real-time sensor data is stressed above, SURF does not have a pre-processing feature as this is a whole other field of research in itself.

In Figure 19 above, XML is the language used to retrieve the thousands of sensor data on a remote server address. The SURF program interfaces with this server using XML script since other internet scripts, such as JavaScript does not support client-side data/file transfer. The black form-boxes at the top right of Figure 19 is filled with sensor data entries and placed into a JavaScript style-container for displaying the streaming data on a web-browser. In this way, by logging into a web-browser, remote real-time data can be accessed.

As in all online simulations, it is necessary to collect measurements from the sensors in Figure 19 with a suitable frequency. For multiphase pipe flow simulation, time-constants are fairly long ~O(10 seconds), and very frequent sampling may not be necessary (Klemp et al., 2002). Nevertheless, even though a sampling rate of 10 seconds can be sufficient for a full multiphase
flow model, use of a simpler model, such as the drift-flux model can furnish a real-time sampling frequency of two-seconds. Moreover, any specified sampling rate must be greater than the minimum cycling time, which is the frequency by which the SCADA system provides a complete set of fresh data representing the true real-time data. These factors above should be considered in determining an appropriate real-time sampling frequency rate. For *SURF*, this rate is two seconds \((n = 2)\) since all calculations including the calibration (parameter estimation) of Section 3.1 can be done within this time-frame.

Finally, the programming requirements for a real-time application are not trivial. The programming environment chosen has to satisfy the following:

1) Hundreds of thousands of incoming real-time data points, which translates to very large storage requirements.
2) Numerical computations cannot be performed using stack array memory due to the very large data-structures and classes/objects. Computations can be performed using dynamically allocated heap memory.
3) The programming environment must support internet browser integration over a network.

The requirements above can be satisfied if using a .NET C++ development environment.

4.1.2. **Real-time Fluid Properties Estimation**

All terms in the pressure drop equation in Section 4.6 are evaluated at in-situ conditions. In a long pipe or well, the pipe is segmented such that the pressure gradient is calculated at several points (or nodes) along the pipe at the pressure and temperature at those points. Thus, in this case, it is required to evaluate various in-situ fluid properties in order to calculate the phase holdups at these points. The fluid properties required to calculate the phase holdups could be obtained from PVT lab analyses, if available. However, since PVT analyses are conducted on the reservoir fluid at reservoir temperature only, they are thus not applicable to facilities piping calculations, since the temperature of the fluid along the piping system is constantly changing. This fact makes it necessary to use fluid property estimations (i.e., correlations) to account for these constantly changing conditions, and in our case, changing in real-time and along the pipe itself.
Appendix A contains the equations and strategies used in SURF for estimating in-situ fluid properties at any point in a well, using minimal operating field data for various operating ranges. The minimal operating field data, which most fluid property correlations are based on, are the stock tank API, separator gas specific gravity, $\gamma_{g,sep}$, and producing GOR, $R_{sb}$. For the example in Section 4.6, the fluid properties are estimated from the minimal operating field data. Since this example has only one segment, the fluids properties estimated in real-time are applied for the whole calibration. If there were multiple segments, then the multi-segmented pipe would have fluid properties changing in real-time at each segment node as well as changing along the segments (or network of segments) themselves.

### 4.2. Gas-in-Water Two-Phase Results

The new model as developed in Chapter 2 was tested against the existing MSWell drift-flux model with default parameters (shown in Table 3 in Section 4.6) and with optimized parameters. The optimized parameters of the MSWell model for all results in this chapter are those suggested by Shi et al. (2005), with the optimized $V_d$ deviation parameters having different values depending on whether the flow is in a small (say, less than 4”) or large (say, 4” and greater) diameter pipe (Figure 13 of Section 2.3). The terms “small” and “large” diameter pipe are defined arbitarily. The experiments chosen for validating the new model were:

1) Experiments carried out in a 6” large diameter pipe at the Schlumberger Cambridge Research (SCR) Centre. The experimental setup and detailed description of these experiments can be found in Oddie et al., (2003). The SCR experiments that exhibited bubbly flows for a wide range of phase flow rates (1,000 to 20,000 bbl/day), i.e., those conducted at 0°, 5° and 45° from vertical, were chosen as applicable for this report.

2) Stanford database set SU_204_3 (Spedding and Nguyen Air-Water data, 1976). This particular small diameter dataset was chosen since the fluid properties matched very closely with the Gas-Water fluid properties of the SCR dataset above. Hence the effect of pipe diameter could be investigated over a wide range of gas and water rates.
3) Experiments of Hill (1992). Once again, this dataset was chosen as its oil-water fluid properties were almost the same as that of the SCR oil-water data. Thus the effect of a larger pipe diameter could be investigated.

A summary of the experiment characteristics are shown in Table 2 below:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe diameter (inches)</td>
<td>6</td>
<td>1.79</td>
<td>7.25</td>
</tr>
<tr>
<td>Inclinations from vertical</td>
<td>0°, 5° and 45°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>Fluids</td>
<td>Gas-Water</td>
<td>Air-Water</td>
<td>Oil-Water</td>
</tr>
<tr>
<td></td>
<td>Oil-Water-Gas</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2: Experiments characteristics used in this report.**

Sections 4.2 to 4.4 display cross plots of the experimental and the predicted steady-state holdup values using MSWell’s default and optimized parameters, and the new drift flux parameter values of this report. For GW flows, the experimental gas holdups of the small diameter (1.79”) SU_204_3 dataset are used to compare the three models above. For OW flows, the experimental oil holdups of the large diameter (7.25”) Hill (1992) dataset are used to compare the models above. In all of the holdup cross-plots, the inner dashed line represents the 10% holdup deviation boundary whereas the outer dashed line represents the 20% holdup deviation boundary. The default MSWell model is hereafter referred to as the “default DFM”; the optimized MSWell model is the “optimized DFM”; and the new drift-flux model is the “new DFM”

Section 4.5 displays the variations of the distribution and drift velocity parameters with holdup for different pipe diameters and different pipe inclinations. The average type of void fraction profile peaking that occurs in these the experiments above are also shown, which was calculated with the new drift-flux model. Lastly, the identification of the average bubble sizes and shapes
that is solved using the new drift-flux model, are displayed over the range of vertical GW, OW and OWG flows of the experiments described above.

For vertical GW flows, Figure 20 shows that the default DFM slightly over-predicts the gas holdup over the range of SCR experiments performed, indicating a higher in-situ slip velocity of the gas in the experiments than that predicted by the model. Application of optimized parameters as $A = 1.0$, $a_1 = 0.05$, $a_2 = 0.21$, $m_0 = 1.85$, $n = 0.95$, and $n_1 = 0.21$, shows a good match to the experimental data, which is interesting since $A = 1.0$ indicates a flat void fraction profile. The new DFM for this range of experimental holdups does not offer an improvement to the default DFM. At gas holdups of 0.15 and less, all three models predict about the same, but beyond this value both the default DFM and the new DFM predicts the gas holdup beyond a 20% deviation from experimental values.

If the pipe diameter is smaller (1.79”), then a clearer trend begins to emerge (Figure 21). In all the models, the prediction of gas holdup decreases with higher gas rates. The new DFM tested with these small diameter GW experiments tends to under-predict the gas holdup. The default DFM tends to also under-predict gas holdup up to a fraction of about 0.75, but above this, it over-predicts the gas holdup.

At 5° to the vertical with the SCR data, Figure 22, there are no major differences from the results of Figure 20. At 45° from the vertical, all three models show a clear tendency for over-prediction of the gas holdup for all experiments at this deviation. For these experiments, the new DFM shows a slightly better gas holdup prediction than the default DFM.

There are not enough GW experiments of the SCR and SU_204_3 datasets shown in Figures (20) – (23) to determine how well the new DFM performs with respect to the default DFM in GW flows. At low deviations from the vertical (0° and 5°), the default DFM shows a slightly better prediction of gas holdup than the new DFM. At a higher deviation from the vertical (45°), prediction of gas holdup between both models is reversed. At all deviations, the optimized DFM predicts better than either of the other two models.
Figure 19: Comparison of default and optimized MSWell models with new model, for gas holdup prediction in gas-water 6-inch (SCR dataset) vertical flows.

Figure 20: Comparison of default and optimized MSWell models with new model, for gas holdup prediction in gas-water 1.79-inch (SU_204_3 dataset) vertical flows.
Figure 21: Comparison of default and optimized MSWell models with new model, for gas holdup prediction in gas-water 6-inch (SCR dataset) deviated flows ($\theta=5^\circ$).

Figure 22: Comparison of default and optimized MSWell models with new model, for gas holdup prediction in gas-water 6-inch (SCR dataset) deviated flows ($\theta=45^\circ$).
4.3. Oil-in-Water Two-Phase Results

For vertical OW flows, Figure 24 shows that the new DFM predicts the oil holdup quite well over the wide range of SCR experiments performed. Application of optimized DFM parameters as $A = 1.0$ and $n' = 1.0$, also shows a good match to the experimental data. As noted in Section 2.3 of chapter 2, the exponent of $\left(1 - \langle\alpha_d\rangle\right)$ in the drift velocity equations of the optimized DFM and the new DFM are equal, i.e., $n-1$ in Eqn. (44) = $n'$ in Eqn. (42) = 1. Hence the differences between the optimized and new DFM lay only in the value of $C_0$. The default DFM under-predicts the oil holdup for these vertical OW (6”) experiments.

In Figure 25, the effect of a larger pipe diameter (7.25”) on the oil holdup prediction on the new and optimized DFM seems to be that the new DFM shows a tendency to over-predict the oil holdup, whereas the opposite is true for the optimized DFM. Both models, however, have about the same results for this set of experiments.

At 5° to the vertical, Figure 26, there are good predictions of the oil holdup for the wide range of oil rates (SCR data), in both the optimized and new DFM. At 45° to the vertical, the default DFM shows a wider scatter of predicted oil holdup than with the lower deviation experiments. The optimized and new DFM shows good prediction of oil holdup at 45° from the vertical.
Figure 23: Comparison of default and optimized MSWell models with new model, for oil holdup prediction in oil-water 6-inch (SCR dataset) vertical flows.

Figure 24: Comparison of optimized MSWell model with new model, for oil holdup prediction in oil-water 7.25-inch (Hill, 1992 dataset) vertical flows.
Figure 25: Comparison of default and optimized MSWell models with new model, for oil holdup prediction in oil-water 6-inch (SCR dataset) deviated flows ($\theta=5^\circ$).

Figure 26: Comparison of default and optimized MSWell models with new model, for oil holdup prediction in oil-water 6-inch (SCR dataset) deviated flows ($\theta=45^\circ$).
4.4. Gas-Oil-Water Three-Phase Results

For OWG flows, the gas and oil holdups are compared separately with experimental values for the SCR three-phase flow experiments for the 6” pipe at 0°, 5° and 45° from the vertical. Results for the optimized DFM were generated using the optimized two-phase parameters.

For vertical OWG flows, Figure 28 shows that the predicted gas holdups all lie below 0.35. There is an over-prediction of the oil holdup with the default DFM in these experiments. The new DFM shows a slightly better prediction than the default DFM, and the optimized DFM has the best prediction of oil holdup. In Figure 29, the new DFM shows the best prediction of the oil holdup for a wide range of oil rates, followed by the optimized DFM and the default DFM. All models show large errors in their oil holdup predictions, caused most probably by the effect of the gas on the flow of oil and water. There are sets of points on Figure 29 aligned horizontally, with the same calculated (but different experimental) values of holdups. For these experiments, there are the same oil and water superficial velocities, but different gas superficial velocities. Since the oil prediction in the three phase model does not use the gas superficial velocity or gas holdup and bases its predictions solely on the water and oil superficial velocities, then this results in the same predicted $\alpha_{ol}$.

At 5° to the vertical, there are no significant differences in both the gas and oil hold predictions from the vertical flows, and the same trends are noted as well. At 45° to the vertical, there is a clear tendency for over-prediction of the gas holdup with the default DFM. The new DFM gives the best predictions of gas holdup at a fraction of 0.2 and below, but is less accurate beyond this range. For the oil holdup prediction, the new DFM predicts better than the other DFMs at higher oil rates.
Figure 27: Comparison of default and optimized MSWell models with new model, for gas holdup prediction in oil-water-gas 6-inch (SCR dataset) vertical flows.

Figure 28: Comparison of default and optimized MSWell models with new model, for oil holdup prediction in oil-water-gas 6-inch (SCR dataset) vertical flows.
Figure 29: Comparison of default and optimized MSWell models with new model, for gas holdup prediction in oil-water-gas 6-inch (SCR dataset) deviated flows ($\theta=5^\circ$).

Figure 30: Comparison of default and optimized MSWell models with new model, for oil holdup prediction in oil-water-gas 6-inch (SCR dataset) deviated flows ($\theta=5^\circ$).
Figure 31: Comparison of default and optimized MSWell models with new model, for gas holdup prediction in oil-water-gas 6-inch (SCR dataset) deviated flows ($\theta=45^\circ$).

Figure 32: Comparison of default and optimized MSWell models with new model, for oil holdup prediction in oil-water-gas 6-inch (SCR dataset) deviated flows ($\theta=45^\circ$).
4.5. Distribution Parameter, Drift Velocity and Bubble Distribution Results

The variations of the distribution parameter to the dispersed phase void fraction at 0°, 5° and 45° to the vertical are shown below. In cases of OW flows, Hg actually signifies Ho. Figure 34 shows two effects of varying pipe diameters and different flow systems. For GW flows there’s a clear tendency for high C0 values, whereas with OW and OWG flows C0 is always close to 1, i.e., a flat profile. In the particular case of the large 7.25” pipe, C0 takes values slightly less than 1 beyond about 0.5, where phase inversion effects are expected to play a larger role. The small diameter C0 points show the form of C0 as it approaches a unity dispersed phase void fraction.

Figure 35 represents another C0 variation with dispersed phase void fraction for flows at 5° to the vertical. When compared to Figure 34, the C0 values are all lower for the GW flows, and at 45°, Figure 36, they are still lower. This trend does not apply for the OW and OWG flows, whose values stay close to 1.

Figure 37 shows the variation of the drift velocity parameter with dispersed phase holdup for different pipe diameters and different pipe inclinations. The drift velocities for the vertical flows are a lot less than the drift velocities at 5° and 45° from the vertical. For flows at the range of pipe diameters shown, the drift velocity exhibits the same form in all flows of gradual decrease with increasing void fraction. At 5°, Figure 38, there is a tendency of higher drift velocities at lower void fractions. Figure 39 shows large changes in the drift velocity particularly for OW and GW flows, with OW having higher drift velocities than GW for the same pipe diameter.

Figure 40 shows the average type of void fraction profile peaking that occurs in the OW and GW experiments of the SCR. These profiles were calculated using the new DFM. As is noted OW flows tend to wall peaking while GW flows show a central peaking profile. Figure 41 shows how we can identify average bubble sizes and shapes in various vertical flows in ours experiments discussed here. As can be expected, the OWG and large diameter flows show the highest distribution of different bubble regimes for different experiments. Conversely, the small diameter pipe shows similar bubble sizes and shapes in a particular area on the Grace (1973) chart.
Figure 33: Variation of distribution parameter with void fraction, for GW/OW/OWG vertical flows in pipes of varying diameters (1.79”, 6” and 7.25”).

Figure 34: Variation of distribution parameter with void fraction, for GW/OW/OWG deviated flows (θ=5°) in a 6” pipe.
Figure 35: Variation of distribution parameter with void fraction, for GW/OW/OWG deviated flows ($\theta=45^\circ$) in a 6” pipe.

Figure 36: Variation of drift velocity (m/s) with void fraction, for GW/OW/OWG vertical flows in pipes of varying diameters (1.79”, 6” and 7.25”).
Figure 37: Variation of distribution parameter with void fraction, for GW/OW/OWG deviated flows ($\theta=5^\circ$) in a 6” pipe.

Figure 38: Variation of distribution parameter with void fraction, for GW/OW/OWG deviated flows ($\theta=45^\circ$) in a 6” pipe.
Figure 39: Average radial void fraction distributions in gas-water and oil-water vertical flows of the SU_204_3 (Spedding and Nguyen, 1976), Hill (1992) and SCR (2003) datasets.

Figure 40: Range of bubble sizes and shapes in GW/OW/OWG vertical flows of the SU_204_3 (Spedding and Nguyen, 1976), Hill (1992) and SCR (2003) datasets.
4.6. Real-time Calibration for Oil Well Case with Simulated Field Data

In this example, the calibration algorithm of section 3.1 is adapted in a field setting for a real-time reception rate of 2-seconds. The real-time fluid properties estimation formulae of Appendix A are used to calculate in-situ phase properties for determining the drift-velocity parameters. The field setting chosen is the simple case where there are two pressure sensors 23’ apart at a surface installation’s lower manifold valve (LMV), and at the tubing head (TH). These are common locations for sensor instrumentation. Though the pressure drop is expected to be small, by calibrating the parameters of the three-phase drift-flux flow through a 6” tubing connecting them, the calibration procedure can be tested for its stability when performing real-time calculations. The field data into the calibration is the minimal operating data shown in Table 3 below. A schematic of the placement of the pressure sensors on the oil well tubing head is shown in Figure 41 below. The differential length used in the pressure drop equation is the pipe length itself since the pressure drop between the two locations is small as compared to the well. The sensors data are simulated, randomly changing data with no pre-processing. There are 10,000 sensor measurements. The pressure drop equation is the same as that used in Shi et al., (2004b), neglecting the acceleration component:

\[ \Delta P = \Delta P_h + \Delta P_f \] (89)

Where, the hydrostatic pressure difference is:

\[ \Delta P_h = \rho_m \cdot g \cdot h = (\alpha_g \rho_g + \alpha_o \rho_o + \alpha_w \rho_w) \cdot g \cdot h \] (90)

And \( \rho_m \) is the mixture density defined as:

\[ \rho_m = \alpha_g \rho_g + \alpha_o \rho_o + \alpha_w \rho_w \] (91)

The frictional pressure difference is:

\[ \Delta P_f = \left( \frac{2 f_w \rho_m V_m^2}{D} \right) \cdot \Delta x \] (92)
Where $f_{tp}$ is the friction factor and $D$ is the well/pipe diameter. The Haaland (1981) correlation is used to obtain this parameter:

$$f_{tp}^{0.5} = -3.6 \log \left[ \frac{6.9}{\text{Re}} + \left( \frac{\varepsilon}{3.7D} \right)^{1.11} \right] \quad (93)$$

Where $\varepsilon$ is the pipe roughness and $Re$ is the Reynolds number defined as:

$$Re = \frac{D m V_w D}{\mu_m} \quad (94)$$

With:

$$\mu_m = \alpha_g \mu_g + \alpha_w \mu_w + \alpha_m \mu_m \quad (95)$$

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<tr>
<td>Settling time observed</td>
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<td>separator GOR, scf/STB</td>
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<td>Primary separator gas</td>
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<tr>
<td>specific gravity</td>
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<td>Pipeline diameter,</td>
<td>6</td>
</tr>
<tr>
<td>inches</td>
<td></td>
</tr>
<tr>
<td>Distance between FTHP</td>
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<tr>
<td>and FLMV, ft</td>
<td></td>
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<th>Real-time Sensors Data (starting values):</th>
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<tr>
<td>Flowing tubing head pressure, psia</td>
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<tr>
<td>Flowing tubing head temperature, °F</td>
<td>101.2</td>
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<td>Flowing lower manifold pressure, psia</td>
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<td>Test separator temperature, °F</td>
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<td>Gas-Liquid Parameters:</td>
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<tr>
<td>$A$</td>
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<tr>
<td>$B$</td>
<td>0.3</td>
</tr>
<tr>
<td>$a1$</td>
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<tr>
<td>$a2$</td>
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<td>Oil-Water Parameters:</td>
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<td>$B1'$</td>
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<td>$B2'$</td>
<td>0.7</td>
</tr>
<tr>
<td>$r'$</td>
<td>1</td>
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Table 3: Oil well tubing head data (field and flow model data) required for calibration of the drift-flux parameters to the pressure drop between the FTHP and FLMV sensors.
Implementing the calibration algorithm of section 3.1 is computationally demanding. In testing with the specific example above, several runs stalled due to two problems:

1) Since there are 10,000 measurements per sensor, very large data structures and classes are created in the calculations of the non-linear least-squares optimization routine, i.e. the objective functions are computed several hundred thousand times in calibrating one set of drift-flux parameters. This causes the computer memory resources to be very low and stalling frequently occurs in this case. Greater computing architecture can remedy this reason for stalling.

2) When a series of unfeasible parameters are calculated from Step 7 of Figure 16, the gradient-based, non-linear, least squares optimization routine uses this erroneous parameter set to start in directions away from the local solution. Hence stalling occurs in
these cases. Ensuring a narrow parameter space for the drift-flux parameters-vector (shown in Table 3 above) is seen to decrease the likelihood of stalling due to this reason.

Figure 44 shows a test run of the three-phase flowing oil well example above. The calculations of $C_0$ and $V_d$ lag behind by $N_{\text{meas}}+N_{\text{pars}}+1$ calculations due to the way the calibration algorithm of section 3.1 solves for the matching holdup values for the measured pressure drops between the FLMV and FTHP pressure sensors ($C_0$ and $V_d$ are shown in the lower right boxes). The top left box of Figure 44 shows the calibrated FTHP versus the measured FTHP. The top right box shows the calibrated three-phase gas holdup (lagging) versus the matched holdup (not lagging). The \textit{calibration details} of the lower left box include the real-time fluid properties estimation and a real-time updated display of calibrated drift-flux parameters.

Figure 42 shows an example of stalling due to large computer memory requirements. As the calibration proceeds, the memory requirements continue to increase until the system resources are so low that the program stalls. Note that the vertical streaks on the program interface indicate that these values are set to \textit{NULL}, which is a good indicator that computer system resources are becoming low.

Figure 43 shows another test run in which stalling occurred. As can be seen in the top right box, a series of unfeasible drift-flux parameter vectors causes a large drop in calibrated three-phase gas holdup. This in turn leads to the gradient-based Levenburg-Marquardt starting in a direction away from the local solution. In trying to get back to the local solution, the Levenburg-Marquardt algorithm stalls and prevents further computations. In these cases, the program run is re-started. It was found that by boxing the drift-flux parameters in a narrow range, test runs were able to proceed without stalling. Another solution for addressing this difficulty of using un-preprocessed real-time data in a local, gradient-based, non-linear optimization routine, is to smooth and filter the data by, for example, a wavelet thresh-holding method or smoothing splines. As noted previously in Section 4.11, no smoothing or filtering of real-time sensors data is performed in \textit{SURF}, as this encompasses whole other areas of research beyond the scope of this research.
Figure 44 shows a successful calibration that was obtained after many attempts. In order to achieve this result, as mentioned above, the test run was shortened (to overcome 1) above) and the parameters were boxed in narrow ranges (to overcome 2) above). As can be seen in Figure 44, once the initial $N_{\text{meas}} + N_{\text{pars}} + 1$ calculation period terminates, the algorithm uses the calibrated drift-flux parameters to update the drift-flux model in the subsequent iteration. Note that unlike Figure 42, a shorter run was performed.
Figure 42: Stalling due to large computer memory requirements

Figure 43: Stalling due to the non-linear, least-squares minimization routine (i.e. Levenburg-Marquardt) getting stuck in a series of unfeasible solutions (or directions).
Figure 44: Successful calibration after many attempts. $C_0$ and $V_d$ calculations are lagging behind by $N_{\text{meas}} + N_{\text{pars}} + 1$ sensor measurements.
Chapter 5

5. Conclusions and Recommendations

5.1. Conclusions

In this work a new drift-flux flow model is formulated for upward bubbly flows in small-to-large diameter wells and pipes. This model accommodates for drag and non-drag forces via a bubble coalescence/breakup model and a void fraction evolution model. Average bubble sizes and bubble regimes can also be determined with the new model. The new drift model was tested using limited large (Oddie et al., 2003; Hill (1992)) and small diameter (Spedding and Nguyen, 1987) data. In the experiments used for testing the new drift-flux model, the predictions of void fraction was shown to be satisfactory for gas-water two-phase flows, better for oil-water-gas three-phase flows and best for oil-water two-phase flows. Identification of the average bubble sizes and regimes matched fairly well with visual observations of SCR (Oddie et al., 2003) dataset.

Also proposed in this report is a new real-time calibration algorithm for calibration of a well or pipe flow model (i.e., the drift-flux model) parameters with field sensors data. A procedure for integrating field sensors data on an automation network in a facility to a computer model on a remote desktop computer is proposed. In the calibration algorithm itself, a direct three-phase holdup objective function is formulated in alignment with an existing two-stage holdup calculation procedure for three-phase flows at the department. The calibration routine was implemented in a simple oil well case with simulated real-time sensors data, for illustrating how minimal field operating data could be used to achieve calibration of the drift-flux model’s parameters to pressure sensors data. In the oil well example, the calibration algorithm is seen to be stable under certain situations. A large memory requirement combined with the difficulty of using un-preprocessed real-time data in a gradient-based, non-linear least squares optimization
routine, causes stalling of the calibration routine whenever there is a series of unfeasible calibrated drift-flux parameters present. In this work, running shorter tests and constraining drift-flux parameters in narrow ranges were posed as solutions for these two problems.

5.2. Recommendations for Future Work

The general recommendation for future development of this work is to expand the model for all flow patterns. Only then could it be used for integration in a reservoir simulator for surface-subsurface modeling, rather than for stand-alone surface facilities modeling which the model is currently restricted to.

Other specific recommendations for future work on this research can include:

1) Developing an alternative drift velocity equation to Eqn. (44) for the special case of gas-liquid counter-current flow, by using Eqns. (37) with the calculated \( d_{p,eq} \) in place of the slip velocity of Eqn. (39) which is currently used in MSWell.

2) Comparisons against more experimental data, especially for fully developed flows in larger diameter pipes beyond the SCR dataset.

3) Linkage between bubble descriptions at the bubble scale to flow-field descriptions, i.e., the flow patterns. Development of this linkage for testing flow pattern identification and transitions in upward bubbly flows.

4) Improvement of the calibration algorithm for better stability, for example, by using smoothed and filtered real-time sensors data.

5) Extending the scope of the research beyond automatic parameter correction, i.e., calibration, to include other parts of integration of smart field technologies on the production facility, shown in Figure 45. Figure 45 also shows the scope of the current research.

6) Optimization or tuning or the new drift flux model’s parameters and testing the optimized model with the calibration algorithm.
Figure 45: Automatic parameter updating with real-time sensor measurements.
Nomenclature

\( a_1 \) = drift velocity ramping parameter
\( a_2 \) = drift velocity ramping parameter
\( A \) = area across the pipe
\( A \) = distribution parameter term, for gas-liquid flows
\( A' \) = distribution parameter term, for liquid-liquid
\( B \) = distribution parameter term, gas void fraction at which \( C_0 \) begins to reduce
\( B_1 \) = distribution parameter term, oil void fraction at which \( C_0' \) begins to reduce
\( B_2 \) = distribution parameter term, oil volume fraction at which \( C_0' \) falls to 1.0
\( C \) = input volume fraction
\( C_o \) = distribution parameter
\( D \) = pipe internal diameter
\( D_d \) = dimensionless diameter
\( F_v \) = mixture velocity sensitivity coefficient
\( g \) = gravitational acceleration
\( K_u \) = Kutateladze number
\( L \) = test section length
\( m(\theta) \) = drift velocity multiplier
\( m \) = velocity profile parameter in Eqn. (26)
\( m_o \) = drift velocity multiplier at the vertical
\( n \) = deviation effect exponent
\( n' \) = Hasan and Kabir drift velocity exponent in oil-water flows
\( n_1 \) = deviation effect exponent for water-gas flows
\( n_{1,2,3} \) = void fraction parameters in Eqn. (26).
\begin{align*}
n_1' &= \text{deviation effect parameter for oil-water flows} \\
n_2' &= \text{deviation effect parameter for oil-water flows} \\
n_3' &= \text{deviation effect parameter for oil-water flows} \\
N &= \text{number of experimental points} \\
Q &= \text{volumetric flow rate} \\
V &= \text{velocity} \\
V_b &= \text{velocity of a small bubble} \\
V_c &= \text{characteristic velocity} \\
V_d &= \text{drift velocity} \\
V_m &= \text{mixture velocity} \\
f &= \text{arbitrary quantity} \\
V_p &= \text{volume of particle} \\
d_{p,eq} &= \text{volume equivalent particle diameter} \\
Eo &= \text{Eotvos number} \\
Re &= \text{Reynolds number} \\
Mo &= \text{Morton number} \\
u &= \text{in-situ velocity} \\
\bar{j} &= \text{superficial velocity, or flux} \\
r &= \text{pipe radius} \\
L_1 &= \text{particle diameter at which there is a change in the type of peaking flow.} \\
N_{pars} &= \text{number of drift-flux model parameters} \\
a &= \text{quadratic coefficient of Eqn. (66)} \\
b &= \text{quadratic coefficient of Eqn. (66)} \\
c &= \text{quadratic coefficient of Eqn. (66)} \\
C_{D,Total} &= \text{total drag coefficient} \\
\bar{d} &= \text{characteristic particle size} \\
\bar{u} &= \text{characteristic velocity} \\
\sigma_{dc} &= \text{interfacial tension between phase-d (dispersed) and phase-c (continuous)} \\
C_{D,Limit} &= \text{drag coefficient limit} \\
Re_{p,Limit} &= \text{Re}_p \text{ number limit}
\end{align*}
\[ \sum_{k=1}^{\infty} C_k \] = new distribution parameter coefficients

\[ \sum_{k=1}^{\infty} N_k \] = new distribution parameter coefficients

\[ V_{ref} \] = reference velocity used in PCAF drift-flux equation

\[ k \] = parameter of PCAF drift-flux equation

\[ n \] = parameter of PCAF drift-flux equation

\[ \alpha_o' \] = oil holdup at max superficial oil flux

\[ GOR_{sep} \] = primary separator GOR (scf/STB)

\[ GOR_{total} \] = total flash separation GOR, (scf/STB), also called initial GOR – includes separator gas and stock tank gas

\[ API^\circ \] = stock tank oil gravity

\[ P_{sep} \] = primary separator pressure, psia

\[ T_{sep} \] = primary separator temperature, °F

\[ \gamma_o \] = stock tank oil specific gravity

\[ \gamma_{g,sep} \] = primary separator gas specific gravity (air = 1.0)

\[ \gamma_{FG,total} \] = total free gas specific gravity

\[ \gamma_{g,ST} \] = stock tank gas specific gravity

\[ P_{init} \] = initial average reservoir pressure

\[ MW_{o} \] = molecular weight of stock tank oil

\[ T_{fr} \] = flowing temperature in °R.

\[ T_{sf} \] = flowing temperature in °F.

\[ P_{fl} \] = flowing pressure, psia.

\[ y_{N_2} \] = mole fraction of well stream or free gas impurities

\[ y_{CO_2} \] = mole fraction of well stream or free gas impurities

\[ y_{H_2S} \] = mole fraction of well stream or free gas impurities

\[ P_{PC} \] = pseudo-critical pressure
\( T_{PC} \) = pseudo-critical temperature

\( \gamma_{g,\text{total}} \) = total gas specific gravity, same as weighted average of all surface gas specific gravities

\( \gamma_{g,\text{well}} \) = well or pipe stream gas specific gravity, same as \( \gamma_{g,\text{total}} \)

\( P_b \) = Bubble point pressure, psia

\( DS_{\text{total}(ppm)} \) = total dissolved solids in produced water (brine), in ppm.

\( H_g \) = holdup for gas phase = \( \alpha_g \)

\( H_o \) = holdup for oil phase = \( \alpha_o \)

\( N_{\text{meas}} \) = number of pressure sensor measurements used in calibration routine

\( R_{\text{sl}} \) = producing gas-oil ratio

**Subscripts**

\( f \) = flooding

\( g \) = gas

\( l \) = liquid

\( m \) = mixture

\( o \) = oil

\( ol \) = oil in the liquid phase

\( ow \) = oil-water

\( s \) = superficial

\( w \) = water

\( wl \) = water in the liquid phase

\( k \) = arbitrary phase

\( p \) = particle

\( d \) = dispersed phase

\( c \) = continuous phase

\( Vd \) = dispersed phase moving relative to volume center
\( Cd \) = dispersed phase moving relative to continuous phase
\( Cd,\infty \) = dispersed phase moving relative to continuous phase in an infinite carrier phase
\( mm \) = millimeters
\( Fd \) = dispersed phase in fluctuating flow
\( Md \) = dispersed phase moving relative to mass center
\( Ld \) = dispersed phase moving relative to local conditions
\( \Gamma \) = interfacial mass transfer
\( g,f \) = gas flooding
\( o,max \) = maximum oil flow
\( m0 \) = mixture at pipe centerline
\( d0 \) = void fraction at pipe centerline
\( D \) = drag
\( app \) = apparent
\( pm \) = packing maximum
\( sd \) = superficial rate of dispersed phase
\( sg \) = superficial rate of gas phase
\( sg \) = superficial rate of oil phase
\( sw \) = superficial rate of water phase

**Superscripts**

\( h \) = horizontal
\( v \) = vertical
\( k \) = iteration level \( k \)
\( k+1 \) = iteration level \( k+1 \)
Greek

\( \alpha \) = holdup or in situ void fraction
\( \alpha \) = parameter in Eqn. (57)
\( \beta \) = distribution parameter term
\( \beta \) = parameter in Eqn. (57)
\( \sigma \) = interfacial tension/surface tension
\( \sigma \) = parameter in Eqn. (64)
\( \rho \) = density
\( \theta \) = deviation from vertical
\( \kappa \) = ratio of the dispersed phase viscosity to the continuous phase viscosity
\( \mu \) = viscosity

Abbreviations

Abs = Absolute
Rel = Relative
FTHP = Flowing tubing head pressure = FTP
FLMV = Flowing pressure at lower manifold valve
FBHP = Flowing bottom-hole pressure
FTHT = Flowing tubing head temperature
TSP = Test separator pressure
TST = Test separator temperature
FTT = Flowing tubing temperature
FBHT = Flowing bottom-hole temperature
GW = Gas-in-Water flows
OW = Oil-in-Water flows
OWG = Oil-Water-Gas flows
DFM = Drift-flux model
$SURF = \text{Stanford University real-time facilities}$

$SCR = \text{Schlumberger Research Centre}$

$MSWell = \text{Multi-segmented Well}$

$SU = \text{Stanford University}$

$GOR = \text{Gas-Oil ratio}$

$BPP = \text{Bubble point pressure}$

$FVF = \text{Formation Volume Factor}$

$SCADA = \text{Supervisory control and data acquisition}$

$NUI = \text{Normally unmanned installation}$

$OODA = \text{Observing, orienting, deciding and acting}$

$PVT = \text{Pressure-Volume-Temperature}$

$API^0 = \text{Stock-tank oil gravity}$

$LMV = \text{Lower manifold valve}$

$TH = \text{Tubing head}$
References


Appendix A

A. Determining Fluid Properties from Field Data

A.1. Gas-Oil Ratio Correction (GOR)

If the producing GOR is calculated using sales gas, then an estimate of stock tank gas is added to the separator GOR to get $R_{sb}$. Two stock tank GOR correlations are used for the above purpose. The McCain (1990) GOR is valid for separator temperatures less than 140 °F, while the Rollins et al., (1990) stock tank GOR estimate accommodates for temperatures beyond this range. The equations for GOR calculations in SURF are of the form below:

$$GOR_{McCain,ST} = f(API, P_{sep}, T_{sep})$$  

(96)

And:

$$GOR_{Rollins,ST} = f(\gamma_o, \gamma_{g,sep}, P_{sep}, T_{sep})$$  

(97)

A.2. Total Gas Specific Gravity

The total gas specific gravity is the GOR-weighted average specific gravities of the surface gases, i.e., the primary separator gas gravity and the stock tank gas gravity. If the stock tank gas specific gravity is unknown, it can be estimated with McCain (1990)’s correlation. If there is free gas in the reservoir, then the free gas specific gravity can also be estimated by a correlation of McCain (1990). In this case, the free gas gravity would be included in the total gas specific gravity correction as shown below:

$$\gamma_{g, total} = \frac{(\gamma_{FG,sep} + \gamma_{g,sep}) GOR_{sep} + \gamma_{g,ST} GOR_{ST}}{GOR_{sep} + GOR_{ST}}$$  

(98)
Where:

\[ \gamma_{FG, res} = f\left(P_{init}, T_{r}, API^*, GOR_{sep}, \gamma_{g, sep}\right) \tag{99} \]

And,

\[ \gamma_{g, ST} = f\left(P_{sep}, T_{sep}, API^*, GOR_{sep}, \gamma_{g, sep}\right) \tag{100} \]

**A.3. Bubble Point Pressure (BPP)**

Knowledge of the BPP is one of the most important factors in the primary and subsequent developments of Black Oil fields. There are in the literature sundry Black Oil BPP empirical correlations available that essentially estimates the BPP as a function of typically available field operating data at the surface, for instance, \(R_{sbs}, \gamma_{sep}, \gamma_{o, API}, T_{flowing}, P_{sep}\) and \(T_{sep}\). In \textit{SURF}, the BPP is predicted from six of the most accurate available BPP correlations in the literature (Figure 47 below). The default BPP correlation is the Glaso (1980) correlation. There is also the ability to periodically specify a known BPP in the \textit{SURF} program.

<table>
<thead>
<tr>
<th>BPP Correlation</th>
<th>Notes</th>
</tr>
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<tbody>
<tr>
<td>Standing (1977)</td>
<td>Based on California oil fields. Valid up to 325 deg F, and no impurities considered, valid for crudes &lt; 15 deg API.</td>
</tr>
<tr>
<td>Al Marhoun (1988)</td>
<td>Based on Middle East oils</td>
</tr>
<tr>
<td>Glaso (1980)</td>
<td>Based on Mostly North Sea oils</td>
</tr>
<tr>
<td>McCain (1998)</td>
<td>Based on regression analysis of large volume of measured data.</td>
</tr>
<tr>
<td>Vasquez-Beggs (1980)</td>
<td>Based on regression analysis of large volume of measured data.</td>
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**Figure 46:** BPP correlations used in the \textit{SURF} program.
Where:

\[ P_{B,\text{Lasater}} = f\left(GOR_{sep}, MW_o, \gamma_o, API^*, T_v, \gamma_{g,\text{total}}\right) \]  \hspace{1cm} (101)

And,

\[ P_{B,\text{Marhoun}} = f\left(GOR_{sep}, \gamma_o, T_v, \gamma_{g,\text{total}}\right) \]  \hspace{1cm} (102)

And,

\[ P_{B,\text{Standing}} = f\left(GOR_{sep}, API^*, \gamma_{g,\text{sep}}, T_v\right) \]  \hspace{1cm} (103)

And,

\[ P_{B,\text{Glaso}} = f\left(GOR_{sep}, \gamma_{g,\text{total}}, API^*, T_v, \gamma, Y_{N_2}, Y_{CO_2}, Y_{H_2S}\right) \]  \hspace{1cm} (104)

And,

\[ P_{B,\text{McCain}} = f\left(GOR_{\text{total}}, API^*, \gamma_{g,\text{sep}}, T_v\right) \]  \hspace{1cm} (105)

And,

\[ P_{B,\text{Vasques}} = f\left(GOR_{\text{total}}, API^*, T_v, \gamma_{g,\text{sep}}, \gamma_o, P_{sep}, T_{sep}\right) \]  \hspace{1cm} (106)

A.4. Well/Pipe Stream Gas Deviation Factor (Z factor)

In calculating the Z factor of the well or pipe stream gas, the cases considered are rich gas existing in two phases (Rayes et al., 1992; Standing, 1977), dry gas (Dranchuk et al., 1975; Hankinson et al., 1969) and gas with impurities (Wichert and Aziz, 1972; Carr et al., 1954).

Where:

\[ Z = f\left(P_{\text{flowing}}, T_{\text{flowing}}, P_{PC}, T_{PC}, \gamma_{g,\text{well}}, Y_{N_2}, Y_{CO_2}, Y_{H_2S}\right) \]  \hspace{1cm} (107)
A.5. Coefficient of Isothermal Oil Compressibility

The coefficient of isothermal oil compressibility is calculated for both the under-saturated (Vasquez et al., 1980), and saturated conditions in the well or pipe stream (McCain et al., 1988).

Where:

\[ C_{oil > BPP} = f(GOR_{total}, T_{g}, API^*, P_{flowing}, \gamma_{g, sep}, \gamma_o, P_{sep}, T_{sep}) \]  

(108)

And,

\[ C_{oil \leq BPP} = f(P_{flowing}, P_{BP} T_{g}, API^*, GOR_{total}) \]  

(109)

A.6. Formation Volume Factor (FVF)

The formation volume factor is calculated for both the under-saturated and saturated conditions in the flow-stream and is used in obtaining the superficial rates into the well/pipe at flowing conditions given surface rates. The oil FVF is calculated using Glaso (1980)’s correlation and the water FVF is calculated from a correlation from McCain (1990).

Where:

\[ B_w = f(P_{flowing}, T_g) \]  

(110)

And,

\[ B_{oil, \leq BPP} = f(GOR_{total}, T_g, \gamma_o, \gamma_{g, total}) \]  

(111)
A.7. Density

The oil density is calculated for both the under-saturated and saturated conditions in the well/pipe stream using a McCain (1990) correlation with the Standing and Witte (1987) P-T adjustment. The water density is posed in a form that depend only the dissolved solids content of the formation brine.

Where:

\[ \rho_o = f(GOR_{total}, \gamma_o, \gamma_{g, total}, \gamma_{flowing}) \]  \hspace{1cm} (112)

And:

\[ \rho_w = f(DS_{total, ppm}) \]  \hspace{1cm} (113)

A.8. Viscosity

The viscosity model in the SURF program is unique, in the sense that, in addition to the viscosity of the crude being calculated at, above and below the BPP, the model also accounts for the formation of emulsions during oil production. This can be particularly helpful when modeling oil-water flows. Produced water in these flows occurs as either free water or as emulsions or more commonly, as both. The program accounts for the most common type of emulsion – water-in-oil, i.e. water droplets in a continuous oil phase, and variations of this type of emulsion. The emulsion type is generally defined according the volume fraction of one phase compared to the other.

The viscosity of emulsions can be substantially higher than the viscosity of either the oil or the water at certain watercuts in the emulsion. Phase inversion, i.e., change in emulsion from a water-in-oil to oil-in-water also occurs at higher watercuts. The program uses the model of Kokal (1987) that provides a viscosity correction to the oil viscosity for various watercuts. This viscosity data used in this model is summarized as a periodic input to the program as shown in figure 45.
Figure 47: Emulsion model used in SURF (Kokal, 1987).

Where:

\[ \mu_g = f(\gamma_{g, \text{total}}, T_{\gamma_g}, \rho_g) \]  \hspace{1cm} (114)

And,

\[ \mu_{\mu, \text{at } BPP} = f(GOR_{\text{total}}, \gamma_o, T_{\gamma_o}, \gamma_{g, \text{total}}) \]  \hspace{1cm} (115)

And:

\[ \mu_{\mu, \text{<or}>BPP} = f(\mu_{\mu, \text{at } BPP}, P_{B_P}, P_{\text{flowing}}) \]  \hspace{1cm} (116)
And,

\[ \mu_w = f(T_p, P_{flowing}, DS_{total, ppm}) \]  \hspace{1cm} (117)

**A.9. Interfacial Tension**

The correlations of Baker *et al.* (1956) and Hough (1951) is used to calculate the Oil/Gas and Gas/Water interfacial tension. The Oil/Water interfacial tension is calculated as the difference between the two (Geoquest, 2001).

Where:

\[ \sigma_{GW} = f(T_p, P_{flowing}, \sigma_{GW, 74}, \sigma_{GW, 280}) \]  \hspace{1cm} (118)

And:

\[ \sigma_{OG} = f(T_p, P_{flowing}, \sigma_{OG, 68}, \sigma_{OG, 100}, API^+) \]  \hspace{1cm} (119)
Appendix B

B. Derived Coefficients of New C₀ Equation

Coefficients of \( \sum_{k=1}^{5} C_k \):

\[
C_1 = \frac{m^2}{(1+m)(1+2m)} \left(1 - \frac{n_2}{n_2 + n_3} \right)
\]

(120)

\[
C_2 = \frac{1}{n_1} \left(1 - \frac{n_2}{n_2 + n_3 - n_1} \right) e^{-n_1} \left(1 + \frac{1}{mn_1} + \frac{1-m}{m^2 n_1^2} \right)
\]

(121)

\[
C_3 = -\frac{1}{n_1} \left(1 - \frac{n_2}{n_2 + n_3 - n_1} \right) e^{-n_1} \left(1 + \frac{1+m}{mn_1} + \frac{1+m}{m^2 n_1^2} \right)
\]

(122)

\[
C_4 = \frac{1}{n_2 + n_3} \left( \frac{n_2}{n_2 + n_3} \left(1 - \frac{n_1}{n_2 + n_3 - n_1} \right) \right) e^{-(n_2+n_1)} \left(1 + \frac{1}{mn_1} + \frac{1-m}{m^2 (n_2 + n_3)^2} \right)
\]

(123)

\[
C_5 = -\frac{1}{n_2 + n_3} \left( \frac{n_2}{n_2 + n_3} \left(1 - \frac{n_1}{n_2 + n_3 - n_1} \right) \right) e^{-(n_2+n_1)} \left(1 + \frac{1+m}{mn_1} + \frac{1+m}{m^2 (n_2 + n_3)^2} \right)
\]

(124)

Coefficients of \( \sum_{k=1}^{5} N_k \):

\[
N_1 = \frac{1}{2} \left(1 - \frac{n_2}{n_2 + n_3} \right)
\]

(125)

\[
N_2 = \frac{1}{n_1} \left(1 - \frac{n_2}{n_2 + n_3 - n_1} \right) (e^{-n_1} - 1)
\]

(126)

\[
N_3 = \frac{1}{n_2 + n_3} \left( \frac{n_2}{n_2 + n_3} \left(1 - \frac{n_1}{n_2 + n_3 - n_1} \right) \right) (e^{-(n_2+n_1)} - 1)
\]

(127)

\[
N_4 = \frac{1}{n_1^2} \left(1 - \frac{n_2}{n_2 + n_3 - n_1} \right) (1 - e^{-n_1} (1+n_1))
\]

(128)
\[ N_s = \frac{1}{(n_2 + n_3)^2} \left( \frac{n_2}{n_2 + n_3} \left( \frac{n_1}{n_2 + n_3 - n_1} \right) \right) \left( 1 - e^{-(n_2 + n_3)} \right) \left( 1 + n_2 + n_3 \right) \] 

(129)
Appendix C

C. Derivation of Maximum Superficial Oil Velocity

In this work, the maximum superficial oil velocity is defined as that oil velocity reached in either fully developed pure core-annular oil-in-water flow (PCAF), or, that oil velocity at the point of phase inversion, i.e., at the point that oil is no longer the dispersed phase. Joseph et al., (1997) derived the maximum oil holdup in PCAF oil-in-water flow, shown in Figure 48, as:

\[ \langle \alpha_o \rangle_{PCAF, QIW \ flow} = \frac{1}{2 - \frac{\mu_o}{\mu_d}} \] (130)

Decarre et al., (1997) developed a correlation for the phase inversion point of any two-phase system with one phase dispersed in the other as:

\[ \langle \alpha_o \rangle_{phase, inversion} = \left[ 1 + \left( \frac{\mu_c}{\mu_d} \right)^{\frac{1}{6}} \left( \frac{\rho_c}{\rho_d} \right)^{\frac{5}{6}} \right]^{-1} \] (131)

Hence, the maximum oil flux occurs at the minimum of these oil holdups:

\[ \langle \alpha_o \rangle_{At \ max \ oil \ flux} = \langle \alpha_o' \rangle = \min \left( \langle \alpha_o \rangle_{phase, inversion} \ , \langle \alpha_o \rangle_{PCAF, QIW \ flow} \right) \] (132)

The holdup in Eqn. (132) is used in the drift-flux equation for PCAF oil-in-water flow, developed by Banwart et al., (1998) to determine \( j_{o,max} \), as:

\[ |j_{o,max}| = \langle \alpha_o' \rangle |j_o| + \left( V_{rej} k \langle \alpha_o' \rangle ^2 (1 - \langle \alpha_o' \rangle ^p) \right) \] (133)
Where:

\[ V_{\text{ref}} = \frac{1}{16} \frac{g(\rho_c - \rho_d)D^2}{\mu_c} , k = 0.0194 , n = 1.75 \]  

(134)

Figure 48: Derivation of the maximum superficial oil velocity in OW flows, i.e., the maximum oil flux in PCAF oil-in-water flow. Figure taken from Prada et al., (2001).