SGP-TR-112

Optimization of Injection Scheduling in Geothermal Fields

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May 1987

Financial support was provided through the Stanford Geothermal Program under Department of Energy Contract Nos. DE-AS03-80SF11459 and DE-AS07-84ID12529 and by the Dept. of Petroleum Engineering, Stanford University

Stanford Geothermal Program
Interdisciplinary Research in
Engineering and Earth Sciences
STANFORD UNIVERSITY
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Abstract

This study discusses the application of algorithms developed in Operations Research to the optimization of brine reinjection in geothermal fields. The injection optimization problem is broken into two sub-problems: (1) choosing a configuration of injectors from an existing set of wells, and (2) allocating a total specified injection rate among chosen injectors. The allocation problem is solved first. The reservoir is idealized as a network of channels or arcs directly connecting each pair of wells in the field. Each arc in the network is considered to have some potential for thermal breakthrough. This potential is quantified by an arc-specific breakthrough index, b_{ij}, based on user-specified parameters from tracer tests, field geometry, and operating considerations. The sum of b_{ij}-values for all arcs is defined as the fieldwide breakthrough index, B. Injection is optimized by choosing injection wells and rates so as to minimize B subject to constraints on the number of injectors and the total amount of fluid to be produced and reinjected. The study presents four computer programs which employ linear or quadratic programming to solve the allocation problem. In addition, a program is presented which solves the injector configuration problem by a combination of enumeration and quadratic programming. The use of the various programs is demonstrated with reference both to hypothetical data and an actual data set from the Wairakei Geothermal Field in New Zealand.

Acknowledgement

I would like to acknowledge the many good ideas and the good humor of my advisor, Roland Horne. I have also had very fruitful discussions with Paul Kruger and Irwin Remson in deciding on an approach to the problem of optimizing injection in geothermal fields. Stanford University has been generous m its financial support by making funds available to me from several sources, including the Orcutt and Rhoades Funds and a fellowship from the chevron Corporation. My work has also been supported as pan of the Stanford Geothermal Program under contracts No. DE-AS03-80SF-11459 and No. DE-AS07-84ID-12529 from the U. S. Department of Energy. I am very grateful to Iris Tommelein for her support and for her insights into combinatorics. This work is dedicated with much love to my parents, Osgood and Louise Lovekin.

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1. INTRODUCTION

As the geothermal industry matures, the need for a method to optimize a program of reinjection becomes more important. In the petroleum industry, injection of water into reservoirs has been used for decades to maintain reservoir pressures, to dispose of produced brine, and to increase recovery of hydrocarbons in place. Similar needs exist in the geothermal industry. Reservoir pressures need to be maintained, and produced brine and condensate from power plants need to be disposed of in an environmentally acceptable manner. Although petroleum and geothermal resources differ in nature, fluid injection holds the promise of increased recoveries from geothermal fields as well, by providing a medium to absorb a greater percentage of the heat in the reservoir rock.

Unfortunately, injection into geothermal fields also has potential for decreasing thermal recovery by extracting heat unevenly. If the injected fluid travels too directly to producing wells without contacting a large volume of the reservoir rock, premature thermal breakthough may occur and the economic life of the field may be cut short. This is all the more Likely because flow patterns in geothermal reservoirs are often controlled by fractures. Tracer surveys provide a powerful tool for gaining insight into these flow patterns. However, because of fracturing, the results of these surveys often seem anomalous. It is not uncommon for more distant wells to show stronger tracer response than wells which are closer to the injector.

The purpose of this study is to provide a systematic approach for using such tracer data, together with information about field geometry and operating conditions, in the optimization of injection scheduling in geothermal fields. Before giving an overview of the approach, it is useful to consider the typical situation in which an operator tries to make decisions about an injection program. Even if the field is new, most if not all of the wells planned for start-up will already have been drilled. From well testing, maximum producing rates will be known for individual wells. The size of the power plant or direct use facility will have been decided, so the total required production will be known Likewise, the total amount of fluid to be reinjected will be known, at least within a range, based either on regulatory requirements for brine disposal or on an operating decision as to how much recharge to the reservoir is desirable. There will perhaps have been some injectivity testing to give an idea of the rates and pressures at which individual wells will take fluid. For an older field which is just Starting a reinjection program, these various constraints will be known with greater certainty, based on years of operating experience.

In this situation, the operator of a geothermal field is interested in two questions:

(1) Which wells should be made injectors?

(2) How should the total required injection rate be distributed?

The first question may be called the problem of *configuration,* and the second the problem of *allocation*. In practise, the operator typically solves the configuration problem first. Often, operational considerations dictate the solution. Some wells may have been drilled as injectors, and they may have been completed in a manner which makes them unsuitable for any other function. Their designation as injectors may have been based on their location relative to existing surface facilities. Most likely, they were simply poor producers. It turns out, however, that the solution to the allocation problem provides a Straightforward approach to solving the configuration problem for an unrestricted case. For this reason, this study will address the allocation problem first.

The inspiration for this study's approach to the allocation problem grows directly out of the observation that, because of fracturing in geothermal fields, tracer response between *two* wells is often unrelated to how close the wells are to each other. In this approach, the geothermal reservoir is idealized as a network of direct connections or arcs between every pair of wells. Each arc is presumed to have some potential for thermal breakthrough. This potential is assigned a numerical value called a breakthrough index, b_{ij}, which is some function of operating rates for the wells on either end of the arc and an arc cost c_{ij}, based on tracer test parameters and field geometry. In the simplest case,

$$b_{ij} = c_{ij} \, q_{ri},$$

where b_{ij} = breakthrough index between wells i and j,

c_{ij} = arc cost between wells i and j, and

q_{ri} = reinjection rate into well i.

The sum of b_{ij}-values for all arcs is defined as the fieldwide breakthrough index, B. Injection is optimized by choosing injection rates so as to minimize B, subject to constraints on individual well capacities and total injection requirements. Presented in this format, the allocation problem bears a striking resemblance to problems of linear programming (LP) which are studied in the field of Operations Research. The similarity is not accidental. The thesis of this paper is that algorithms from Operations Research, used in conjunction with tracer tests, provide a useful method of optimizing geothermal injection.

One major assumption should be highlighted at the outset. Using tracer data to predict thermal breakthrough assumes that tracer fronts and thermal fronts propagate in similar patterns, even though the mechanisms of propagation are different. Propagation of a tracer front involves transportation of a chemical species through the reservoir by convection and dispersion. Propagation of a thermal front involves not just the transportation of the injected fluid through the reservoir but the conduction of heat to this fluid from the rock. The different mechanisms of propagation form the basis of the usefulness of tracers as a predictive tool: one hopes that the tracer gets there first. However, multi-phase behavior in the reservoir may cause the two propagation patterns to diverge. Therefore, the approach proposed here is probably most applicable to the case of single-phase, hot-water-dominated fields.

The structure of this report will be as follows. The second section will review previous work in injection optimization and tracer studies and will summarize applications of techniques from Operations Research to reservoir engineering and related problems. The third section will discuss the formulation of the injection allocation problem and will compare several algorithms for solving it. The fourth section will present an algorithm for choosing the optimal injector configuration. The fifth section will describe the results of applying these algorithms to tracer data from the Wairakei Geothermal Field in New Zealand. The last section will summarize the conclusions of the study.

2. PREVIOUS WORK

The usual approach to studying the effects of injection in geothermal fields is to construct analytical or numerical models. Analytical models apply in the case of simple reservoir geometries and serve as useful points of reference in verifying numerical models. Lauwerier,[1] Bodvarsson,[2] and Gringarten and Sauty [3] have presented analytical models for predicting the movement of thermal fronts in porous media of uniform thickness, porosity, and permeability. However, for the common case of fractured geothermal reservoirs, these models are not applicable. An analytic description of the advance of a thermal front in a reservoir with horizontal fractures was presented by Bodvarsson and Tsang,[4] who then used this model to verify a numerical routine. O'Sullivan and Pruess [5] described the similarity method (which they called "quasi-analytical") for analyzing pressure increases due to injection of cool fluids in a geothermal reservoir, and they compared this method with another numerical routine. Witherspoon *et al.* [6] presented a more elaborate analytical model which incorporated a network of horizontal and vertical fractures. The analytical solutions of both O'Sullivan and Pruess and of Witherspoon *et al.* indicated that the propagation of a cool front along fractures in early times would give way at later times to a uniform thermal sweep. An analytical model for geothermal reservoirs with major vertical fractures was discussed by Pruess and Bodvarsson? who also presented results of numerical simulations for the same case.

Numerous numerical models of the injection of cool fluids into fractured reservoirs have been presented in the literature, including work by Lippmann *et al.*,[8] Garg and Pritchett,[9] Hunsbedt *et al.*,[10] Pruess and Narasimhan,[11] and Bodvarsson *et al.*.[12] Bodvarsson *et* al. [13] provided a good overview of modeling techniques available for geothermal systems and included a discussion of injection modeling. These numerical models have considered a variety of different reservoir geometries (one-, two-, or three-dimensional, linear or radial) and reservoir fluids (vapordominated, liquid-dominated, or two-phase). Maddock *et al.* [14] coupled a numerical model of a geothermal reservoir undergoing injection with a numerical model of a power plant. A sequence of studies of injection in the Cerro Prieto Field in Mexico by Tsang *et al.*[15,16,17,18] illustrated the progressive refinement of a numerical model to meet the geologic constraints of a particular field. Other fields for which numerical modeling has been used to study the effects of injection include the Larderello Field in Italy,[19,20] the Baca Field in New Mexico,[21] the Krafla Field in Iceland,[22] and the Los Azufres Weld in Mexico.[23]

Although numerical modeling permits consideration of more complex geothermal systems than analytical models can handle, it does not in itself answer the question of how geothermal fields should be most effectively developed. Starting with Lee and Aronofsky,[24] a number of authors in the petroleum industry have proposed ways of enhancing field development by combining reservoir simulation with optimization techniques from Operations Research. See and Home [25] summarized this work and presented an algorithm for using linear programming, in conjunction with a series of simulation "experiments," to optimize a schedule of injection rates and producing pressures for a secondary recovery project. Authors in groundwater hydrology have also used combinations of numerical simulation and optimization techniques to determine appropriate injection rates for waste disposal projects,[26,27,28,29] as discussed in a review paper by Gorelick.[30] The success of these combined simulation/optimization approaches clearly depends on the accuracy of the numerical model of the reservoir. Unfortunately, because of fracturing and non-isothermal conditions, such models are difficult to construct for geothermal reservoirs. This is especially true early in a reservoir's productive life, before there is enough production history to perform a satisfactory history match.

Tracer testing provides a powerful tool for predicting the thermal response of a geothermal reservoir under injection. The use of tracers as a "pre-warning system" in geothermal reservoirs was advocated by Vetter.[31] Hanson and Kasameyer [32] presented an analytical method for calculating producing temperatures based on fluid residence times determined from tracer tests. Horne[33] showed that tracer data give at least a qualitative indication of a geothermal reservoir's degree of fracturing, and he suggested that, independent of any flow model for the reservoir, one could characterize the connections between injection and production wells by a "connectability map" based on tracer transit times. The correlation between rapid tracer breakthrough and a potential for premature thermal breakthrough was questioned by Pruess and Bodvarsson,[7] who showed by numerical simulation that, for certain fracture geometries, thermal breakthrough could occur first through fractures with slower tracer transit times. However, the bulk of field experience indicates that rapid thermal breakthrough and large tracer recoveries both correlate strongly with subsequent thermal breakthrough.[34,35,36] Tracer test results have been published for a number of fields, including the Wairakei and Broadlands Fields in New Zealand,[37] the Geysers Field in California,[38] the Larderello Field in Italy,[39] the Kakkonda, Onuma, Hatchobaru, and Otake Fields in Japan,[40] and the Klamath Falls Field in Oregon.[41]

In treating a geothermal field as a network of direct connections, the current study builds on Horne's idea of a connectability map. The reservoir is considered as a network of pipes, each with some physical parameter (analagous to a diameter or a Reynold's number) expressing the ease with which a tracer slug or a thermal front could pass through. To gain insight into the optimization of such a system, the literature of optimizing pipe networks was reviewed. Linear programming (LP) was found to be a commonly employed technique.[42,43,44,45,46,47] In these studies of pipe network optimization, the objective function to be minimized was typically some combination of installment costs and discounted operating costs, while the system constraints were provided by linearized flow equations, network geometry, water supply limitations, and outlet flow requirements. The decision variables in these formulations were usually the dimensions (lengths, diameters, or both) of the pipes to be installed.

Bhave[45] described an approach to the optimization of pipe networks that differed from the others in that his algorithm used two stages of LP. The first stage was based on an analogy to the transportation problem.[48] In this stage, the decision variables were the flow rates in the pipes, and the "transportation costs" associated with each pipe were expressed not in monetary terms but in terms of head loss per unit of water transmitted. In the process of minimizing these transportation costs, the first stage of Bhave's algorithm eliminated certain pipes from consideration and converted what had been a looped network to a branching network. The second stage of the algorithm then solved for the dimensions of the remaining pipes in a manner similar to that of the other optimization routines.

The current study's approach to optimizing geothermal injection also draws an analogy to the transportation problem. As will be discussed in greater detail later, the cost associated with each arc is based in part on parameters from tracer tests. Several authors have discussed method4 of inferring fracture apertures and other reservoir properties from tracer tests by applying a non-linear, least-squares method of curve-fitting to plots of produced tracer concentration versus time.[49,50,51] As pointed out by Walkup,[51] all these methods are attempts to solve the "inverse problem," *i.e.*, to infer the properties of an unknown system based on the way it changes known inputs into known outputs. The method proposed here allows the use of any reservoir properties so inferred. However, since it builds on the assumption (supported by field experience) that tracer response and thermal response are strongly correlated, it does not require a solution of the inverse problem. Rather than make inferences about what the geothermal reservoir actually is, the proposed method makes operational decisions directly based on what the reservoir actually does.

3. ALLOCATION PROBLEM

3.1 Analogy to Transportation Problem

The problem of allocating a required total injection rate among specified injection wells resembles the classical transportation problem from the field of Operations Research. To illustrate this similarity, the transportation problem will be briefly discussed. In the transportation problem, a set of factories supplies a set of stores. Each factory can only produce a certain amount of goods, and each store requires a certain amount of goods to meet demand. Goods are transported from factories to stores over various routes. This is illustrated by an idealized network of routes or arcs (Figure 1), in which nodes 1 and 2 may represent factories and nodes 3, 4, and 5 may represent stores. Each mute has associated with it a cost per unit of goods shipped. The problem is to decide how to distribute the goods from the factories to the stoles so as to minimize total transportation costs, subject to the constraints that no factory can ship more than its capacity and each store must receive at least its minimum requirement.

The LP formulation of the transportation problem for N_1 factories and N_2 stores is as follows:

$$
\begin{aligned}
&\text{Minimize} && C = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} c_{ij}\, x_{ri} && \\
&\text{Subject to} && \sum_{j=1}^{N_2} x_{ij} \le S_i, && i = 1, N_1 \\
& && \sum_{i=1}^{N_1} x_{ij} = D_j, && j = 1, N_2 \\
& && x_{ij} \ge 0, && \text{for all } i,j
\end{aligned}
\qquad (3.1)
$$

The components of this formulation include: (1) the decision variables, x_{ij}, which are the amounts of goods shipped from factory i to store j; (2) the arc costs, c_{ij}, which are the an-sportation costs per unit of goods shipped; (3) the objective function, C, which is the total transportation cost to be minimized; (4) the supply constraints, which require that the sum of goods supplied by factory *i* be less than its capacity, S_i; (5) the demand constraints, which require that the sum of goods received by store *j* be at least as great as its demand, D_j; and (6) the non-negativity constraints, which require that goods be shipped in only one direction, from factories to stores. In solving this problem by LP, the problem requirements are reduced to a set of linear equations, which are then solved simultaneously by an algorithm such as the Simplex method.[52]

Figure 1 may also be used to illustrate the analogy to the injection optimization problem. In this analogy, nodes 1 and 2 could represent injection wells, and nodes 3, 4, and 5 could represent production wells. The arcs in the network represent the potential fluid flow paths from each injector to each producer. However, these arcs do not imply anything about the actual geometry of fluid flow. Each arc has associated with it some "cost" per unit of fluid transmitted, where the cost is an expression of the increased likelihood of thermal breakthrough, as assessed based on tracer tests, field geometry, and operational considerations. The problem is to minimize the likelihood of thermal breakthrough throughout the field, while meeting constraints on the injection capacity of individual wells and satisfying total injection requirements.

The following LP formulation for the injection optimization problem illustrates the parallels with the transportation problem for the case of N_1 injectors and N_2 producers:

$$
\begin{array}{lllll}
\textbf{Minimize} & B = \displaystyle\sum_{i=1}^{N_1}\sum_{j=1}^{N_2} b_{ij} = \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} c_{ij}\, q_{ri} & & \\
\textbf{Subject to} & q_{ri} \le q_{rimax}, & i = 1, N_1 & \\
& \displaystyle\sum_{i=1}^{N_1} q_{ri} = Q_{rtot} & & (3.2)\\
& q_{ri} \ge 0, & i = 1, N_i &
\end{array}
$$

The decision variables, q_{ri}, are the reinjection rates for each injection well, *i*. The arc costs, c_{ij}, express the increased chance of thermal breakthrough resulting from movement of a unit of fluid from each injector to each producer. The product of an injection rate and an arc cost constitutes the breakthrough index, b_{ij}, for a particular arc. The summation of breakthrough indices for all arcs constitutes the fieldwide breakthrough index, B, which is the objective function to be minimized. The supply constraints for the injection optimization problem simply express the requirement that each injector has to operate at a rate less than its capacity, q_{rimax}. The demand constraint requires that the summation of all injection rates must equal the specified fieldwide total injection rate, Q_{rtot}. Finally, the non-negativity constraint ensures that none of the injectors are operating at "negative rates", *i.e.*, that they are not acting as producers.

Despite the similarity between the formulations for the transportation problem and the injection optimization problem, the two formulations differ in several respects. The most important difference is that the transportation problem solves for the amount of goods shipped across *each arc*, while the injection optimization problem solves for injection rates *at* each *injection well.* In other words, the decision variables for the transportation problem are arc-specific, while for the injection optimization problem they are well-specific. This difference stems from the fact that, unlike the manager choosing routes for shipping goods from factories to stores, a geothermal developer has no direct control over which paths fluids take in the reservoir. The developer does have *indirect* control over the paths of reinjected fluids, because these paths are influenced by the rates at which offsetting production wells are operated. Later, this study will present other formulations of the injection optimization problem which account for the mutual dependence of injection and production rates. However, in practise, the rates at which production wells are operated may be fixed by other considerations. Therefore, as a simplest case, it is reasonable to formulate the injection optimization problem with injection rates as the only decision variables.

One consequence of this is that the supply constraints for the injection optimization problem do not involve a summation of injection rates. In a sense, the injection rate into each well already represents a summation of flows on all pathways leading away from the injector in the reservoir. A second consequence is that the demand constraint in the injection optimization problem is not expressed as a summation of flows into particular production wells. Rather, the demand constraint is expressed in terms of the total injection rate that the entire reservoir "demands," in the judgement of the geothermal developer. Further, in the LP formulation presented here, the demand constraint is given as a strict equality rather than an inequality. Because the optimization routine always seeks to inject at the lowest total rate possible, this simplification does not affect the final rate allocations. Finally, unlike the transportation problem, there is no necessary correspondence between the amount of "goods" shipped out from the factories and the amount of "goods" received at the stores, *i.e.*, between the amount of fluid injected and the amount of fluid produced. The LP formulation of the injection optimization problem does not require a material balance between what is put into the reservoir and what is taken out. The production rates may influence the choice of injection rates, but the total amount of injection is determined only by what the developer specifies. In this respect, the LP formulation follows the situation in real life, in which a developer may choose to reinject only some fraction of the fluid produced or to supplement produced fluid with additional injection water from some outside source.

3.2 Definition of Arc Costs

In the injection optimization problem, the arc costs are not expressed in monetary terms, but in terms of increased likelihood of premature thermal breakthrough. The relation between known reservoir parameters and thermal breakthrough is difficult to quantify. For instance, it would be difficult to calculate the time required for a given percentage drop in the enthalpy of produced fluids without detailed knowledge of reservoir properties and operating conditions.

For the purposes of optimization, however, such detailed knowledge is not necessary. All that is required is a *relative* assessment of the "cost" of injection into different wells. For the optimization process to be valid, it is only necessary that the likelihood of thermal breakthrough be assessed on the same terms for each injector/producer pair. On this basis, one can consider any known parameter that relates injectors and producers and decide, in relative terms, whether lit has a direct or an inverse relationship with the likelihood of thermal breakthrough. This allows one to weight one's definition of arc costs according to whatever data are available or whatever factors one considers important.

In the computer algorithms prepared for this study, the weighting factors composing the arc costs have been drawn from three sources: tracer tests, field geometry, and operating conditions. Table 1 describes the various weighting factors used. Each of these factors will be discussed in the following paragraphs.

The tracer test parameters considered in this study are based on a slug-type tracer test. In this type of test, a certain quantity or "slug" of tracer is released instantaneously in an injection well. This gives rise to a characteristically spiked tracer response profile at the production wells, as illustrated in Figure 2. One parameter which may be directly interpreted from such a profile is the initial tracer response time, t_i. Intuitively, t_i should be inversely correlated with the likelihood of thermal breakthrough. That is, the longer it takes for the tracer to break through from a given injector to a given producer, the less likely it is that premature thermal breakthrough will be a problem between those two wells. Therefon, t_i enters into the calculation of arc costs as a reciprocal, as shown in Table 1.

A similar relation holds for the peak tracer response time, t_p. Of the two forms of tracer response time, t_p is usually easier to pick, because it involves a clear change from rising to falling tracer concentrations. In the case of t_i, background levels of tracer recovery may make the onset of true tracer response difficult to identify. However, if one defines an initial breakthrough concentration as some percentage of the peak concentration (*e.g.*, 10 %, as suggested by McCabe *et al.*[37]), then the identification of t_i is straightforward. In some cases, the choice of t_p may also not be clear, as when there are multiple peaks (possibly representing different flow paths within the reservoir) or when tracer concentrations climb slowly to a broad plateau. In such cases, t_p may be defined as the time to the first of multiple peaks (since this would represent the most direct flow path) or the time to the beginning of the plateau. Rapid tracer decay (as in the case of dye tracers at high temperatures) may also cause the choice of t_p to be suspect. In such cases, t_i would be an appropriate substitute for t_p as an indicator of potential for thermal breakthrough. It should be noted that the calculation of arc costs would generally not use both t_i and t_p. Depending on the quality and type of the tracer data, either one or the other would be used.

Often, studies reporting actual tracer tests have used tracer transit times to calculate an apparent tracer velocity, v_a, based on the straight-line, horizontal distance between wells. Because the actual flow paths of fluids in the reservoir are not known and are likely to be more convoluted than a simple straight line, such v_a-values represent a lower bound on the actual velocities of tracers in the reservoir. However, since experience with fractured geothermal reservoirs has shown that horizontal distance alone bears no predictable relation to the speed of either tracer or thermal response, this study has used tracer response times directly as weighting factors, without converting them into apparent velocities.

Two other weighting factors which are available from a tracer response profile are the peak tracer concentration, C_p, and the fractional tracer recovery, f. C_p is simply the concentration at time t_p. To obtain a value for f, one must first calculate the mass of tracer recovered by integrating the area under the tracer response curve and multiplying by the producing rate (assumed constant) during the tracer test. (If the producing rate was not constant during the tracer test, one may multiply each concentration measurement by its respective producing rate to produce a curve of the amount of tracer recovered per unit time, and then calculate the total tracer recovered by integrating under this curve.) The f-value is then just the mass of tracer produced divided by the mass of tracer injected. Both C_p and f are positively correlated with the likelihood of premature thermal breakthrough, and they may therefore be used directly as weighting factors in calculating arc costs.

The injection and production rates during the tracer test (q_{rt} and q_{pt}) may also be used as weighting factors. If a well which injects at a low rate during a tracer test causes a stong response at an offsetting producer, this response is a more serious indicator of potential for thermal breakthrough than a similar response caused by a well injecting at a high rate. Thus, q_{rt} is inversely correlated with the likelihood of thermal breakthrough, and it enters into the calculation of arc costs as a reciprocal. The Same logic applies in the case of q_{pt}: a well which exhibits a strong tracer response with a low q_{pt} is more likely to experience subsequent thermal breakthrough than a well with a similar tracer response but a higher q_{pt}. Another way of looking at q_{rt} and q_{pt} is to think of them as factors which normalize arc costs to account for possible differences between rates during tracer tests and rates under operating conditions. For the formulation in which the operating injection rate, q_r, is the decision variable, the inclusion of q_{rt} as a reciprocal in the arc cost means that the breakthrough index ($b = cq_r$) is proportional to the ratio q_r/q_{rt}. When this ratio is large, premature thermal breakthrough is more likely. Similarly, if the producing rate under operating conditions, q_p, is included in the arc costs (as discussed below), the ratio q_p/q_{pt} is also positively correlated with thermal breakthrough.

In the category of weighting factors from field geometry, the most accessible parameter is the horizontal distance between wells (L). As already discussed, L has no predictable relation with thermal breakthrough in the case of fractured reservoirs. However, in the case of porous-media-type reservoirs or reservoirs in which high permeability zones approximate horizontal planes (*e.g.*, at contacts between lava flows), the flow of injected fluid away from injection Cells in the reservoir may be radial. In this case, the surface area available for heat exchange from the rock to the cooler fluid grows in proportion to the square of L.[7] Therefore, the likelihood of thermal breakthrough may be considered inversely proportional to L^2, which may enter into the calculation of arc costs as a reciprocal. It should be emphasized, though, that distance between wells is not a reliable substitute for tracer test data in the common case of fractured geothermal fields.

The other accessible parameter in terms of field geometry is the difference in elevation (H) between producing and injecting zones. Tracer test data from Wairakei suggest that tracer breakthrough is much more likely in deep producing wells?' This makes physical sense, because Cooler injected fluids are more dense than reservoir fluids and would be expected to sink within the reservoir. However, H itself is not appropriate as a weighting factor, because it may be either positive or negative. To calculate a weighting factor based on H, this study has used an exponential function, because it is strictly positive and because it increases or decreaset the arc cost based on whether H is positive (producing zones below injecting zones) or negative (producing zones above injecting zones). When H is zero (producing and injecting zones at the same elevation), the exponential of H is 1, and the arc cost is unaffected. To keep the exponential term from dominating all other weighting factors, the exponent, H, has been multiplied by a scaling factor, S. Thus, elevation enters into the calculation of arc costs as the weighting factor e^{SH}. For elevation differences on the order of hundreds of meters, an S-value of 10^{-3} keeps this weighting factor in a range between 0.37 and 2.72.

In the category of weighting factors based on operating conditions, the expected producing rates of individual wells, q_p, have already been mentioned. Large producing rates increase the pressure drawdown near production wells and increase the likelihood of thermal breakthrough. Ideally, the process of optimizing injection would allow the selection of both injection and production rates. However, producing rates may be already determined by other considerations (such as total production requirements or surface facilities). In such cases, expected producing rates may be directly incorporated as weighting factors in the arc costs of the objective function.

The following equation illustrates how the various weighting factors discussed so far could be combined in calculating the breakthrough index for each arc:

$$b = c\,q = \left\{ \frac{1}{t_i}\,\frac{1}{t_p}\,C_p\,f\,L^2\,e^{SH}\,\frac{q_p}{q_{pt}}\,\frac{1}{q_{rt}} \right\} q_r \tag{3.3}$$

The expression in parentheses represents one formulation of the arc cost in expanded form. Because the different weighting factors apply in different situations, an actual optimization run would probably use only some subset of these factors. For example, as mentioned earlier, t_i and t_p Would usually not both be used. It should also be noted that the list of weighting factors is not exhaustive: other weighting factors could be included, based on the developer's knowledge of the reservoir and operating requirements. Further, although this study has applied a scaling factor only in the case of elevation differences, scaling factors could easily be applied to other arc cost components as well, depending on which factors the developer considers important.

3.3 Cdmputer Program Descriptions

3.3.1 General

This study has developed four computer programs to solve the problem of allocating injection rates among pre-chosen injectors and an additional program to solve the problem of choosing an optimal injector configuration. Table 2 lists these programs as well as supporting programs which facilitate data entry. The program for solving the configuration problem will be discussed in Section 4. Of the injection allocation programs, the first three make use of linear programming (LP), building on the analogy with the transportation problem and applying the arc costs discussed in Section 3.2. The LP solver employed by all three of these programs is called ZXOLP and is part of the IMSL library.[53] The last allocation program uses quadratic programming (QP) and employs a QP solver called QPSOL, developed by the Department of Operations Research at Stanford University.[54]

The three LP allocation programs differ principally in the way they account for the dependency of injection rates on production rates in surrounding wells. The first program (LPAL1) solves for injection rates only and allows fixed production rates (if known) to be used as weighting factors in the arc costs. The second program (LPAL2) solves simultaneously for both injection and production rates. The third program (LPAL3) also solves for both injection and production rates, but in an alternating fashion: production rates are used as weighting factors for the injector solutions and vice versa, until successive solutions match. This section will discuss these three routines in detail and compare the solutions they provide. For the purpose of this comparison, a hypothetical set of data has been constructed (Table 3) to match the the 5-well, idealized geothermal field shown in Figure 1.

3.3.2 LP Solution for Injection Rates Only

The LP formulation for LPAL1 has already been presented in Equation 3.2. To facilitate comparison with the other LP allocation models, this formulation is restated in the first section of Table 4. As was mentioned before, the LPAL1 formulation consists of a linear objective function, B (in which the decision variables, q_{ri}, are multiplied by arc costs, c_{ij}), and a set of linear constraints (which place bounds on the feasible range of q_{ri}-values and require that a specified total injection rate be achieved). Appendix A shows the steps for reducing this formulation to a Simplex tableau, which is the format required for processing by ZXOLP. The computer code for LPAL1 and its data-entry program (LPIN1) are included as Appendix

D. Figure 3 is a flow chart for the LPAL1 program. Figure 8 shows an example output from the execution of LPAL1 using the data in Table 3. This sample output may be used to illustrate several aspects of how the program works.

The program first summarizes the conditions for which the optimization was run. It prints out the number of injectors and producers, and it lists which weighting factors were specified by the user to calculate the arc costs. In the example shown, all possible weighting factors were specified in order to show the options currently available in the program. As was previously mentioned, however, it is unlikely that all these factors would be used together in an actual optimization, since they apply in different situations.

Next, the program calculates the arc costs and lists them for each injector/producer pair. These arc costs are simply the product of all the specified weighting factors. The arcs are identified using well names provided by the user. For each injector, the program then calculates a *cost coefficient*, which is a sum of arc costs for all the arcs connecting that injector to producing wells. The program includes a routine to scale the cost coefficients up or down, to insure that they fall within numerical bounds which can be handled by ZX0LP without excessive rounding error. If a scaling factor is applied (as in the example shown), the program prints what this factor is.

The origin of the term "cost coefficient" is illustrated by the following equation:

$$B = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} c_{ij} q_{ri} = \sum_{i=1}^{N_1} \left[c_{i1} + c_{i2} + \cdots + c_{iN_2} \right] q_{ri} \tag{3.4}$$

This shows that the double summation in the original objective function may be broken down into a single summation of all injection rates, each with a coefficient which is itself a sum of individual arc costs. These cost coefficients play a central role in the LP optimization because they provide a ranking of the decision variables in the objective function. It is according to this ranking that the program makes its rate allocation: the injection rates with the highest cost coefficients are made as small as possible. In another sense, the cost coefficients provide a ranking of the injection wells themselves: those with the the highest cost coefficients are the most damaging to operate and have the greatest potential for causing premature thermal breakthrough.

The program next summarizes both the rate information provided by the user and the rate allocations calculated by the LP solver. If producing rates under operating conditions have been specified, the program tabulates these and sums them to determine a fieldwide production rate, which is printed along with the required fieldwide injection rate. The injection rates allocated to individual wells are tabulated in two columns representing two phases of the LP solution. In Phase I, the program simply identifies a feasible solution set, *i.e.*, a set that satisfies all the constraints. In the example, Phase I of the program assigns rates of 90 and 50 kg/s to Wells No. 1 and No. 2, respectively, which satisfies the requirement that the total rate be 140 kg/s and keeps each well at or below its maximum allowable rate. If the fieldwide injection requirement had been greater than the sum of maximum allowable rates, the program would lhave terminated with an error message. In Phase XI, the program proceeds from the feasible solution to an optimal solution, based on the cost coefficients. In the example, Phase I1 reverses the rate assignments of Wells No. 1 and No. 2 because Well No. 1 has a much larger cost coefficient. (The contrast between the two cost coefficients is somewhat exaggerated in this example because of the compounded effect of using all weighting factors.) The tabulation of assigned rates also includes a listing of each well's *slack*, *i.e.*, the difference between its assigned rate and its maximum allowable rate.

The last portion of the output lists objective function values for the different phases of the LP solution. If a feasible solution exists, the Phase I objective function should equal the sum df the right-hand sides (RHS's) of the original constraints (see Appendix A). In the example, these constraints are the maximum injection rates for both wells (90 kg/s each) and the fieldwide injection requirement of 140 kg/s, all of which sum to the output value of 320. The Phase I and Phase II values of the fieldwide breakthrough index (B) illustrate the improvement achieved in going from the initial feasible solution to the final optimal solution. The absolute value of B is not significant and will depend on the weighting factors used. What is significant is that the Phase II value is less than the Phase I value. Further, for a given set of weighting factors, the value of B sums up in a single number the potential for thermal breakthrough of the entire field and thus permits a comparison in relative terms with other injector configurations.

Now that the use of weighting factors has been demonstrated for a simple LP algorithm, several additional points should be made about them. First, weighting factors may be either *arc–specific* or well–specific. Arc-specific factors (such as those based on tracer response, distance, or elevation change) describe the connections between injectors and producers, while well-specific factors (such as injecting or producing rates) describe parameters that apply only at individual wells. As discussed in Section 3.2, injection or production rates may be regarded as normalizing factors in the calculation of arc costs. However, since the optimization process is based on assigning different costs to individual arcs, using well-specific factors alone does not allow the program to make a meaningful allocation of injection rates. For instance, if producing rates under operating conditions were the only weighting factors used, then all arcs into a given producer would be the same. Further, since each injector is connected to every producer, the cost coefficients for all injectors would be identical, and no ranking of decision variables could take place. To prevent this occurring, all the LP allocation programs include data-checking routines that cause execution to terminate if no arc-specific weighting factors are applied.

Another point regarding weighting factors concerns the use of sparse data sets. Tracer test parameters may be unknown for several injector/producer pairs, either because no tracer is recovered for certain arcs or because certain wells are not tested or monitored. In the case of no recovery, the parameters which are directly proportional to thermal breakthrough (*e.g.*, C_p or f) may be entered as zeros, which causes the corresponding arc cost to be calculated as zero and effectively tells the program that thermal breakthrough along this arc is not possible. However, parameters which are inversely related to thermal breakthrough (*e.g.*, t_i or t_p) may *not* be entered as zeros, because this will lead to division by zero in the calculation of the arc cost. In this study, inversely related parameters have been entered as arbitrarily large numbers, which causes the arc cost to be negligibly small. In the case where certain producing well$ were simply not monitored (or where no tracer tests were conducted for certain injectors), the missing tracer test parameters must be entered in analogous fashion, depending on whether they are directly or inversely related to thermal breakthrough. Unfortunately, the allocation program has no way of distinguishing between the cases of no response and missing information. Thus, the effect of sparse data is to favor those injector/producer pairs about which least is known. On the other hand, if the field being optimized is close to its *total* capacity, the allocation program will attempt to *shut* in the worst wells based on whatever actual data are available. A sparse data set may cause ambiguity at lower fieldwide rates, but the allocation routine has the virtue of using all available data first.

3 3 3 Simultaneous LP Solution for Injection and Production Rates

Because the likelihood of thermal breakthrough depends both on injection rates and producing rates, a second LP allocation routine (LPAL2) has been developed which allows the user to solve simultaneously for both types of rates as part of the injection optimization process. The second section of Table 4 presents the LP formulation for LPAL2. The reduction of this formulation to a Simplex tableau is shown in Appendix B, and the code for LPAL2 and its data-entry program (LPIN2) are included in Appendix E. A flow chart of the program is shown in Figure 4.

A comparison of the LP formulations for LPAL1 and LPAL2 in Table 4 illustrates several significant differences between the two programs. First, LPAL2 includes both injection and production rates as decision variables in the objective function by summing the two rates for each injector/producer pair and multiplying each sum by its respective arc cost This approach insures that the objective function remains linear, because no two decision variables are combined as cross-products.

A second difference between LPAL2 and LPAL1 is that the number of constraints is expanded. Not only must each well operate below its maximum capacity, but it must also maintain some specified minimum operating rate *greater than zero*. This is necessary because only active arcs should be allowed to contribute to the "cost" of injection, as measured by the objective function value, B. If either q_{ri} or q_{pj} were permitted to go to zero for a particular arc, that arc would still be making a contribution to the objective function, even though it was inactive. (It should be noted that LPAL1 avoided this problem entirely by defining N_2 in the LP formulation as the number of *active* producers.) Because each well has a positive minimum rate, the non-negativity constraint is not necessary for the LPAL2 formulation.

Finally, just as the sum of all injection rates must meet some specified fieldwide injection rate, so must the production rates sum to some fieldwide total. It should be pointed out, however, that the problem formulation still does not require that these two total rates (Q_{rtot} and Q_{ptot}) be the same, *i.e.*, there is no requirement of material balance within the reservoir.

Figure 9 presents a sample output from the program LPAL2 using the data from Table 3. This output is similar in format to the output from LPAL1 (Figure 8), but there are several important differences. First, because producing rates under operating conditions are now decision variables, they have been removed from the list of potential weighting factors. (As before, this run invoked all possible weighting factors, to illustrate which were available.)

Second, cost coefficients are now listed for both injection and producing wells. Third, the output value of fieldwide required production rate is no longer calculated as a sum of individual producing rates but represents a fieldwide constraint specified by the user. Fourth, the tabulations of assigned injection and production rates include columns for both maximum and minimum specified rates for each well. In addition, the tables show not only each well's slack (the difference between its assigned rate and its maximum) but also its surplus (the difference between its assigned rate and its minimum). For both the injectors and the producers, note that Phase II of the program again assigns the lowest possible rates to the wells with the highest cost coefficients. As before, the Phase I objective function represents the sum of the RHS's of the original constraints, and the change in B from Phase I to Phase II shows that the likelihood of thermal breakthough has been decreased by optimizing the rate allocation.

33.4 Alternating LP Solution for Injection and Production Rates

Because LPAL2 quires that a non-zero, minimium rate be specified for all wells, it effectively guarantees that all wells considered will be active in the final solution. However, from an operational point of view, it is probably more desirable to be able to shut in problem wells entirely. In order to allow this possibility while still solving for both injection and production rates, a third LP allocating program (LPAL3) has been devised. This program applies the same basic algorithm as LPAL1 in what may be called an "alternating" approach. That is, it uses allocated production rates from one iteration as weighting factors in the allocation of injection rates in the next iteration, and vice versa. The iterations continue until convergence is achieved, *i.e.*, until successive rate allocations match. The LP formulation for LPAL3 is presented in the third section of Table 4, and the flow chart in Figure 5 illustrates the structure of the algorithm. On first appearances, it is not clear that this algorithm will necessarily converge. However, extensive sensitivity testing has shown that the LPAL3 algorithm is, in fact, quite stable. Convergence is usually achieved in three iterations, and four iterations (with a data set deliberately intended to cause cycling) is the maximum number that has been obsewed. The computer codes for LPAL3 and its data-entry program (LPIN3) are included in Appendix F.

Again, a sample of program output (Figure 10) helps illustrate how the program works. The same hypothetical data set is used as in the previous two examples (Table 3), and all available weighting factors are invoked. In this example, LPAL3 achieves convergence in three iterations, and the output basically looks like a sequence of three LPAL1 runs. The summary of input data at the start of the output includes one new item: the maximum allowable number of iterations specified by the user. This is a safeguard against cycling, although in practise cycling has not proved to be a problem. The iterations are labeled as to whether injection or production rates are being determined. For the first iteration, maximum production rates (q_{pjmax}) are used as weighting factors in solving for injection rates, since no previous production rate solutions are available. As before, the first injector iteration identifies Well No. 1 as the most damaging and assigns it the lowest possible rate (consistent with meeting fieldwide requirements). In contrast to the LPAL2 results, however, the second iteration (solving for producers) shuts in Well No. 3 entirely and apportions the required total production rate among the remaining two producers. The impact of this allocation is seen in the arc costs for the third iteration: since Well No. 3 is now inactive, the inclusion of its zero rate as a weighting factor causes arcs 1-3 and 2-3 to have a zero cost. This in turn has a strong impact on the cost coefficients for Wells No. 1 and 2: although No. 1 is still slightly more prone to cause thermal breakthrough, the two wells are much closer to being equivalent in this regard. In the final injection rate allocation, the initial feasible solution identified in Phase I happens to be the optimal solution, so the values of the fieldwide breakthrough index are the same for Phases I and II. The final assigned rates also happen to be the same as in the first iteration, so no further iterations are executed.

335 QP Solution for Injection and Production Rates

The LPAL3 program just described represents an attempt to account for the mutual dependence of injection and production rates by alternately exchanging their roles as decision variables and weighting factors. The motive behind this rather elaborate iterative process is to maintain the linearity of the objective function, thus allowing comparatively simple LP solvers (such as Simplex) to be applied. Another approach, however, is to explicitly acknowledge the interdependence of injection and production rates by treating both as decision variables and including them in the objective function as a product (rather than as a sum, as in LPAL2). The objective function then becomes a quadratic, and the solution of the problem entails the use of a quadratic programming (QP) solver.

The last of the allocation programs developed in this study takes this appmach. It is called QPAL, and, as mentioned earlier, it uses a QP solver named QPSOL. The theory behind QPSOL's method of solution is presented in the User's Manual[54] and will not be discussed here. The final section of Table 4 shows the QP formulation of the allocation problem, and Appendix C presents the steps necessary to put the problem in a format that QPSOL can handle. A flow chart for QPAL is shown in Figure 6, and computer codes for QPAL and its data-entry program (QPIN) are included in Appendix G.

A sample of output from QPAL using the data from Table 3 is presented in Figure 11. The most significant departure from the outputs of the LP programs is that the cost coefficients are absent. Instead of being combined as coefficients in a linear objective function, the arc costs are fed to QPSOL as elements of a Hessian matrix of second derivatives, as shown in Appendix C. Thus, QPAL does not provide an explicit ranking of wells in terms of their potential to cause thermal breakthrough.

On the other hand, it should be noted that the actual allocations provided by QPAL exactly duplicate those of LPAL3. Numerous runs with both programs on a variety of data sets have shown that this is almost always the case. The only exceptions are cases in which the cost coefficients shown by LPALS are exactly equal for two or more wells. In such cases, the allocations provided by LPAL3 appear to be a function of the order of the constraints in the Simplex tableau, while the allocations provided by QPAL are essentially random. Fortunately, exact equality of cost coefficients is rare for actual data sets, except in degenerate cases (*e.g.*, when the fieldwide required production rate is set to zero) or in cases where data sets are sparse (*e.g.*, when there are only enough tracer data to characterize a few of the arcs). In fact, a comparison of the problem formulations for LPALS and QPAL in Table 4 shows that the two formulations are equivalent once LPAL3 has converged, because the cost coefficients for each iteration of LPAL3 have the other set of decision variables embedded within them. Thus, QPAL really provides verification of the alternating LP method, since it arrives at the same answers by a totally different solution method.

Although QPAL does not yield an explicit ranking of wells by cost coefficients, it does have one advantage over LPAL3, in that it provides an assessment of the quality of each solution. * the example shown, the output describes the final rate allocation as the "optimal QP solution." If the problem had been a case in which LPAL3 would calculate equal cost coefficients for more than one well, then QPAL would have issued a warning, labeling the solution as a "weak local minimum." For the same case, LPAL3 would have proceeded to make an arbitrary allocation of rates, and, unless the user happened to notice that several of the cost coefficients were the same, this allocation might mistakenly be considered bona fide. This becomes more of a problem as more wells are considered and the list of cost coefficients grows longer. However, as already mentioned, the occurrence of exactly equal cost coefficients is rather rare for actual cases.

3.3.6 Comparison of Allocation Programs

The preceding sections have compared the four allocation programs developed for this study with reference to a particular data set. A general summary of the similarities and differences between the programs is provided in Table 5. As the table shows, all the LP programs operate by providing an explicit ranking of wells based on cost coefficients and throttling back one well at a time, from most to least damaging, until total rate requirements are just met. QPAL does not provide an explicit ranking of wells, but it uses a quadratic programming solver which generally yields the same rate allocations as LPAL3. All the programs except LPAL1 allocate both injection and production rates. All the programs except LPAL2 allow wells to be shut in, *i.e.*, to be assigned a zero rate. The well rankings provided by LPAL1 and LPAL2 do not vary with changes in total operating rates, because the arc costs used in calculating cost coefficients are all fixed. Effectively, this means that neither LPAL1 nor LPAL2 can take into account the mutual dependence of injection and production rates in determining the likelihood of thermal breakthrough. For this reason, LPAL3 is the most realistic of the LP programs presented. (Note that if producing well rates are predetermined, LPAL3 can be used to generate the same injection allocation as LPAL1 by simply setting the required total producing rate equal to the sum of the known well rates.) QPAL also accounts for the mutual dependence of injection and production rates and is the only program presented which explicitly assesses the quality of the solution by identifying indeterminate cases.

4. CONFIGURATION PROBLEM

4.1 Enumeration Approach

The computer programs discussed so far have addressed the problem of how to allocate a specified total injection rate among a pre-chosen set of injection wells. The configuration problem concerns how to choose this set of injectors from a group of pre-existing wells. The solution of the allocation problem provides a straightforward approach to the configuration problem, The end result of the allocation routines is not only a set of injection rates but a minimized value of the fieldwide breakthrough index, B. For a particular configuration, this value expresses in a single number the likelihood of premature thermal breakthrough for *the* entire field under optimal loading. Therefore, the configuration problem may be approached by enumeration, *i.e.*, by applying an allocation algorithm to each possible injector configuration and selecting the configuration with the lowest minimized B-value as optimal.

The theoretical upper limit on the number of configurations which an enumerative approach would have to consider is given by the expression

$$\frac{N!}{N_1!\,N_2!}$$

where N = total number of wells

N_1 = number of injectors

N_2 = number of producers

For number of wells in a typical geothermal field, this value is small enough that consideration of all configurations would not require excessive computer time. Moreover, certain configurations could usually be removed from consideration because the *sum* of maximum rates for the wells involved would be insufficient to meet the total rates required. Thus, *the* actual number of configurations for which the allocation routine would need to be run would usually be less than the theoretical maximum.

It should be noted that the data requirements for such a configuration-choosing routine are much more extensive than for the allocation routines previously discussed. The data must characterize not just the arcs between designated injector/producer pairs (as in Figure 1) but between all well pairs (as in Figure 13). For directional information, data should be supplied in both directions. A complete set of tracer data, for example, requires that a separate tracer test be conducted on each well and that tracer response be monitored in all other wells. Further, rate limitations must be specified for each well both as an injector and as a producer.

4.2 Computer Program Description

This study has developed a configuration-choosing program called INCON, which uses the enumerative approach in conjunction with the QP allocation algorithm. The computer codes for INCON and its associated data-entry program (CONDAT) are included in Appendix H. The input parameters for INCON include the total number of wells, the maximum allowable number of injectors, and the required fieldwide production and injection rates. For each possible injector configuration, the program checks to insure that the required fieldwide rates can be met, It then runs the QP allocation algorithm on each feasible configuration, and selects the configuration with the lowest B-value. A flow chart for INCON is presented in Figure 7.

To illustrate how INCON works, the hypothetical data set in Table 3 has been incorporated into an expanded data set (Tables 6 through 14) which matches the idealized field network shown in Figure 13. This expanded data set has been deliberately constructed so that the longest arc (1-3) exhibits the strongest tracer response by all measures. The responses for the other arcs have been chosen to be more ambiguous, *i.e.*, strong by some measures and weak by others. This permits a demonstration of the sensitivity of the program to the choice of different weighting factors. The initial and *peak* tracer response times have been chosen to exhibit similar relative weightings for the various arcs, though of course the *peak* times are always greater than the corresponding initial times.

A sample output from INCON is shown in Figure 12. The program prints out the total number of wells and the maximum number of injectors specified by the user. It also prints out the total production and injection rates required, as well as a list of the weighting factors invoked. The program then lists the calculated arc *costs.* In the example, note that the arc cost for arc 1-3 is larger than the others by several orders of magnitude. *Also* note that the arcs with no tracer response (3-1 and 5-3) have arc *costs* of zero, *i.e.*, the program considers premature thermal breakthrough along these arcs to be impossible. Finally, the program prints out a set of allocated injection and production rates for the optimal configuration, as well as the minimized B-value associated with *this* configuration. If the same B-value is obtained for more than one configuration, the program gives the number of equivalent configurations and lists the optimal rate allocations for each.

Table 15 illustrates the sensitivity of INCON to the selection of different weighting factors using the expanded hypothetical data set. In a succession of runs, each of the four tracer test parameters was used individually, followed by a run in which all four parameters were used together. Each run required a maximum of two injectors. The maximum individual well rates for injectors and producers were 90 and 75 kg/s, respectively, and fieldwide injection and production rates were both set at 140 kg/s. Both the initial and the *peak* arrival times yielded the same optimal configuration and the same rate allocations. This reflects the parallelism that was built into the data set for these two factors. However, the *peak* tracer concentration and the fractional recovery each yielded different configurations. Interestingly, the configuration yielded by the combination of all four factors was different from that of any of the factors individually. A final point to note is that all configurations avoided the "bad arc," *i.e.*, the combination of Well No. 1 as an injector and Well No. 3 as a producer.

5. APPLICATIONS TO WAIRAKEI

5.1 Background

The Wairakei Geothermal Field is a liquid-dominated field located near the town o Taupo on the North Island of New Zealand. A series of tracer tests were conducted in the field in 1979 and 1980. These tests took advantage of a downflow of cooler fluid in several wells from a zone above the reservoir. The tests were intended to determine where this cooler fluid was going and whether production from offsetting wells was being adversely affected. Glass vials of radioactive tracer (Iodine-131) were lowered into the downflowing wells and broken below the point of cool fluid entry. producing wells were monitored continuously for tracer response. Measured tracer concentrations were normalized by dividing them by the amount of tracer injected to account for variation in the size of the tracer slugs in the different tests. McCabe et *al.* provide a detailed description of the testing procedures and results.[37]

The Wairakei tracer tests are a classic example of fracture-controlled flow in a geothermal reservoir. Figure 14 shows the results of three tests involving injection into WK-107 March, 1979), WK-101 (June, 1979), and WK-80 (February, 1980). The solid lines indicate a fractional recovery greater than 1.0%, while the longdashed and short-dashed lines indicate fractional recoveries of 0.1-1.0% and less than 0.1%, respectively. In several instances, wells which were further away from the tracer injection wells exhibited stronger response than closer wells. Because the Wairakei Field illustrates so well the notion of a geothermal reservoir as a network of direct connections between wells, and because it has several sets of tracer data to quantify these connections, it is an ideal test case to demonstrate the use of the injection optimization programs presented in this study.

Tables 16 through 23 summarize the Wairakei data that have been used for the allocation programs. The tracer test parameters (t_i, t_p, C_p, and f) are those reported by

McCabe *et al.*[37] Production rates during the test (q_{pt}) were estimated from actual production rates as of December, 1976, as reported by Pritchett *et al.*[55] Injection rates during the tests (q_{rt}) for Wells WK-101 and WK-107 are those reported by Bixley.[56] Since no q_{rt}-value was available for Well WK-80, a value of 50 kg/s was estimated. These values of q_{pt} and q_{rt} were also used as the maximum well capacities (q_{rmax} and q_{pmax}). Values of the horizontal distance (L) between wells were determined by measurement from Figure 14. To calculate values of the elevation change (H) between producing and injecting zones, the following assumptions were made: (1) The depth of the injection zone was taken as the depth at which the tracer was released. (2) The depth of the production zone was taken as the depth of the lowest fissure indicated on drill logs,[57] or, in the absence of reported fissures or logs, as the midpoint of the open interval.[55] (3) For Well WK-121, the elevation of the uphole perforations at 975 m (-532 m sub-sea) was used, since this was reported to be the primary source of production.[37] A list of reservoir zone elevations (with reference to mean sea level) based on these assumptions is presented in Table 24.

For the configuration-choosing program, the maximum well capacities were assumed to apply for all wells both as injectors and as producers. A computer program (HGEN) was written to generate a set of elevation changes for a complete set of arcs using the injection and production zone elevations just described. A second computer program (LGEN) was written to calculate a complete set of horizontal distances from the surface well coordinates reported by Pritchett.[55] These calculated L-values differed only slightly from the measured L-values used with the allocation programs. Codes for these two programs are included in Appendix I.

5.2 Optimal Rate Allocations

5.2.1 Sensitivity to Total Rate

All four allocation programs were run on the Wairakei data to investigate how varying total injection and production rates would affect the optimal rate allocation. The wells involved in the tracer tests included the three wells with cool fluid downflow (the "injection" wells) and nineteen producing wells. The fieldwide capacities for injection and production were 140 and 689 kg/s, respectively, based on the sum of individual well capacities (Tables 20 and 21). Sensitivity studies were run using a single weighting factor ($1/t_i$) to calculate cost coefficients. These sensitivity studies entailed fixing one of the total rates (either Q_{rtot} or Q_{ptot}) at a value below total capacity and varying the other from total capacity to a low rate.

For all the sensitivity studies, LPAL1 and LPAL2 established well rankings which were invariate with total rates. This was as expected for these two programs, because neither of them incorporates variable well rates into their calculations of cost coefficients. LPAL1 ranked WK-107 as the most prone to thermal breakthrough, followed by WK-101 and WK-80. LPAL2 ranked the injectors the same way and also provided a ranking of the producers. This ranking is presented in Table 25. As total rates were cut back in the sensitivity studies, these programs throttled back one well at a time in the order of the predetermined rankings. In contrast, the ranking of wells by LPAL3 varied with total rates, and the rate allocations for one category of wells (producers or injectors) depended on the rates of wells in the other category. Further, the rate allocations from QPAL generally agreed with those from LPAL3, as expected. These points are illustrated by the following three sensitivity studies.

In the first sensitivity study, Q_{ptot} was fixed at 550 kg/s, and Q was varied from a capacity rate of 140 kgls to 50 kg/s. Table 26 shows the sequence in which LPAL3 and QPAL shut wells in. (This table and all subsequent tables of sensitivity data present ranking for all three injectors; however, for the sake of brevity, the only producers listed are those with curtailed rates.) Several points are worth noting from Table 26. First, the cost coefficients of the producers shift continuously as the injection rate drops. Second, for marginal changes in the injection rate, the relative ranking of the producers stays the same. For example, as Q_{rtot} goes from 140 to 100 kg/s, the producers maintain their relative ranking and their allocated rates. As long as the producing rate allocations remain unchanged, the cost coefficients for the injectors also stay the same. However, when Q_{rtot} drops to the point that WK-107 is shut in entirely (90 kg/s), the ranking of the producers shift, which changes their allocated rates and alters the cost coefficients of the injectors. Table 26 also illustrates that the allocations by LPAL3 and QPAL generally agree. This agreement breaks down when Q_{rtot} is reduced to 50 kg/s, because the ranking of producers becomes indeterminate. LPAL3 chooses to curtail WK-121, but this choice is arbitrary because all remaining producers have the same cost coefficient (0.005). QPAL curtails a different set of producers, but labels the solution as non-unique. It should be noted, however, that even with a sparse data set, the problem of indeterminacy does not occur until only one injector remains active. This illustrates that LPAL3 and QPAL make use of all available data first in deciding which wells to cut back.

The second sensitivity study fixed Q_{rtot} at 100 kg/s and decreased Q_{ptot} from a capacity rate of 689 kg/s to 400 kg/s. Table 27 presents a summary of this sensitivity study. In this case, the cost coefficients for the injectors shift continuously as Q_{ptot} is reduced. However, because WK-107 is ranked as the most damaging injector at all levels of Q_{ptot}, the injection rate allocation stays the same, and the cost coefficients for the producers stay the same as well. As Q_{ptot} is reduced, the producers are throttled back one at a time, according to rank. The allocations by QPAL agree with those by LPAL3, as before. At a Q_{ptot} of 400 kg/s, an indeterminate condition is reached when LPAL3 elects arbitrarily to curtail WK-68, which has the same cost coefficient as WK-70. QPAL makes the same allocation, but labels it as non-unique.

In the third sensitivity study, Q_{itot} was fixed at a lower rate of 70 kg/s, and Q_{ptot} was again reduced gradually from an initial rate of 689 kg/s. The results of this sensitivity study are presented in Table 28. The first point to note is that the ranking of producers differs from that of the previous sensitivity study because the injection allocation has changed (*i.e.*, WK-107 has been shut in). As additional producing wells are shut in, the ranking of the injectors shifts so that WK-80 rather than WK-101 is curtailed. This causes a corresponding realignment of the producing wells. At a Q_{ptot} of 500 kg/s, the problem becomes doubly indeterminate, first because the two remaining injectors have the same cost coefficient (0.05). and second because all the producers after WK-108 also have identical cost coefficients (0.007). For this indeterminate case, LPAL3 and QPAL make different allocations in both the injector and the producer categories.

In *summary,* the three sensitivity cases showed that LPAL3 and QPAL could optimize injection for a fixed injector configuration in a way that accounted for the interdependence of injection and production rates. The rate allocations provided by LPAL1 and LPAL2 were less satisfying because they were based on a fixed well ranking. The ranking provided by LPAL3 depended on total rates, though for marginal changes in total rates the relative ranking remained the same. The rate allocations of LPAL3 and QPAL agreed in all cases except when *the* optimal allocation was indeterminate.

5.23 Sensitivity to Different Weighting Factors

To investigate the impact of different weighting factors on the choice of an optimal rate allocation, a series of runs was performed with both LPAL3 and QPAL. In these runs, Q_{itot} and Q_{ptot} were fixed at 100 and 500 kg/s, respectively. The various weighting factors were applied first individually, then in various combinations. The results from these runs are summarized in Table 29.

The following points are worth noting. First, the tracer test parameters (t_i, t_p, C_p, and f), when used individually (Runs 1-4), all tended to shut in the Same wells, though not always in exactly the same order. All the tracer test parameters provided the same ranking for injectors. Among the producers, four wells (WK-24, WK-48, WK-116, and WK-121) were usually the first to be shut in. (The only exception was in the run using f alone, which ranked WK-76 above WK-48). In addition, several combinations of the tracer test parameters (Runs 8-10) caused the same wells to be shut in as when the parameters were used individually. In these runs, the four "problem" producers were always the first to be shut in, and always in the same order.

A second point is that using the elevation parameter alone (e^{SH}) yielded a rate allocation which was quite similar to the allocations from tracer test parameters. In this run (No.5), the injectors received the same ranking as before, and three of the four "problem" producers were among the first four producers to be shut in. Further, using e^{SH} in combination with the tracer test parameters (Run No. 11) duplicated the rankings of the previous tracer parameter combinations. In contrast, using the horizontal distance parameter alone ($1/L^2$) yielded a totally different rate allocation. In this run (No. 6), the ranking of the second and third injectors was reversed, and only one of the four "problem" producers (WK-48) was even partially cut back.

A final point concerns the use of the reciprocal of q_{pt} as the only weighting factor. Since there was not enough information to distinguish different producing rates during the three tracer tests, the producing rates were considered the same for all tests. In effect, this made $1/q_{pt}$ a well-specific weighting factor. Run No. 7 showed that the impact of using such a weighting factor alone was to cause all wells in the opposite category (*i.e.*, the injectors) to have identical cost coefficients, thus making the problem indeterminate. As in previous indeterminate cases, LPAL3 and **QPAL** provided different injection allocations.

In summary, the sequence of runs applying a variety of different weighting factors to Wairakei data showed that tracer test parameters tended to yield similar rate allocations, whether used singly or in combinations. Further, elevation changes alone could be used to calculate allocations which were similar to those from tracer test parameters. On the other hand, using just the horizontal distance between wells yielded totally different rate allocations. This suggests that, for fractured reservoirs such as Wairakei, elevation changes are much more important than horizontal distances in determining optimal injection allocations.

5.3 Optimal Configuration

To demonstrate the application of the configuration-choosing program **(INCON)** to the Wairakei Field, two runs were made, the first based on elevation changes between producing and injecting zones (H-based), the second based on horizontal distances between wells (*L*-based). These two data sets were selected because, despite all the tracer data available for Wairakei, H and L were the only parameters that could provide a characterization for each arc. The runs assumed that Q_{ptot} and Q_{rtot} were to be *550* and 100 kg/s, respectively. The maximum number of injectors was specified as three. With a total of 22 wells, this meant that the maximum number of configurations to be considered was **1,540.** However, because not all the combinations of wells could achieve the required total rates, the number of combinations for which **INCON** actually performed a rate allocation was only 1,122. Each execution of the program with these data sets required about 50 minutes of real time using a **DEC** VAX **11/750** computer.

The optimal injector configurations for the H-based and *L*-based cases are shown in Figures **15** and 16, respectively. The rate allocations associated with these two configurations are listed in Tables 30 and 31. As would be expected, the two configurations are quite different. The H-based configuration places injection in deep wells near the center of the field, while the L-based configuration places injection in isolated wells at the field's southeast comer. Based on the parallels between H-based and tracer-based rate allocations discussed in the previous section, it might be reasonable to expect that the H-based injector configuration would be better in practise. However, it should be noted that the final H-based configuration depend$ not just on elevations but on rate constraints. If **INCON** had optimized on the basis of elevation alone, it would have simply chosen the three deepest wells (WK-48, WK-121, and WK-18) as injectors. Because these wells could not collectively produce 100 kg/s, the program designated WK-121 and WK-18 as inactive producers and chose the next two deepest wells (WK-24 and WK-55) as injectors instead. The combined maximum rates of these three injectors happened to be exactly 100 kg/s. It is clear, though, that slight changes in either the estimated capacities for individual wells or the required total rate could cause the optimal H-based configuration to change significantly.

6. CONCLUSIONS

6.1 The optimization of injection scheduling in geothermal fields may be accomplished by working in relative terms with data directly available from tracer tests, field geometry, and operating considerations.

6.2 Linear and quadratic programming may be used to allocate a specified total injection rate among pre-chosen wells. Such methods should allow for the interdependence of injection and production rates in determining the likelihood of thermal breakthrough.

6.3 The optimization techniques described in this study make use of all available data first in deciding which wells to eliminate as injectors. The techniques are not a substitute for efforts to understand reservoir behavior in a more physical sense, but they allow a geothermal developer to make beneficial use of whatever tracer return data are available.

6.4 For the Wairakei Geothermal Field, several different combinations of tracer test data yield the same allocations of injection and production rates. This suggests that the design of an optimal injection strategy does not depend critically on fine details of tracer response.

6.5 For fractured reservoirs such as Wairakei, elevation differences between production and injection zones are much more important than horizontal distances between wells in determining the optimal allocation of injection rates. The fact that large elevation differences tend to correlate with strong tracer response supports the theory that reinjected water moves rapidly downward within the reservoir.

6.6 The choice of an optimal injector configuration may be made by enumerating all feasible configurations, optimizing the rate allocation for each, and selecting the configuration with the lowest potential for premature thermal breakthrough. However, the solutions provided by such an approach are very dependent on specified rate constraints.

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TABLES

FACTOR	DEFINITION	RELATION TO ARC COST		
t_i	Initial tracer amval time	c	a	$1/t_i$
t_p	Peak tracer arrival time	c	a	$1/t_i$
C_p	Peak tracer concentration	c	a	C_p
f	Fractional tracer recovery	c	a	f
q_{pt}	Producing rate during tracer test	c	a	$1/q_{pt}$
q_{rt}	Injection rate during tracer test	c	a	$1/q_{rt}$
q_p	Producing rate under operating conditions	c	a	q_p
L	Horizontal distance between wells	c	a	L^2
H	Elevation change from producing zone to injection zone (**S** = scaling factor)	c	a	e^{SH}

Table 1. Weighting factors for arc costs.

PROGRAM NAME	APPLICATION	DATA **ENTRY** PROGRAM
LPAL1	Linear programming allocation for injection rates **only**	LPIN1
LPAL2	Simultaneous linear programming allocation for **both** injection and production rates	LPIN2
LPAL3	Alternating linear programming allocation for **both** injection **and** production rates	LPIN3
QPAL	Quadratic programming allocation for **both** injection and production rates	QPIN
INCON	Injector configuration chooser	CONDAT

Table 2. Summary of computer programs to optimize injection

INJECTION WELL	WEIGHTING FACTOR	PRODUCERS		
		3	4	5
1	t_i	0.5	4.0	4.5
	t_p	1.0	10.0	15.0
	C_p	5000	50	35
	f	0.035	0.001	0.002
	q_{pt}	25	50	35
	q_{rt}	55	55	55
	q_p	70	30	40
	L	140	115	50
	H	+500	+250	+300
2	t_i	2.0	5.0	3.0
	t_p	7.0	9.0	12.0
	C_p	30	40	25
	f	0.012	0.005	0.003
	q_{pt}	30	40	45
	q_{rt}	60	60	60
	q_p	70	30	40
	L	75	90	95
	H	+50	-200	-150

Table 3. Hypothetical data for injection allocation programs.

Table 4
Summary of Injection Allocation Models

Definition of Variables

B = fieldwide breakthrough index
N_1 = number of injectors
N_2 = number of producers
i = subscript for injection wells
j = subscript for producing wells
c_{ij} = weighting factor between wells i and j
q_{ri} = injection rate into well i
q_{pj} = producing rate from well j
q_{rimax} = maximum permissible injection rate into well i
q_{rimin} = minimum permissible injection rate into well i
q_{pjmax} = maximum permissible producing rate from well j
q_{pjmin} = minimum permissible producing rate from well j
Q_{rtot} = total required injection rate
Q_{ptot} = total required producing rate

Linear Programming (LP)

1. Model to allocate injection rates only (LPAL1):

Minimize
$$B = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} c_{ij}\, q_{ri}$$

Subject to
$$q_{ri} \le q_{rimax}, \qquad i = 1, N_1$$
$$\sum q_{ri} = Q_{rtot}$$
$$q_{ri} \ge 0, \qquad i = 1, N_1$$

2. Model to allocate both injection and production rates - simultaneous solution (LPAL2):

Minimize
$$B = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} c_{ij}\,(q_{ri} + q_{pj})$$

Subject to
$$q_{ri} \le q_{rimax}, \qquad i = 1, N_1$$
$$q_{ri} \ge q_{rimin}, \qquad i = 1, N_1$$
$$\sum q_{ri} = Q_{rtot}$$
$$q_{pj} \le q_{pjmax}, \qquad j = 1, N_2$$
$$q_{pj} \ge q_{pjmin}, \qquad j = 1, N_2$$
$$\sum q_{pj} = Q_{ptot}$$

Table 4 (Cont.)
Summary of Injection Allocation Models

3. Model to allocate both injection and production rates - alternating solution (LPAL3):

A, Minimize

$$B_1 = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} c_{ij}\, q_{ri}$$

Subject to

$$q_{ri} \le q_{rimax}, \qquad i = 1, N_1$$
$$\sum q_{ri} = Qrtot$$
$$q_{ri} \ge 0, \qquad i = 1, N_1$$

Note: c_{ij} includes q_{pj}-term from previous producer iteration

B. Minimize

$$B_2 = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} c_{ij}\, q_{pj}$$

Subject to

$$q_{pj} \le q_{pjmax}, \qquad j = 1, N_2$$
$$\sum q_{pj} = Qptot$$
$$q_{pj} \ge 0, \qquad j = 1, N_2$$

Note: c_{ij} includes q_{ri}-term from previous injector iteration

Quadratic Programming **(QP)**

1. Mode(to allocate both injection and production rates (QPAL):

Minimize

$$B = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} c_{ij}\, q_{ri}\, q_{pj}$$

Subject to

$$q_{ri} \le q_{rimax}, \qquad i = 1, N_1$$
$$q_{pj} \le q_{pjmax}, \qquad j = 1, N_2$$
$$\sum q_{ri} = Qrtot$$
$$\sum q_{pj} = Qptot$$
$$q_{ri} \ge 0, \qquad i = 1, N_1$$
$$q_{pj} \ge 0, \qquad j = 1, N_2$$

Program Feature	LPAL1	LPAL2	LPAL3	QPAL
Provides ranking of injectors	Yes	Yes	Yes	No
Solves for both injection and production rates	No	Yes	Yes	Yes
Allows wells to be shut in	Yes	No	Yes	Yes
Well ranking varies with total rates	No	No	Yes	NA
Assesses quality of solution	No	No	No	Yes

Table 5. Comparison of allocation programs.

	PRODUCERS				
INJECTORS	**1**	**2**	**3**	**4**	**5**
1	-	**12.0**	**0.5**	**4.0**	**4.5**
2	**3.5**	-	**2.0**	**5.0**	**3.0**
3	**NR**	**6.5**	-	**4.3**	**7.2**
4	**3.7**	**2.9**	**9.0**	-	**4.8**
5	**5.4**	**2.5**	**NR**	**4.1**	-

Table 6. Time to initial tracer response for hypothetical data set (t_i in days; *NR* = no recovery).

	PRODUCERS				
INJECTORS	**1**	**2**	**3**	**4**	**5**
1	-	**28.0**	**1.0**	**10.0**	**15.0**
2	**11.0**	-	**7.0**	**9.0**	**12.0**
3	**NR**	**16.0**	-	**10.5**	**19.0**
4	**14.0**	**9.5**	**25.0**	-	**13.5**
5	**17.0**	**13.0**	**NR**	**8.0**	-

Table 7. Time to *peak* tracer response for hypothetical data set (t_p in days; *NR* = no recovery).

	PRODUCERS				
INJECTORS	1	2	3	4	5
1	-	2200	5000	50	35
2	39	-	30	40	25
3	0	27	-	500	3500
4	42	45	48	-	54
5	52	36	0	70	-

Table 8. Peak tracer response for hypothetical data set (C_p, normalized concentrations in l^{-1}).

	PRODUCERS				
INJECTORS	1	2	3	4	5
1		0.024	0.035	0.001	0.002
2	0.007	-	0.012	0.005	0.003
3	0	0.006		0.004	0.028
4	0.009	0.005	0.007		0.004
5	0.011	0.001	0	0.008	-

Table 9. Fractional tracer recovery for hypothetical data set (f, dimensionless: fraction of tracer injected).

	PRODUCERS				
INJECTORS	1	2	3	4	5
1	-	37	25	50	35
2	28	-	30	40	45
3	16	35	-	36	42
4	34	36	47	-	52
5	25	34	43	41	-

Table 10. Producing rate during tracer tests for hypothetical data set (q_{pt}, kg/s).

INJECTOR	RATE
1	55
2	60
3	45
4	50
	48

Table 11. Injection Rate during tracer tests for hypothetical data set (q_{rt}, kg/s).

	PRODUCERS				
INJECTORS	1	2	3	4	5
1	-	80	140	115	SO
2		-	75	90	95
3				60	120
4				-	80

Table 12. Horizontal distance between wells for hypothetical data set. Note: Matrix is symmetric, so only upper triangular elements are listed. (L, meters)

	PRODUCERS				
INJECTORS	1	2	3	4	5
1	-	+450	+500	+250	+300
2	-450	-	+50	-200	-150
3	-500	-50	-	-250	-200
4	-250	+200	+250	-	+50
5	-300	+150	+200	-50	-

Table 13. Elevation change from producer to injector for hypothetical data set (H, meters).

Individual Well Constraints	
Maximum Injection Rate (q_{rmax})	90
Maximum Production Rate (q_{pmax})	75
Minimum Injection Rate (q_{rmin})	0
Minimum Production Rate (q_{pmin})	0
Fieldwide Constraints	
Required Total Injection (Q_{rtot})	140
Required Total Production (Q_{ptot})	140

Table 14. Constraints on injection and production rates for hypothetical data set (kg/s).

WEIGHTING FACTOR(S)	INJECTORS CHOSEN	ASSIGNED RATES	PRODUCERS CHOSEN	ASSIGNED RATES
t_i	4	50	1	65
	5	90	2	0
			3	75
t_p	4	50	1	65
	5	90	2	0
			3	75
C_p	3	90	1	75
	4	50	2	65
			5	0
f	1	90	3	0
	2	50	4	65
			5	75
t_i, t_p, C_p, f	3	90	1	65
	5	50	2	75
			4	0

Table 15. Sensitivity of INCON to different weighting factors.

	INJECTORS		
PRODUCERS	WK-80	WK-101	WK-107
WK-18	***NR***	***NR***	
WK-22	-	***NR***	-
WK-24	***NR***	***NR***	0.2
WK-30	-		4.5
WK-44	***NR***	***NR***	-
WK-48	***NR***	***NR***	0.3
WK-55	***NR***	***NR***	5.5
WK-67	-	-	2.2
WK-68	-	-	4.0
WK-70	***NR***	-	4.0
WK-74		***NR***	-
WK-76	4.0	2.5	-
WK-81	-	-	4.8
WK-83	***NR***	-	4.5
WK-88	***NR***	***NR***	
WK-103	-	2.0	
WK-108	5.5	-	10.0
WK-116	3.3	2.5	
WK-121	-	1.2	

Table 16. **Initial** tracer response time for Wairakei Field (t_i, days).
NR = no recovery; "-" = producers not monitored for tracer response.

	INJECTORS		
PRODUCERS	WK-80	**WK-101**	**WK-107**
WK-18	***NR***	***NR***	
WK-22		***NR***	
WK-24	***NR***	NR	0.4
WK-30	-	-	**9.0**
WK-44	***NR***	***NR***	
WK-48	***NR***	***NR***	0.7
WK-55	***NR***	***NR***	15.7
WK-67	-	-	15.3
WK-68			15.0
WK-70	***NR***		***9.5***
WK-74	-	***NR***	-
WK-76	8.7	7.0	
WK-81	-	-	***9.5***
WK-83	***NR***	-	11.0
WK-88	***NR***	***NR***	-
WK-103	-	**5.0**	
WK-108	10.0		23.0
WK-116	7.6	7.5	
WK-121	-	2.5	

Table 17. Peak tracer response time **for** Wairakei **Field** (t_i, days).
NR = no recovery; "-" = **producers** not monitored for tracer response.

	INJECTORS		
PRODUCERS	WK-80	WK-101	WK-107
WK-18	***NR***	***NR***	-
WK-22		***NR***	
WK-24	***NR***	***NR***	10,000
WK-30	-	-	***55***
WK-44	***NR***	***NR***	-
WK-48	***NR***	***NR***	2360
WK-55	***NR***	***NR***	29
WK-67		-	46
WK-68	-		39
WK-70	***NR***		43
WK-74		***NR***	-
WK-76	**88**	10	-
WK-81		-	21
WK-83	***NR***	-	53
WK-88	***NR***	***NR***	
WK-103	-	30	-
WK-108	16		17
WK-116	230	23	-
WK-121	-	10,500	

Table 18. Peak tracer concentration **for** Wairakei (C_p, l^{-1}).
NR = no recovery; **"-" = producers** not monitored for tracer **response.**

	INJECTORS		
PRODUCERS	**WK-80**	WK-101	WK-107
WK-18	***NR***	***NR***	-
WK-22	-	***NR***	
WK-24	***NR***	***NR***	0.0373
WK-30	-	-	0.0028
WK-44	***NR***	***NR***	-
WK-48	***NR***	***NR***	0.0133
WK-55	***NR***	***NR***	0.0018
WK-67	-	-	0.0032
WK-68	-	-	0.0007
WK-70	***NR***	-	0.0025
WK-74	-	***NR***	
WK-76	0.0024	0.0005	-
WK-81	-	-	**0.0009**
WK-83	***NR***		0.0034
WK-88	***NR***	***NR***	
WK-103	-	**0.0009**	
WK-108	0.0006	-	0.0001
WK-116	**0.0040**	0.0005	
WK-121	-	**0.0580**	-

Table 19. Fractional **tracer** recovery **for** Wairakei Field (f, fraction).
NR = no recovery; **"-" = producers** not monitored for tracer response.

PRODUCERS	RATES
WK-18	11
WK-22	13
WK-24	33
WK-30	44
WK-44	34
WK-48	20
WK-55	47
WK-67	50
WK-68	10
WK-70	40
WK-74	49
WK-76	45
WK-81	52
WK-83	52
WK-88	53
WK-103	28
WK-108	51
WK-116	38
WK-121	19

Table $0. Producing rates during tracer tests for Wairakei Field (q_{pt}, kg/s). NR = no recovery; "-" = producers not monitored for tracer response. Source: Average 1976 production from Pritchen *et al.* [55] Rate for WK-121 based on McCabe *et al.* [37]

INJECTORS	RATE
WK-80	50 (est.)
WK-101	40
WK-107	50

Table 21. Injection rates during tracer tests for Wairakei Field (q_{rt}, kg/s). Source: Bixley. [56]

	INJECTORS		
PRODUCERS	WK-80	WK-101	WK-107
WK-18	536	557	345
WK-22	204	354	342
WK-24	325	227	209
WK-30	387	299	238
WK-44	482	338	427
WK-48	304	272	117
WK-55	355	427	216
WK-67	323	340	126
WK-68	324	316	124
WK-70	350	294	174
WK-74	130	249	321
WK-76	142	139	81
WK-81	337	394	178
WK-83	453	343	326
WK-88	631	499	524
WK-103	317	168	336
WK-108	229	188	84
WK-116	499	350	499
WK-121	641	489	622

Table 22. Horizontal distances between wells in Wairakei Field (L, meters).

	INJECTORS		
PRODUCERS	WK-80	WK-101	WK-107
WK-18	+365	+360	+391
WK-22	+165	+160	+191
WK-24	+363	+358	+389
WK-30	+210	+205	+236
WK-44	+264	+259	+290
WK-48	+591	+586	+617
WK-55	+264	+259	+290
WK-67	+222	+217	+248
WK-68	+188	+183	+214
WK-70	+156	+151	+182
WK-74	+142	+137	+168
WK-76	+119	+114	+145
WK-81	+244	+239	+270
WK-83	+141	+136	+167
WK-88	+233	+228	+259
WK-103	+145	+140	+171
WK-108	- 33	- 38	- 7
WK-116	+97	+ 92	+123
WK-121	+590	+585	+616

Table 23. Elevation changes from production zones to injection zones in Wairakei Field (L, meters).

WELLS	ELEVATIONS	WELLS	ELEVATIONS
WK-18	-307	WK-76	-61
WK-22	-107	WK-80	+58
WK-24	-305	WK-81	-186
WK-30	-152	WK-83	-83
WK-44	-206	WK-88	-175
WK-48	-533	WK-101	+53
WK-55	-206	WK-103	-87
WK-67	-164	WK-107	+84
WK-68	-130	WK-108	+91
WK-70	-98	WK-116	-39
WK-74	-84	WK-121	-532

Table 24. **Reservoir zone elevations for wells** in Wairakei **Field (meters above sea level).**

PRODUCERS	COST COEFFICIENTS
WK-24	16.7
WK-48	11.1
WK-121	2.8
WK-116	2.3
WK-76	2.2
WK-103	1.7
WK-67	1.5
WK-108	0.9
WK-70 [a]	0.8
WK-68 [a]	0.8
WK-83 [b]	0.74
WK-30 [b]	0.74
WK-81	0.70
WK-55	0.61
WK-18 [c]	0.001
WK-22 [c]	0.001
WK-44 [c]	0.001
WK-74 [c]	0.001
WK-88 [c]	0.001

Table 25. **Ranking of producing wells in** Wairakei **Field by** LPAL2. **Cost coefficients are exactly equal for wells with same letter superscript.**

Qrtot kg/s	LPAL3 Injectors	cost Coefficient	Curtailed Producers	cost Coefficient	QPAL Injectors	Curtailed Producers
140	107	8.10E+01	24 a	2.50E+02	107	24 a
	101	2.00E+01	48 a	1.67E+02	101	48 a
	80	1.30E+01	121 a	3.30E+01	80	121 a
			116 a	3.10E+01		116 a
			76 b	2.90 E+O1		76 b
100	107 b	8.10E+01	24 a	5.00E+01	107 b	24 a
	101	2.00E+01	48 a	3.33E+01	101	48 a
	80	1.30E+01	121 a	3.33E+01	80	121 a
			116 a	3.12E+01		116 a
			76 b	2.85E+01		76 b
90	107 a	3.12E+02	121 a	3.30E+01	107 a	121 a
	80	8.00E+00	116 a	3.10E+01	80	116 a
	101	5.50E-02	76 a	2.90E+01	101	76 a
			103 a	2.00E+01		103 a
			108 b	9.00E+00		108 b
70	107 a	3.12E+02	121 a	3.30E+01	107 a	121 a
	80 b	8.00E+00	116 a	2.50E+01	80 b	116 a
	101	5.50E-02	76 a	2.40E+01	101	76 a
			103 a	2.00E+01		103 a
			108 b	5.00E+00		108 b
50	107 a	3.08E+02	116 a	1.50E+01	80 a	18 a,d
	101 a	2.60E+01	76 a	1.30E+01	101 a	22 a,d
	80	5.50E-02	108 a	9.00E+00		24 a,d
			121 b,c	5.00E-03		30 a,d
						121 a,d
						116 b,d

a Total Curtailment
b Partial Curtailment
c Arbitrary allocation by LPAL3
d Non-unique allocation by QPAL

Table 26. Sensitivity of Wairakei well ranking to total rate.
Case I. Qptot = 550 kg/s; Qrtot varies.

	LPAL3				QPAL	
ptot g/s	Injectors	cost Coefficients	Curtailed Producers	cost Coefficients	Injectors	Curtailed Producers
89	107 b	3.13E+02			107 b	
	101	6.30E+01			101	
	80	3.20E+01			80	
50	107 b	1.27E+02	24 a	5.00E+01	107 b	24 a
	101	6.30E+01	48 b	3.33E+01	101	48 b
	80	3.20E+01			80	
00	107 b	8.10E+01	24 a	5.00E+01	107 b	24 a
	101	4.00E+01	48 a	3.33E+01	101	48 a
	80	2.60E+01	121 a	3.33E+01	80	121 a
			116 b	3.12E+01		116 b
50	107 b	8.10E+01	24 a	5.00E+01	107 b	24 a
	101	2.00E+01	48 a	3.33 E+O1	101	48 a
	80	1.30E+01	121 a	3.33E+01	80	121 a
			116 a	3.12E+01		116 a
			76 b	2.85E+01		76 b
00	107 b	8.00E+01	24 a	5.00E+01	107 b	24 a
	101	8.00E+00	48 a	3.33 E+O1	101	48 a
	80	5.00E-02	121 a	3.33E+01	80	121 a
			116 a	3.12E+01		116 a
			76 a	2.85E+01		76 a
			103 a	2.00 E+O1		103 a
			108 b	1.01E+01		**108 b**
450	107 b	7.40E+01	24 a	5.00E+01	107 b	24 a
	101	4.50E-02	48 a	3.33E+01	101	48 a
	80	4.50E-02	121 a	3.33 E+O1	80	121 a
			116 a	3.12E+01		116 a
			76 a	2.85E+01		76 a
			103 a	2.00E+01		103 a
			108 a	1.01E+01		108 a
			67 b	4.55E+00		67 b
400	107 b	5.20E+01	24 a	5.00E+01	107 b	24 a
	101	4.00E-02	48 a	3.33E+01	101	48 a
	80	4.00E-02	121 a	3.33E+01	80	121 a
			116 a	3.12E+01		116 a
			76 a	2.85E+01		76 a
			103 a	2.00E+01		103 a
			108 a	1.01E+01		108 a
			67 a	4.55E+00		67 a
			68 b,c	2.51 E+00		68 b,d
			70 e,c	2.51E+00		

a Total Curtailment
b Partial Curtailment
c Arbitrary allocation by LPAL3
d Non-unique allocation by QPAL
e Uncurtailed producer with same cost coefficient as WK-68

Table 27. Sensitivity of Wairakei well ranking to total rate.
Case II. Qrtot = 100 kg/s; Qptot varies.

Qptot kg/s	LPAL3 Injectors	cost Coefficients	Curtailed Producers	Cost Coefficients	QPAL Injectors	Curtailed Producers
689	107 a	3.13E+02			107 a	
	101 b	6.30E+01			101 b	
	80	3.20E+01			80	
650	107 a	3.13E+02	116 a	2.32E+01	107 a	116.a
	101 b	4.70E+01	76 b	2.05E+01	101 b	76 b
	80	2.00E+01			80	
600	107 a	3.13E+02	116 a	2.32E+01	107 a	116 a
	101 b	2.50E+01	76 a	2.05E+01	101 b	76 a
	80	9.00E+00	121 b	1.67E+01	80	121 b
550	107 a	3.12E+02	121 a	3.33E+01	107 a	121 a
	80 b	8.00E+00	116 a	2.51E+01	80 b	116 a
	101	5.50E-02	76 a	2.35E+01	101	76 a
			103 a	2.00E+01		103 a
			108 b	5.50E+00		108 b
500	107 a	8.00E+01	116 a	2.32E+01	107 a	116 a
	101 b,c	8.00E+00	76 a	2.05E+01	80 b,d	76 a
	80	5.00 E-02	121 a	1.67E+01	101	121 a
			103 a	1.00E+01		103 a
			108 a	9.10E+00		108 a
			88 b,c	7.00E-03		24 b,d
a	Total Curtailment		c	Arbitrary allocation by LPAL3		
b	Partial Curtailment		d	Non-unique allocation by QPAL		

Table 28. Sensitivity of Wairakei well ranking to total rate.
Case III. Qrtot = 70 kg/s; Qptot varies.

Veighting Factor	LPAL3 Injectors	LPAL3 cost Coefficients	LPAL3 Curtailed Producers	LPAL3 cost Coefficients	QPAL Injectors	QPAL Curtailed Producers
. 1/ti	107 b	8.05E+01	24 a	5.00E+01	107 b	24 a
	80	8.20E+00	48 a	3.33E+01	80	48 a
	101	5.00E-02	121 a	3.33E+01	101	121 a
			116 a	3.12E+01		116 a
			76 a	2.85E+01		76 a
			103 a	2.00E+01		103 a
			108 b	1.01E+01		108 b
!. 1/tp	107 b	2.82E+01	24 a	2.50E+01	107 b	24 a
	80	4.50E+00	121 a	1.60E+01	80	121 a
	101	5.00E-02	48 a	1.43E+01	101	48 a
			116 a	1.19E+01		116 a
			76 a	1.15E+01		76 a
			103 a	8.00E+00		103 a
			108 b	5.40E+00		108 b
3. cp	107 b	1.28E+04	121 a	4.20E+05	107 b	121 a
	80	7.20E+02	24 a	1.09E+05	80	24 a
	101	0.00E+00	48 a	2.36E+04	101	48 a
			116 a	1.24E+04		116 a
			76 a	4.80E+03		76 a
			103 a	1.20E+03		103 a
			108 b	9.70E+02		108 b
1 f	107 b	5.80E-01	121 a	2.32E+00	107 b	121 a
	80	3.00E-02	24 a	3.73E-01	80	24 a
	101	0.00E+00	116 a	2.20E-01	101	116 a
			76 a	1.40E-01		76 a
			48 a	1.33E-01		48 a
			103 a	3.60E-02		103 a
			83 b	3.40E-02		83 b
5. e^SH	107 b	5.98E+02	48 a	1.81E+02	107 b	48 a
	80	5.86E+02	121 a	1.81E+02	80	121 a
	101	5.79E+02	18 a	1.44E+02	101	18 a
			24 a	1.44E+02		24 a
			44 a	1.30E+02		44 a
			55 a	1.30E+02		55 a
			81 b	1.28E+02		81 b
6. 1/L^2	107 b	1.10E-02	76 a	6.30E-04	107 b	76 a
	101	4.00E-03	74 a	3.90E-04	101	74 a
	80	3.00E-03	108 a	3.70E-04	80	108 a
			103 a	2.10E-04		103 a
			48 b	1.90E-04		48 b

Table 29. Sensitivity of Wairakei well ranking to different weighting factors.
Qrtot = 100 kg/s; Qptot = 500 kg/s
(p. 1 of 2)

	LPAL3				QPAL	
Weighting Factor	Injectors	cost Coefficients	Curtailed Producers	cost Coefficients	Injectors	Curtailed Producers
7. 1/qpt	107 b,c	1.05E+01	68 a	1.00E+01	80 b,d	68 a
	80	1.05E+01	18 a	9.10E+00	101	18 a
	101	1.05E+01	22 a	7.70E+00	107	22 a
			121 a	5.30E+00		121 a
			48 a	5.00E+00		103 a
			103 a	3.60E+00		24 a
			24 a	3.00E+00		44 a
			116 b	2.60E+00		116 b
3. 1/ti, Cp, & f	107 b	5.39E+00	24 a	2.03E+04	107 b	24 a
	101	3.80E-01	121 a	2.03E+04	101	121 a
	80	9.00E-03	48 a	1.05E+03	80	48 a
			116 a	1.40E+01		116 a
			76 a	3.00E+00		76 a
			67 b	1.00E+00		67 b
3. 1/tp, Cp, & f	107 b	2.26E+00	24 a	1.02E+04	107 b	24 a
	101	5.00E-02	121 a	9.74E+03	101	121 a
	80	0.00E+00	48 a	4.48E+02	80	48 a
			116 a	6.10E+00		116 a
			30 a	1.70E-01		30 a
			83 b	1.60E-01		83 b
10. 1/tp, 1/tp, Cp, & f	107 b	6.04E-01	24 a	5.08E+04	107 b	24 a
	80	9.00E-03	121 a	8.12E+03	80	121 a
	101	0.00E+00	48 a	1.50E+03	101	48 a
			116 a	1.90E+00		116 a
			76 a	3.00E-01		76 a
			103 a	1.00E-01		103 a
			67 b	4.00E-02		67 b
11. 1/tp, 1/tp, Cp, f, & e^SH	107 b	7.50E-01	24 a	7.50E+04	107 b	24 a
	80	9.00E-03	121 a	1.46E+04	80	121 a
	101	0.00E+00	48 a	2.77E+03	101	48 a
			116 a	2.00E+00		116 a
			76 a	4.00E-01		76 a
			103 a	1.00E-01		103 a
			67 b	6.00E-02		67 b

a Total Curtailment c Arbitrary allocation by LPAL3

Table 29 (cont.) Sensitivity of Wairakei well ranking to total rate.
Qrtot = 100 kg/s; Qptot = 500 kg/s.
(p. 2 of 2)

INJECTOR NAME	MAXIMUM INJECTION RATE	ASSIGNED INJECTION RATE
WK-24	33	33
WK-48	20	20
WK-55	47	47

PRODUCER NAME	MAXIMUM PRODUCING RATE	ASSIGNED PRODUCING RATE
WK-18	11	0
WK-22	13	13
WK-30	44	44
WK-44	34	0
WK-67	50	40
WK-68	10	10
WK-70	40	40
WK-74	49	49
WK-76	45	45
WK-80	50	50
WK-80	52	0
WK-83	52	52
WK-88	53	0
WK-101	40	40
WK-103	28	28
WK-107	50	50
WK-108	51	51
WK-116	38	38
WK-121	19	0

Table 30. Rate allocation for optimal well configuration in Wairakei, based on elevation differences between wells.
Q_{itot} = 100 kg/s; Q_{ptot} = 550 kg/s

INJECTOR NAME	MAXIMUM INJECTION RATE	ASSIGNED INJECTION RATE
WK-88	53	53
WK-116	38	28
WK-121	19	19
PRODUCER NAME	**MAXIMUM PRODUCING RATE**	**ASSIGNED PRODUCING RATE**
WK-18	11	0
WK-22	13	13
WK-24	33	22
WK-30	44	0
W K 4	34	0
WK-48	20	20
WK-55	47	47
WK-67	50	50
WK-68	10	10
WK-70	40	40
WK-74	49	49
WK-76	45	45
WK-80	50	50
WK-80	52	52
WK-83	52	0
WK-101	40	40
WK-103	28	0
WK-107	50	50
WK-108	51	51

Table 31. Rate allocation for optimal well configuration in Wairakei, based on horizontal distance between wells.
$Q_{rtot} = 100$ kg/s; $Q_{ptot} = 550$ kg/s