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Criteria for Determining Times for End of Transient Flow and Start of Pseudosteady State Flow

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Abstract

This report describes criteria for determining the time of transition in well pressure behavior, i.e., the end of transient flow and start of pseudosteady state flow. Transition times were determined for the pressure at a well in various locations within either a closed or a constant pressure drainage area. The corresponding transition times were also determined from pressure derivative behavior.

The basic criteria for transition are that pressure or pressure derivative deviate from the behavior of fullydeveloped transient flow, or pseudosteady state flow. This can be reduced to finding linear relationships of pressure or pressure derivative with time, or with the logarithm of time. Deviation from the linear relationships can be determined by graphical analysis, or by calculation using the appropriate analytic solutions. In order to determine dimensionless transition times for various well locations and drainage shapes, well pressure behavior was calculated using superposition of line source solutions to generate the appropriate boundaries. Pressure derivatives were developed by a similar superposition of the derivative of the continuous line source solutions. The derivative of the line source was also used to develop the derivative of pressure buildup behavior for various well locations in rectangular drainage shapes.

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1. Introduction

The pressure in a well producing at constant flowrate from a homogeneous and finite reservoir has at least two distinct pressure regimes. The two regimes are characterized by pressure changes as functions of time. During the early period, commonly referred to as the transient period, pressure is proportional to the logarithm of time. This can be represented as a straight line on a semilog graph. During the later period, referred to as pseudosteady state, pressure is proportional to time, which is equivalent to a straight line on a Cartesian graph. The two flow periods, transient and pseudosteady state, are separated by a transition period, sometimes referred to as the late transient period (Matthews and Russell, 1967).

The two pressure regimes are significant for well test analysis in that physical characteristics of a reservoir can be determined from the relationship between pressure and time, and by the fact that each period may be recognized on an appropriate graph. In the transient flow period, the well behaves as if it were in an infinite reservoir. The end of the transient period occurs when a boundary or discontinuity is encountered. The duration of the transient period can be used to determine the distance to the nearest boundary or discontinuity. Similarly, pseudosteady state starts when all boundaries have been encountered, and the well pressure reflects removal of fluid from a finite drainage (closed) volume. The relationship between pressure and time in pseudosteady state can be used to determine the reservoir volume drained by the well.

The relationship between pressure and time for flow of fluid in a porous medium can be expressed efficiently in terms of dimensionless variables, which are proportional to the real variables. The factors of proportionality are constants for a given system, and can be defined in terms of physical and dynamic characteristics of the system and fluid. The objective of well test pressure analysis is to determine the constants of proportionality by comparing the real time (field) pressure behavior with a theoretical model expressed in terms of the corresponding dimensionless variables.

The various pressure regimes which can be observed in the real time field data should also be observed in the dimensionless model data, if the model is selected correctly. Each pair of pressure and time in the real data correspond to exactly one pair of dimensionless pressure and time. Therefore, by identifying characteristics in the pressure behavior and by studying the dimensionless model solution, a well test analysis can be performed by identifying the same characteristics in the real time data. The transition times, time for end of transient behavior and time for start of pseudosteady state, are easy to identify by simple graphical analysis. Furthermore, the dimensionless model data can be used to design a well test.

The criterion used to determine transition times has an effect on the precision of the analysis, because of the one-to-one relationship between real and dimensionless variables. Well test analysis is usually performed by graphical displays, either hand or computer generated. The criterion for determining a transition time is that pressure behavior deviates from a linear relationship which exists during the corresponding pressure-time regime.

Several criteria have been used to determine transition times in analysis of dimensionless pressure data. Earlougher *et al.* (1968), and Ramey and Cobb (1971) used as criteria that dimensionless pressure deviates from a linear relationship for constant-rate production. Their method of analysis was analogous to the method used to analyze real time data.

Dietz (1965) determined the time for start of pseudosteady state from graphs published by Matthews *et al.* (1954), of the pressure correction function, p^*. He used as criterion for start of pseudosteady state, the time at which the pressure correction function became a linear function of the logarithm of time. Matthews *et al.* had shown that after some time of production, the pressure correction function became a linear function of the logarithm of the producing time.

Jones (1961), and Odeh and Nabor (1966) determined transition times for a well in the center of a closed cylindrical reservoir. Their criteria were based on a relationship developed by Jones for radius of average pressure and effective radius of drainage. The criterion for the end of the transient period was that the ratio of the radius of average pressure to the radius of drainage remains constant during transient pressure behavior, but changes as external boundary effects are detected at the well. The criterion for start of pseudosteady state was that the effective radius of drainage equals the external boundary radius.

Muskat (1937*a*) presented an expression for a time that he called the readjustment time. This was based on the assumption that for pressure to reach steady state after a change of rate, a certain excess volume of fluid had to be removed. The readjustment time was defined as the time required to remove the excess volume at the constant producing rate.

The various methods and criteria for determining transition times have resulted in much variation in, and confusion about, the results reported. Some times have been determined for production, and others for pressure buildup after shut in.

Unfortunately, "transition" or "readjustment" times, or "radius of investigation" formulas are often applied blindly to the wrong field problem. Other problems have occurred because of neglecting precision. Van Everdingen and Hurst (1949) stated that the continuous line source solution (proper for many multiwell interference tests) for pressure at an observation well became a linear function of the logarithm of time at a dimensionless time t_D/r_D^2 of about 25. This was interesting because it identified the start of the transient period, or the minimum time to establish a semilog straight line. Later Ramey (1975) reported that field data precision often indicated a much shorter time of about 5. Unfortunately, the dawning age of the digital computer encouraged those interested in accuracy to seek precision in computing model solutions that could not be matched in either field data measurements, or in graphical analysis on 8 ½ by 11 inch graph paper. An example of the effect of computer precision on the time to start of pseudosteady state can be seen in papers by Earlougher *et al.* (1968), and Earlougher and Ramey (1973). On at least two separate occasions, times reported for the start of pseudosteady state for identical systems were quite different. The reason for the differences was that the computed precision was different for the various cases. There may also be a source of confusion when model dimensionless transition times are determined by different criteria than the transition times for field data.

The transition times described in the literature have usually referred to a transition which can be observed in pressure-time behavior. Presumably the same transitions should also appear in the behavior of a pressure-time derivative. Apparently the pressure derivative transition times have not been reported. Furthermore, field pressure-time derivative data often appear to indicate transitions that are not apparent in field pressuretime data. It is therefore useful to determine dimensionless transition times for pressure-time derivatives, and establish uniform criteria for determining transition times in either pressure or pressure derivative behavior. This is the objective of this study.

2. Literature Review

Some important early works in development of the theory for transient flow of fluid in porous media were presented by Moore *et al.* (1933), Theis (1935), Muskat (1937*a*), and van Everdingen and Hunt (1949). Muskat, working from general solutions to the diffusivity equation, developed particular solutions for various boundary conditions. Van Everdingen and Hunt used the technique of Laplace transformation to solve the diffusivity equation with various boundary conditions. Of the solutions developed, one lhas had particular importance for subsequent developments in transient theory. That is the continuous line source for pressure dismbution in an infinite slab reservoir with fluid withdrawn at a constant rate from a single well (Theis (1935) and van Everdingen and Hurst (1949)).

Theis (1935) used the continuous line source solution from Lord Kelvin's theory of heat transfer. He presented the solution in a format which made it readily useful for well test analysis in groundwater hydrology.

It was expected that the line source solution would have limitations when applied to real systems. The line source assumes fluid withdrawal from a well of zero radius, and results in infinite pressure drop at the source. Mueller and Witherspoon (1965) compared the line source solution with data published by Mortada (1955) for a finite radius source. Their conclusion was that the line source solution was accurate within 1% for many practical situations involving interference between wells, or oilfields.

The line source solution is a powerful tool for generating solutions for pressure distributions in finite reservoirs. Matthews *et al.* (1954) determined pressure behavior at a well in finite rectangular reservoir shapes by superposition of the line source solution. Their applications emphasized pressure buildup. The effects of reservoir boundaries were created with an infinite array of image wells, each of which created a pressure distribution according to the line source solution. The resulting pressure distributions were obtained by summation of the effect of all wells, real and images. This, of course, was an extension of Muskat's (1937*a*) use of superposition for bounded, steady flow shapes.

Others have expanded the method of superposition. Earlougher *et al.* (1968) used superposition to develop pressure distributions in rectangular reservoirs. As part of this work, they also showed that the pressure distribution for rectangular shapes can be obtained from superposition of the solution for pressure distributions in a closed square. Kumar and Ramey (1974) presented a solution for the pressure behavior of a well in the center of a constant pressure square. Ramey *et al.* (1973) showed that superposition of line sources could be used to determine pressures in rectangular reservoirs with mixed outer boundaries, closed or no flow, and constant pressure. Larsen (1981) presented a general method for superposition to generate various outer boundary shapes;.

It is not the purpose of this review to consider all applications of line source superposition. However, it is notable that Caudle (1967) and others have extended line source superposition to generation of fluid injection patterns with determination of streamlines, streak lines and isotime lines, as well as to generate approximate irregular outer boundary shapes. This method produced a generation of simple reservoir models which could even consider thickness and other variations.

With the development of transient flow theory, simple techniques became available for analyzing flow data from wells to determine important reservoir characteristics. Theis (1935), and Homer (1951) published a method using production history and pressure buildup after shut-in to determine permeability and average reservoir pressure at shut-in. Miller *et al.* (1950) published a related technique which only used buildup pressures. The techniques were simple to use, because the underlying model had a linear relationship between pressure and a logarithmic function of shut-in time. Unfortunately, different assumptions and graphs were involved, frequently yielding different results.

Early pressure analysis methods for petroleum reservoirs used pressure buildup data, rather than pressure drawdown production data. Russell (1963) presented a method for two-rate drawdown analysis which was analogous to the technique presented by Horner (1951) for buildup. Matthews and Russell (1967) identified three distinct pressure: regimes in drawdown data: transient, late transient, and pseudosteady state. The transient period was as described previously. The late transient period started when boundaries or discontinuities began affecting pressure behavior, and formed the transition to pseudosteady state behavior. In the pseudosteady state period, all boundaries had been felt by the well, and pressure behavior was in accordance with removal of fluid from a finite volume of constant compressibility fluid.

Ramey and Cobb (1971), in an analysis of pressure buildup theory for a well in the center of a closed square, presented equations defining transient and pseudosteady state behavior. The late transient period did not appear significant for this drainage shape.

Since data in the various pressure regimes provide different information, and require different analytical techniques, it is of interest to determine the time for start and end of the various regimes. Several authors have provided somewhat different results, as discussed by Ramey and Cobb (1971), and Kumar and Ramey (1974).

Recent additions to the pressure analysis literature have been methods to analyze pressure-time derivatives to obtain information similar to that obtained from pressure-time analysis. The pressure derivative was introduced by Jaeger (1956) for interference testing, but apparently did not attract much attention. Tiab and Kumar (1980) developed a pressure derivative of the line source solution, and applied the method to interference analysis. They established the range in time and space, for which the derivative of the line source is a valid representation for the derivative of field reservoir pressures. At the time, it was believed that pressure derivative methods would require unusually accurate pressure data, and the method did not receive the interest it deserved.

Bourdet *et al.* (1983) presented the use of pressure derivative analysis for single well data by developing type *curves* of both the pressure derivative and pressure vs. time. This ingenious presentation used the single well wellbore storage and skin effect data of Agarwal *et al.* (1970) as cross-plotted by Gringarten *et al.* (1979), with the pressure derivative data by Ramey and Agarwal (1972). The importance of the combined type curve presentation (Bourdet *et al.*, 1983) is extraordinary. The derivative type curve permits identification of effects not evident in pressure-time data alone. Practical use requires computer processing of data, but fortunately, does not necessarily require data of high precision. It is often necessary to adjust field data for practical field phenomena. Thus, cataloging pressure derivative characteristics, appears of great potential utility.

Bourdet *et al.* (1984) also showed how the pressure derivative can be useful to interpret two-porosity, or fissured, or layered system data.

We turn now to the main objective of this study.

3. Solution for Pressure Behavior

In order to study criteria for determining the various pressure regimes for a well in a finite reservoir, it is necessary *to* have an algorithm for pressure behavior at the wellbore as a function of time. Matthews *et al.* (1954) and Earlougher *et al.* (1968) have shown that the pressure at a well producing from a finite, rectangular reservoir, can be derived from the superposition of the solutions for pressure in a well producing from an infinite reservoir.

3.1 Pressure Distribution in an Infinite Reservoir

The pressure distribution in a homogeneous reservoir of constant thickness, containing a single, slightly compressible fluid is governed by the diffusivity equation, van Everdingen and Hurst (1949):

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\,\frac{\partial p}{\partial r}\right\} = \frac{\phi \mu c_t}{k}\frac{\partial p}{\partial t} \tag{1}$$

with the initial condition:

$$p = p_i\,,\ at\quad t = 0\,,\ \text{and}\quad r \rightarrow \infty\,,\ t \geq 0 \tag{2}$$

and the boundary condition at the well:

$$r\frac{\partial p}{\partial r} = -\frac{q\mu}{2\pi kh}\,,\ at\quad r = 0\quad \text{and}\quad t > 0 \tag{3}$$

We will use the: following dimensionless variables:

Dimensionless radius:

$$r_D = \frac{r}{r_w} \tag{4}$$

Dimensionless pressure:

$$p_D = \frac{2\pi kh}{q\mu} \; (p_i - p) \tag{5}$$

which is equivalent to:

$$p_D = \frac{\Delta p}{2\, m} \tag{6}$$

where m is constant in terms of time and pressure and defined as:

$$4 \pi k h \tag{7}$$

Dimensionless time:

$$t_D = \frac{kt}{\phi \mu c_t \, r_w^{\,2}} \tag{8}$$

which is equivalent to:

$$t_D = \eta \, \frac{t}{r_w^{\,2}} \tag{9}$$

where η is the hydraulic diffusivity:

$$\eta = \frac{k}{\phi \, \mu \, c_t} \qquad (10)$$

with all dimensions in Darcy units.

Eq. 1 can be written in dimensionless form as:

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left\{ r_D \frac{\partial p_D}{\partial r_D} \right\} = \frac{\partial p_D}{\partial t_D} \qquad (11)$$

with the initial condition:

$$p_D = 0 \, , \; at \; \; t_D = 0 \, , \; \text{and} \; \; r_D \rightarrow \infty \, , \; t_D \geq 0 \qquad (12)$$

and with the inner boundary condition:

$$\frac{\partial p_D}{\partial r_D} = -1 \, , \; \; at \; \; r_D = 0 \; \; \text{and} \; \; t_D > 0 \qquad (13)$$

The solution is the continuous line source solution:

$$p_D = -\tfrac{1}{2} \, Ei \left\{ -\frac{r_D^{\,2}}{4 t_D} \right\} \qquad (14)$$