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AN EVALUATION OF THE CAPACITANCE PROBE AS A TECHNIQUE FOR DETERMINING LIQUID SATURATIONS IN LABORATORY FLOW EXPERIMENTS

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ABSTRACT

Of all the methods reported to date, the capacitance probe comes closest to meeting the constraints of measuring liquid saturations of cores at high temperatures and pressures (i.e. 250° C and 500 psi). Changes in liquid saturation in the inner .4 inches of porous core are reflected in a one to four percent change in the resonant frequency of the probe. The higher the temperature at which the measurement takes place, the smaller the change for a given change in liquid saturation. The depth of penetration of the probe is only abut one sixth the core thickness, and the properties of the core must be uniform throughout for the measurements to be meaningful.

INTRODUCTION

The problem of determining liquid saturations in porous cores was given much attention in the late forties[1,2,3,4] and briefly in the mid-sixties[5,6]. The techniques which were developed were all concerned with low to moderate pressures, and room temperatures. The methods are each suited to a different situation and are listed together in Table 1.

When working at high temperatures and pressures an additional constraint, the necessity of a core casing which can withstand these conditions, is required. Metal is the cheapest and most readily available material, but automatically eliminates techniques such as microwave attenuation, for which the casing acts as an effective shield to the signal.

The constraints which need to be satisfied here are as follows:

(1) the measurement must be made outside of the porous rock sample under study without interrupting the fluid flow pattern,

(2) the saturation indication must be independent of fluid distribution in the volumne scanned by the measuring device,

(3) the sensing element should have a small field of definition enabling several independent measurements to be made along the length of the core sample,

(4) the properties of the fluid phases should not be influenced by the addition of any required tracer substances,

(5) the device must be modified to accomodate high temperatures and pressures in the core.

To date, no one has published results of a technique satisfactorily meeting all these requirements.

It is the purpose of this paper to describe and evaluate a technique which does meet all the above criteria, and compare it with methods already in use.

THE CAPACITANCE PROBE

Description of the Method:

This technique measures the difference in capacitance between water filled porous media and steam filled porous media. Water has a dielectric constant of abut 78 at 25°C due to the dipolar nature of the water molecule but decreases to about 26. at 250°C.[7] However this still represents a significant difference from the value for steam at about 2 for most temperatures.

As the content of pore space changes from water to steam, the resultant decrease in dielectric constant for the material reduces the signal measured by the probe.

The dielectric permittivity, or constant, of the material at a measuring frequency ω is given by*

$$\epsilon = \epsilon_r - j\left(\frac{\sigma}{\epsilon_r \omega}\right)$$

* Terms are defined in Appendix I.

The values for ϵ and ϵ_r will increase as the amount of water in the material increases. The conductivity σ is that of the material and changes both with the degree of liquid saturation and with changes in the content of any impurities in the pore water of the sample[6]. As the measurements are to be made isothermally, it is assumed that changes in ϵ are due totally to changes in liquid saturation. These changes are non-linear, and require a separate calibration for each core sample.

Apparatus:

A block diagram of the apparatus in use is as shown in Fig. 1. with the core casing and probe itself illustrated in Figures 2 and 3. The probe is a capacitor which is part of an oscillating circuit. The resonant frequency of the circuit depends upon the probes capacitance, A small percentage of this capacitance depends upon the dielectric constant of the narrow interval of core being measured, and this is a function of the liquid saturation.

The probe oscillating circuit is housed with an identical crystal stabilized oscillator in a small grounded metal box attached to the probe. Both oscillators are tuned to about 7.5 MHz with about a 1 kHz difference. Thus any differences in beat frequency between the two oscillating circuits is 'sampled' at 1/1000 times the beat frequency. Calculations of the field measured by the probe indicate that changes in the resonant frequency due to changes in liquid saturation are of the order of 1/100 of the beat frequency and are therefore measurable.

TABLE

TECHNIQUES FOR MEASURING FLUID SATURATIONS IN POROUS CORES

TECHNIQUE	METHOD OF MEASUREMENT	SE	AG	
Magnetic Susceptibility	Uses a magnetic tracer in one of the fluid phases to determine liquid saturation of that phase.	oil saturations, water saturations, with gas present. (two-phase flow)	cannot easily differentiate between oil and water, core casimg must be non-magnetic.	4.
Neutron Scattering	Difference in scattering of neutron beam by substances containing hydrogen is qualitatively different from substances with less or no hydrogen.	oil saturation, water saturation, possibly steam/water mixtures, metal core casing o.k.	cannot discriminate water, oil easily; need consolidated core.	3.
Nuclear Magnetic Resonance	Resonating coil recieves signal from precessing hydrogen nuclei in the core fluids.	three phase fluid saturations; gas, water, oil, or any combination.	non-magnetic core casing required, recieving coil and leads must be sheilded	5.
Microwave Attenuation	Core placed in path of microwave beam produces an attenuation of the beam which is a function of the liquid saturation in the core.	cannot easily distinguish between water and oil, best for soil moisture measurements.	non-magnetic, non-conducting core casing required which will not absorb a large amount of the beam.	9.
Capacitance Probe	Utilizes the difference in capacitance between phases in core.	liquid saturation of core, can be modified easily to high T and P, good for water and steam.	Circuits must be well sheilded against all stray capacitances, should have common ground	

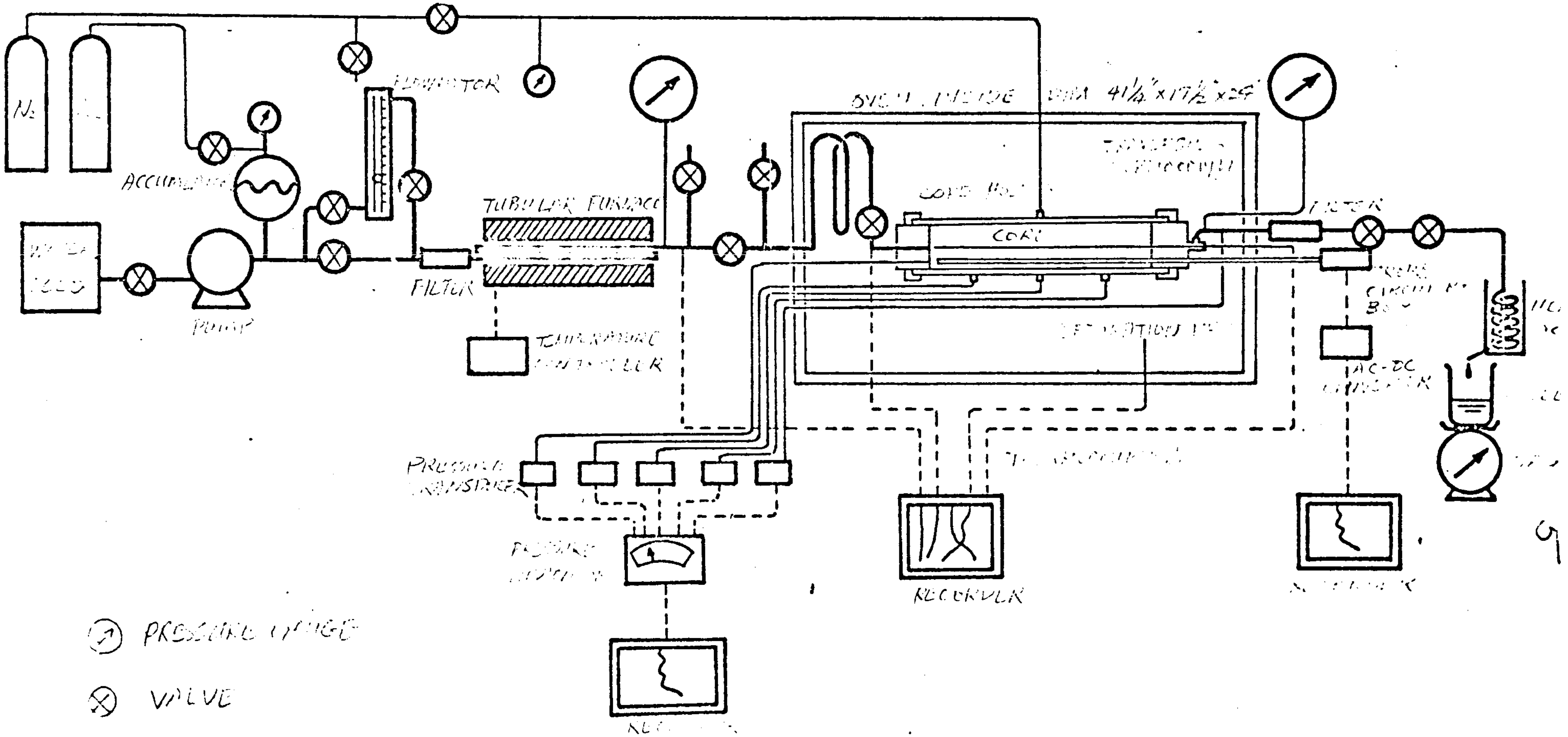


FIGURE 1. SCHEMATIC DIAGRAM OF APPARATUS

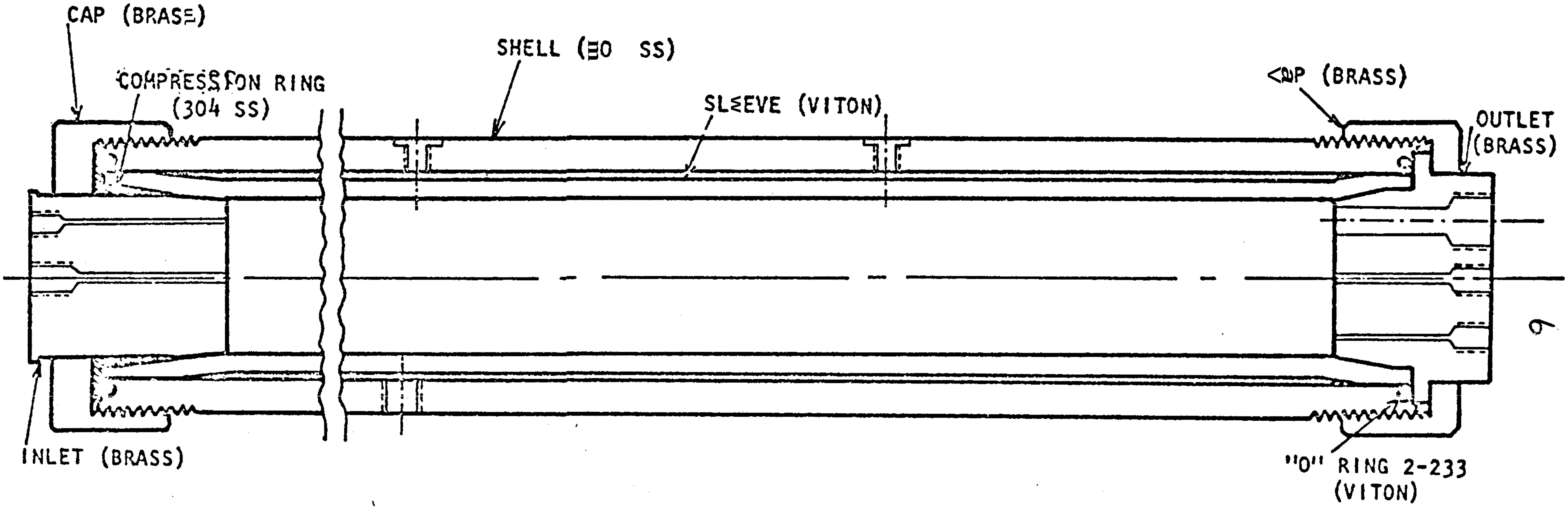


FIGURE 2. CORE HOLDER

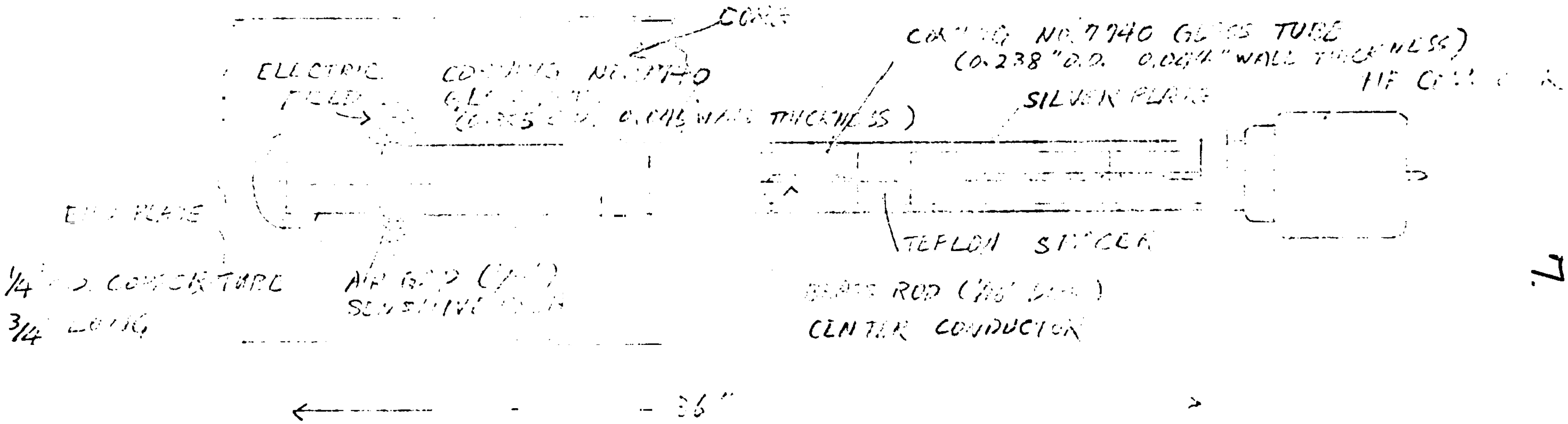
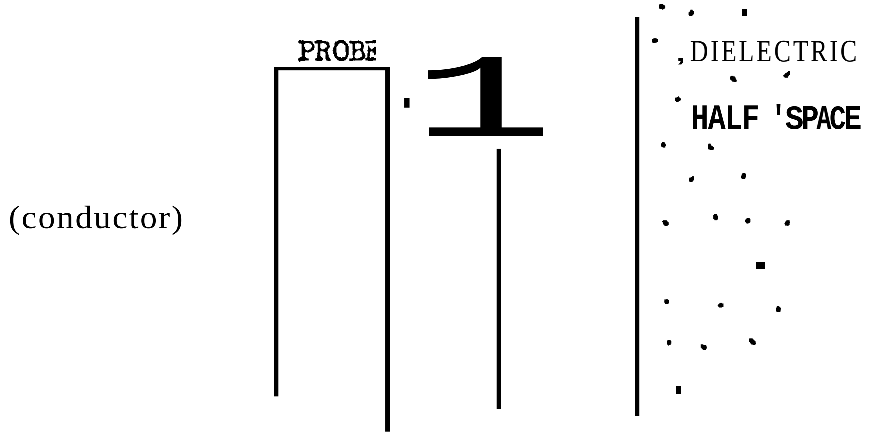


FIGURE 3. CAPACITANCE PROBE

The frequency difference is measured by heterodyning and amplifying the resulting frequency difference signal, This signal is then rectified and the d.c. output recorded on a 10 mamp chart recorder. The circuits are shown in Figures 4 and 5.

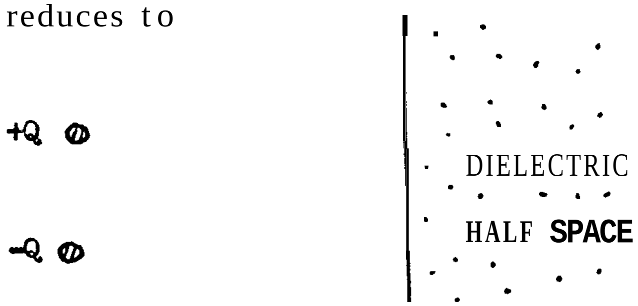
Calculation of the Field Equations for the Capacitance Prober

Since the measurements are taken at a frequency of about 30 MHz, the signal travels on the outer surface of the capacitance probe. In calculating the field configuration then, the probe may be modeled with;

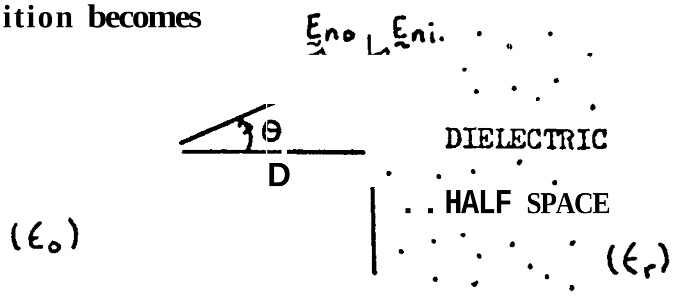


Since the capacitance is a function of geometry only, I will model the static case. The impedance of the probe at high frequencies will then depend upon its capacitive reactance, which is a function of frequency. This will determine the resonance condition of the probe, with the d.c. capacitance appearing as a scale factor.

The static case reduces to



Since *the* geometry is symmetric and divisible by two, *the* result by superposition becomes



The normal component of E = $E_r \cos\theta$

$$\cos\theta = \frac{D}{r}$$

$$E_r = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r^2}\right)$$

$$E_n = \frac{Q}{4\pi\epsilon_0}\left(\frac{D}{r^3}\right) = \left(\frac{Q}{4\pi\epsilon_0}\right)\left(\frac{D}{(s^2+D^2)^{3/2}}\right)$$

and the normal component is discontinuous, (due to the induced surface charge).

The field due *to* the induced surface charge must be added to the normal component. From Gauss' Law, this *is* $\frac{1}{2}\frac{\sigma_b}{\epsilon_0}$ where σ_b = the surface charge density.

The sum of these two fields is

$$E_{ni} = \left(-\frac{Q}{4\pi\epsilon_0}\right)\left(\frac{D}{(s^2+D^2)^{3/2}}\right) - \frac{\sigma_b}{2\epsilon_0}$$

the normal component of the electric field just inside *the* boundary.

Now $\sigma_b = \underset{\sim}{P} \cdot \underset{\sim}{n}$ ($\underset{\sim}{n}$ = the outward normal *to* the surface)

$$= \epsilon_0(\epsilon_r - 1)\, E_{ni} \qquad (P_n)$$

Therefore $\sigma_b = -\epsilon_0(\epsilon_r - 1)\, E_{ni} = -\epsilon_0(\epsilon_r - 1)\left[\frac{Q}{4\pi\epsilon_0}\,\frac{D}{(s^2+D^2)^{3/2}} + \frac{\sigma_b}{2\epsilon_0}\right]$

or $\sigma_b = -\dfrac{(\epsilon_r - 1)\,Q D}{2\pi(\epsilon_r+1)(s^2+D^2)^{3/2}}$

The induced charge and Q have opposite signs, as expected.

Two approaches are now possible;

(1) Calculate V, E at all points, using Coulombs law, by integrating over the surface charge distribution, and then add the field due to the point charge Q.

(2) Use the method of images and the fact that the normal compnent of D is continuous. This is the approach I will use.

Using equations I. and 2.:

$$E_{ni} = - \left[1 - \frac{(\epsilon_r - 1)}{(\epsilon_r + 1)}\right] \frac{QD}{4\pi\epsilon_0 (s^2 + D^2)^{3/2}}$$

$$= - \frac{2}{(\epsilon_r + 1)} \frac{QD}{4\pi\epsilon_0 (s^2 + D^2)^{3/2}}$$

just outside the dielectric $|E| = |D| = E_{,}$ such that

$$E_{no} = \epsilon_r E_{ni}$$

the normal conponent of $\underset{\sim}{D}$ is continuous across the boundary since there is a zero free charge density.

Far image charges, the field you calculate must always be outside the region in which you have the image charge. Note that both E„ and E_{ni} point in the same direction, outward form the dielectric, (since the fields are added such that - Q appears).

Now E, is as if the dielectric is replaced by the image charge

$$Q' = - \frac{(\epsilon_r - 1)}{(\epsilon_r + 1)} Q$$

located a distance D behind the boundary.

For points inside the dielectric, E_{ni} gives rise to two alternate sets of point charges:

(a) A charge Q together with an image charge

$$Q' = \frac{(\epsilon_r - 1)}{(\epsilon_r + 1)}\, Q$$

at a distance D from the boundary.

(b) a single *charge*

$$Q'' = Q + Q' = \left(\frac{2\epsilon_r}{\epsilon_r + 1}\right) Q$$

which replaces Q, the dielectzic extending in this case on both sides of the boundary (i.e. to all space).

Choice (a) is ruled out by *the* constraint that image charges must always be outside the region in which the field is required. Therefore the problem reduces to a field of a point charge

$$Q'' = \left(\frac{2\epsilon_r}{\epsilon_r + 1}\right) Q$$

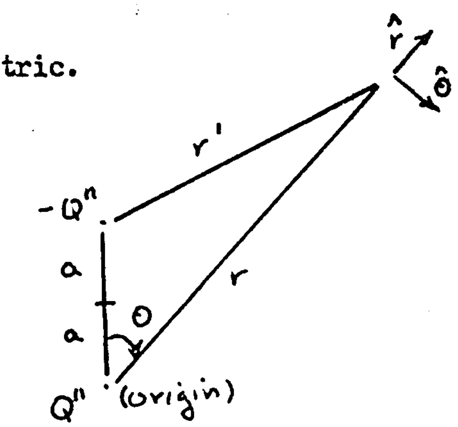
which replaces Q for all points inside the dielectric. *The* potential due to two opposite charges is then

$$V = \frac{Q''}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{r'}\right)$$

and the electric field is

$$E_r = -\frac{\partial V}{\partial r} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{Q''}{r^2} - \frac{Q''(r - 2a\cos\theta)}{(r')^3}\right)$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \left(\frac{-1}{4\pi\epsilon_0}\right)\left(\frac{2Q''\, a\sin\theta}{r\cdot(r')^3}\right)$$



$$r' = \sqrt{r^2 + 4a^2 - 4ar\cos\theta}$$

Using these equations, I have plotted the field lines dues to the capacitance probe with an applied d.c. voltage using the method *of* free hand field mapping[8] and curvilinear squares. The result is as shown in Figure 6.

ELECTRIC FIELD MAP FOR
THE CAPACITANCE PROBE

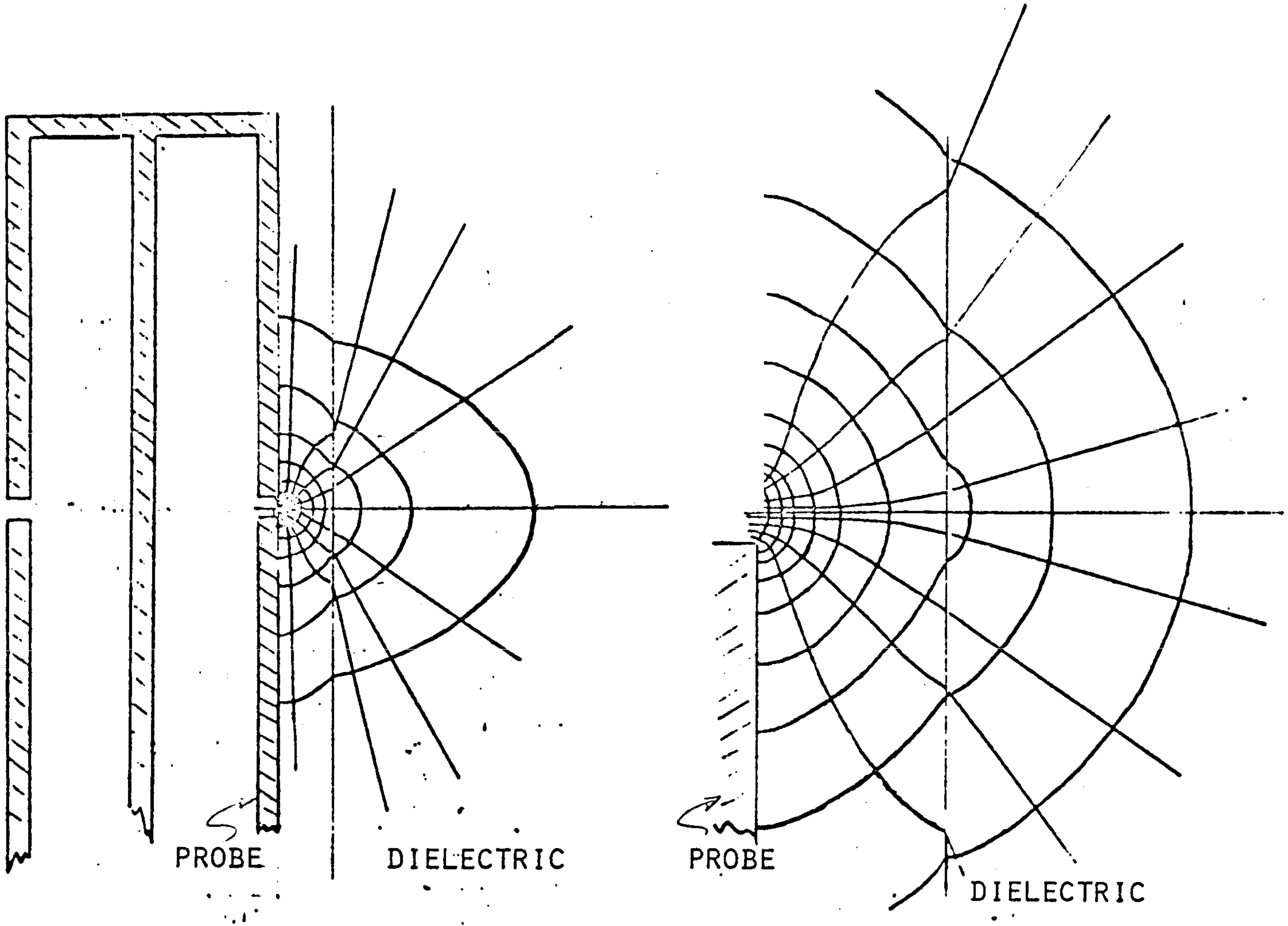


FIGURE 6.

KEY

FLUX LINES

POTENTIAL LINES

Capacitance Calculation:

The electrostatic capacitance per unit length can now readily be calculated from the field plot. By Gauss's law, the charge induced on a surface is equal to the flux ending there. This is the number of flux tubes N_f multiplied by the flux per tube extending into the dielectric. The capacitance per unit length is[8].

$$C = \frac{N_f}{N_p} \text{ farads/meter.}$$

Plugging in values from the field map the result is

$$C = \frac{\epsilon}{4} \text{ pf/inch}$$

where the "length" is the circumference of the inner surface of the dielectric. Multiplying by this value gives

$$C = C_o(s) \text{ pf} \simeq 1.0 \text{ pf} \quad (\text{for typical values of } \epsilon_r).$$

The capacitance of the probe itself may be calculated from the equation for the capacitance of a coaxial capacitor

$$C = \frac{2\pi\epsilon_o}{\ln(r_2/r_1)}$$

The result of this calculation, using the probe dimensions is

$$C = 24.88 \text{ picofarads.}$$

Therefore the percentage of the total signal due to capacitance changes in the core material is of the order of one to four percent.

Capacitive Reactance of the Circuit:

The resonance properties of the probe, upon which the output signal depends, depend themselves on the capacitive reactance

$$X_C = \frac{1}{2\pi f C}$$

Differentiating this equation at a constant frequency yields

$$\delta \chi_c / \chi_c = - \frac{\delta C}{C}$$

Therefore changes in the capacitance of the medium result in changes of the same order of magnitude in the capacitive reactance of the probe, and hence in the signal output of the probe.

Depth of Penetration of the Signal:

The divergence of the electric field calculated for points inside the dielectric indicates that the field falls to 1% of its value within .4" inside the boundary of the dielectric. This may either be calculated directly by taking the divergence of E, or by calculating the decrease in the value of E as you increase the value of r, (i.e. calculate the value of E at the surface of the dielectric and then at a distance r within the dielectric.). A similar calculation reveals that the a similar attenuation occurs about one-eigth of an inch on either side of a dividing line between the two charges. The area measured by the probe can then be approximated by a rectangle 1/4 X 1/2 inches extending into the dielectric.

CONCLUSION

The dielectric probe is potentially an effective means of quantitatively determing liquid saturations in porous media. It is easily modified to fit the experimental conditions of high temperature and pressure (i.e. 250° C and 500 p.s.i.).

The field measured by the probe is of the order of 1/25 to 1/100 of the probes' beat frequency. Since changes in the beat frequency are sampled at periods of 1/1000 of the beat frequency, the changes in beat frequency due to changes in liquid saturation are easily measurable.

The divergence of the electric field shows the maximum depth of penetration to be about .4 inches. Since the core is about 3 inches thick, this requires that for the measurements to meaningful, the porosity and permeability of this part of the core must be equivalent to the rest of the core.

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APPENDIX I

DEFINITION OF TERMS USED IN THE TEXT

ϵ_r = Real permittivity of dielectric of porous core + some water

ϵ = Complex dielectric permittivity of core

σ = Conductivity of porous core + some water

ω = f = frequency of probe signal voltage

ϵ_0 = Real permittivity of air

D = distance from probe to core

E_r = radial component of the electric field (polar coordinates)

E_θ = angular component of the electric field

E_n = component of the electric field normal to the surface of the dielectric

σ_b = surface charge density induced on the inner surface of the dielectric

$\underset{\sim}{P}$ = polarization vector in the dielectric

$\underset{\sim}{n}$ = unit normal vectar perpendicular to the inner wall of the dielectric

$\underset{\sim}{D}$ = electric. displacement vector = $\epsilon_0 \underset{\sim}{E} + \underset{\sim}{P}$

V = potential of the electric field

C = capacitance (farads or picofarads)

N_f = number of flux lines in material of core

N_p = number of potential lines in material of core

r_2 = outer radius of probe

r_1 = inner radius of probe

X_c = capacitive reactance of probe

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