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# A Comparative Analysis of Energy Management Strategies for Hybrid Electric Vehicles

*This paper presents a formalization of the energy management problem in hybrid electric vehicles and a comparison of three known methods for solving the resulting optimization problem. Dynamic programming (DP), Pontryagin's minimum principle (PMP), and equivalent consumption minimization strategy (ECMS) are described and analyzed, showing formally their substantial equivalence. Simulation results are also provided to demonstrate the application of the strategies. The theoretical background for each strategy is described in detail using the same formal framework. Of the three strategies, ECMS is the only implementable in real time; the equivalence with PMP and DP justifies its use as an optimal strategy and allows to tune it more effectively.*
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## 1 Introduction

Hybrid electric vehicles (HEVs) have the potential to reduce fuel consumption and emissions in comparison to conventional vehicles, thanks to the presence of a reversible energy storage device and one or more electric machines. The addition of these devices offers idle off capability, regenerative braking, power assist ability, and potential for engine downsizing. These abilities are due to the fact that the electric actuators and the storage system can operate bidirectionally, storing part of the energy produced by the engine and by the vehicle deceleration, and use it when needed. The presence of an additional energy storage device gives rise to new degrees of freedom since at each time the total power request for moving the vehicle can be delivered by one of the on-board energy sources or their combination. With the additional degrees of freedom comes the problem of finding the most efficient way of splitting the power demand between the engine and the battery. The *energy management strategy* is the control layer to which this task is demanded. The main objective of the energy management strategy [1] is to minimize fuel consumption and, possibly, emissions over a driving cycle without compromising the performance of the vehicle. In this work, minimization of fuel consumption (neglecting drivability considerations) is considered; however, other optimization objectives could be assumed, such as minimization of pollutant emissions or maximization of battery life [2].

The energy management closed-loop scheme in a HEV is shown in Fig. 1. The vehicle speed controller is the human driver in a real vehicle and is typically modeled by a simple feedback controller in simulation. The speed controller decides the total power request $P_{\text{req}}$ that the powertrain must deliver in order to follow the prescribed velocity profile. The energy management strategy decides how to split the total power request between the energy sources present on-board. Typically, being these a battery (or other electrical storage device) and the fuel tank, it decides the value of the engine power $P_{\text{ice}}$ and the battery power $P_{\text{batt}}$. If the powertrain has more than one degree of freedom, other control variables are also computed: for example, the repartition of torque between the electric machines, if more than one, or the transmission ratio, etc.

For the purposes of this study, the overall vehicle is seen as a dynamic system with two states, the vehicle speed and the state of charge (SOC) of the storage device. The two states are decoupled: This allows us to consider the state of charge as the only state variable in the energy management problem, while the vehicle speed is controlled independently. Faster dynamics, such as the speed transients of the various powertrain components, are neglected because they are much faster than the state of charge variations and do not affect the fuel consumption sensibly. This steady-state model of the powertrain is sufficient to represent with reasonable accuracy the overall efficiency and fuel consumption, yet simple enough to quickly simulate a complete driving cycle.

The HEV energy management problem can be cast into an optimal control problem since its objective is to minimize a performance index defined over an extended period of time (the trip or the driving cycle) by using a sequence of instantaneous control actions (the instantaneous values of power split). Several strategies have been proposed in literature to solve this problem (see, for example, the overviews in Refs. [1,3]). The following classification can be proposed based on the techniques used.

*Numerical methods for global optimization* assume the knowledge of the entire driving cycle and find the global optimal control numerically; dynamic programming (DP) [4–6] and numerical search methods (e.g., genetic algorithms [7]) belong to this category. These methods give the optimal solution over the prescribed driving cycle but are not implementable due to the necessity of knowing a priori the driving cycle.

*Numerical methods for local optimization* are based on optimization techniques similar to the previous category but consider a short-term optimization horizon extending into the future, during which the driving cycle is predicted. These techniques are implementable online but tend to require high computational capabilities. Among them are model predictive control [8–10] and stochastic dynamic programming [11–13], in which statistical methods are used to predict the most likely future driving cycle.

*Analytical optimization methods* consider the entire driving cycle but use an analytical problem formulation to find the solution in a closed, analytical form. They may also provide an analytical formulation that makes the numerical solution faster than the purely numerical methods, sometimes at the cost of oversimplifying the problem in order to obtain a suitable description. Among these methods are Pontryagin's minimum principle (PMP) [14,15] and the Hamilton–Jacobi–Bellmann equation.

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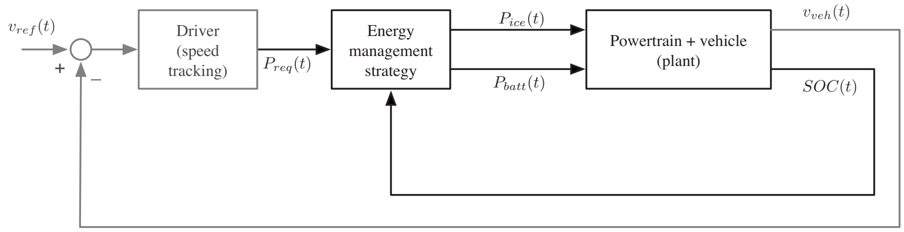


**Fig. 1 The role of energy management control in a hybrid electric vehicle**

*Instantaneous minimization methods* consist in the minimization, at each time step of the optimization horizon, of an appropriately defined instantaneous cost function. If the instantaneous cost function is suitably defined, the result is close to the global optimum. The most common of these strategies is the equivalent consumption minimization strategy (ECMS), originally introduced in Ref. [16] and further developed by several authors [17–19].

*Heuristic methods* do not involve explicit minimization or optimization; instead, the energy management is implemented with rules and algorithms based on engineering intuition. Rule-based control [20] and fuzzy logic [21,22] are part of this category. These strategies are robust and computationally efficient, requiring lower computational load than minimization-based methods. However, they may fail to fully exploit the potential of the hybrid electric architecture due to the lack of formal optimization. Blending heuristic and instantaneous minimization strategies has also been proposed [23].

This paper does not introduce new energy management strategies; instead, it aims at presenting a thorough formalization of the energy management problem and provides a comparative analysis of three techniques: DP, PMP, and ECMS. The first step for the study of energy management strategies is a suitable problem formulation, presented in Sec. 2. After the presentation of a case study in Sec. 3, DP is described in Sec. 4, PMP in Sec. 5, and ECMS in Sec. 6. The strategies are compared in Sec. 7, pointing out their similarities, with the objective of showing how an implementable strategy based on the ECMS framework can, in fact, be seen as the implementation of the optimal solution obtained with DP: The link is represented by the minimum principle, as shown in Sec. 8.

This link implies that strategies based on equivalent consumption minimization can be regarded as an implementation of the optimal solution to the energy management problem and therefore can be used with more confidence; furthermore, the coherent description of the optimal control problem and the ties between the three strategies allow for a more effective tuning of the ECMS.

Since this paper aims at giving a generic point of view on HEV control strategies, the theoretical results are generic and valid for any hybrid electric architecture. However, for illustrative purposes, numerical examples are developed using a series hybrid architecture as a case study. Also, for simplicity, we refer to batteries as energy storage device, but the same considerations remain true for other devices as well, for example, supercapacitors.

## 2 The Energy Management Problem

The optimal control problem in a hybrid electric vehicle consists in finding the sequence of controls $u(t)$ that leads to the minimization of the performance index $J$, defined as:

$$J(x(t_0),u(t),x(t_f)) = \phi(x(t_0),x(t_f)) + \int_{t_0}^{t_f} L(x(t),u(t),t)dt \quad (1)$$

where $t$ represents the time, $u(t)$ is the control action, $x(t)$ is the state variable, $[t_0,t_f]$ is the optimization horizon, $L(\cdot)$ is the instantaneous cost function, and $\phi(\cdot)$ is the terminal cost (cost due to the final value of the state). The optimal control law is denoted as $u^*(t)$, and the corresponding state trajectory is denoted as $x^*(t)$. By definition, the optimal control is such that

$$J(x(t_0),u(t),x(t_f)) \geq J(x(t_0),u^*(t),x^*(t_f)) \;\forall\; u(t) \neq u^*(t) \quad (2)$$

The state variable $x(t) \in \mathbb{R}$ is the battery state of charge, which is the amount of charge stored in the battery at a given moment, expressed as a fraction of the total amount of charge that the battery can accumulate.

$u(t) \in \mathbb{R}^m$ is the vector of control variables. The number $m$ of control variables in the energy management problem depends on the powertrain architecture and, in particular, on the number of energy paths between the energy sources and the wheels. A general definition of the control variables is

$$u(t) = \{P_{\text{batt}}(t), \rho_1(t), \ldots, \rho_{m-1}(t)\} \quad (3)$$

where $P_{\text{batt}}(t)$ is the total power output of the battery and $\rho_i(t)$ are additional variables that express how the battery power is split among the electric actuators if there is more than one degree of freedom. If the powertrain only has one degree of freedom, i.e., $m=1$, then $u(t)=\{P_{\text{batt}}(t)\}$.

The instantaneous cost is the fuel consumption: $L(\cdot)=\dot{m}_f(u(t))$. If pollutant emissions are also to be minimized, then $L(\cdot)$ can be defined as a weighted average of fuel and pollutant mass flow rates; similarly, if battery life is of concern, $L(\cdot)$ may include a term accounting for battery wear.

Considering a quasi-static engine model, the fuel consumption is only a function of the engine torque $T_{\text{ice}}(t)$ and speed $\omega_{\text{ice}}(t)$. Using a powertrain model, these variables are related to the control $P_{\text{batt}}(t)$, the driver's power demand $P_{\text{req}}(t)$, and the vehicle speed $v_{\text{veh}}(t)$ in order to express the fuel consumption as $\dot{m}_f(P_{\text{batt}}(t), P_{\text{req}}(t), v_{\text{veh}}(t))$.

The vehicle speed and the power demand are considered as measured external inputs, and the dynamics of the powertrain components are neglected because they are much faster than the battery state of charge dynamics and do not affect significantly the vehicle energy balance [24].

The constraints to which the optimization is subject are

(a) system dynamics
(b) initial state value
(c) terminal state value
(d) instantaneous state limitations
(e) instantaneous control limitations

*(a) System dynamics.* During the entire optimization horizon, the system evolves according to its dynamic equation

$$\dot{x}(t) = f(x(t),u(t)) \;\forall\; t \in [t_0 t_f] \quad (4)$$

In the HEV case, this equation represents the evolution of the battery state of charge as a function of the battery power. By definition, the variation of the SOC is proportional to the current at the battery terminals:

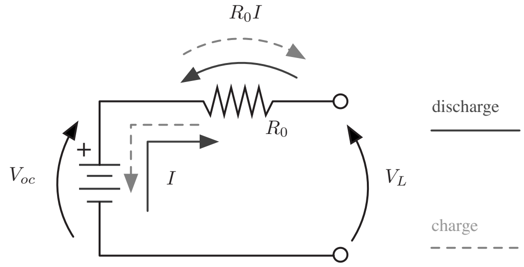


**Fig. 2 Simple circuit model of a generic battery**

$$\dot{x}(t) = -\frac{1}{Q_{\rm nom}} I(t) \tag{5}$$

where $I(t)$ is the current (positive during discharge) and $Q_{\rm nom}$ is the nominal charge capacity of the battery. The SOC variation can be expressed as a function of the battery power by using a simple circuit model, such as the one shown in Fig. 2. In this circuit,

$$V_L(t)I(t) = P_{\rm batt}(t) = V_{\rm oc}(x)I(t) - R_0(x)I^2(t) \tag{6}$$

where $V_{\rm oc}(x)$ is the open-circuit voltage of the battery, $R_0(x)$ is the equivalent internal resistance (both are functions of the state of charge), $V_L(t)$ is the load voltage at the terminals, and $I(t)$ is the current flowing at the terminals of the device.

Solving Eq. (6) for the current and replacing it into Eq. (5) yields

$$\dot{x}(t) = -\frac{1}{Q_{\rm nom}} \frac{V_{\rm oc}(x) + \sqrt{V_{\rm oc}^2(x) - 4R_0(x)P_{\rm batt}(t)}}{2R_0(x)} = f(x, P_{\rm batt}) \tag{7}$$

Equation (7) is the form of the state equation assumed in the rest of the paper. Note that any other form of the function $\dot{x} = f(x, P_{\rm batt})$ is equally acceptable since the conclusions drawn later in the paper do not depend on the specific expression of this equation. However, it is important to note the explicit dependence of the state equation on the value of the state itself through the open-circuit voltage $V_{\rm oc}$ and the internal resistance $R_0$. An example of their variation with state of charge is shown in Fig. 3, which is referred to the case study introduced in Sec. 3.

*(b) Initial state value.* The system state at the beginning of the optimization horizon must assume the initial value $x_0$:

$$x(t_0) = x_0 \tag{8}$$

*(c) Terminal state value.* The terminal value of the state must satisfy the equality constraint (*hard constraint*),

$$x(t_f) = x_f \tag{9}$$

For charge-sustaining vehicles, $x_f = x_0$, which is the terminal condition assumed in this work. In some cases, the terminal state constraints are not defined explicitly and are replaced by *soft constraints* introduced with the terminal cost $\phi(x(t_0), x(t_f))$ that appears in Eq. (1). This may be useful, for example, in plug-in hybrid vehicles, in which the battery can be discharged to a low state of charge value, and the terminal cost accounts for the cost of the electricity needed to recharge the battery. In this paper, however, we assume no terminal cost, i.e., $\phi(x(t_0), x(t_f)) = 0,\ \forall x(t_0), x(t_f)$.

*(d) Instantaneous state limitations.* At each time $t \in [t_0, t_f]$, the state of charge must remain within lower and upper bounds:

$$x_{\min} \le x(t) \le x_{\max} \tag{10}$$

For a more compact notation, the instantaneous state constraints can also be written as follows:

$$G(x(t),t) \le 0, \quad G(x(t),t) = \begin{cases} x(t) - x_{\max} \\ x_{\min} - x(t) \end{cases} \tag{11}$$

*(e) Instantaneous control limitations.* At each time, the control variable(s) must be in the set of admissible controls:

$$u(t) \in \mathcal{U}(t) \tag{12}$$

where $\mathcal{U} \subseteq \mathbb{R}^m$ is a compact set in all the control variables. The definition of the admissible control set $\mathcal{U}(t)$ is specific to each architecture: The general guidelines are that the controls must be such that the torque or power delivered by each machine does not exceed their intrinsic limitations, while at the same time, the total torque or power demand at the wheels is satisfied (to the highest degree possible).

At this point, the optimal control problem is completely defined, although not particularized explicitly for any specific architecture. The following sections show how three different methods can be used to solve it. A case study is first described, which will be used in the rest of the paper as an example of the explicit formulation of the problem and its constraints. The reason for the comparison of different strategies is to show the similarities between the different approaches, improve their understanding, and provide a method for the online implementation of the optimal solution to the problem.

## 3 Case Study

**3.1 Powertrain Architecture.** A series hybrid vehicle is presented as a simple case study to show the application of the strategy with a numerical example using a medium-size sport-utility vehicle (SUV) with Li-ion batteries. The powertrain architecture is shown in Fig. 4, and the data used for simulation are listed in Table 1.

The total power request $P_{\rm req}(t)$ is the sum of the power request from the traction motor, $P_{\rm em,e}(t)$, and the electrical accessories, $P_{\rm acc}(t)$:

$$P_{\rm req}(t) = P_{\rm em,e}(t) + P_{\rm acc}(t) \tag{13}$$

The power request is satisfied using either the generator or the battery, i.e.:

$$P_{{\rm gen},e}(t) + P_{\rm batt}(t) = P_{\rm req}(t) \tag{14}$$

The set of admissible values for the battery power is defined as the range that satisfies both the following conditions (i.e., the intersection of the two):

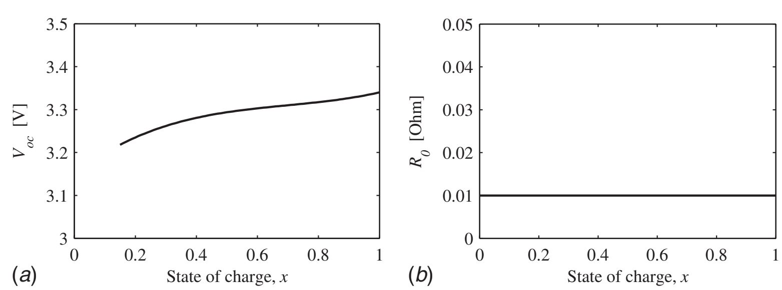


**Fig. 3 Effect of the state of charge on the circuit parameters of Fig. 2**

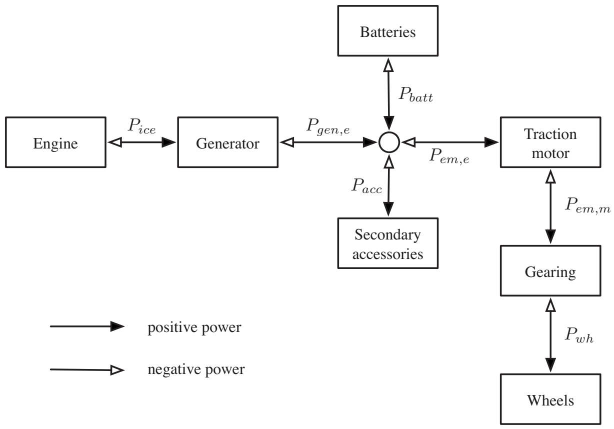


**Fig. 4 Series hybrid architecture used as application example**

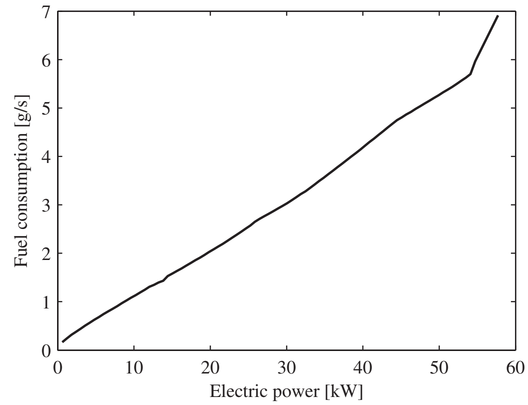


**Fig. 6 Fuel consumption of the genset as a function of net electric power generated (assuming operation along the optimal line OOL shown in Fig. 5)**

$$P_{\mathrm{batt,min}}(x) \le P_{\mathrm{batt}}(t) \le P_{\mathrm{batt,max}}(x) \tag{15}$$

$$P_{\mathrm{req}}(t) - P_{\mathrm{gen},e,\mathrm{max}}(t) \le P_{\mathrm{batt}}(t) \le P_{\mathrm{req}}(t) - P_{\mathrm{gen},e,\mathrm{min}}(t) \tag{16}$$

where $P_{\mathrm{batt,min}}(x)$ and $P_{\mathrm{batt,max}}(x)$ are the minimum (peak recharge) and maximum (peak discharge) power of the battery, which depend on its state of charge, and $P_{\mathrm{gen},e,\mathrm{max}}(t)$ and $P_{\mathrm{gen},e,\mathrm{min}}(t)$ are the maximum and minimum electric power that the generator can produce. The battery power is assumed to be positive during discharge and negative during recharge. The generator power is positive if generating and negative if motoring. However, the minimum generator power is set to zero because the generator cannot be used as a motor.

Since in a series HEV there is no mechanical connection between the engine and the wheels, the engine speed can be set independently from the vehicle speed. In principle, this means that the speed can be chosen as the value corresponding to the maximum efficiency for a given output power; i.e., the genset can be operated along its *maximum efficiency line* or *optimal operating line* (OOL), as shown in Fig. 5. This is a strong assumption but is justified by the fact that a quasi-static modeling approach is used, with relatively low time resolution: This means that the genset controller brings the machines at their speed setpoint in the time interval between successive simulation steps (1 s in this case).

**Table 1 Data used for simulation**

| | |
|---|---|
| Vehicle mass | 2000 kg |
| Genset maximum power | 60 kW |
| Motor maximum power | 110 kW |
| Battery maximum power | 110 kW |
| Battery energy capacity | 3.5 MJ |

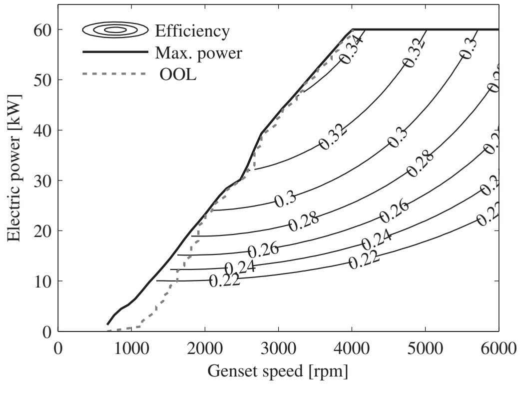


**Fig. 5 Efficiency map of engine-generator set**

Under this condition, the fuel consumption is only a function of the net electric power produced by the generator, as shown in Fig. 6:

$$\dot{m}_f = \dot{m}_f(P_{\mathrm{gen},e}) = \dot{m}_f(P_{\mathrm{req}} - P_{\mathrm{batt}}) \tag{17}$$

## 4 Numerical Optimization: Dynamic Programming

DP generates a numerical solution to the optimal control problem defined in Sec. 2. In other words, it gives sufficient conditions for the global optimality. DP is based on Bellman's principle of optimality:

> An optimal control policy has the property that no matter what the previous decisions (i.e., controls) have been, the remaining decisions must constitute an optimal policy with regard to the state resulting from those previous decisions [25].

In order to apply DP, the system dynamics is written in a discrete-time form, and the state and control domain are also discretized; thus, a finite (albeit very large) number of possible solutions are considered. Let us define the control policy $\pi$ as the sequence of controls during the optimization horizon: $\pi = \{u_0, u_1, \ldots, u_{N_t-1}\}$, where $N_t$ is the number of time steps in which the optimization horizon is subdivided and a subscript indicates the time instant. The total cost of the control policy $\pi$ is

$$J_0(\pi) = \phi(x_N) + \sum_{0}^{N-1} L(x_k, u_k, t_k) \tag{18}$$

and the optimal policy is

$$\pi^* = \arg\min_{\pi} J_0(\pi) \tag{19}$$

The cost-to-go $J_k(\pi)$ is defined as the cost incurred in moving from the time step $k$ to the end of the optimization horizon, following the policy $\pi$. Bellman's principle is applied by computing the optimal cost-to-go, $J_k^*$, in an iterative fashion, starting from the final instant of the optimization horizon. At $t=t_N$:

$$J_N^*(x) = \phi(x_N) \tag{20}$$

and then for each instant $t_k$, $k=N-1, N-2, \ldots, 0$:

$$J_k^*(x) = J_{k+1}^*(x) + \min_{u_k \in \mathcal{U}_k} L(x_k, u_k, t_k) \tag{21}$$

where $x_k$ and $u_k$ are the values of the state and control at time $t_k$, and $J_k^*$ is the value of the optimal cost-to-go at the same time. The optimal cost-to-go is computed for each value of the state in the admissible range. The control policy that generates the optimal cost-to-go at a given time is $\pi_k^*(x_k)$. After the backward iterations have been completed, the law $\pi_k^*(x_k)$ is defined for each value of time and is used to compute the optimal control sequence starting from the initial time and state values using a forward iteration of the algorithm. For a more detailed description of the DP algorithm, see Ref. [26] or [27].

DP is capable of determining the optimal solution to the discretized problem. This solution is generally suboptimal for the continuous problem because of the approximation introduced with the discretization; however, if the grid is fine enough, the approximation is negligible. The need for a backward procedure means that the solution can be obtained only offline, for a driving cycle known a priori, and therefore it is not possible to use DP for an online implementable solution. Furthermore, the high computational load makes any DP optimization prohibitive on typical onboard microcontrollers.

## 5 Analytical Optimization: Pontryagin's Minimum Principle

Pontryagin's minimum principle [28] states that if the control law $u^*(t)$ is optimal for the problem defined in Sec. 2, the following conditions are satisfied.

1. $u^*(t)$ minimizes at each instant the Hamiltonian of the optimal control problem:

$$H(x(t),u(t),t,\lambda(t)) \geq H(x(t),u^*(t),t,\lambda(t)), \qquad \forall\, u \neq u^*$$

where the Hamiltonian is defined as

$$H(x(t),u(t),t,\lambda(t)) = \lambda^T(t)\cdot f(x(t),u(t)) + L(x(t),u(t),t)$$

with $f(x(t),u(t))$ being the system dynamic equation, $L(x(t),u(t))$ being the instantaneous cost, and $\lambda(t)$ being a vector of auxiliary variables called *co-states* of the system. $\lambda$ has the same dimension as the state vector $x$ and therefore is a scalar in our problem.

2. The co-state variable satisfies the following dynamic equation:

$$\dot{\lambda}(t) = -\frac{\partial H(x(t),u(t),t,\lambda(t))}{\partial x} \tag{22}$$

The conditions given by the minimum principle are *necessary*, not sufficient. Every solution that satisfies the necessary conditions is called an *extremal* solution. If the optimal solution exists, then it is also extremal. In general, the opposite is not true: A solution may be extremal without being optimal. However, if the problem has a unique optimal solution and the application of the minimum principle gives only one extremal solution, then this is the optimal solution.

In practical applications, the minimum principle can be used to find solution candidates by computing and minimizing the Hamiltonian function at each instant, which generates, by construction, extremal controls. If the Hamiltonian is a convex function of the control, then there is only one extremal solution, which is therefore optimal.

In the HEV energy management problem, the Hamiltonian is

$$H(x(t),P_{\mathrm{batt}}(t),\lambda(t)) = -\lambda(t)\cdot f(x(t),P_{\mathrm{batt}}(t)) + \dot{m}_f(P_{\mathrm{batt}},P_{\mathrm{req}}(t)) \tag{23}$$

where $f(x,P_{\mathrm{batt}})$ is given by Eq. (7), and the control $P_{\mathrm{batt}}(t)$ is obtained at each instant as the value that minimizes Eq. (23):

$$P^*_{\mathrm{batt}}(t) = \arg\min_{P_{\mathrm{batt}}} H(x(t),P_{\mathrm{batt}}(t),\lambda(t)) \tag{24}$$

The co-state variable $\lambda(t)$ appearing in Eq. (23) is obtained from the dynamic Eq. (22), which, considering that $\dot{m}_f(P_{\mathrm{req}},P_{\mathrm{batt}})$ is not a function of the state of charge $x$, becomes

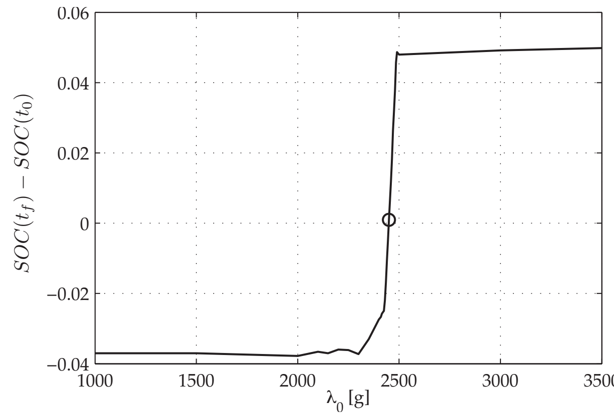


**Fig. 7 Effect of initial co-state value on SOC variation**

$$\dot{\lambda}(t) = -\lambda(t)\frac{\partial f(x(t),u(t))}{\partial x} \tag{25}$$

Note that the co-state variation is different from zero only if the system dynamic equation $f(x(t),u(t))$ is indeed a function of the state itself. As mentioned in Sec. 2, in the case of a battery this is true in the measure that the open-circuit voltage $V_{\mathrm{oc}}$ and the internal resistance $R_0$ depend on the state of charge. Their variation is typically limited, especially in charge-sustaining HEVs where the battery is used in a narrow SOC range: Therefore, the co-state variation is also very small and sometimes completely neglected.

Equation (25), together with Eq. (7), represents a system of two differential equations with two variables, $x(t)$ and $\lambda(t)$. The solution requires two boundary conditions. For the problem definition considered in Sec. 2, these are the initial and final values of the state: $x(t_0)=x_0$ and $x(t_f)=x_f$.

Despite being completely defined, this two-point boundary value problem can be solved numerically only using an iterative procedure because one of the boundary conditions is defined at the final time. The procedure is known as the *shooting method* and consists in replacing the two-point boundary value problem with a conventional initial-condition problem, starting from an initial guess for $\lambda(t_0)$. The solution of the problem is then obtained by integration in time of Eqs. (25) and (7), replacing at each time the value of $P_{\mathrm{batt}}$ resulting from the minimization (Eq. (24)). If the final value of the state does not match the desired terminal condition $x^*(t_f)=x_f$, the value of $\lambda(t_0)$ is adjusted iteratively until the terminal condition on the state is met. A bisection procedure can be used to obtain convergence in few iterations, making the minimum principle sensibly faster than dynamic programming. The solution is very sensitive to the initial co-state value, as shown in Fig. 7.

The existence and uniqueness of the solution cannot be proved formally in the general case, but it is reasonable to assume that at least one optimal solution exists for the energy management problem in the sense that there must necessarily be at least one sequence of controls giving the lowest possible fuel consumption. If the minimum principle generates only one extremal solution, that can be considered the optimal solution; if there is more than one extremal solution, they are all compared (i.e., the total cost resulting from the application of each is evaluated) and the one yielding the lowest total cost is chosen.

## 6 Real-Time Control: ECMS

The ECMS was introduced by Paganelli et al. [16,29] as a method to reduce the global optimization problem to an instantaneous minimization problem to be solved at each instant, without use of information regarding the future.

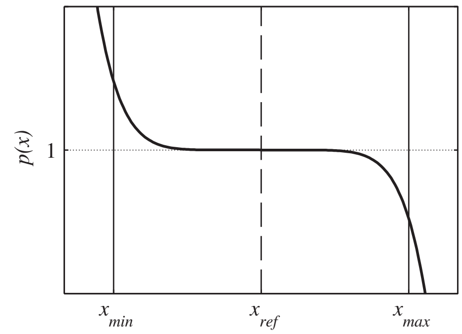


**Fig. 8 SOC correction term for ECMS, $p(x)$**

This strategy is based on the concept that in charge-sustaining vehicles, the battery is used only as an energy buffer, and all the energy ultimately comes from fuel. Thus, the battery can be seen as an auxiliary, reversible fuel tank that is never refilled using energy from outside the vehicle. In order to keep the vehicle charge-sustaining, the electricity used during the battery discharge phase must be replenished later using the fuel from the engine (either directly or indirectly through a regenerative path). In both charge and discharge phases, a virtual fuel consumption can be associated with the use of electrical energy and can be summed to the actual fuel consumption to obtain the instantaneous *equivalent* fuel consumption,

$$\dot{m}_{\text{eqv}}(t) = \dot{m}_f(t) + \dot{m}_{\text{batt}}(t) = \dot{m}_f(t) + \frac{s}{Q_{\text{lhv}}} P_{\text{batt}}(t) \cdot p(x) \qquad (26)$$

where $\dot{m}_f(t)$ is the engine instantaneous fuel consumption, $Q_{\text{lhv}}$ is the fuel lower heating value (energy content per unit of mass), $\dot{m}_{\text{batt}}(t)$ is the virtual fuel consumption associated with the use of the battery, $P_{\text{batt}}(t)$ the battery power, and $p(x)$ is a correction function that takes into account the deviation of the current SOC from the reference (nominal) SOC, as shown in Fig. 8. The factor $s$ is called *equivalence factor* and is used to convert electrical power into equivalent fuel consumption; it plays an important role in the ECMS, as will be shown later. Depending on the sign of $P_{\text{batt}}(t)$ (i.e., on whether the battery is charged or discharged), the virtual fuel flow rate can be either positive or negative; therefore, the equivalent fuel consumption can be higher or lower than the actual fuel consumption. The correction term $p(x)$ is shown in Fig. 8: Its value is unitary if the SOC is at the reference value $x_{\text{ref}}$, but it changes if the SOC becomes higher or lower, in such a way as to compensate for the deviation from the condition $x = x_{\text{ref}}$. In fact, $p(x) < 1$ when $x > x_{\text{ref}}$, which means that a lower cost is attributed to the battery energy, thus making the discharge more likely when the SOC is above the reference value. On the other hand, $p(x) > 1$ when $x < x_{\text{ref}}$ (SOC below reference value): In this condition, the cost of battery energy is increased to make its discharge less likely.

At each time, the equivalent fuel consumption is calculated using Eq. (26) for several candidate values of the control variable $P_{\text{batt}}(t)$; the value that gives the lowest equivalent fuel consumption is selected.

In standard ECMS, the equivalence factor is a constant, or rather a set of constants: Because it represents the chain of efficiencies through which fuel is transformed into electrical power and vice versa, it changes for each operating condition of the powertrain. In particular, there are at least two equivalence factors, one to apply during battery charge, and another during battery discharge; there can be more if the powertrain has more than one mode of operation (for example, if it is a series/parallel architecture). In each mode, the equivalence factor can be interpreted as the average overall efficiency of the electric path during a specific driving cycle.

The values of the equivalence factors affect the vehicle fuel consumption and the trend of the battery state of charge. The battery tends to be discharged if the equivalence factor is too low (charge-depleting behavior) or to be charged if it is too high (charge-increasing behavior). In order to obtain a charge-sustaining solution and to minimize the total fuel consumption during a driving cycle, it is necessary to tune all the equivalence factors for the specific driving cycle. For example, in the series HEV case study, it is possible to define charge and discharge equivalence factors ($s_{\text{chg}}$ and $s_{\text{dis}}$), corresponding respectively to negative and positive values of battery power $P_{\text{batt}}$. The effect that these have on the fuel consumption and the charge sustainability of the solution is shown in Fig. 9.

## 7 Strategy Comparison

A comparison of the results of all three strategies is shown in Fig. 10 using the regulatory driving cycle U.S.06 as a test.

By observing the SOC profile in Fig. 10(*b*), it is apparent that the dynamic programming and the minimum principle give essentially the same solution, with only minor differences (justified by slightly different implementation details, such as the discretization steps for the control variable). The global nature of the DP and PMP solutions is evident in the trend of the state of charge, which first increases, then decreases steadily during the high-power phase of the cycle, and finally increases to reach just the desired terminal value. By contrast, the ECMS solution, even though perfectly charge-sustaining by virtue of the appropriate choice of equivalence factors (point A in Figs. 9(*a*) and 9(*b*)), is clearly different from the formally optimal solution.

The difference between the results obtained with the three strat-

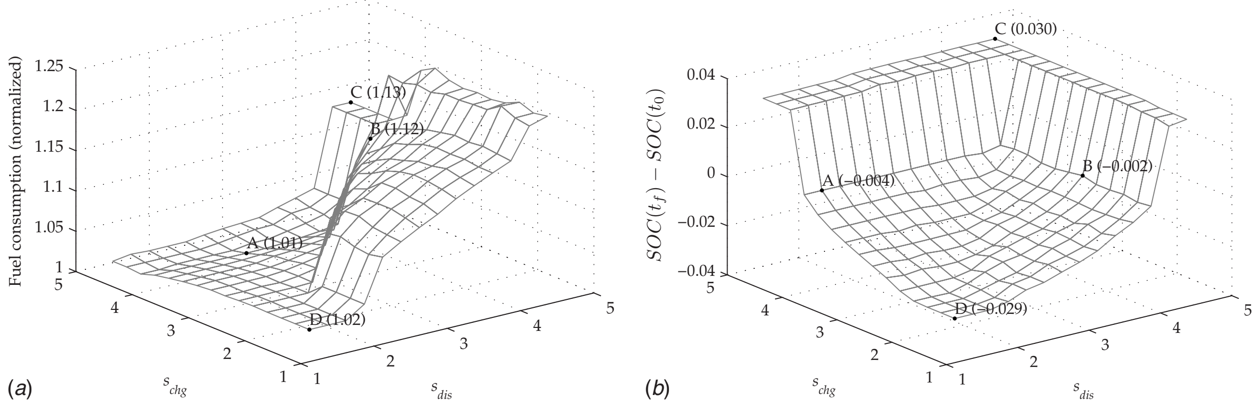


**Fig. 9 Effect of ECMS equivalence factors**

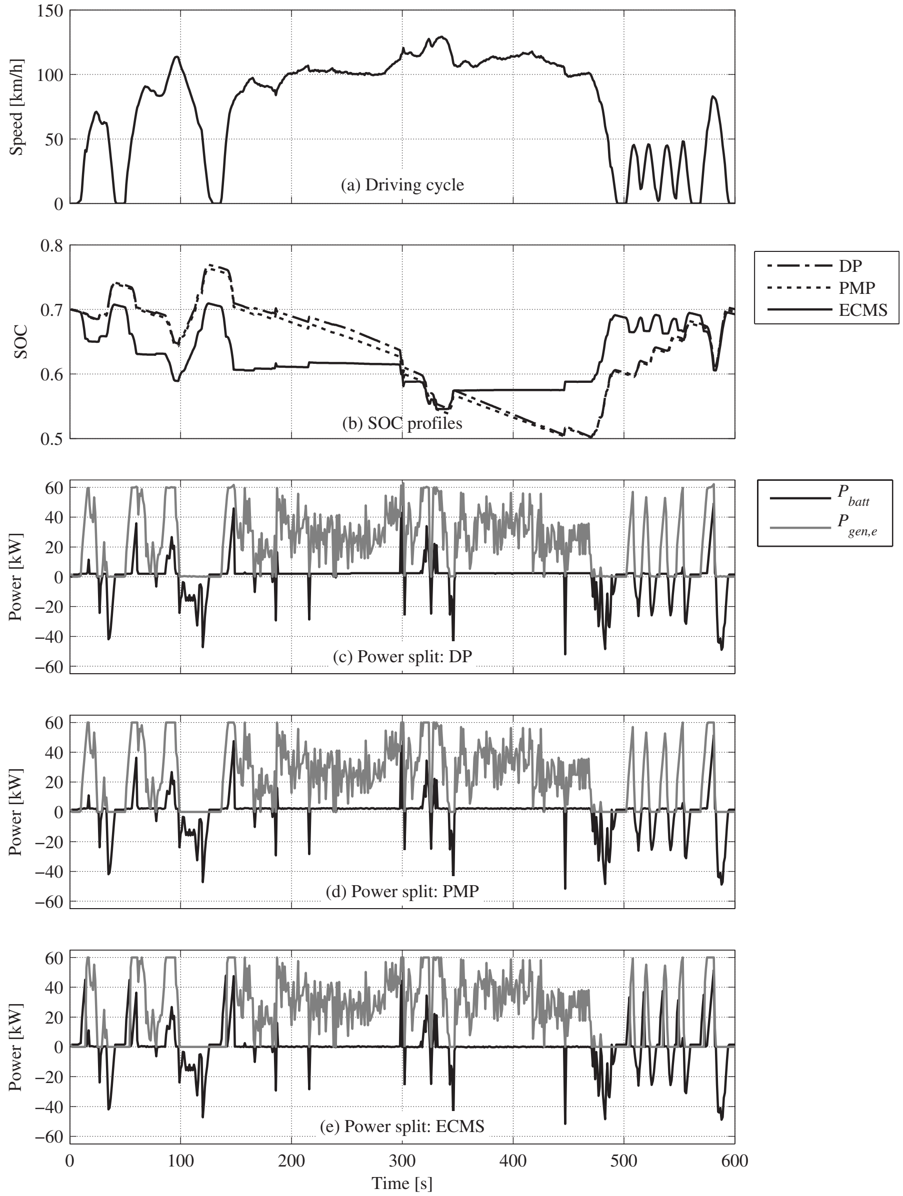


**Fig. 10 Comparison of the three strategies on the cycle U.S.06**

egies can be better appreciated by looking at the comparison of the battery power $P_{\mathrm{batt}}(t)$ and genset power $P_{\mathrm{gen}}(\mathrm{t})$ in the three cases (Figs. 10(*a*)–10(*c*)). While DP and PMP tend to steadily discharge the battery at constant power (in the order of 2 kW) for an extended period of time (for example, in the interval between $t=150$ s and $t=400$ s), the ECMS tends to discharge it at the beginning of an acceleration phase, and then—when a low SOC level is reached and the correction function $p(x)$ becomes important—it stops discharging, and only the genset is used to satisfy the power demand. This is especially visible in the first 60 s and in the time interval between 500 s and 560 s. This behavior is clear evidence of the local nature of the ECMS solution, which tends to react in a similar way to similar power demand trends, as opposed to DP, which, by using the a priori knowledge of the entire cycle, can modify the power split trends in different phases of the cycle to achieve the global optimum. It is significant to observe how PMP, when tuned with the appropriate co-state value, can give a solution practically identical to DP despite being implemented with an instantaneous minimization. Of course, the globally optimum behavior of PMP is achievable only with optimal tuning of the co-state, which is done iteratively offline and requires the knowledge of the driving cycle. In all three cases, the overall fuel consumption is very close, as shown in Table 2.

## 8 Optimal ECMS Based on PMP

Observing the ECMS equivalent fuel consumption (Eq. (26)) and the Hamiltonian (Eq. (23)), the two appear to be very similar. In fact, both are the sum of two terms, one of which is the instantaneous cost, i.e., the fuel consumption $\dot{m}_f$, while the other is proportional to battery power (in ECMS) or SOC derivative (in

**Table 2 Fuel consumption for the three strategies. All values are normalized with respect to the DP solution.**

| Driving cycle | DP | PMP | ECMS |
|---|---|---|---|
| UDDS | 1 | 1.000 | 1.017 |
| U.S.06 | 1 | 1.001 | 1.017 |
| FTP highway | 1 | 1.000 | 1.009 |

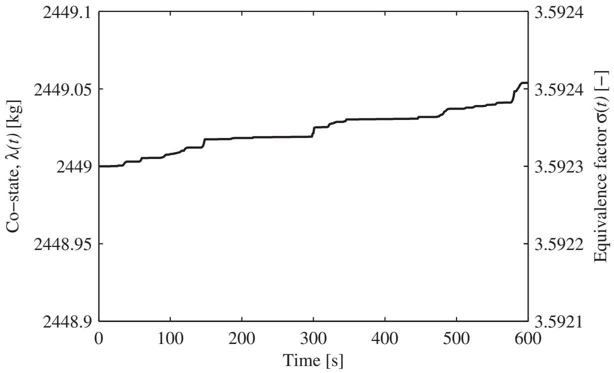


**Fig. 11 Evolution of $\lambda(t)$ and $\sigma(t)$ during the driving cycle of Fig. 10 (note the small scale of the vertical axis)**

PMP). By comparing these two terms, it is possible to rewrite the PMP expression in order to match the ECMS physical meaning as

$$\dot{m}_{\text{eqv}} = \dot{m}_f + \sigma(t)\frac{f(x,P_{\text{batt}})E_{\text{batt}}}{Q_{\text{lhv}}}p(x) \tag{27}$$

where

$$\sigma(t) = -\lambda(t)\frac{Q_{\text{lhv}}}{E_{\text{batt}}} \tag{28}$$

is an a-dimensional equivalence factor, just like $s$ in Eq. (26).

The equivalence factor $\sigma(t)$ in expression (27) is valid for all the powertrain operating conditions, without distinction between charge and discharge. In fact, instead of using a set of equivalence factors to represent the average efficiencies in different operating conditions, an explicit model of the power flow is introduced with the function $f(x,P_{\text{batt}})$, covering the entire operating range. This is true also for powertrain architectures including several operating modes since the different efficiency characteristics of each mode are included in the function $f(x,P_{\text{batt}})$ and in the relation between $P_{\text{batt}}$ and the powertrain output (e.g., torque at the wheels). The equivalence factor is therefore a tuning parameter related to the driving cycle, being the same as the co-state of the minimum principle. Since $\sigma(t)$ is directly proportional to the co-state $\lambda(t)$, its variation during a trip can be modeled using the co-state differential Eq. (25), thus reducing the tuning to the determination of its initial value.

Reformulated in this way, the ECMS becomes the implementation of the optimal solution of PMP [30], giving results identical to PMP and therefore very close to the DP optimal solution, with an improvement with respect to the traditional ECMS. Moreover, it is more easily tuned due to the fact that there is only one free parameter, $\sigma$. In particular, the tuning parameter is the initial value of the equivalence factor, $\sigma_0=\sigma(t_0)$, while the instantaneous value is obtained by the integration of Eq. (25). However, the variation of the co-state during a driving cycle is very small, as shown in Fig. 11, and therefore the equivalence factor $\sigma$ can be considered constant with very good approximation:

$$\sigma(t) \simeq \sigma_0$$

## 9 Online Implementation

The three strategies presented are not directly implementable online because they require complete knowledge of the driving cycle for explicit minimization (DP) or appropriate tuning (PMP and ECMS). Furthermore, the computational load of DP makes its real-time implementation impossible. On the other hand, the ECMS has the potential of being implemented online because it uses an instantaneous minimization. However, for obtaining optimal results, the ECMS must be tuned for each driving cycle to find the most appropriate value of the equivalence factor $\sigma_0$.

The presence of the correction term $p(x)$ in Eq. (27) makes the ECMS solution less likely to reach extremely high or low SOC values but is still not sufficient to guarantee that the state of charge remains around the nominal level, and the solution is close to the optimum. A higher correction achieves the first objective at the expense of the optimality of the solution, while a lower correction allows for a solution closer to the optimum given by PMP, but in which charge sustainability is achieved only with accurate tuning of $\sigma_0$.

In order to realize an implementable strategy that is also close to the optimal solution, it is necessary to adapt online the value of the equivalence factor using adaptive-ECMS (A-ECMS) algorithms.

In A-ECMS, the nominal value of equivalence factor $\sigma_0$ is changed during vehicle operation, according to an adaptation law. Some methods to achieve this are feedback of the state of charge to ensure charge sustainability [31], pattern recognition algorithms to identify the type of driving cycle the vehicle is following and to select the optimal $\sigma_0$ from a precomputed database [19], some form of driving cycle prediction to attempt a priori determination of the most appropriate equivalence factor during short-term future [18].

The approach presented in this paper is useful to devise more effective A-ECMS approaches, thanks to a deeper understanding of the meaning of the equivalence factor. These approaches are object of current research by the authors.

A different method to implement an optimal solution originated from DP is to derive heuristic rules from the observation of the optimal solution [32], which can be easily coded to be executed in real time. This approach is also currently under investigation by the authors and leads to the design of a rule-based strategy.

## 10 Conclusion

The application of three energy management strategies to a hybrid electric vehicle has been presented, showing how the optimal solution obtained with DP can also be computed by applying PMP, which in turn leads to a real-time, implementable strategy in the form of an appropriately formulated ECMS, presented in Sec. 8. Despite still requiring ad hoc tuning for the specific driving cycle, this ECMS results in a more effective (i.e., closer to the optimum) strategy, which can also be tuned more easily and rapidly because it only needs one tuning parameter (even for complex hybrid architectures with several operating modes). The development of adaptive ECMS strategies is also facilitated due to a more fundamental understanding of the equivalence factor.

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