A control-oriented lithium-ion battery pack model for plug-in hybrid electric vehicle cycle-life studies and system design with consideration of health management

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HIGHLIGHTS

- Control-oriented methodology to model aging propagation in battery packs.
- Systematic methodology for system-level SOH assessment of battery systems.
- Pack SOH based on individual cells SOH, electrical topology and equalization approach.
- Understanding of lithium-ion battery pack aging under realistic PHEV operation.
- Capacity and power fade of battery packs containing cells with NMC-LMO cathodes.

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ABSTRACT

A crucial step towards the large-scale introduction of plug-in hybrid electric vehicles (PHEVs) in the market is to reduce the cost of its battery systems. Currently, battery cycle- and calendar-life represents one of the greatest uncertainties in the total life-cycle cost of battery systems. The field of battery aging modeling and prognosis has seen progress with respect to model-based and data-driven approaches to describe the aging of battery cells. However, in real world applications cells are interconnected and aging propagates. The propagation of aging from one cell to others exhibits itself in a reduced battery system life. This paper proposes a control-oriented battery pack model that describes the propagation of aging and its effect on the life span of battery systems. The modeling approach is such that it is able to predict pack aging, thermal, and electrical dynamics under actual PHEV operation, and includes consideration of random variability of the cells, electrical topology and thermal management. The modeling approach is based on the interaction between dynamic system models of the electrical and thermal dynamics, and dynamic models of cell aging. The system-level state-of-health (SOH) is assessed based on knowledge of individual cells SOH, pack electrical topology and voltage equalization approach.

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1. Introduction

A crucial step towards the large-scale introduction of plug-in hybrid electric vehicles (PHEVs) in the market is to reduce the cost of their energy storage devices. Lithium-ion (Li-ion) batteries are the preferred energy storage technology in PHEVs due to their high energy and power density [1]. One of the goals of US Department of Energy (DOE) Vehicle Technologies Program for hybrid electric systems is to, by 2022, reduce the production cost of Li-ion batteries by nearly 75% from 2012 costs. Currently, battery cycle and calendar life represents one of the greatest uncertainties in the total life-cycle cost of advanced energy storage systems [2].

A battery pack in a PHEV is a collection of modules, which are in turn made up of series/parallel combinations of individual cells. Cells are electrically connected in parallel to satisfy high capacity requirements and in series to provide the desired system voltage. Fig. 1(c,d) depicts a sketch of two possible electrical configurations [3]. The one on the left is termed PS because it consists of a parallel string of \( n_p \) cells in series. The one on the right is termed SP because it consists of \( n_e \) elements in series, each consisting of \( n_e \) elements.

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1 Part of this research was conducted at the time she was Research Scientist at OSU-CAR.
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>heat transfer surface area ([\text{m}^2])</td>
</tr>
<tr>
<td>Ah</td>
<td>total ampere-hour throughput ([\text{Ah}])</td>
</tr>
<tr>
<td>(c_p)</td>
<td>specific heat capacity at constant pressure (\text{[J g}^{-1} \text{K}^{-1}])</td>
</tr>
<tr>
<td>C</td>
<td>electrical capacitance ([\text{F}])</td>
</tr>
<tr>
<td>(d_h)</td>
<td>hydraulic diameter ([\text{m}])</td>
</tr>
<tr>
<td>(E_{ac})</td>
<td>cell activation energy for capacity fade (\text{[J mol}^{-1}])</td>
</tr>
<tr>
<td>(E_{air})</td>
<td>cell activation energy for resistance growth (\text{[J mol}^{-1}])</td>
</tr>
<tr>
<td>(h)</td>
<td>convective heat transfer coefficient (\text{[W m}^{-2} \text{K}^{-1}])</td>
</tr>
<tr>
<td>I</td>
<td>current ([\text{A}])</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity (\text{[W m}^{-1} \text{K}^{-1}])</td>
</tr>
<tr>
<td>L</td>
<td>channel length ([\text{m}])</td>
</tr>
<tr>
<td>m</td>
<td>mass ([\text{g}])</td>
</tr>
<tr>
<td>(Nu)</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>(Pr)</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>(q_{air})</td>
<td>volumetric flow rate of air moving through channel (\text{[m}^{-3} \text{s}^{-1}])</td>
</tr>
<tr>
<td>Q</td>
<td>heat transfer rate ([\text{W}])</td>
</tr>
<tr>
<td>R</td>
<td>thermal resistance (\text{[KW}^{-1}]) or Electrical resistance ([\text{[Ohm]}])</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>(R_g)</td>
<td>Universal gas constant, 8.314 ([\text{JK}^{-1} \text{mol}^{-1}])</td>
</tr>
<tr>
<td>(R_{int})</td>
<td>battery internal resistance increase ([%])</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density (\text{[g}^{-3} \text{m}^{-1}])</td>
</tr>
<tr>
<td>S</td>
<td>battery capacity ([\text{Ah}])</td>
</tr>
<tr>
<td>(S_{loss})</td>
<td>capacity loss ([%])</td>
</tr>
<tr>
<td>T</td>
<td>temperature ([\text{°C}]), ([\text{K}])</td>
</tr>
<tr>
<td>(u_{air})</td>
<td>velocity of main airflow ([\text{m s}^{-1}])</td>
</tr>
<tr>
<td>v</td>
<td>kinematic viscosity (\text{[m}^{2} \text{s}^{-1}])</td>
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<tr>
<td>V</td>
<td>voltage ([\text{V}])</td>
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Subscripts

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<th>Symbol</th>
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<tbody>
<tr>
<td>air</td>
<td>air</td>
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<tr>
<td>c</td>
<td>battery cell</td>
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<tr>
<td>C</td>
<td>capacity fade</td>
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<td>cc</td>
<td>conduction among battery cells</td>
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<tr>
<td>ch</td>
<td>cooling channel</td>
</tr>
<tr>
<td>i</td>
<td>battery cell or cooling channel index number</td>
</tr>
<tr>
<td>(j, k)</td>
<td>battery cell index pair</td>
</tr>
<tr>
<td>(ku)</td>
<td>surface convection</td>
</tr>
<tr>
<td>oc</td>
<td>open circuit</td>
</tr>
<tr>
<td>p</td>
<td>string of cells</td>
</tr>
<tr>
<td>(P)</td>
<td>battery pack</td>
</tr>
<tr>
<td>R</td>
<td>resistance growth</td>
</tr>
<tr>
<td>s</td>
<td>element of cells in parallel</td>
</tr>
<tr>
<td>u</td>
<td>convection</td>
</tr>
<tr>
<td>0</td>
<td>initial or nominal conditions</td>
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Superscripts

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(T)</td>
<td>transpose</td>
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</table>

Temperature and its gradient within the battery system strongly affect battery performance, life and safety [4]. Therefore, a battery pack includes a battery thermal management system (BTM) to keep the battery at optimal average temperature, minimizing temperature differences among cells [5]. BTMs systems include: air-cooling BTM is chosen for the model development. Under forced convection condition, cooling air is drawn into the battery pack through the inlet and guided into the cooling channels surrounding the cells, deflector plates and cooling fan, etc. Under forced convection condition, cooling air is drawn into the battery pack through the inlet and guided into the cooling channels by the lower air deflector. Similarly, the heated air is guided out the battery pack through the upper air deflector.

Cell parameters such as battery capacity and internal resistance, varies from cell-to-cell due to manufacturing variability, different operational conditions and aging [13], [14]. In a battery pack, cell-to-cell imbalances such as voltage, capacity and state-of-charge (SOC), negatively affect the available energy of a battery system reducing their performance and life [15–17]. Therefore, a battery pack typically includes a voltage equalizer (VE). VE strategies include: active equalization techniques (AE), in which the voltages are balanced by transferring the excess of stored energy to other cells [18–21]; and passive equalization techniques (PE), in which the excess of energy is dissipated in the form of heat [15], [22].

The United States Advanced Battery Consortium (USABC) defines two operational modes for PHEVs: Charge-Depleting (CD) and Charge-Sustaining (CS) [1]. During CD the battery is depleted starting from a battery SOC of SOC\(_{\text{max}}\) and until reaching a pre-defined SOC\(_{\text{min}}\). During CS the battery SOC is kept within a window UEF\(_{CS}\) with an average value of SOC\(_{\text{min}}\) [1], see Fig. 3. The Ratio of CD to the total operating time is defined as the ratio of \(t_{\text{CD}}\) to \((t_{\text{CD}} + t_{\text{CS}})\):

\[
\text{Ratio} = \frac{(t_{\text{CD}} + t_{\text{CS}})}{t_{\text{CD}}} = \frac{t_{\text{CD}}}{t_{\text{CD}} + t_{\text{CS}}}
\]

which indicates the fraction of time spent in CD mode over the total operation time [23]. Therefore, Ratio = 1 corresponds to CD operation. Ratio = 0 corresponds to CS operation. Ratios such that 0 < Ratio < 1 correspond to mixed operation i.e. the total operating time is divided between CD and CS. For example, the SOC profile shown in Fig. 3 corresponds to mixed operation with a Ratio of 1/2.

Battery charging is typically done through CC-CV protocol [24]. That is, a constant current (CC) is used until the battery voltage reaches a predetermined limit, followed by a constant voltage (CV) until the current declines to a predetermined value. In this work we express the current in terms of C-rate. We refer to the CC expressed in C-rate as charging rate (CR).

Aging is the reduction in performance, availability, reliability, and life span of a system or component. Generally, battery aging manifest itself in a reduction in the ability to store energy and deliver power, performance metrics correlated with loss in capacity and increase in internal resistance [25,26]. Among the micro-mechanisms of Li-ion battery aging we cite active particle loss and metal sediment or SEI film accumulation. A review of today’s knowledge on the mechanics of aging in Li-ion batteries can be found in Refs. [25,27]. These physical-chemical mechanisms are enhanced by stress factors [25,26]. The stress factors for PHEV

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\(^{2}\) A C-rate is a measure of the rate at which a battery is charged/discharged relative to its maximum capacity. Operationally, C-rate = \(i(t)/S_0\), where \(i(t)\) is battery input current and \(S_0\) is the battery nominal capacity.
The state-of-health (SOH) of a battery cell, which is used to describe its physical condition, is commonly characterized by a cell parameter that is correlated with its aging. Depending on the application, the SOH of a battery cell may be characterized by loss in capacity, increase in internal resistance, or a combination of both. The capacity and internal resistance are cell parameters correlated with the ability to store energy and deliver power respectively. In this work, we refer as capacity state of health (SOH) to the figure of merit that is correlated with the battery’s ability to store energy; and, as power state of health (SOH) to the figure of merit that is correlated with the battery’s ability to deliver power.

In a battery pack the ability to store energy and deliver power, and therefore the system level SOH, highly depends on individual cell performances and SOH, electrochemical topology, and voltage equalization. From the best of our knowledge there is no work available in the literature that deals with the system level SOH assessment of interconnected systems and in particular its application to advanced battery systems.

Battery aging models are able to predict capacity and/or power loss in response to operating conditions such as C-rate, temperature, SOC. The field of battery aging modeling, has seen progress with respect to physics-based and semi-empirical models to describe the aging of cells. However, in real world applications battery cells are interconnected and aging propagates. The propagation of aging from one cell to others exhibits itself in a reduced battery system life span. Aging propagation has a profound effect on the accuracy of battery systems SOH assessment and prognosis.

The field of battery pack modeling is sparse. Although the field of battery modeling has seen progress in recent years, describing the battery pack life remains an open problem. In particular a model that enables the prediction of battery electro-thermal performance and life. Despite the fact that this topic has been pointed out as critical for the field of lithium-ion batteries, the problem has not yet been explored in the available literature. The complexities of battery systems make the development of efficient models and simulation tools challenging.

This paper proposes a systematic methodology for modeling the propagation of aging in advanced battery systems. The modeling approach is such that is a control-oriented multi-time scale modeling that enables the prediction of electro-thermal dynamics, aging dynamics while considering electrical topology and cell-to-cell variability. The proposed model allows for a more complete understanding of the impacts and trade-offs of cell-to-cell variability, electrical topology and battery thermal management, on battery performance and life under actual PHEV operation. The model may be also used for verification and validation of control algorithms such as estimation and identification, in particular for battery management systems including health management. This modeling approach constitutes the first step towards an integrated system design with ‘a priori’ consideration of health management.

2. Battery cell model

Fig. 5 shows a control oriented block diagram of the model of a single cell subject to aging. The model is composed of three interconnected submodels: the electrical submodel is used to predict battery cell voltage and SOC in response to current and temperature; the thermal submodel is used to predict the cell temperature in response to current, voltage and ambient temperature; and the aging submodel to predict capacity and power fade in response to the cell operating conditions charge sustaining/depleting, SOC, temperature, and charging rate.

2.1. Electrical submodel

The electrical dynamics of a battery cell are modeled by the 1st order Randle equivalent circuit shown in Fig. 6. The circuit is composed of an ideal voltage source Voc to model the cell open circuit voltage, a resistance R to model the electrolyte resistance and an RC circuit in series configuration to model the cell electric dynamics (R1, C1). In a cell, the open circuit voltage (OCV) is defined as the voltage that is measured with a voltmeter at the terminals of a cell, when there is no current drawn into the battery. The cell electrical model is given by:

\[
\begin{align*}
\frac{dV_c(t)}{dt} & = \frac{-1}{R_1C_1}V_c(t) + \frac{I(t)}{C_1} \\
\frac{dSOC(t)}{dt} & = \frac{-I(t)}{S(t)} \\
V_{cell}(t) & = V_{oc} - R(t)I(t) - V_c(t)
\end{align*}
\]

where SOC(t) is the cell state of charge, S(t) is the cell capacity, I(t) is the input current, V_c(t) is the voltage across the capacitor C_1 and V_{oc}
is the OCV. The current convention is such that negative current sign is used for charging and positive current for discharging. The OCV is a function of SOC and temperature. All the electrical circuit parameters depend on operating conditions (i.e., current, temperature, SOC, charge/discharge), the cell capacity \( S(t) \) and resistance \( R(t) \) vary with the age of the battery.

2.2. Thermal submodel

Fig. 7(a) shows a schematic of a lithium-ion pouch cell. For the battery cell thermal submodel, the heat generation and temperature within the cell are assumed to be uniformly distributed [41]. According to the energy conservation law, the temperature change for a single battery cell is given by:

\[
\frac{dT}{dt} = \frac{Q_g}{C_0} - \frac{Q_d}{C_0}
\]  

(3)

where, \( T \) is the temperature within the cell; \( Q_g \) is the rate of heat generated by the single cell; and \( Q_d \) is the rate of heat removed from the cell by the cooling channels.

The rate of heat generation for a battery cell can be approximated by Ref. [42]:

\[
Q_g = I \cdot \left( V_{oc} - V_{cell} - \frac{dV_{oc}}{dT} \cdot T \right)
\]  

(4)

where the dependence of the cell open circuit voltage on temperature may be neglected as a first approximation [40].

The rate of heat dissipated by the cooling channels is given by:

\[
Q_d = Q_{ku.1} + Q_{ku.2}
\]  

(5)

where, \( Q_{ku.1} \) and \( Q_{ku.2} \) are the rate of heat removed from the cell by the cooling channels 1 and 2 respectively.
Heat is removed from the battery cell by surface convection, when air enters the channel at a uniform temperature less than the battery temperature. For a rectangular channel with constant wall temperature, the local heat transfer coefficient can be estimated using the analytical correlation for laminar forced convection proposed in Ref. [43] as:

$$ h(y) = \frac{6 \cdot k_{\text{air}} \cdot d_H}{d_H} \sqrt{\frac{Re \cdot Pr \cdot d_H}{120 \cdot y}} $$  \hspace{1cm} (6)

where, $k_{\text{air}}$ is the air thermal conductivity and $d_H$ is the cooling channel hydraulic diameter, $Pr$ is the Prandtl number of air, $y$ is the channels longitudinal coordinate value and, the Reynolds number is given by

$$ Re = \frac{u_m \cdot d_H}{\nu_{\text{air}}} \quad u_m = \frac{2 \cdot u_{\text{air}}}{3} $$  \hspace{1cm} (7)

Since the cooling channel length is much larger that its hydraulic diameter (i.e. $L_{\text{ch}} \gg d_H$), we can say that the fully developed region length is much larger than the thermal entrance region length, and we can simplify the model by making the reasonable assumption that the local convection coefficient is approximately constant along the cooling channel and takes the value of the average local heat transfer coefficient over the length of the channel, $\overline{h}$. Therefore,

$$ \overline{h} = \frac{1}{L_{\text{ch}}} \int_0^{L_{\text{ch}}} h(y) \, dy $$  \hspace{1cm} (8)

Since the local heat transfer coefficient is higher at the inlet of the channel and channel’s mean temperature is rapidly increases along the channel, the air temperature at the outlet of the channel is used to express the rate of heat removed by surface convection. Therefore, the rate of heat removed by surface convection by the cooling channel (index number $i$, $i = 1, 2$) can be expressed using Newton’s law of cooling as.

$$ Q_{\text{air},i} = \overline{h} \cdot A_{\text{ch}} \cdot (T - T_{\text{ch},i}) $$  \hspace{1cm} (9)

where, $A_{\text{ch}}$ is the cooling channel heat transfer surface area and, $T_{\text{ch},i}$ is the air temperature at the outlet of channel $i$.

The energy balance equation for the air stream flowing through the channel (index number $i$, $i = 1, 2$) is given by.

$$ Q_{\text{air},i} - Q_{\text{air},i} = 0 $$  \hspace{1cm} (10)

where,

$$ Q_{\text{air},i} = \rho_{\text{air}} \cdot c_{p,\text{air}} \cdot q_{\text{air},i} \cdot (T_{\text{ch},i} - T_{\text{air},i}) $$  \hspace{1cm} (11)

A schematic of the thermal circuit diagram for a single cell is shown in Fig. 8.

The estimation of convection heat transfer rate between battery cooling surface and cooling air and other parameters can be verified by measuring battery cooling plate temperature changes following

Fig. 7. Schematic view of (a) a battery cell, (b) battery cell and cooling channels, (c) battery cell and cooling channel notation.

Fig. 8. Thermal circuit diagram for an isolated (i.e not interconnected) battery cell.

Fig. 9. Severity factor (SF) surfaces for an LMO-NMC Lithium-ion battery cell: (a) Capacity SF (Equation (14)) (b) Resistance SF (Equation (18))[23].
a designated testing procedure such as the one proposed in Refs. [36], [44] using a CFD model such as the one proposed in Refs. [7], [36] or using system identification techniques [45]. Similarly, the proposed methodology can be used for liquid-cooled TMS by estimating the corresponding parameters using similar approaches as the ones previously mention. For example, in Ref. [46] a CFD model is used to calculate the flow rate and pressure drop on channels in a fluid cooling system.

2.3. Aging submodel

The aging submodel describes the capacity and power fade as function of the influencing PHEV stress factors and the battery charge throughput [23]. In a battery, the total ampere-hour throughput [Ah] in both charge and discharge is given by

\[ \text{Ah} = \int_0^t |I(t)| \, dt, \]

where \( I(t) \) is the input current to the battery [47].

The aging submodel is composed of two elements: the capacity fade model and a resistance growth model.

2.3.1. Capacity fade

The cell capacity loss [%] after \( Ah > 0 \) charge throughput is given as:

\[ S_{\text{loss}}(Ah) = 100 \cdot \frac{S_0 - S(Ah)}{S_0} \tag{12} \]

where \( S_0 \) is the cell nominal capacity and \( S(Ah) \) is the cell capacity after \( Ah > 0 \) charge throughput. The capacity loss of a single cell is described by Ref. [23],

\[ S_{\text{loss}}(Ah) = a_c(SOC_{\text{min}}, \text{Ratio}) \cdot \exp \left( -\frac{E_{\text{c}}}{{R_g}T_{\text{avg}}} \right) \cdot Ah^2 \tag{13} \]

where, \( a_c(\cdot) \) is the capacity severity factor function given by,
where the dimensionless constant coefficients $a_C, b_C, c_C, d_C, e, SOC_0,$ and the dimensional constant coefficient $E_{ac}$ are obtained from experimental data. Fig. 9(a) shows the surface $a_C(\cdot)$ that describes the dependence of capacity fade on SOCmin and Ratio.

We adopt the definition of the battery cell capacity state of health as the ratio of the cell capacity at ampere-hour throughput $Ah > 0$ to the cell nominal capacity $S_0$. Therefore, the SOHC of a cell is given by:

$$\text{SOHC}(Ah) = \frac{100 - S_{loss}(Ah)}{100}$$

2.3.2 Resistance growth

The resistance growth [%] after $Ah > 0$ charge throughput is given by Ref. [23]:

$$R_{inc}(Ah) = 100 \cdot \frac{R(Ah) - R_0}{R_0}$$

where $R_0$ is the cell nominal internal resistance and $R(Ah)$ is the cell internal resistance after $Ah > 0$ charge throughput. The resistance growth of a single cell is described by:

$$R_{inc}(Ah) = a_R(SOC_{min}, \text{Ratio, CR}) \cdot \exp\left(-\frac{E_{ar}}{RT_{air}}\right) \cdot Ah$$

where $a_R(\cdot)$ is the resistance severity factor function given by:

$$a_R(\cdot) = a_R + \beta_R \cdot (SOC_{min} - SOC_0)^c + \gamma_R \cdot \exp\left[d \cdot (CR_0 - CR_{eq})\right] + e \cdot (SOC_{min} - SOC_0)$$

and, the dimensionless constant coefficients $a_R, b_R, \gamma_R, CR_0, d, e, SOC_0$ and the dimensionless constant coefficient $E_{ar}$ are obtained from experimental data. Fig. 9(b) shows the resistance severity factor surface $a_R(\cdot)$ that describes the dependence of resistance growth on SOCmin and $CR_{eq}$.

In extended-range PHEV applications, high power and high capacity to store energy are required. Therefore in addition to capacity loss, the SOH needs to be characterized with a system parameter correlated with power fade. The internal resistance is commonly used as a cell parameter correlated with power loss [28, 29]. For this reason, we define the power SOH of a single cell as the ratio of the cell internal resistance at ampere-hour throughput $Ah > 0$ to the cell nominal resistance $R_0$. Therefore, the SOHR of a cell is given by:

$$\text{SOHR}(Ah) = \frac{100 + R_{inc}(Ah)}{100}$$

For a fresh cell, the power SOH is equal to one (SOHR = 1) and increases with aging. For automotive applications from a power performance stand point, the cell end-of-life (EOL) of a battery is reached when the SOHR reaches 1.2 (i.e. 20% in resistance increase, approximately 20% in power loss).

2.3.3 Aging submodel dynamics

Combining the two aging elements described before, the aging dynamics for a single cell are given by:

$$\begin{bmatrix}
\frac{d\text{SOHC}}{d(Ah)} \\
\frac{d\text{SOHR}}{d(Ah)}
\end{bmatrix} = \begin{bmatrix}
\phi_1(w) \cdot Ah^{-1} \\
\phi_2(w)
\end{bmatrix}$$

where:

$$\phi_1(w) = \frac{-z}{100} \cdot a_C(SOC_{min}, \text{Ratio}) \cdot \exp\left(-\frac{E_{ac}}{RT_{air}}\right)$$

$$\phi_2(w) = \frac{1}{100} \cdot a_R(SOC_{min}, \text{Ratio, CR}) \cdot \exp\left(-\frac{E_{ar}}{RT_{air}}\right)$$

with:

$$w = [SOC_{min}, \text{Ratio, CR}, T]$$

2.4 Battery cell model

Combining the three submodels previously described, a battery cell subject to aging can be described by fast dynamics coupled with slow or aging dynamics [47].

$$\dot{x}(t) = \begin{bmatrix}
\hat{V}_c \\
\hat{T}
\end{bmatrix} = \begin{bmatrix}
\frac{-1}{R_1 C_1} V_c + \frac{I}{C_1} \\
\frac{-I}{SOHC - S_0}
\end{bmatrix}
\begin{bmatrix}
l (1 - SOHR) \cdot R_0 - V_c \\
(\alpha_1 T + \alpha_2 T_{air,1} + \alpha_3 T_{air,2})
\end{bmatrix}
\begin{bmatrix}
m_{Cp,c} \\
m_{Cp,c}
\end{bmatrix}
\begin{bmatrix}
\phi_1(w) \cdot Ah^{b-1} \\
\phi_2(w)
\end{bmatrix}$$

$$y(t) = [V_c = V_{oc} - \text{SOHR} \cdot R_0, I - V_c]$$

$$\text{Ah} = \int_0^t \left| l(t) \right| dt$$

where:

- $x(t) \in \mathbb{R}^3$ is the set of state variables associated with the fast dynamic behavior of the cell which correspond to the electrical and thermal dynamics;
- $\text{SOH}(Ah) \in \mathbb{R}^2$ is the set of variables that describe the state of health the cell: the capacity state-of-health SOHC(Ah) and the power state-of-health SOHR( Ah) (slowly vary with aging);
- $l(t) \in \mathbb{R}$ and $T_{air}(t) \in \mathbb{R}$ are the inputs acting on the cell;
- $w \in \mathbb{R}^4$ is the set of stress factors given by $w = [SOC_{min}, \text{Ratio, CR, T}]$;
- $V_{oc} \in \mathbb{R}$ is the cell output;
- $\phi_1(w)$ and $\phi_2(w)$ are nonlinear functions given by Equation (22).
- $a_1, a_2, a_3$ are system parameters given by $a_1 = \frac{1}{R_{u1} + R_{u1}}, a_2 = \frac{1}{R_{u2} + R_{u2}}, a_3 = \frac{1}{R_{u3} + R_{u3}}$. 
• Battery pack model

Fig. 10 shows a control oriented block diagram of the model of a battery pack composed of N battery cells subject to aging. The model is composed of three interconnected submodels: the electrical submodel is used to predict battery cells voltages and SOCs in response to pack current and ambient temperature; the thermal submodel is used to predict cells temperatures in response to battery input power/current; and the aging submodel is used to predict battery cells capacities and power fade in response to battery input currents, voltages and ambient temperature; and the aging submodel is used to predict battery cells health, SOHs, and cell power state of health, SOHs are the capacity state of health, $R_{b_{(j,k)}}$ is the capacity state of health, $r_{c_{(j,k)}} ~ \sim \mathcal{N}(1, \sigma_r)$, $r_{E_{(j,k)}} ~ \sim \mathcal{N}(1, \sigma_E)$, $I_{(j,k)}$ are a normally distributed numbers, $\sigma_C$, $\sigma_R$ are the standard deviations for the capacity and internal resistance respectively, $R_{b_{(j,k)}}$ and $S_{b_{(j,k)}}$ are the resulting varied scheduled parameters. For simplicity, for the rest of the discussion, $\sigma_C$ and $\sigma_R$ are assumed to be the equal, and equal to $\sigma$.

3.1. Electrical submodel

The electrical submodel is composed of $N-1$ elements: $N$ cell electrical elements are used to predict battery cells voltages and SOCs in response to cells input currents; and the electrical interconnection model is used to predict the input current to the cells in response to battery input power/current.

3.1.1. Electrical elements

Each cell in the pack is modeled by a 1st order Randle model (Equation (2)). To model manufacturing variability, within each cell the parameters resistance $R$ and capacity $C$ are assumed to be normally distributed [3]. Therefore, for a cell index order pair $(j,k)$, the parameters are varied using the following expression.

$$R_{b_{(j,k)}} = SOHR_{b_{(j,k)}} \cdot T_{R_{b_{(j,k)}}} \cdot R_{b_{(j,k)}}$$

$$S_{b_{(j,k)}} = SOHS_{b_{(j,k)}} \cdot T_{C_{b_{(j,k)}}} \cdot S_{b_{(j,k)}}$$

(25)

where, $R_{b_{(j,k)}}$ and $S_{b_{(j,k)}}$ are the base scheduled parameters resistance and capacity respectively, $SOHR_{b_{(j,k)}}$ is the cell power state of health, $SOHS_{b_{(j,k)}}$ is the capacity state of health, $r_{b_{(j,k)}} ~ \sim \mathcal{N}(1, \sigma_R)$, $r_{c_{(j,k)}} ~ \sim \mathcal{N}(1, \sigma_C)$, $I_{(j,k)}$ are a normally distributed numbers, $\sigma_C$, $\sigma_R$ are the standard deviations for the capacity and internal resistance respectively, $R_{b_{(j,k)}}$ and $S_{b_{(j,k)}}$ are the resulting varied scheduled parameters. For simplicity, for the rest of the discussion, $\sigma_C$ and $\sigma_R$ are assumed to be the equal, and equal to $\sigma$.

3.1.2. Electrical interconnection element

The input current $I_{(j,k)}$ for a cell index order pair $(j,k)$, is calculated using Kirchhoffs voltage and current laws. For each electrical topology, $I_{(j,k)}$ is calculated in terms of the battery pack input current $I_{pack}$ and cells electrical parameters. Therefore, the electrical interconnection element (IEE) for a module $\mathcal{P}(n_e, m_e, \mathcal{E} = (j_0 \cdots j_{N-1}))$ is given by the following linear set of equations:

$$\begin{align*}
\mathbf{A} \mathcal{E} \cdot I_{\mathcal{E}} = & \mathbf{B} \mathcal{E} \cdot \left[ \mathbf{V}_{oc} - \mathbf{V}_{e} \right] + \mathbf{C} \mathcal{E} \cdot I_{\rho} \\
I_{\rho} = & \frac{\mathbf{P}_{\rho}}{\mathbf{V}_{\rho}}
\end{align*}$$

(26)

where:

• $I_{\rho}(t) \in \mathbb{R}$ is the pack input current;

The set of inputs to the IEE are:

![Fig. 11. Schematic of the thermal model heat transfer modes.](image)

![Fig. 12. Nomenclature for the lumped-capacitance thermal modeling, $N = 9$.](image)

![Fig. 13. Thermal circuit diagram for a battery cell (index number, 2 ≤ $i$ ≤ $N-1$).](image)
\[ P_{i}(t) \in \mathbb{R} \text{ is the pack input power request;} \]
\[ V_{i}(t) \in \mathbb{R} \text{ is the pack output voltage;} \]
\[ V_{oc}(t) \in (\mathbb{R}^{N} \times 1), \text{ is the vector of cells OCVs,} \]
\[ V_{oc}(t) = [V_{oc1}(t), V_{oc2}(t), \ldots, V_{ocN-1}(t), V_{ocN}(t)]^T \]
\[ V_{c}(t) = [V_{c1}(t), V_{c2}(t), \ldots, V_{cN-1}(t), V_{cN}(t)]^T \]

- The EIE output is:
  - \( I(t) = [I_1(t), I_2(t), \ldots, I_{N-1}(t), I_N(t)]^T \)
- and the EIE system parameters are:
  - \( A_{\gamma}, B_{\gamma}, C_{\gamma} \in (\mathbb{R}^N \times \mathbb{R}^N), \) and \( D_{\gamma} \in (\mathbb{R}^N \times 1), \) are the matrices which entries depend on \( n, m, e, f, (j) \), and \( SOHR \), and are given by:
    \[
    (A_{\gamma}, B_{\gamma}, C_{\gamma}) = \begin{cases} \text{Equations (47)} & \text{if } \gamma = SP \\ \text{Equations (48)} & \text{if } \gamma = PS \end{cases}
    \]

3.2. Thermal submodel

The thermal submodel is composed of \( N + 1 \) elements: \( N \) cell thermal elements that are used to predict cell temperature in response to cell input current, cell output voltage and heat dissipated from the cell; and the thermal interconnection model is used to predict the heat dissipated from the cells in response to battery cells temperatures and air input temperature.

3.2.1. Thermal elements

The temperature change for a battery cell (index number \( i \), \( 1 \leq i \leq N \)) is modeled according to the energy conservation law (Equation (3)).

\[
m_{cP_c} \frac{dT_i}{dt} = Q_{g,i}(t) - Q_{d,i}(t)
\]

where:
\[ Q_{g,i}(t) \text{ is the rate of heat generated by the cell (index number } i), \]
\[ Q_{d,i}(t) \text{ is given by:} \]
\[
Q_{d,i}(t) = Q_{cc,i}(i-1)(t) - Q_{cc,i}(i+1)(t) + Q_{ku,i}(i)(t) + Q_{ku,i}(i+1)(t)
\]

where:
\[ Q_{cc,i}(i-1) \text{ and } Q_{cc,i}(i+1) \text{ are the rate of heat transfer by conduction from the cell index number } (i) \text{ to the cells index numbers } (i - 1) \text{ and } (i + 1) \text{ respectively and,} \]
\[ Q_{ku,i}(i) \text{ and } Q_{ku,i}(i+1) \text{ are the rate of heat removed by surface convection by the cooling channels with index numbers } (i) \text{ and } (i + 1) \text{ respectively from the cell with index number } (i). \]

A schematic of the model heat transfer modes is shown in Fig. 11. A schematic with the model nomenclature is shown in Fig. 12.

The energy balance equation for the air stream flowing through the cooling channel (index number \( i \)), is given by.

\[
\frac{dT_i}{dt} = \frac{Q_{g,i}}{m_{cP_c}} - \frac{Q_{d,i}(T_i - T_{air,i} - T_{i-1})}{m_{cP_c}} + \frac{Q_{ku,i}(i)(t) + Q_{ku,i}(i+1)(t)}{m_{cP_c}}
\]

where: \( (1 \leq i \leq N + 1) \), \( Q_{ku(i-1)} \text{ and } Q_{ku(i+1)} \text{ are the heat transferred by surface conduction from cells index number } (i) \text{ and } (i-1) \text{ to channel (index number } i \text{) respectively, and } Q_{ku(i)} \text{ is the heat removed from cooling channel (index number } i \text{) by convection.} \]

The rate of heat generation for a single battery cell is approximated using Equation (4). The rate of heat transfer by convection from cell index number \( i \) to cells index number \( (i-1) \) and \( (i+1) \) can be expressed using the conduction thermal resistance of the material located between two cells \( R_{cc} \), as given by:

\[
Q_{cc,i(i-1)} = \begin{cases} \frac{T_i - T_{ch,i}}{R_{cc}} & \text{for } 1 \leq i \leq N \\ 0 & \text{for } i = N + 1 \end{cases}
\]

(30)

\[
Q_{cc,i(i+1)} = \begin{cases} \frac{T_i - T_{ch,i}}{R_{cc}} & \text{for } 1 \leq i < N + 1 \\ 0 & \text{for } i = N + 1 \end{cases}
\]

(31)

where: \( T_{ch,i} \) and \( T_{ch,i+1} \) is the outlet air temperature for the cooling channels with index numbers \( (i) \) and \( (i + 1) \) respectively.

Similarly, the rate of heat transfer by surface conduction from cell index number \( (i-1) \) to channel (index number \( i \)), is given by:

\[
Q_{ku(i-1)} = \begin{cases} \frac{T_{i-1} - T_{ch,i}}{R_{ku,i}} & \text{for } 1 \leq i \leq N + 1 \\ 0 & \text{for } i = N + 1 \end{cases}
\]

(32)

where \( T_{ch,i} \) is the outlet air temperature for the cooling channel with index number \( (i) \).

The heat removed from cooling channel (index number \( i \)) by convection can be expressed using the convection thermal resistance \( R_{ku} \), as given by:

\[
Q_{ku} = \frac{T_{i} - T_{air,i}}{R_{ku,i}} \text{ for } 1 \leq i \leq N + 1
\]

(33)

where \( T_{air,i} \) is the air temperature at the inlet of the channel (index number \( i \)).

A schematic of the thermal circuit diagram for a single cell upon interconnection is shown in Fig. 13.

3.2.2. Interconnected thermal element

From the previous discussion and making the reasonable assumption that the temperature at the inlet of all channels is the same (\( T_{inj} = T_{air} \) [7]), upon interconnection, the temperature change rate for a cell (index number \( i \)) is given by:

\[
\frac{dT_i}{dt} = \frac{Q_{g,i}}{m_{cP_c}} - \frac{Q_{d,i}(T_i - T_{air,i} - T_{i-1} - T_{i+1})}{m_{cP_c}}
\]

(34)
where $Q_{d}(\cdot)$ is a linear function of $T_{e}$, $T_{air}$, $I_{i-1}$ and $I_{i+1}$, given by:

$$Q_{d}(\cdot) = a_{1i}I_{i} + a_{2i}I_{air,i} + a_{3i}I_{i-1} + a_{4i}I_{i+1}$$  \hspace{1cm} (35)$$

where, $a_{1i}$, $a_{2i}$, $a_{3i}$, $a_{4i}$ are given by Equation (50).

Therefore, the thermal interconnection element (TIE) for a module $\mathcal{H}(n_{e}, m_{e}, f, \epsilon, f_{i(j)\rightarrow(j)}, \sigma)$ is given by the following algebraic set of equations:

$$Q_{d} = D \cdot T + E \cdot T_{air}$$  \hspace{1cm} (36)$$

where, the set of inputs to the TIE are:

- $T_{air}(t) \in \mathbb{R}$, is the air temperature at the deflector plate input;
- $T \in (\mathbb{R}^{N} \times 1)$, is the vector of cells temperatures,

$$T(t) = [T_{1}(t), T_{2}(t), ..., T_{N-1}(t), T_{N}(t)]^{T}$$

the output of the TIE is,

- $Q_{d}(t) \in (\mathbb{R}^{N} \times 1)$, is the vector of rates of heat dissipated from cells,

$$Q_{d}(t) = [Q_{d1}(t), Q_{d2}(t), ..., Q_{dN-1}(t), Q_{dN}(t)]^{T}$$

and the TIE system parameters are given by,

- $D \in (\mathbb{R}^{N} \times \mathbb{R}^{N})$, $E \in \mathbb{R}^{N}$, are the matrices which entries are given by Equation (49) (See appendix).

### 3.3. Aging submodel

The aging submodel is composed of $N + 1$ elements: $N$ cell aging elements are used to predict capacity and power fade in response of cell operating conditions charge sustaining/depleting, SOC, temperature, charging rate; and a system level SOH element to predict the pack state of health based on knowledge of individual cells SOH, electrical topology and voltage equalization approach.

#### 3.3.1. Aging elements

The aging dynamics for a cell (index number $i$) are described by Equation (21), where the interconnected stress factor vector $w_{i} = [SOC_{min}, Ratio, CR_{i}, T]_{i}$ is used as an input for the aging submodel, and the interconnected charge throughput is given by $Ah_{i}$, see Equation (24), where $I(t)$ is the input current to the battery cell (index number $i$) is calculated using Equation (26).

The input current at cells is a highly nonlinear function of: individual cells electrical parameters, which depend on individual cells operating conditions; individual cells capacity and power SOH; as well as battery pack electrical topology and thermal management, see Equation (26). Therefore, there is not a bijective mapping from the pack ampere-hour throughput ($Ah_{p}$) to the battery individual cells ampere-hour throughput ($Ah_{i}$).

#### 3.3.2. System level SOH element

##### 3.3.2.1. Pack capacity SOH

In a battery pack the available capacity depends on individual cells capacities, battery system electrical topology and voltage equalization approach [16,48]. For a string of cells in series $p$ (Fig. 1(a)) with passive equalization (PE), where the voltage of series connected cells is balanced by dissipating the excess of stored energy to other cells, the available pack capacity is given by the average of the cells capacities [16]. Therefore, the available capacity for a string $p$ with PE after $Ah_{p} > 0$ charge throughput is given by:

$$S_{p}(Ah_{p}) = \frac{1}{m_{e}} \sum_{j=1}^{m_{e}} S_{j}(Ah_{j})$$  \hspace{1cm} (38)$$

where $S_{j}(Ah_{j})$ is the capacity of the $j$-th cell in the string after $Ah_{j} > 0$ charge throughput.

Due to the self balancing nature of the parallel configuration, disregarding the equalization technique used, the available capacity of an element of cells in parallel $s$ (Fig. 1(b)) is given by $n_{e}$ times the average of the cells capacities. Therefore, the available capacity for an element $s$ after $Ah_{s} > 0$ charge throughput is given by:

$$S_{s}(Ah_{s}) = \sum_{k=1}^{n_{e}} S_{k}(Ah_{k})$$  \hspace{1cm} (39)$$

where $S_{k}(Ah_{k})$ is the capacity of the $k$-th cell in the element after $Ah_{k} > 0$ charge throughput.

Extending the concept of cell level SOHC to battery pack level, we define the capacity SOH of a pack $\mathcal{H}(n_{e}, m_{e}, f, \epsilon, f_{i(j)\rightarrow(j)}, \sigma)$ as the ratio of the pack available capacity at $Ah_{p} > 0$ ampere-hour throughput to the pack nominal capacity $S_{0,p}$, as follows:

$$SOH_{C,p}(Ah_{p}) = \frac{S_{p}(Ah_{p})}{S_{0,p}}$$  \hspace{1cm} (40)$$

where $S_{p}(Ah_{p})$ is the pack available capacity at $Ah_{p}$, and the pack nominal capacity $S_{0,p}$ is given by:

$$S_{0,p} = n_{e} \cdot S_{0}$$  \hspace{1cm} (41)$$

where, $S_{0}$ is nominal capacity of the cells that compose the pack. Therefore, the capacity state of health of a pack $\mathcal{H}(n_{e}, m_{e}, f, \epsilon, f_{i(j)\rightarrow(j)}, \sigma)$ at $Ah_{p} > 0$ is given by Equation (44).

##### 3.3.2.2. Pack power SOH

In a battery pack with voltage equalization, the battery pack power loss may be correlated with the pack equivalent internal resistance. For this purpose, the reasonable assumption that the OCVs of the cells in a battery pack with equalization are similar and may be consider equal is made. Therefore, we define the power state of health of a pack as the ratio of the pack equivalent internal resistance at $Ah_{p} > 0$ ampere-hour throughput to the pack nominal internal resistance $R_{0,p}$ as follows:

$$SOH_{R,p}(Ah_{p}) = \frac{R_{p}(Ah_{p})}{R_{0,p}}$$  \hspace{1cm} (42)$$

where $R_{p}(Ah_{p})$ is the pack equivalent resistance given and the pack nominal resistance is given by:

$$R_{0,p} = \frac{m_{e}}{n_{e}} R_{0}$$  \hspace{1cm} (43)$$

where $R_{0}$ is the nominal internal resistance of the cells that
compose the pack. Therefore, the power state of health of a pack 
\(\mathcal{P}(n_e, m_e, r, e, V, E, f_j, \ldots, j_k, \sigma)\) is given by Equation (45).

3.3.2.3. Battery pack state-of-health equations.

\[
SOH_{C, \rho}(Ah, \rho) = \begin{cases} 
\frac{1}{n_e} \sum_{j=1}^{m_e} \left( \min_{k=1}^{n_d} \left( SOH_{C, j}(Ah, j) \right) \right) & \text{for PS with PE} \\
\frac{1}{n_e} \min_{j=1}^{m_e} \left( \sum_{k=1}^{n_d} SOH_{C, j}(Ah, j) \right) & \text{for SP with PE} \\
\frac{1}{n_e m_e} \sum_{j=1}^{m_e} \sum_{k=1}^{n_d} SOH_{C, j}(Ah, j) & \text{for SP/PS with AE}
\end{cases}
\]

(44)

where \(SOH_{C, j}(Ah, j)\) is the capacity state of health for cell index pair \((j, k)\) at \(Ah_{j(k)} > 0\).

\[
SOH_{R, \rho}(Ah, \rho) = \begin{cases} 
\frac{n_e}{m_e} \sum_{j=1}^{m_e} \left( SOH_{R, j}(Ah, j) \right) & \text{for SP} \\
\frac{n_e}{m_e} \sum_{j=1}^{m_e} \left( SOH_{R, j}(Ah, j) \right) & \text{for PS}
\end{cases}
\]

(45)

where \(SOH_{R, j}(Ah, j)\) is the power state of health for cell index pair \((j, k)\) at \(Ah_{j(k)} > 0\), and the operator \(\|\) is defined as \(SOH_1 \| SOH_2 = \frac{SOH_{R, j}(Ah, j)}{SOH_{C, j}(Ah, j)}\).

3.4. SOC and thermal balancing

The equalization of the SOCs of different cells in the pack is implemented using both passive and active voltage equalization. The voltage equalization is implemented in the model by bringing the SOC of all cells to the lowest SOC in the pack when simulating passive voltage equalization or to the average of the cells when simulating active voltage equalization. The implementation of the voltage equalization in the pack model assumes that the equalization occurs instantaneously at the end of the CC-CV charging voltage equalization in the pack model.

The equalization of the SOCs of different cells in the pack is implemented using both passive and active voltage equalization. The voltage equalization is implemented in the model by bringing the SOC of all cells to the lowest SOC in the pack when simulating passive voltage equalization or to the average of the cells when simulating active voltage equalization. The implementation of the voltage equalization in the pack model assumes that the equalization occurs instantaneously at the end of the CC-CV charging voltage equalization in the pack model. Therefore, the electrical and thermal dynamics occurring during the balancing process are not taken into account.

Additionally, to simulate a real case scenario in which the battery is thermally balanced each time the vehicle is not in operation, the battery pack is thermally balanced at the end of each driving cycle, \(t = t_{CC} + t_{CV} + t_{charging}\), by resetting the individual cell temperatures to the ambient temperature.

3.5. Battery model

Summarizing the previous discussion, a battery module \(\mathcal{P}(n_e, m_e, r, e, V, E, f_j, \ldots, j_k, \sigma)\) can be described by dynamic system of order \(5n_m\), composed of the fast dynamics coupled with an slow or aging dynamics given by:

where:

- \(N = n_e m_e\) is the number of cells in the pack;
Fig. 14. Simulated battery electrical topologies: (left) Parallel strings of cells, (right) Strings of parallel cells.

1 ≤ i ≤ N;
- \( P_{s,t}(t) \in \mathbb{R} \) is the input power to the pack;
- \( I_{s,t}(t) \in \mathbb{R} \) is the input current to the pack;
- \( T_{air}(t) \in \mathbb{R} \) is the air temperature at the deflector plate input;
- \( \chi_i(t) \in \mathbb{R} \), is the set of state variables associated with the fast dynamic behavior of cell (index number i);
- \( \text{SOH}_i \in \mathbb{R}^2 \), is the set of variables that describe the state of health cell (index number i); the capacity state of health \( \text{SOH}_c \); and the power state of health \( \text{SOH}_p \); (slowly vary with aging);
- \( w_i \in \mathbb{R}^4 \), are the set of stress factors for cell (index number i).

\[ w_i = \left[ \text{SOC}_{\text{min,i}}, \text{Ratio, CR}_i, T_i \right] \]

- \( V_{p} \in \mathbb{R} \), is the pack output voltage;
- \( \phi_1(\cdot) \) and \( \phi_2(\cdot) \) are nonlinear functions given by Equation (22);
- \( I_i(t) \in \mathbb{R} \), and \( Q_{d,i}(t) \in \mathbb{R} \), are the inputs acting on the cell (index number i);
- \( I(t) \in (\mathbb{R}^N \times 1) \), is the vector of input currents to the cells,
  \[ I = [I_1, I_2, ..., I_{N-1}, I_N]^T; \]
- \( Q_{d}(t) \in (\mathbb{R}^N \times 1) \), is the vector of heat dissipated from the cells,
  \[ Q_{d} = [Q_{d,1}, Q_{d,2}, ..., Q_{d,N-1}, Q_{d,N}]^T; \]
- \( V_{oc}(t) \in (\mathbb{R}^N \times 1) \), is the vector of cells OCV,
  \[ V_{oc} = [V_{oc,1}, V_{oc,2}, ..., V_{oc,N-1}, V_{oc,N}]^T; \]
- \( T(t) \in (\mathbb{R}^N \times 1) \), is the vector temperature at cells

\[ T = [T_1, T_2, ..., T_{N-1}, T_N]^T \]

- \( V_{c}(t) \in (\mathbb{R}^N \times 1) \), is the vector of cells voltages across the capacitor, \( V_c = [V_{c,1}, V_{c,2}, ..., V_{c,N-1}, V_{c,N}]^T; \)
- \( D \in (\mathbb{R}^N \times \mathbb{R}^N) \), \( E \in \mathbb{R}^N \), are the matrices given by Equation (49);
- \( A_{\Psi}, \text{SOH}_R \in (\mathbb{R}^N \times \mathbb{R}^N) \), \( B_{\Psi} \in (\mathbb{R}^N \times \mathbb{R}^N) \), \( C_{\Psi} \in \mathbb{R}^N \) are the matrices given by,

\[ (A_{\Psi}, B_{\Psi}, C_{\Psi}) = \begin{cases} \text{Equations (47)} & \text{if } \mathcal{E} = \text{SP} \\ \text{Equations (48)} & \text{if } \mathcal{E} = \text{PS} \end{cases} \]

Fig. 15. (a) Voltage variation at cells in elements in series (s1, s2, s3) and (b) Current variation at s3 during a CD micro-cycle for pack M2,α=0.05 (topology 333P).

Table 1

<table>
<thead>
<tr>
<th>NMC-LMO Lithium-ion battery cell specifications.</th>
<th>Value (without laminated coat)</th>
<th>Value (with laminated coat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>0.19 [m]</td>
<td>0.225 [m]</td>
</tr>
<tr>
<td>Width</td>
<td>0.145 [m]</td>
<td>0.165 [m]</td>
</tr>
<tr>
<td>Thickness</td>
<td>(5 \times 10^{-3} ) [m]</td>
<td>(5 \times 10^{-3} ) [m]</td>
</tr>
<tr>
<td>Mass</td>
<td>384 [g]</td>
<td>384 [g]</td>
</tr>
<tr>
<td>Nominal capacity</td>
<td>15 [Ah]</td>
<td>15 [Ah]</td>
</tr>
<tr>
<td>Nominal voltage</td>
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<td>3.75 [V]</td>
</tr>
<tr>
<td>( c_p )</td>
<td>800 Jkg(^{-1})K(^{-1})</td>
<td>800 Jkg(^{-1})K(^{-1})</td>
</tr>
</tbody>
</table>
4. Simulation results

In this section, simulation results are presented and discussed. The model developed in Section 3 is used to describe two battery modules composed of $N = 9$ cells each one. The battery cells are pouch cells with composite LMO-NMC positive electrode and graphite negative electrode, for which an experimentally validated fast and slow dynamic model has been developed [23]. Each cell has a nominal capacity of 15 Ah and a nominal voltage 3.75 V, dimensions and other cell properties are presented in Table 1. The first module ($M_1$) is such that $\mathcal{P}_{M_1}(n_c, m_c, I, f_{ij}^{(i)}, \sigma) = (3, 3, PS, f_{M_1}, \sigma)$, while the second module ($M_2$) is such that $\mathcal{P}_{M_2}(n_c, m_c, I, f_{ij}^{(i)}, \sigma) = (3, 3, PS, f_{M_2}, \sigma)$. Where, the bijective mapping functions $f_{M_1}$ and $f_{M_2}$ are defined as shown in Fig. 14. Both packs with an air cooling arrangement as the one shown in Fig. 12. Since the pack is composed of 9 cells, the reasonable assumption that all channels are identical is made, see Ref. [7] (cooling arrangement configuration Type IV). Two simulators are developed in Matlab one for each topology.

Simulation results of the two packs under different initial cell-

<table>
<thead>
<tr>
<th>Cell#</th>
<th>Capacity variability $r_{C_i}$</th>
<th>Initial capacity $S_{0_i}$</th>
<th>Resistance variability $r_{R_i}$</th>
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<td>1</td>
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<td>0.9999</td>
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<tr>
<td>7</td>
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<td>0.9864</td>
</tr>
<tr>
<td>8</td>
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<td>14.8846</td>
<td>1.0004</td>
</tr>
<tr>
<td>9</td>
<td>1.0094</td>
<td>15.1403</td>
<td>1.0069</td>
</tr>
</tbody>
</table>

$^a r_{C_i} = N(1, 0.0125), r_{R_i} = N(1, 0.0125)$, see Equation (25).
to-cell variability are presented. When the standard deviation of $\sigma = 0$, the initial capacity and internal resistance of all cells in the pack are identical. For the other cases, to facilitate the comparison of the results, a random set of initial conditions is generated as follows: first, a set of numbers $N(0, 1)$ is generated, then this set is multiplied by 0.025 or 0.05 respectively, and the nominal cell parameters are modified accordingly. The initial capacity and resistance values for the simulation results presented in this paper are shown in Tables 2 and 3.

To compare the fast and slow dynamics of a single cell to the behavior of the cells in a battery module (i.e. interconnected cells vs. not interconnected), we use nominally the same cycle conditions for both scenarios. To have nominally the same cycling conditions, the charge depleting and charge sustaining micro-cycles shown in Fig. 4, which are defined for a single cell, are scaled up in such a way that if all the cells in the pack were identical, and subject to the same operational conditions, each cell would be subject to the same power cycling as the single cell. This convention is used throughout the simulation results.

4.1. Battery pack electrical dynamics

Fig. 15(a) shows the voltage variation at the elements in series ($s_1, s_2, s_3$) while Fig. 15(b) shows the current variation at the third element in series ($s_3$, cells # 7, 8 and 9) for module $M_{2,0.05}$ during ta CD micro-cycle.

Fig. 16 shows the state of charge profiles of the cells with the highest and lowest SOC in packs $M_{1,0.05}$ and $M_{2,0.05}$ during a cycle of Ratio $= 0.5$, SOC$_{min} = 25\%$, CR $= 3C/2$ and $T = 15^\circ C$. Due to the self-balancing nature of the parallel configuration of elements ($s_1 s_2 s_3$) in $M_{1,0.05}$, the SOC imbalance is lower for $M_{1,0.05}$ compared to $M_{2,0.05}$.

4.2. Battery pack thermal dynamics

Fig. 17 shows the temperature profile of the hottest and coldest cells in packs $M_{1,0.05}$ and $M_{2,0.05}$ during a cycle of Ratio $= 0.5$, SOC$_{min} = 25\%$, CR $= 3C/2$, $T = 15^\circ C$. The difference between the maximum and minimum temperatures is greater for pack $M_{2,0.05}$ for which the electrical topology allow a higher cell-to-cell current imbalance; and therefore a higher cell-to-cell heat generation imbalance compared to $M_{1,0.05}$.

Fig. 18 (a,b) show the battery pack temperature variation at cells over time for packs $M_{1,0.05}$ and $M_{2,0.05}$ during a cycle of Ratio $= 0.5$, SOC$_{min} = 25\%$, CR $= 3C/2$, $T = 15^\circ C$. Similarly, Fig. 18(c,d) show the battery pack temperature variation at cells over time for packs $M_{1,0.05}$ and $M_{2,0.05}$ during the same cycle at $T = 15^\circ C$ When the nominal capacity and resistance of all the cells in the pack are identical ($\sigma = 0$), and no aging has taken place, the temperature variation at cells is similar for the two topologies. However, when
Fig. 19. Capacity loss and resistance growth evolution of the most and least aged cells compared to an isolated cell (i.e not interconnected) under the same nominal conditions for pack topology $33P$ during cycling under a $\text{Ratio} = 1, \text{SOC}_{\text{min}} = 45\%$ and $\text{CR} = 3\text{C}/2$ profile, $T = 15 \, ^\circ\text{C}$.

Fig. 20. Difference in the capacity loss and resistance increase attained at cells and packs $M_1$ and $M_2$ with compared to single cell at $t = 310$ days of continuous cycling under a nominal cycle of $\text{Ratio} = 1, \text{SOC}_{\text{min}} = 45\%$ and $\text{CR} = 3\text{C}/2$, $T = 15 \, ^\circ\text{C}$. 
cell-to-cell manufacturing variability is considered, the temperature variation in the pack strongly depends on electrical topology. The temperature variation at cells is greater for topology $M_{2,0}$ for which its electrical topology allows a higher cell-to-cell current imbalance, and therefore a higher cell-to-cell heat generation imbalance.

4.3. Battery pack aging dynamics

Fig. 19 show the capacity loss and resistance increase profiles of the most and least aged cells in packs $M_{2,0.05}$, $M_{2,0.025}$, $M_{2,0.05}$ compared to an isolated cell (i.e not interconnected) under a cycling of Ratio = 1, SOC$_{min}$ = 45%, CR = 3C/2, $T = 15$ °C. As shown in the figures, all the cells in the pack age faster compared to the single cell reducing the battery system life-span. Similarly, Fig. 20 show bar plots with the difference in the capacity loss attained at cells and pack compared to an isolated cell at the same nominal cycling conditions when the isolated cell has reached its end-of-life (20% in capacity fade, approximately $t = 310$ days of continuous cycling). The first columns in the x axes of each bar plot correspond to the cell number position within the pack (see Fig. 14) while the last two columns correspond to the pack under active equalization (AE) and passive equalization (PE). The difference in capacity loss is expressed in percentage compared to the capacity loss attained by the isolated cell at $t = 310$ days (20% of capacity loss). For example, the capacity loss attained by cell #7 in pack $M_{2,0.05}$ at $t = 310$ days is approximately 23% (15% more compared to the isolated cell).

The simulation results suggest that the capacity and power fade cell-to-cell imbalance (i.e. aging imbalance) increases while increasing the manufacturing standard deviation ($\sigma$). The results also suggest that the distribution of the cell-to-cell aging imbalance depends on the pack electrical topology. The pack difference in capacity loss and internal resistance increase depend on the cell-to-cell aging imbalance as well as on the voltage equalization approach. However, the simulation results suggest that the influence of the manufacturing standard deviation ($\sigma$) on the pack aging is not significant. This latter statement requires an in-depth analysis as it may imply that stringent requirements currently imposed for the manufacturing of the battery cells may be possibly reduced, reducing the battery system cost.

5. Conclusions

The propagation of aging, which has a profound effect on battery system life-span, depends on battery electrical topology, thermal management system, cell-to-cell manufacturing variability and voltage equalization technique. This paper proposed a novel methodology for modeling aging propagation in battery packs. The methodology describes the propagation of aging and its effect on the life-span of advanced energy storage systems using a control oriented representation. The model is able to predict battery pack aging, thermal, and electrical dynamics under actual PHEV operational conditions. The model includes consideration of random variability of the cells, electrical topology and thermal management. The proposed model allow for a more complete understanding of the impacts and trade-offs of cell-to-cell variability, electrical topology and battery thermal management, on battery performance and life under actual PHEV operation.

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Appendix

This section contains model equations, which for simplicity, were not included within the main text but are relevant components of the proposed battery pack modeling approach.

\[
\begin{align*}
A_{[j][k]} &= \begin{cases} 
1, & \text{if } (j = l), (1 \leq k \leq n_e), (1 \leq l \leq m_e) \\
R_{e(j,k)}, & \text{if } (1 \leq j \leq m_e), (1 \leq k \leq n_e - 1), (l = m_e + (j - 1)(n_e - 1) + k) \\
-R_{e(j,k)}, & \text{if } (1 \leq j \leq m_e), (2 \leq k \leq n_e), (l = m_e + (j - 1)(n_e - 1) + (k - 1)) \\
0, & \text{else where}
\end{cases} \\
B_{[j][k]} &= \begin{cases} 
1, & \text{if } (1 \leq j \leq m_e), (1 \leq k \leq n_e - 1), (l = m_e + (j - 1)(n_e - 1) + k) \\
-1, & \text{if } (1 \leq j \leq m_e), (2 \leq k \leq n_e), (l = m_e + (j - 1)(n_e - 1) + (k - 1)) \\
0, & \text{else where}
\end{cases} \\
C_{[l]} &= \begin{cases} 
1, & \text{if } (1 \leq l \leq m_e) \\
0, & \text{else where}
\end{cases}
\end{align*}
\]
$A_{l,j(k)} = \begin{cases} 
1, & \text{if } (j = 1), (1 \leq k \leq n_e),(l = 1) \\
1, & \text{if } (1 \leq j \leq m_e), (1 \leq k \leq n_e), (l = 1 + (k - 1)(m_e - 1) + j) \\
1, & \text{if } (2 \leq j \leq m_e), (1 \leq k \leq n_e), (l = 1 + (k - 1)(m_e - 1) + (j - 1)) \\
-R_{c(k)}, & \text{if } (1 \leq j \leq m_e), (1 \leq k \leq n_e), (l = (1 - n_e) + m_k n_k + (k - 1)) \\
0, & \text{else where} 
\end{cases}$

$B_{l,j(k)} = \begin{cases} 
1, & \text{if } (1 \leq j \leq m_e), (1 \leq k \leq n_e - 1), (l = 1 + m_k n_k - n_e + k) \\
-1, & \text{if } (1 \leq j \leq m_e), (2 \leq k \leq n_e), (l = (1 - n_e) + m_k n_k + (k - 1)) \\
0, & \text{else where} 
\end{cases}$

$C_{l,1} = \begin{cases} 
1, & \text{if } (l = 1) \\
0, & \text{else where} 
\end{cases}$

$D_{l,i} = \begin{cases} 
\alpha_{1,i} & \text{if } (1 \leq i \leq N), (j = i) \\
\alpha_{3,i} & \text{if } (2 \leq i \leq N), (j = i - 1) \\
\alpha_{4,i} & \text{if } (1 \leq i \leq N - 1), (j = i + 1) \\
0, & \text{else where} 
\end{cases}$

$E_{l,1} = \begin{cases} 
\alpha_{2,i} & \text{if } (1 \leq i \leq N) \\
0, & \text{else where} 
\end{cases}$

where $\alpha_{1,i}$, $\alpha_{2,i}$, $\alpha_{3,i}$, $\alpha_{4,i}$ are given by,

*$\alpha_{1,i} = \frac{-R_{eq,i} - R_{eq,i+1} - R_{cc,(i+1)} + 1}{R_{ku,i} + R_{ku,i+1} + R_{cc,(i+1)} + 1} + \frac{1}{R_{ku,i}}$ if $i = 1$ 

$\alpha_{2,i} = \frac{-R_{eq,i} - R_{eq,i+1} - R_{cc,(i+1)} + 1}{R_{ku,i} + R_{ku,i+1} + R_{cc,(i+1)} + 1} + \frac{1}{R_{ku,i}}$ if $2 \leq i < N - 1$ 

$\alpha_{3,i} = \frac{-R_{eq,i} - R_{eq,i+1} - R_{cc,(i+1)} + 1}{R_{ku,i} + R_{ku,i+1} + R_{cc,(i+1)} + 1} + \frac{1}{R_{ku,i}}$ if $i = N$ 

$\alpha_{4,i} = \frac{-R_{eq,i+1} - 1}{R_{ku,i} + R_{cc,(i+1)} + 1} + \frac{1}{R_{ku,i} + R_{cc,(i+1)} + 1}$ if $1 \leq i < N$ 

$\alpha_{4,i} = \frac{-R_{eq,i+1} - 1}{R_{ku,i} + R_{cc,(i+1)} + 1} + \frac{1}{R_{ku,i} + R_{cc,(i+1)} + 1}$ if $i = N$ 

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