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An optimal regulation strategy with disturbance rejection for energy management of hybrid electric vehicles[☆]

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ABSTRACT

The energy management problem of finding the optimal split between the different sources of energy in a charge-sustaining parallel HEV, ensuring stability and optimality with respect to a performance objective (fuel consumption minimization over a driving cycle), is addressed in this paper. The paper develops a generic stability and optimality framework within which the energy management problem is cast in the form of a nonlinear optimal regulation (with disturbance rejection) problem and a control Lyapunov function is used to design the control law. Two theorems ensuring optimality and asymptotic stability of the energy management strategy are proposed and proved. The sufficient conditions for optimality and stability are used to derive an analytical expression for the control law as a function of the battery state of charge/state of energy and system parameters. The control law is implemented in a simplified backward vehicle simulator and its performance is evaluated against the global optimal solution obtained from dynamic programming. The strategy performs within 4% of the benchmark solution while guaranteeing optimality and stability for any driving cycle.

1. Introduction

A generic hybrid electric vehicle (HEV), regardless of the architecture considered, has two sources onboard that can supply the torque/power requested by the driver. HEVs use batteries, electric motors, regenerative braking and reduction of engine idling time to enhance a conventional internal combustion engine, thus achieving better fuel economy. The electric motor provides a portion of the power for propulsion, especially at high-load conditions when the vehicle is accelerating; some of the vehicle's inertial energy can be recaptured through regenerative braking systems and stored in vehicle batteries. According to Pesaran (2011), a possible classification of today's vehicles in the market can be given based on internal combustion engine size and electric machine size, as follows:

(1) Conventional internal combustion engine (ICE) vehicles;
(2) Micro hybrids (start/stop);
(3) Mild hybrids (start/stop + kinetic energy recovery);
(4) Full hybrids (mild hybrid capabilities + electric launch);
(5) Plug-in hybrids (full hybrid capabilities + electric range);
(6) Electric vehicles (battery or fuel cell).

In particular:

- In conventional vehicles, the ICE is the only source of power. For this type of vehicle the total power request at the wheel is entirely satisfied by the ICE An, Stodolsky, and Santini (1999), Miller (2003).
- A start–stop system automatically shuts down and restarts the ICE to reduce the amount of time the engine spends idling, thereby reducing fuel consumption and emissions. This feature is present in hybrid electric vehicles, but has also appeared in vehicles which lack of a hybrid electric powertrain. Non-electric vehicles featuring start–stop system are called micro-hybrids An et al. (1999), Miller (2003).
- In a mild hybrid vehicle generally an ICE is equipped with an electric machine (one motor/generator in a parallel configuration) allowing the engine to be turned off whenever the car is coasting, braking, or stopped An et al. (1999), Miller (2003).
- A full hybrid vehicle can run only using the engine, the batteries, or a combination of both. A large, high-capacity battery pack is needed for battery-only operation in the electric launch. In these vehicles a supervisory control is needed to provide

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coordination among the actuators in order to minimize fuel consumption An et al. (1999), Guzzella and Sciarretta (2007), Miller (2003). The objective of the present paper is to propose a new optimal regulation energy management strategy for a medium duty full hybrid truck.

- A plug-in hybrid electric vehicle (PHEV) utilizes rechargeable batteries that can be restored to full charge by connecting to an external electric power source Guzzella and Sciarretta (2007).
- A fuel cell vehicle (or electric vehicle) uses a fuel cell (or battery) to produce electricity and power its on-board electric motors Guzzella and Sciarretta (2007).

The objective of the energy management strategy in a HEV is to find the optimal torque/power split between the primary and secondary energy sources that minimizes a given objective function over an entire driving cycle.[2] The energy management problem in a charge-sustaining[3] HEV has been studied in the literature for over a decade (see, for instance, Brahma, Guezennec, and Rizzoni (2000); Pisu and Rizzoni (2007) and references therein).

In general, the energy management strategies can be categorized based on the feasibility of implementation in a real vehicle. The first category involves the use of classical optimal control techniques guaranteeing global/local optimality of the solution. Dynamic Programming (DP) assumes *a-priori* knowledge of the driving cycle and solves the problem backwards in time, considering all possible power split choices at each instant leading to the global optimal solution Brahma et al. (2000), Koot et al. (2005), Lin, Peng, Grizzle, and Kang (2003). On the other hand, Pontryagin's Minimum Principle (PMP) formulates and minimizes a Hamiltonian function (a function of the instantaneous cost and the state constraint) at each instant to obtain the optimal solution Anatone, Cipollone, and Sciarretta (2005), Delprat, Lauber, Guerra, and Rimaux (2004), Kim, Cha, and Peng (2011), Serrao, Onori, and Rizzoni (2009).

The second category of strategies consists of algorithms that are implementable in a real-vehicle, but they do not necessarily guarantee optimality. Equivalent Consumption Minimization Strategy (ECMS), adaptive energy management strategies and rule-based control strategies are in this category. The basic idea of ECMS is to reformulate the global optimization problem into a local optimization problem with tuning parameters. This method can give very good results, but the optimal equivalence factor, which depends on the driving cycle, must be determined *a-priori* using offline methods Paganelli, Ercole, Brahma, Guezennec, and Rizzoni (2001). To overcome this problem, adaptive ECMS methods have been proposed in the literature, for example, by adapting the tuning parameter of ECMS by predicting the driving cycle Musardo, Rizzoni, Guezennec, and Staccia (2005), or using pre-computed driving cycle-optimal equivalent factor correlations Gu and Rizzoni (2006), or using the correlation between equivalence factor and battery state of charge Chasse, Sciarretta, and Chauvin (2010), Kessels, Koot, van den Bosch, and Kok (2008), Onori, Serrao, and Rizzoni (2010). Rule-based strategies have also been very popular because of their simplicity in real-time implementation. The rules can be derived, for example, from the DP solution to the optimization problem Bianchi et al. (2010), Lin et al. (2003), Jalil, Kheir, and Salman (1997).

The following shortcomings in the HEV literature motivate the main contributions of the paper:

- Because the strategies that are based on classical optimal control techniques require *a-priori* knowledge of the driving cycle, they cannot be implemented in a real vehicle. They can be either used as benchmark solutions to perform comparative analysis of other implementable energy management strategies or to derive rules for rule-based strategies.
- The second category of strategies which can be implemented in a real vehicle need to be tuned for the intended driving conditions to perform close to the optimal solution.[4]

In this paper, we depart from these approaches and present a novel idea to design an energy management controller for a parallel HEV.

Linear-quadratic optimal control theory has been developed extensively over the past century; extension to nonlinear optimal control has broadened the effectiveness of such techniques. Because nonlinear controllers can effectively model the nonlinearities in the system and hence perform better than linear controllers for nonlinear systems, it is not surprising that significant effort has been devoted to developing the theory of nonlinear optimal regulation Bass and Webber (1966), Lukes (1969), Willemstein (1977).

In this paper, we develop a stability and optimality framework for charge sustaining HEVs, based on the results on nonlinear optimal regulation in feedback control problems involving non quadratic cost functionals, found in Bernstein (1993), Haddad and Chellaboina (2008). We aim at finding an analytical energy management strategy that can be easily implemented in a real vehicle assuring optimality and stability. It is shown that by suitably casting the energy management problem into a nonlinear optimal regulation problem and using an appropriate Lyapunov function candidate, it can be proved that the state-feedback based optimal control law (with respect to minimum fuel consumption) produces a charge-sustaining behavior. The control Lyapunov function[5] is also used in deriving an analytical closed-form expression for the optimal control law. The paper is an extension of the work proposed in Sampathnarayanan, Onori, and Yurkovich (2012) in the presence of external disturbances for the pre-transmission parallel HEV.

The paper is organized as follows: Section 2 describes the energy management problem in a pre-transmission parallel HEV along with the battery state of charge dynamics, integral and instantaneous constraints and Willans line based engine fuel consumption rate model. Section 3 casts the energy management problem into a nonlinear optimal regulation problem, where a set of sufficient conditions is proposed for the asymptotic stability of the origin and optimality of the control law with respect to the fuel consumed. Next, the optimal control law is implemented in a vehicle simulator and the results are evaluated against the benchmark solution from DP. Section 5 lists the main contributions of the paper and the intended future work.

[2] In this work, we limit our focus to the problem of fuel consumption minimization, with no inclusion of drivability considerations. Typically, the gear shifting optimization pertains to the transmission control and it is not an objective of the supervisory control. The optimization of the gear shifting strategy would require the formulation of an optimal control problem which includes both continuous time and discrete time dynamics. In this work, we assume that the transmission controller operates independently to the supervisory controller, therefore the gear shifting strategy is treated as a known external input to the energy management system.

[3] A HEV in which the battery can be recharged/discharged only using the vehicle and not externally Miller (2003).

[4] Though rule-based energy management strategies are relatively easy to develop and implement in a real vehicle, a significant amount of calibration effort is required to improve its performance over a driving cycle. Furthermore these sub-optimal strategies are not necessarily scalable to other powertrain architectures and different component sizes.

[5] A control-Lyapunov function Pontryagin, Boltyanskii, Gamkrelidze, and Mishchenko (1962) is a function $V(x, u)$ that is continuous, positive-definite ($V(x, u) > 0\ \forall x \neq 0$), proper ($V(x) \to \infty$ *as* $|x| \to \infty$), and such that

$$\forall x \neq 0, \exists u \quad \dot{V}(x, u) < 0.$$

2. Energy management problem in a charge-sustaining HEV

Unlike in a conventional vehicle, the additional degree of freedom offered by the hybrid powertrain presents a challenging optimization problem. The objective of the energy management strategy in a HEV is to find the optimal torque/power split, between the primary (fuel) and secondary (battery) energy sources that minimizes a given objective function over an entire driving cycle. The minimization can be performed with respect to several objectives, such as fuel consumption, emissions, battery aging, etc., or a combination of these objectives Musardo, Staccia, Mohler, Guezennec, and Rizzoni (2005); Serrao, Onori, Sciarretta, Guezennec, and Rizzoni (2011). In this paper, we consider the problem of minimizing the total mass of fuel, m_f [g] during a driving cycle, or equivalently, minimize the following integral cost J:

$$J = \int_{t_0}^{t_f} \dot{m}_f(u(t))dt \tag{1}$$

where $\dot{m}_f$ is the instantaneous fuel consumption rate expressed in [g/s], $u(t)$ is the control input, and $t_f - t_0$ is the length of the driving cycle. The energy management problem, by its very nature, is a constrained optimal control problem, where the objective function (1) is minimized under system dynamics, instantaneous (local) and integral (global) constraints on the state and control variables, as outlined in the following.

2.1. System dynamics

In order to study the energy management problem, a quasi-static model of the vehicle is used. The battery state of charge (SOC) is the scalar state variable of the energy management problem whose dynamics can be expressed as

$$\dot{SOC}(t) = -\alpha \frac{I(t)}{Q_{\max}} \tag{2}$$

where α represents the Coulombic efficiency Guzzella and Sciarretta (2007), $I(t)[A]$ is the current flowing in (negative) and out (positive) of the battery and $Q_{max}[Ah]$ is the maximum battery charge capacity. Numerous battery models have been developed in the HEV literature depending on the intended level of accuracy Hu, Yurkovich, Guezennec, and Yurkovich (2009); Johnson, Pesaran, and Sack (2001). However, the energy management problem places more importance on the efficiency and losses in the battery pack, which allows the use of a zero-th order equivalent circuit based model shown in Fig. 1. The parameters of the zero-th battery model are the equivalent resistance[6] R_{eq} $[\Omega]$ and the open circuit voltage[7] V_{oc} $[V]$. In general these parameters depend on several factors such as SOC and temperature. Typically, in a charge-sustaining HEV, the battery is used only over a limited range of SOC (typically between 0.5–0.8). Experimental evidence, Hu et al. (2009), shows that over this range of SOC operation, the model parameters (R_{eq}, V_{oc}) do not vary significantly as a function of SOC and are therefore assumed to be known constants in this paper. The effect of temperature on battery parameters is not investigated here and it is left to future studies. With reference to Fig. 1, the voltage at the battery pack terminals is given by

$$V_L(t) = V_{oc} - I(t)R_{eq}, \tag{3}$$

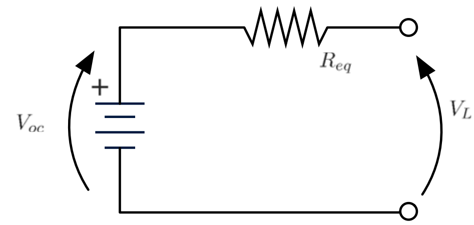

Fig. 1. Zero-th order electrical circuit model of the battery.

where $V_L(t)$ is the instantaneous terminal voltage. Multiplying (3) by current $I(t)$ on both sides, battery power $P_{batt}(t)$ is expressed as,

$$P_{batt}(t) = V_L(t)I(t) = V_{oc}I(t) - I^2(t)R_{eq}. \tag{4}$$

Solving the algebraic quadratic Eq. (4), the battery current $I(t)$ is expressed as a function of $P_{batt}(t)$ as:

$$I(t) = \frac{V_{oc} - \sqrt{(V_{oc})^2 - 4R_{eq}P_{batt}(t)}}{2R_{eq}}. \tag{5}$$

This result can then be substituted into the definition of $\dot{SOC}(t)$ of Eq. (2) to generate the nonlinear mapping

$$\begin{cases} \dot{SOC}(t) = -\alpha \dfrac{V_{oc} - \sqrt{(V_{oc})^2 - 4R_{eq}P_{batt}(t)}}{2R_{eq}Q_{max}}, \\ \dot{SOC}(t) = f_{SOC}(SOC(t), P_{batt}(t)). \end{cases} \tag{6}$$

2.2. Integral constraints

In a charge sustaining HEV, the net energy from the battery is zero over a given driving cycle, which means that the SOC at the end of the driving cycle should be the same as that in the beginning of the driving cycle, or

$$SOC(t_f) = SOC(t_0), \tag{7}$$

where $SOC(t_0)$, $SOC(t_f)$ represent the battery SOC at the beginning and end of the driving cycle. This integral constraint over the state variable must be satisfied for any given driving cycle.

2.3. Instantaneous constraints

Instantaneous constraints imposed on the state and control variables concern physical operation limits of components. In a pre-transmission parallel HEV powertrain (shown in Fig. 2) those are

$$\begin{cases} P_{batt,\min} \le P_{batt}(t) \le P_{batt,\max}, \\ SOC_{\min} \le SOC(t) \le SOC_{\max}, \\ T_{x,\min} \le T_x(t) \le T_{x,\max} \quad \forall t \in [t_0, t_f] \\ P_{x,\min} \le P_x(t) \le P_{x,\max}, \\ \omega_{x,\min} \le \omega_x(t) \le \omega_{x,\max}, \quad x = ice,\ mot, \end{cases} \tag{8}$$

where the last three inequalities represent instantaneous limitations on the engine, *ice*, and electric motor, *mot*, torque, power and speed respectively; $(\cdot)_{min}$, $(\cdot)_{max}$ are the minimum and maximum values of power, SOC, torque and speed at each instant. Moreover, constraints are also enforced at each instant to ensure that the total power demand at the wheels is satisfied. As shown in Fig. 2, the engine is connected in parallel with the electric motor and battery pack and can be engaged or disengaged from the wheels using a clutch.

To summarize, the optimal energy management problem consists of finding the control law $u(t)$ that minimizes (1) under

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[6] The equivalent resistance in a zero-th circuit model is a single lumped resistance of the battery pack estimated from experimental data Hu et al. (2009).

[7] The open circuit voltage is the output voltage of the battery pack when it is not loaded and it is determined from experimental data Hu et al. (2009).

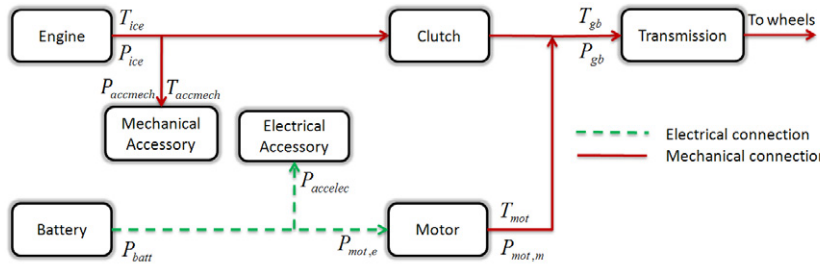

Fig. 2. Power flow diagram of pre-transmission parallel HEV.

the constraints:

$$\begin{cases} \dot{SOC}(t) = f_{SOC}(SOC(t), u(t)) \quad \forall t \in [t_0, t_f], \\ SOC(t_0) = SOC_{init}, \\ |SOC_{t_f} - SOC_{ref}| - \epsilon_{SOC} \leq 0, \\ SOC_{min} - SOC \leq 0 \quad \forall t \in [t_0, t_f], \\ SOC - SOC_{max} \leq 0 \quad \forall t \in [t_0, t_f], \\ u \in [u_{min}, u_{max}] \quad \forall t \in [t_0, t_f], \end{cases} \tag{9}$$

where $SOC_{init}, SOC_{ref}, SOC_{min}, SOC_{max}$ represent the predefined initial value, reference value, minimum and maximum limits of the battery SOC respectively; ϵ_{SOC} is the maximum allowable deviation of the battery SOC at the end of the driving cycle ($SOC(t_f)$) from SOC_{ref}; and, $u(t)$ is the control input selected depending on the vehicle architecture. In a parallel HEV, the control input can be either battery power ($u(t) = P_{batt}(t)$) or the engine power ($u(t) = P_{ice}(t)$) depending on the vehicle mode of operation. The vehicle can operate in three different modes depending on the status of clutch and gear position. These modes are described next.

2.3.1. All electric mode

With the clutch open, the engine can be switched off and the vehicle uses only the battery and electric motor for propulsion. The torque/power requested by the driver at the wheels is satisfied using the battery and electric motor (mot). The torque/power balance equations that must be satisfied are

$$\begin{cases} T_{mot}(t) = T_{gb}(t), \\ P_{batt}(t) = P_{mot,e}(t) + P_{accelec}(t), \quad \forall t \in [t_0, t_f] \\ \omega_{mot}(t) = \omega_{gb}(t), \end{cases} \tag{10}$$

where $T_{gb}(t), \omega_{gb}(t)$ represent the instantaneous gearbox torque and speed; $T_{mot}(t), \omega_{mot}(t)$ represent the instantaneous electric motor and speed; $P_{accelec}(t)$ represents the instantaneous electrical accessory power and $P_{mot,e}(t)$ represent the instantaneous electrical power at input/output terminals of the electric motor.

2.3.2. Parallel mode with neutral gear

In this mode of operation, the vehicle is stopped with the clutch closed and gearbox in neutral position. Though both the devices are connected to the transmission, because the gear is in neutral condition, the engine can be operated at any desired speed. This mode of operation mimics the real world situation of the vehicle being stopped at a traffic signal when the gearbox torque requested by the driver is zero. The engine is kept on and used in conjunction with the battery and electric motor to charge/discharge the battery. The torque/power balance equations that must be satisfied are

$$\begin{cases} T_{mot}(t) + T_{ice}(t) = T_{accmech}(t), \\ P_{batt}(t) = P_{mot,e}(t) + P_{accelec}(t), \quad \forall t \in [t_0, t_f] \\ \omega_{mot}(t) = \omega_{ice}(t) = \omega_{ice,opt}(t), \end{cases} \tag{11}$$

where $T_{ice}(t), \omega_{ice}(t)$ represent the instantaneous engine torque and speed, $T_{accmech}(t)$ represents the instantaneous mechanical accessory torque and $\omega_{ice,opt}(t)$ represents the instantaneous *optimal* engine speed based on the maximum efficiency operating line of the engine. Assuming a constant efficiency for the electric motor (η_{mot}), the engine power, $P_{ice}(t)$, can be represented as a function of battery power, $P_{batt}(t)$, electrical, $P_{accelec}(t)$ and mechanical $P_{accmech}(t)$ accessory power as

$$P_{ice}(t) = \frac{1}{\eta_{mot}} P_{accelec}(t) + P_{accmech}(t) - \eta_{mot} P_{batt}(t). \tag{12}$$

2.3.3. Parallel mode

With the clutch closed, the parallel mode of operation uses both the devices to propel the vehicle and their speed is directly determined by the vehicle velocity. In this mode of operation, the vehicle is moving and the gear is free to operate in any condition. The vehicle speed determines the speed of the devices and the torque/power requested by the driver is supplied by the parallel configuration. The only degree of freedom available in this mode is the engine torque, $T_{ice}(t)$ or electric machine torque, $T_{mot}(t)$. The torque/power balance equations that must be satisfied are

$$\begin{cases} T_{mot}(t) + T_{ice}(t) = T_{gb}(t) + T_{accmech}(t), \\ P_{batt}(t) = P_{mot,e}(t) + P_{accelec}(t), \quad \forall t \in [t_0, t_f] \\ \omega_{mot}(t) = \omega_{ice}(t) = \omega_{gb}(t). \end{cases} \tag{13}$$

The battery power can be represented as a function of engine power and the requested power, $P_{req}(t)$, as

$$\begin{cases} P_{batt}(t) = -\frac{1}{\eta_{mot}} P_{ice}(t) + \frac{1}{\eta_{mot}} P_{req}(t), \\ P_{req}(t) = P_{gb}(t) + \frac{1}{\eta_{mot}} P_{accelec}(t) + P_{accmech}(t). \end{cases} \tag{14}$$

2.4. Engine fuel consumption rate model

The optimal control problem in HEV minimizes the fuel consumption (1) over a driving cycle, which is generally modeled as a map for every possible combination of engine speed and torque. The fuel consumption map of the engine used in this paper is shown in Fig. 11. The engine fuel consumption rate can be expressed as a function of the engine torque/power and speed using an appropriate Willans line model Sciarretta and Guzzella (2007). In general, for any energy conversion device, the efficiency of the device can be modeled by representing the input power as an affine function of the output power and losses. At a given engine speed, the output power, $P_{ice}(t)$, can be written as an affine function of the input chemical power, $P_{chem}(t)$. The slope and intercept of each of the Willans lines can be expressed as polynomial functions (Figs. 3 and 4) of the engine speed, by

$$\begin{cases} P_{chem}(t) = e_0(\omega_{ice}(t)) + e_1(\omega_{ice}(t)) P_{ice}(t), \\ e_0(\omega_{ice}(t)) = e_{00} + e_{01}\omega_{ice}(t) + e_{02}\omega_{ice}^2(t), \\ e_1(\omega_{ice}(t)) = e_{10} + e_{11}\omega_{ice}(t) + e_{12}\omega_{ice}^2(t), \end{cases} \tag{15}$$

where $e_{ij}, i, j = 0, 1, 2$ are constant coefficients, $P_{chem} = \dot{m}_f Q_{LHV}$ (Q_{LHV} is the lower heating calorific value of diesel in kJ/kg) is the

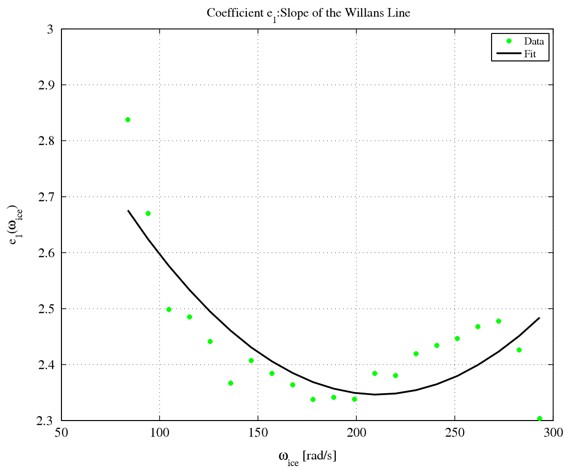

Fig. 3. Quadratic fit for slope of Willans line model.

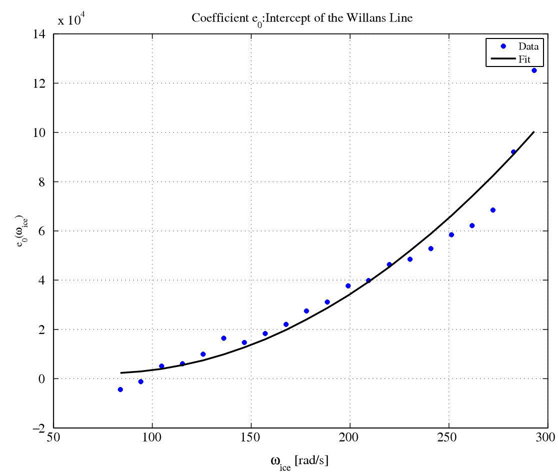

Fig. 4. Quadratic fit for intercept of Willans line model.

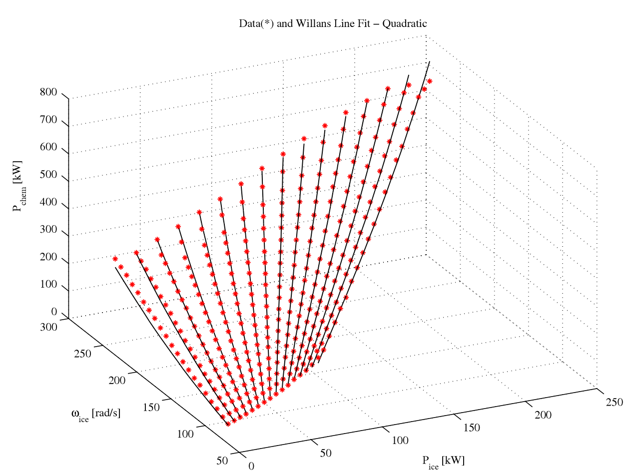

Fig. 5. Effectiveness of Willans line model.

chemical power input to the engine and $P_{ice}(t) = T_{ice}(t)\omega_{ice}(t)$ is the engine power output. Given the engine torque $T_{ice}(t)$ and speed $\omega_{ice}(t)$, the fuel consumption rate can be written as

$$\dot{m}_f(t) = \frac{1}{Q_{LHV}}[e_0(\omega_{ice}(t)) + e_1(\omega_{ice}(t))P_{ice}(t)]. \tag{16}$$

The effectiveness of the Willans line model in approximating the fuel consumption rate of the engine is shown in Fig. 5.

2.4.1. Parallel mode with neutral gear

In this mode, the engine speed $\omega_{ice}(t)$ is independent of the vehicle speed and can be chosen to operate the engine in its most efficient region. The *optimal* engine speed, $\omega_{ice,opt}(t)$, can be calculated by minimizing the chemical power, $P_{chem}(t)$, as

$$\begin{cases} \dfrac{\partial P_{chem}}{\partial \omega_{ice}} = 0 \Rightarrow \omega_{ice,opt} = -\dfrac{1}{2}\dfrac{e_{01} + e_{11}P_{ice}}{e_{02} + e_{12}P_{ice}}, \\ \dfrac{\partial^2 P_{chem}}{\partial \omega_{ice}^2} > 0, \\ \omega_{ice} \in [\omega_{ice,idle}, \omega_{ice,max}], \end{cases} \tag{17}$$

where $\omega_{ice,idle}$, $\omega_{ice,max}$ represent the idle and maximum speed of the engine. The fuel consumption rate can be expressed as an affine function of $P_{ice}(t)$, as

$$\dot{m}_f(t) = m_0 + m_1 P_{ice}(t) \tag{18}$$

where m_0 and m_1 are known constants obtained from (16) and (17). Moreover, because $P_{ice}(t)$ is a function of the control input $P_{batt}(t)$ as given by (12), ultimately the fuel consumption rate $\dot{m}_f(t)$ can be expressed as a direct function of the control input, $P_{batt}(t)$, i.e.,

$$\dot{m}_f(t) = p_0 + p_1 P_{batt}(t), \tag{19}$$

through coefficients p_0, p_1 expressed as follows:

$$\begin{cases} p_0 = m_0 + m_1\left(P_{accmech} + \dfrac{1}{\eta_{mot}}P_{accelec}\right), \\ p_1 = -m_1\eta_{mot}. \end{cases} \tag{20}$$

2.4.2. Parallel mode

In this mode, engine and electric motor speed are directly determined from the vehicle speed. The fuel consumption rate model can be written as

$$\dot{m}_f(t) = \frac{1}{Q_{LHV}}[e_0(\omega_{gb}(t)) + e_1(\omega_{gb}(t))P_{ice}(t)]. \tag{21}$$

Depending on the control input chosen, the fuel consumption rate can be expressed as a function of the control input using (14). Under the assumption that the slope and intercept of the Willans line model are independent of the engine speed, the fuel consumption rate can be expressed as an affine function of engine power $P_{ice}(t)$ in the manner

$$\dot{m}_f(t) = p_2 + p_3 P_{ice}(t), \tag{22}$$

where p_2, p_3 are known constants calculated by using the $\omega_{gb}(t)$ into (15). The coefficients p_0, p_1, p_2, p_3 are in general a function of time as they depend on the vehicle mode of operation. The Willans line based engine fuel consumption rate model and the battery *SOC* dynamics described in this section are used in the remainder of the paper to cast the energy management problem as a nonlinear optimal regulation problem. The closed-form expression for the fuel consumption rate using the Willans line model is used in finding the analytical optimal control law.

3. Stability framework for HEV optimal control problem

In this section, we focus on creating a generalized stability and optimality framework to analyze and design energy management strategies that can be implemented in a real vehicle. The framework developed can be adapted to various powertrain architectures and is scalable with respect to different component sizes. The asymptotic stability of the origin and optimality of the control law are developed by formulating the energy management problem into nonlinear optimal regulation theory with disturbance rejection. This section provides the mathematical preliminaries

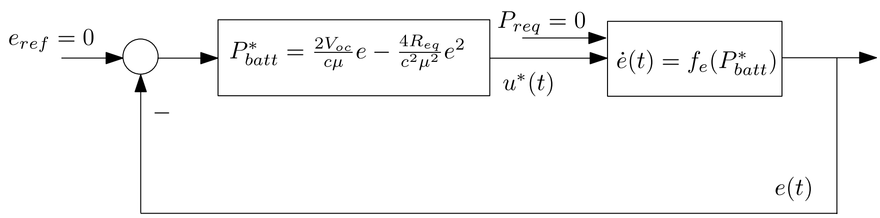

Fig. 6. Energy management strategy as a nonlinear optimal regulation problem.

necessary for the framework and we propose and prove a series of theorems on how the state feedback control law guarantees optimality and asymptotic stability, both with and without external disturbances, thereby leveraging results from nonlinear optimal regulation theory Bernstein (1993), Haddad and Chellaboina (2008). Section 3.1 provides a theorem to find an analytical expression for a state-feedback based control law that asymptotically stabilizes the origin while minimizing the fuel consumption over an infinite time horizon in the absence of external disturbances. Section 3.2 extends the theorem in the presence of external disturbances.

3.1. Nonlinear optimal regulation of HEV

This subsection deals with formulating a theorem that uses a control Lyapunov function to develop a state-feedback based control law that minimizes the fuel consumption over infinite horizon and stabilizes the battery SOC in the absence of external disturbances. We formulate the energy management problem assuming that the power requested at the wheels is zero as a nonlinear optimal regulation problem as shown in Fig. 6. The error in battery SOC ($e(t) = SOC_{ref} - SOC(t)$) and battery power are considered as the state and control variables of the system. The battery $SOC(t)$ error dynamics are defined as

$$\begin{cases} e(t) = SOC_{ref} - SOC(t), \\ \dot{e}(t) = \alpha \dfrac{V_{oc} - \sqrt{(V_{oc})^2 - 4R_{eq}P_{batt}(t)}}{2R_{eq}Q_{max}} = f_e(P_{batt}(t)) \end{cases} \tag{23}$$

where $P_{batt}(t)$ is the control input of the system. Some mathematical preliminaries for the scalar system (23) with single control input, which are instrumental to the following discussion, are presented next Bernstein (1993).

Consider an open set $\mathcal{D} \subset \mathbb{R}$ such that $e \in \mathcal{D}$, an arbitrary set $\mathcal{U}_1 \subset \mathbb{R}$ such that $P_{batt} \in \mathcal{U}_1$ and $0 \in \mathcal{D}, 0 \in \mathcal{U}_1$. In the HEV problem, the state domain and control domain can be defined as

$$\begin{cases} e \in \mathcal{D} = [SOC_{ref} - SOC_{max}, SOC_{ref} - SOC_{min}], \\ P_{batt} \in \mathcal{U}_1 = [P_{batt,min}, P_{batt,max}]. \end{cases} \tag{24}$$

Furthermore, let $f_e : \mathcal{U}_1 \to \mathbb{R}$ satisfy $f_e(0) = 0$. Now consider the controlled system

$$\dot{e}(t) = f_e(P_{batt}(t)), \quad e(0) = e_0, \; t \geq 0, \tag{25}$$

where the control input $P_{batt}(\cdot)$ is restricted to the class of functions such that

$$P_{batt}(t) \in \Omega_1, \quad t \geq 0, \tag{26}$$

where the control constraint set $\Omega_1 \subseteq \mathcal{U}_1$ is compact and $0 \in \Omega_1$. The control input constraint Ω_1 is defined the maximum and minimum battery power depending on the battery parameters at each instant. Let the *optimal control law* P^*_{batt} be a measurable mapping $P^*_{batt} : \mathcal{D} \to \Omega_1$ satisfying $P^*_{batt}(0) = 0$. Now the system (23) with feedback control $P_{batt} = P^*_{batt}(e)$, has the form

$$\dot{e}(t) = f_e(P^*_{batt}(e(t))), \quad e(0) = e_0, \; t \geq 0. \tag{27}$$

In order to address the problem of characterizing feedback controllers that minimize a performance functional, let $\mathcal{H}_1 : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $\dot{m}_f : \mathbb{R} \to \mathbb{R}$ and $\lambda \in \mathbb{R}$ such that,

$$\mathcal{H}_1(e, P_{batt}, \lambda) \triangleq \dot{m}_f(P_{batt}) + \lambda \cdot f_e(P_{batt}), \tag{28}$$

where $\mathcal{H}_1(\cdot, \cdot, \cdot)$ is the Hamiltonian function, $\dot{m}_f(\cdot)$ is the instantaneous cost function expressed in (19) and λ is the co-state variable.

Finally, without loss of generality, a new Hamiltonian function $\bar{\mathcal{H}}_1$ is defined to take on zero-value when evaluated at the optimal control, as

$$\begin{cases} \mathcal{H}_1(e, P_{batt}, \lambda) = \dot{m}_f(P_{batt}) + \lambda f_e(P_{batt}), \\ \bar{\mathcal{H}}_1(e, P_{batt}, \lambda) \triangleq \mathcal{H}_1(e, P_{batt}, \lambda) - p_0, \end{cases} \tag{29}$$

where p_0 is a parameter of the engine fuel consumption rate model from (20).

The result that follows gives sufficient conditions under which the origin $e(t) = 0$ can be locally asymptotically stabilized under nonlinear state feedback control, while also assuring optimality of the fuel consumption over an infinite time horizon. Sufficient conditions for stability and optimality are given in the case where no external inputs or disturbances enter the system (25), which corresponds to $P_{gb}(t) = 0 \forall\, t \geq 0$, and the system initial condition different from zero, i.e. $e_0 \neq 0$. In the context of charge-sustaining HEVs, the considered scenario (Fig. 6) corresponds to having the vehicle switched on without any tractive force at the wheels (vehicle velocity = 0) and the battery SOC is not at the reference value, i.e. $SOC_{ref} \neq SOC(0)$. What follows is an original result on stability and optimality in the context of energy management problem in HEVs, that builds upon the main results shown in Bernstein (1993).

Theorem 1. *Consider the system* (25) *with performance functional*

$$J(e_0, P_{batt}(\cdot)) \triangleq \int_0^\infty \dot{m}_f(P_{batt}(t))dt. \tag{30}$$

Then with the feedback control $P_{batt}(t) = P^*_{batt}(e(t))$, *where* $P^*_{batt}(e(t))$ *satisfies*

$$\begin{cases} P^*_{batt}(e(t)) = \dfrac{2V_{oc}}{c\mu}e(t) - \dfrac{4R_{eq}}{c^2\mu^2}e^2(t), \\ c = \dfrac{2R_{eq}Q_{max}p_1}{\alpha}, \end{cases} \tag{31}$$

the solution $e(t) = 0, t \geq 0$ *of the closed-loop system* (27) *is locally asymptotically stable and the optimal feedback control law* $P^*_{batt}(e(\cdot))$ *minimizes* $J(e_0, P_{batt}(\cdot))$

Proof. Considering the candidate Lyapunov function $V(e) = \frac{1}{2}\mu e^2$; $\mu > 0$, local asymptotic stability of the origin $e(t) = 0$ and optimality of P^*_{batt} with respect to $J(e_0, P_{batt}(\cdot))$ are proved using the following conditions given in Bernstein (1993):

(1) The Lyapunov function $V(e)$ has a minimum value of 0 at the origin

$$V(0) = 0. \tag{32}$$

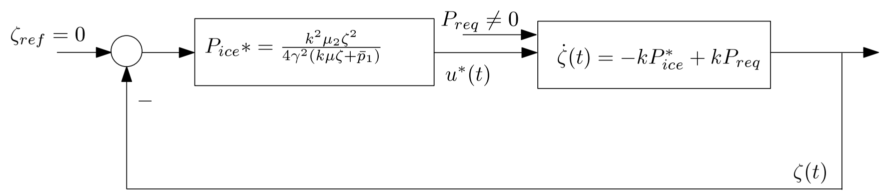

Fig. 7. Energy management strategy as a nonlinear optimal regulation problem with disturbance rejection.

(2) The candidate Lyapunov function $V(e)$ is a positive definite function of e

$$V(e) > 0 \quad \forall\, e \in \mathcal{D}, e \neq 0. \tag{33}$$

(3) The optimal feedback control law is zero at the origin:

$$P^*_{batt}(0) = 0. \tag{34}$$

(4) Asymptotic stability of the origin is achieved when the optimal control law is applied, i.e. $\dot{V}(P^*_{batt}) < 0$:

$$\begin{cases} \dfrac{\partial V}{\partial e} f_e(P^*_{batt}(e)) < 0 \quad \forall e \in \mathcal{D}, e \neq 0, \\ = \mu e f_e(P^*_{batt}(e)) < 0, \\ \Rightarrow \begin{cases} f_e(P^*_{batt}(e)) < 0 \Rightarrow P^*_{batt}(e) < 0 \quad \forall e > 0, \\ f_e(P^*_{batt}(e)) > 0 \Rightarrow P^*_{batt}(e) > 0 \quad \forall e < 0. \end{cases} \end{cases} \tag{35}$$

(5) The Hamiltonian function $\bar{\mathcal{H}}_1$ takes on the minimum value of zero when the optimal control law ($P_{batt} = P^*_{batt}(e)$) is applied:

$$\begin{cases} \bar{\mathcal{H}}_1 \left(e, P^*_{batt}(e), \left(\dfrac{\partial V}{\partial e} \right)^T \right) = 0, \\ \Rightarrow \dot{m}_f(P^*_{batt}(e)) = -\dfrac{\partial V}{\partial e} f_e(P^*_{batt}(e)) \quad \forall e \in \mathcal{D}. \end{cases} \tag{36}$$

From (36), substituting the expression of fuel consumption (19), the optimal control law $P^*_{batt}(e)$ from nonlinear state feedback is:

$$\begin{cases} P^*_{batt} = \dfrac{2V_{oc}}{c\mu} e - \dfrac{4R_{eq}}{c^2\mu^2} e^2, \\ c = \dfrac{2R_{eq}Q_{max}p_1}{\alpha}. \end{cases} \tag{37}$$

(6) The Hamiltonian function $\bar{\mathcal{H}}_1$ takes on a value greater than zero when a control law (P_{batt}) other than the optimal control law (P^*_{batt}) is applied:

$$\begin{cases} \bar{\mathcal{H}}_1 \left(e, P_{batt}, \left(\dfrac{\partial V}{\partial e} \right)^T \right) \geq 0, \\ \Rightarrow \dot{m}_f(e, P_{batt}) \geq \dfrac{\partial V}{\partial e} f_e(P_{batt}) \quad \forall e \in \mathcal{D},\ u \in \Omega_1. \end{cases} \tag{38}$$

All the sufficient conditions are satisfied and the optimal control law ($P^*_{batt}(e)$) as a function of the state variable is obtained.

3.2. Nonlinear optimal regulation of HEV with disturbance rejection

This subsection deals with extending the theorem proved in Section 3.1 in the presence of external disturbances. The theorem formulated in that section assumes that the vehicle is stopped and the requested power at the wheels is zero. The situation considered now corresponds to the vehicle moving and the energy management strategy must find the optimal torque/power split between the engine and electric motor. Thus we formulate the energy management problem with the vehicle operating in parallel mode (Sections 2.3.3 and 2.4.2) as a nonlinear optimal regulation problem with disturbance rejection as shown in Fig. 7. The power requested at the wheels, $P_{req}(t)$, is considered as disturbance to the system. The error in battery state of energy ($\zeta = SOE - SOE_{ref}$) and engine power, $P_{ice}(t)$ are considered as the state and control variables of the system. The battery SOE is used instead of battery SOC because it is more convenient in formulating the theorem and its proof from a control design stand point. The battery SOE is defined as the amount of battery energy stored, relative to the maximum energy capacity of the battery, which can be expressed as

$$\begin{cases} S\dot{O}E = -\eta_{batt} \dfrac{P_{batt}}{E_{max}}, \\ E_{max} = Q_{max} V_{oc,max}. \end{cases} \tag{39}$$

where η_{batt} is the efficiency of the battery, $V_{oc,max}[V]$ is the maximum open-circuit voltage of the battery and $E_{max}[J]$ is the maximum battery energy capacity. The battery SOE can be calculated from SOC using a simple linear transformation,

$$SOE = SOC \frac{V_L}{V_{oc,max}}, \tag{40}$$

where $V_L(t)$ is the terminal voltage of the battery and SOC is the battery state of charge as defined in (2). Define the battery SOE error, ζ, dynamics as a function of the control input, $P_{ice}(t)$, as:

$$\begin{cases} \zeta(t) = SOE_{ref} - SOE(t), \\ \dot{\zeta}(t) = -kP_{ice}(t) + kP_{req}(t), \\ k = \dfrac{\eta_{batt}}{E_{max}\eta_{mot}}, \end{cases} \tag{41}$$

where $k(1/J)$ is a constant dependent on the battery and electric motor parameters, η_{mot} is the electric motor efficiency and $P_{req}(t)$ is the requested power at the gearbox (external disturbance to the system).

Consider an open set $\mathcal{Z} \subset \mathbb{R}$ such that $\zeta \in \mathcal{Z}$, an arbitrary set $\mathcal{U}_2 \subset \mathbb{R}$ such that $P_{ice} \in \mathcal{U}_2$ and $0 \in \mathcal{Z}, 0 \in \mathcal{U}_2$. In this case, the state domain and control domain can be defined as

$$\begin{cases} \zeta \in \mathcal{Z} = [SOE_{ref} - SOE_{max}, SOE_{ref} - SOE_{min}], \\ P_{ice} \in \mathcal{U}_2 = [0, P_{ice,max}]. \end{cases}$$

Also consider the disturbance input to the system as $w \in \mathcal{W}$ such that $\mathcal{W} \subset \mathbb{R}$. In the parallel mode of operation, the power requested at the gearbox, $P_{req}(t)$, as defined in (14), is the disturbance input $w(t) = P_{req}(t)$. Now consider the controlled system

$$\begin{cases} \dot{\zeta}(t) = -kP_{ice}(t) + kP_{req}(t), \quad \zeta(0) = \zeta_0,\ t \geq 0, \\ z(t) = \zeta(t), \end{cases} \tag{42}$$

where z is the performance variable. The control input $P_{ice}(\cdot)$ is restricted to the class of *admissible* controls consisting of measurable functions $P_{ice}(\cdot)$ such that

$$P_{ice} \in \Omega_2, \quad t \geq 0, \tag{43}$$

where the control constraint set $\Omega_2 \subset \mathcal{U}_2$ is compact and $0 \in \Omega_2$. Let the *optimal control law* P^*_{ice} be a measurable mapping $P^*_{ice} : \mathcal{Z} \to \Omega_2$ satisfying $P^*_{ice}(0) = 0$. Now the system (42) with feedback control $P_{ice} = P^*_{ice}(\zeta)$, has the form

$$\dot{\zeta} = -kP^*_{ice} + kP_{req}, \quad \zeta(0) = \zeta_0,\ t \geq 0. \tag{44}$$

In order to address the problem of characterizing feedback controllers that minimize a performance functional, let $\Gamma(\zeta) : \mathcal{Z} \to \mathbb{R}$, $\mathcal{H}_2 : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $\dot{m}_f : \mathbb{R} \to \mathbb{R}$ and $\lambda \in \mathbb{R}$ such that,

$$\mathcal{H}_2(\zeta, P_{ice}, \lambda) \triangleq \dot{m}_f(P_{ice}) + \Gamma(\zeta) + \lambda \cdot (-kP_{ice}), \tag{45}$$

where $\mathcal{H}_2(\cdot,\cdot,\cdot)$ is the Hamiltonian function, $\dot{m}_f(\cdot)$ is the instantaneous cost function expressed in (22), $\Gamma(\zeta)$ is a positive definite function of ζ and λ is the co-state variable. Finally, without loss of generality, a new Hamiltonian function $\bar{\mathcal{H}}_2$ is defined to take on zero-value when evaluated at the optimal control, as:

$$\begin{cases} \mathcal{H}_2(\zeta, P_{ice}, \lambda) = \dot{m}_f(P_{ice}) + \Gamma(\zeta) + \lambda \cdot (-kP_{ice}), \\ \bar{\mathcal{H}}_2(\zeta, P_{ice}, \lambda) \triangleq \mathcal{H}_2(\zeta, P_{ice}, \lambda) - p_2, \end{cases} \tag{46}$$

where p_2 is a parameter of the engine fuel consumption rate model as shown in (22).

The result that follows gives sufficient conditions under which the origin $\zeta = 0$ can be locally asymptotically stabilized under nonlinear state feedback control, while assuring optimality with respect to the fuel consumed over an infinite time horizon in the presence of external disturbances. The feedback controller guarantees stability, minimizes an auxiliary performance functional, and guarantees that the input–output map of the closed-loop system is dissipative, nonexpansive, and passive (Haddad & Chellaboina, 2008) for bounded input disturbances. In the context of charge-sustaining HEVs, the considered scenario corresponds to the vehicle driven for a driving cycle considering the power request at the wheels as an external disturbance to the system (Fig. 7). What follows is an extension of the theorem proposed in Section 3.1 in the presence of external disturbances based on the results given in Haddad and Chellaboina (2008).

Theorem 2. *Consider the system (42) with performance functional*

$$J(\zeta_0, P_{ice}(\cdot)) \triangleq \int_0^\infty \dot{m}_f(P_{ice})dt. \tag{47}$$

Then with the feedback control $P_{ice} = P^*_{ice}(\zeta)$, where P^*_{ice} satisfies

$$P^*_{ice} = \frac{k^2\mu^2\zeta^2}{4\gamma^2(k\mu\zeta + p_3)}, \tag{48}$$

the solution $\zeta(t) = 0$, $t \geq 0$ of the closed-loop system (44) is locally asymptotically stable and the optimal feedback control law $P^*_{ice}(\zeta(\cdot))$ minimizes $J(\zeta_0, P_{ice}(\cdot))$.

Proof. Considering the candidate Lyapunov function $V(\zeta) = \frac{1}{2}\mu\zeta^2$, $\mu > 0$ and functions $\Gamma(\zeta) = \frac{1}{4\gamma^2}\left(\frac{\partial V}{\partial \zeta}\right)^2 k^2$, and $r(\zeta, P_{req}) = \gamma^2 P_{req}^2 - \zeta^2$, $\gamma, k > 0$ the local asymptotic stability of the origin $\zeta(t) = 0$ and optimality of P^*_{ice} with respect to $J(\zeta_0, P_{ice}(\cdot))$ are proved using the following conditions taken from the book Haddad and Chellaboina (2008):

(1) The Lyapunov function $V(\zeta)$ has a minimum value of 0 at the origin

$$V(0) = 0. \tag{49}$$

(2) The candidate Lyapunov function $V(\zeta)$ is a positive definite function. In fact, $V(\zeta)$ is a quadratic function of ζ

$$V(\zeta) > 0 \quad \forall \zeta \in \mathcal{Z}, \zeta \neq 0. \tag{50}$$

(3) The optimal feedback control law is zero at the origin:

$$P^*_{ice}(0) = 0. \tag{51}$$

(4) Asymptotic stability of the origin is achieved when the optimal control law is applied, i.e. $\dot{V}(P^*_{ice}) < 0$:

$$\begin{cases} \dfrac{\partial V}{\partial \zeta}(-kP^*_{ice}(\zeta)) < 0 \quad \forall \zeta \in \mathcal{Z}, \zeta \neq 0, \\ \Rightarrow \begin{cases} P^*_{ice}(e) < 0 \quad \forall e < 0, \\ P^*_{ice}(e) > 0 \quad \forall e > 0. \end{cases} \end{cases} \tag{52}$$

This analysis provides conditions on the sign of state feedback control law $P^*_{ice}(\zeta)$ and because the engine power cannot be negative, the signs of the optimal feedback law can be expressed as:

$$\begin{cases} P_{ice}{}^*(\zeta) = 0, \quad \forall e \leq 0, \\ P_{ice}{}^*(\zeta) > 0, \quad \forall e > 0. \end{cases} \tag{53}$$

(5) The Hamiltonian function $\bar{\mathcal{H}}_2$ takes on the minimum value of zero when the optimal control law ($P_{ice} = P^*_{ice}(\zeta)$) is applied:

$$\begin{cases} \bar{\mathcal{H}}_2\left(\zeta, P^*_{ice}, \left(\dfrac{\partial V}{\partial \zeta}\right)\right) = 0, \\ p_3 P^*_{ice} + \dfrac{1}{4\gamma^2}k^2\mu^2\zeta^2 - k\mu\zeta P^*_{ice} = 0 \quad \forall \zeta \in \mathcal{Z}. \end{cases} \tag{54}$$

The optimal nonlinear state feedback control law (P^*_{ice}) can now be expressed as

$$\begin{cases} P^*_{ice} = \dfrac{k^2\mu^2\zeta^2}{4\gamma^2(k\mu\zeta + p_3)}, \\ k = \dfrac{\eta_{batt}}{E_{\max}\eta_{mot}} > 0, \quad \gamma > 0, \mu > 0. \end{cases} \tag{55}$$

(6) The Hamiltonian function $\bar{\mathcal{H}}_2$ takes on a value greater than zero when a control law (P_{ice}) other than the optimal control law (P^*_{ice}) is applied:

$$\bar{\mathcal{H}}\left(\zeta, P_{ice}, \left(\frac{\partial V}{\partial e}\right)\right) \geq 0. \tag{56}$$

(7) In order to prove the passivity with respect to the disturbance input $P_{req}(t)$, the following condition must be satisfied

$$\begin{cases} \dfrac{\partial V}{\partial \zeta}kP_{req} \leq r(\zeta, Preq) + \dot{m}_f(P^*_{ice}(\zeta)) + \Gamma(\zeta), \\ \Rightarrow k\mu\zeta P_{req} \leq \gamma^2 P_{req}^2 + \zeta^2\left[\dfrac{k^2\mu^2}{4\gamma^2} - 1\right] + p_3. \end{cases} \tag{57}$$

If there exists a constant γ such that $\gamma \geq 3k$, then a minimum bound for μ can be calculated as

$$\frac{\zeta^2}{36}\mu^2 - (k\zeta P_{req})\mu + 9k^2P_{req}^2 - \zeta^2 + p_3 \geq 0. \tag{58}$$

The passivity condition (57) is satisfied only if we can find a suitable γ and μ.

All the conditions are satisfied and the optimal control law ($P^*_{ice}(\zeta)$) as a function of the state variable is obtained.

3.3. Optimal and stabilizing control law

The control law ($P_{ice} = P^*_{ice}(\zeta)$) developed using the theorem can be expressed as

$$P^*_{ice} = \frac{k^2\mu^2\zeta^2}{4\gamma^2(k\mu\zeta + p_3)}, \tag{59}$$

where $\zeta(t)$ is the error in battery SOE, $k, \gamma, p_3 > 0$ are known constants, and μ is the only calibration parameter of the control law.

These parameters and constants depend on the powertrain architecture and the components; for example,

Table 1
Vehicle characteristics.

Vehicle mass	19,878 kg
Engine capacity	6.7 L Diesel
Engine power	194 kW
Motor power	200 kW
Battery energy capacity	7.5 kWh (27 MJ)
Electrical Accessory	7 kW
Mechanical Accessory	4 kW

(1) $k = \frac{\eta_{batt}}{E_{max}\eta_{mot}}$ is a constant depending on the battery capacity, efficiency and electric motor efficiency and the size/type of battery and electric machine;
(2) $\gamma = 3k$ is a constant expressed as a function of k obtained from the passivity condition in (57);
(3) p_3 is a Willans line coefficient of the engine as in (22) which depends on the size and type of the engine used;
(4) μ is the calibration parameter that must be tuned to achieve the best performance.

According to the theorem, the control law P^*_{ice} locally asymptotically stabilizes the origin $e = 0$. This implies that the battery SOE asymptotically converges to the SOE reference value. Because the battery SOC is linearly related to the battery SOE (40), the control law results in asymptotic convergence of the battery SOC to its reference value. In a charge-sustaining HEV, the battery SOC reference value is ideally the initial value with which the trip began. In addition to stabilizing the battery SOC, the control law minimizes the amount of fuel consumed over the infinite time horizon. That is, it is also optimal with respect to the performance functional $J(\zeta_0, P_{ice}(\cdot))$,

$$J(\zeta_0, P^*_{ice}(\zeta)) = \min_{P_{ice}} J(\zeta_0, P_{ice}(\zeta)). \tag{60}$$

Though the theorem guarantees that the control law minimizes the fuel consumed over an infinite time horizon, the energy management problem in a HEV minimizes the amount of fuel consumed over a finite length of a driving cycle (as shown in (9) in Section 2.3). The presence of calibration parameter μ in the control law (59) signifies the application of theoretically developed control law in a real-world application. Thus, for a given driving cycle, the sufficient conditions of asymptotic stability of battery SOE and optimality with respect to fuel consumed, are assured only with the *optimal* μ (μ^*). The *optimal* value of μ for a given driving cycle is obtained by studying the behavior of battery SOC and the fuel consumed over the driving cycle. The calibration of μ and its performance in comparison with the global optimal solution is described in Section 4.

4. Simulation results

This section describes the simulation environment used to implement the optimal control law developed using the theorem in Section 3 and its performance compared to several other energy management strategies. The pre-transmission parallel HEV (Fig. 2) is modeled in the MATLAB/Simulink environment. The characteristics of the vehicle used in this paper are shown in Table 1.

4.1. Vehicle simulator

In general the vehicle model can be simulated using a forward or backward structure. The former is a longitudinal and quasi-static vehicle simulator with standard representation of road load based on inertial, rolling and aerodynamic resistances. It is called a forward simulator because the torque/speed signals are propagated to/from the different components of the vehicle. Based on the vehicle velocity profile to be followed, a simple PID based driver model generates acceleration and brake pedal commands (much like the real driver). Because the forward simulator is primarily used in the analysis of energy management strategies, all the components are modeled using quasi-static map based models with the most relevant dynamics.In order to compare the performance of the proposed control law with the optimal global solution obtained from dynamic programming, a backward vehicle simulator is used. Based on the assumption that the vehicle is supposed to follow the desired velocity trajectory, the torque required at the wheels and subsequently the torque/speed required from the components are calculated using simplified stationary maps.

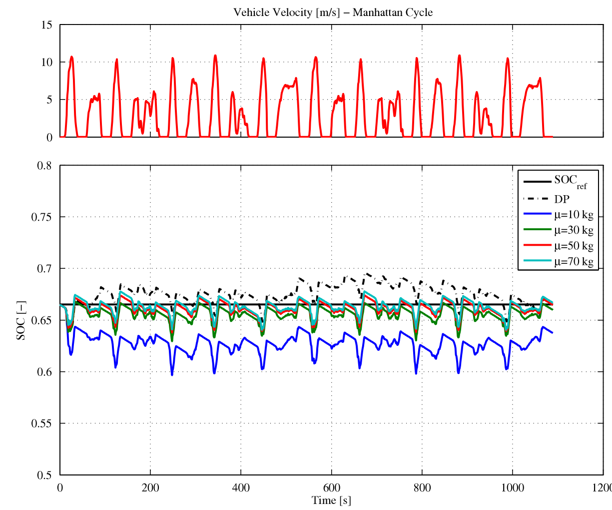

Fig. 8. Effect of μ on SOC.

4.2. Implementation of analytical control Law

In order to evaluate and compare the performance of the analytical control law proposed in Section 3 with DP, it is implemented in a backward vehicle simulator. Though the theorem proved in that section assumes an infinite time horizon, the developed control law has been implemented over a finite length driving cycle. The control law is thus implemented as a solution to the energy management problem described in Section 2.3.

4.2.1. Calibration of parameter-μ

The optimal control law $P^*_{ice}(\zeta)$ shown in (59) has a tuning parameter μ that must be calibrated to obtain the optimal and stable battery SOC. The effect of the calibration parameter μ is shown in Fig. 8.As seen from these plots, the value of μ impacts the convergence of SOC to SOC_{ref} at the end of the driving cycle. The optimal value of μ is selected based on this deviation (ΔSOC) and the equivalent fuel consumed (FC_{eqv}), defined as

$$\begin{cases} \Delta SOC = \dfrac{SOC(t_f) - SOC_{ref}}{SOC_{ref}} \cdot 100, \\ FC_{eqv} = \dfrac{\Delta SOCE_{max}}{\eta_{path} Q_{LHV}}, \end{cases} \tag{61}$$

where $SOC(t_f)$ is the battery SOC at the end of the driving cycle and η_{path} is the approximate efficiency of the drivetrain used in regenerating/discharging the SOC. If ΔSOC is positive, it implies that $SOC(t_f) > SOC_{ref}$ and the excess battery SOC can be used later to save fuel. If ΔSOC is negative, it implies that more fuel is required to recharge the battery SOC to the reference value. The effects of using different values of μ are summarized in Table 2. The *optimal* value of μ ensures convergence of SOC to the reference value SOC_{ref} while consuming the least amount of fuel over the driving cycle.

Table 2
Effect of μ for Manhattan driving cycle.

Strategy	μ (kg)	ΔSOC (%)	FC_{eqv}(%)
Optimal Control Law(OCL)	10	−4.16	106.3
	30	−0.79	105.1
	50	−0.07	104.9
	70	0.25	105.1
DP	–	−0.072	100

Table 3
Performance comparison with DP for Manhattan driving cycle.

Strategy	ΔSOC (%)	FC_{eqv} (%)
Optimal control law ($\mu^* = 53.13$)	−0.002	103.5
DP	−0.072	100

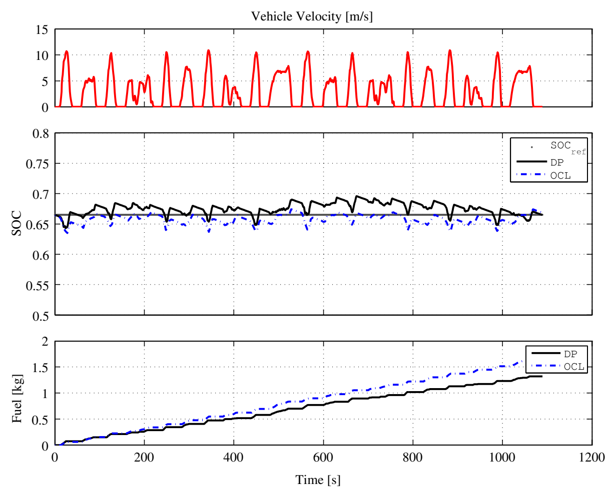

Fig. 9. SOC and equivalent fuel consumed-Manhattan.

4.2.2. Performance comparison with DP

The calibration parameter μ of the control law is optimized for a Manhattan driving cycle by selecting μ that corresponds to minimum value of ΔSOC and FC_{eqv}. In order to find the optimal μ (μ^*) for each driving cycle, an iterative shooting method is used (Serrao, Onori, & Rizzoni, 2011). The results of such a shooting method is shown in Table 3. The performance of the control law (Eq. (59)) with the optimum value of μ is evaluated against the global optimal solution obtained from DP and is shown in Figs. 9–12. The analytical control law consumes 4% more fuel than the global optimal solution and uses the battery SOC similar to the DP solution throughout the driving cycle. The engine and electric motor torque resulting from the analytical control law and DP are compared in Fig. 10. The engine and electric motor are operated mainly in their most efficient regions similar to the DP solution as shown in Figs. 11 and 12.

In addition to urban driving cycle represented by Manhattan driving cycle, the control law is tested on the WVU-Interstate driving cycle. This driving cycle is a representative of the highway driving conditions experienced by heavy-duty HEVs. The control law performs within 3% of the fuel consumed by DP and the charge sustainability is also assured as shown in Table 4. The performance of the optimal control law in comparison with global optimal solution is shown in Figs. 13 and 14.

4.2.3. Sensitivity of analytical control law with μ

Because the optimality and stability properties of the control law developed in Section 3.2 depends on the *optimal* value of μ, it is important to study the sensitivity of the results to μ. In order to generalize the effects of μ, ΔSOC is calculated for a wide range

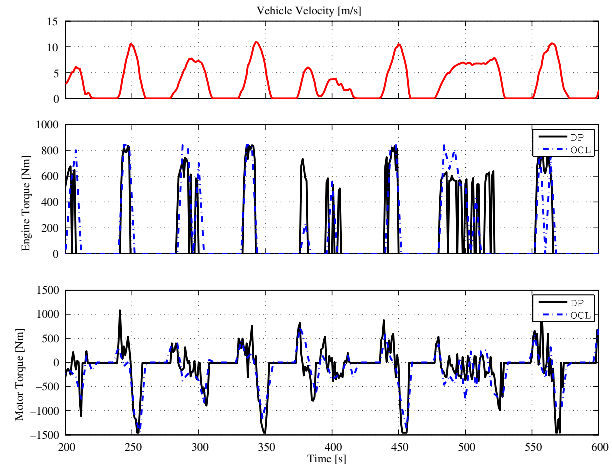

Fig. 10. Engine and electric motor torques — detail-Manhattan.

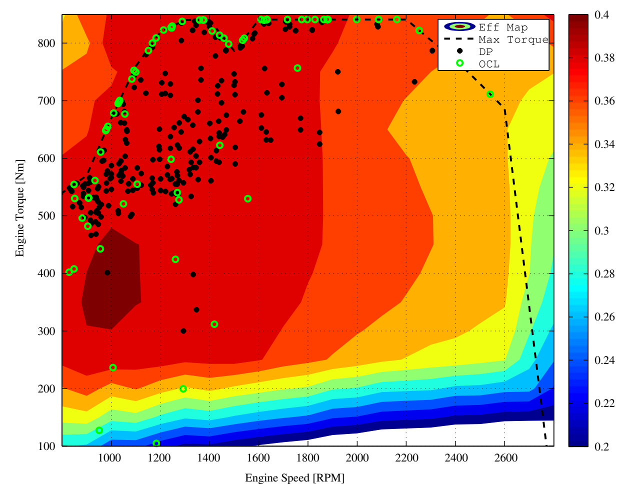

Fig. 11. Engine operating points-Manhattan.

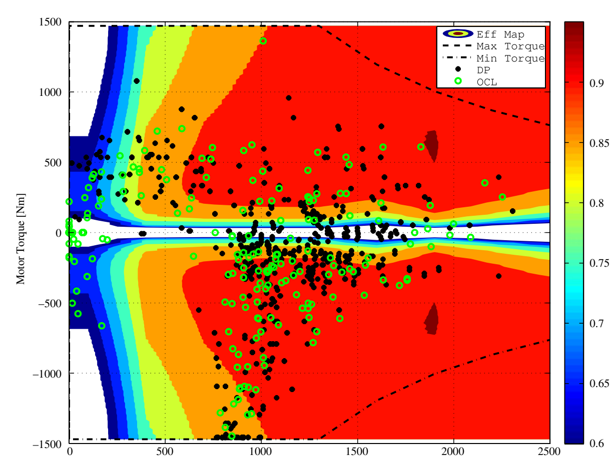

Fig. 12. Electric motor operating points-Manhattan.

Table 4
Performance comparison with DP for WVU-Interstate driving cycle.

Strategy	ΔSOC (%)	FC_{eqv} (%)
Optimal control law ($\mu^* = 58.88$)	−0.035	102.7
DP	−0.069	100

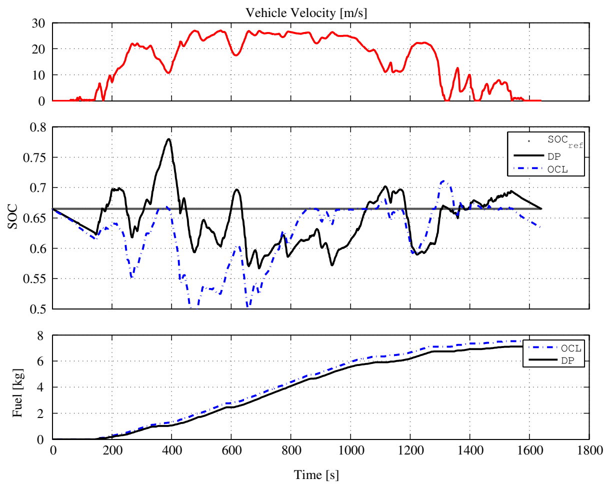

Fig. 13. SOC and equivalent fuel consumed-WVU-Interstate.

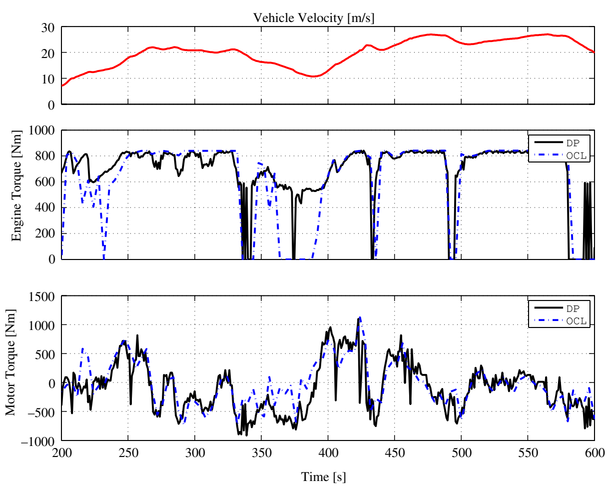

Fig. 14. Engine and electric motor torques — detail-WVU-Interstate.

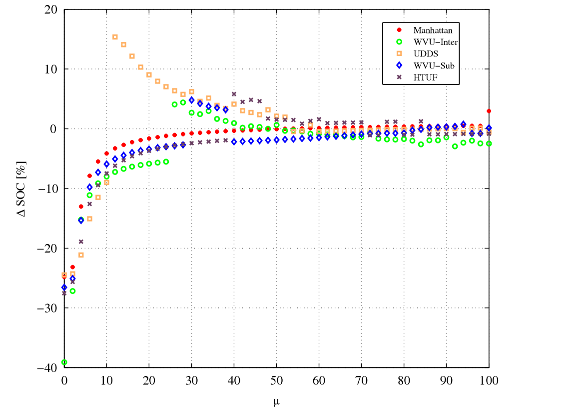

Fig. 15. Effect of μ on ΔSOC for different driving cycles (Manhattan, WVU-Interstate, etc.)

of μ for different representative driving cycles. For example, the Manhattan, WVU-suburban, UDDS truck driving cycles represent the urban driving conditions of heavy-duty vehicles. The WVU-Interstate, HTUF driving cycles represent a combination of urban and highway driving cycles. The effect of different values of μ is shown in Fig. 15. For all the driving cycles, there is a single value of μ that assures charge sustainability ($\Delta SOC = 0$) and consumes the least amount of fuel. The sensitivity of the optimal value of μ (μ^*) for the different driving cycles is shown in Fig. 15. As seen from the plot, the optimal control law developed ensures a minimum amount of ΔSOC as μ reaches a steady state value. This is significant because the optimality and stability properties for a wrong guess of μ are still close to the performance of μ^*.

5. Conclusion

The main contribution of the paper is a new stability and optimality framework for designing analytical energy management strategy. The proposed strategy is designed and developed for a charge sustaining pre-transmission parallel HEV, but the methodology is scalable to different vehicle architectures and component sizes. The paper proves a series of theorems on solving the problem as a nonlinear optimal regulation problem with and without disturbance rejection. The theorems are instrumental in developing a closed-form expression for the nonlinear state feedback based optimal control law. The resulting novel control law is proved optimal with respect to the fuel consumed over an infinite time horizon and guarantees local asymptotic stability of the origin. Though the optimality of the control law and asymptotic stability property of the origin are proved for an infinite time horizon, the paper shows the performance of the optimal control law when applied to a finite time driving cycle. The optimality ensures that minimum fuel is consumed and stability guarantees that battery SOC at the end of the driving cycle converges to SOC_{ref}. The optimal control law is implemented in a simplified backward simulator and its performance is compared with the global optimal solution from DP. The strategy with the optimal value of calibration parameter consumes within 4% of the fuel consumed using DP.

Appendix. Nonlinear optimal regulation

Let $D \subset \mathbb{R}^n$ be an open set and let $U \subset \mathbb{R}^m$ be an arbitrary set, where $0 \in D$ and $0 \in U$. Furthermore, let $f : D \times U \to \mathbb{R}^n$ satisfy $f(0, 0) = 0$. Now consider the controlled system

$$\dot{x}(t) = f(x(t), u(t)), \qquad x(0) = x_0, \quad t \geq 0. \tag{62}$$

The control $u(\cdot)$ in (62) is restricted to the class of *admissible* controls consisting of measurable functions $u(\cdot)$ such that

$$u(t) \in \Omega, \quad t \geq 0, \tag{63}$$

where the control constraint set $\Omega \subset U$ is compact and $0 \in \Omega$. A measurable mapping $\phi : D \to \Omega$ satisfying $\phi(0) = 0$ is called a *control law*. Given a control law $\phi(\cdot)$ and a feedback control $u(t) = \phi(x(t))$, the *closed-loop system* has the form

$$\dot{x}(t) = f(x(t), \phi(x(t))), \qquad x(0) = x_0, \quad t \geq 0. \tag{64}$$

In order to address the problem of characterizing feedback controllers that minimize a performance functional, let $L : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}$, $L : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ and $p \in \mathbb{R}^n$ such that,

$$H(x(t), u(t), p) \triangleq L(x(t), u(t)) + p^T f(x(t), u(t)). \tag{65}$$

Furthermore, define the set of asymptotically stabilizing controllers $S(x_0)$ for each initial condition $x_0 \in D$, that is, $S(x_0) \triangleq u(\cdot) : u(\cdot)$ is admissible and $x(\cdot)$ given by (62) satisfies $x(t) \to 0$ as $t \to \infty$.

Theorem 3 (Bernstein, 1993). *Consider the controlled system* (62) *with performance functional*

$$J(x_0, u(\cdot)) \triangleq \int_0^\infty L(x(t), u(t))dt. \tag{66}$$

Assume that there exists a C^1 function $V : D \to \mathbb{R}$ and a control law $\phi : D \to \Omega$ such that

(1)

$$V(0) = 0, \tag{67}$$

(2)

$$V(x(t)) > 0, \quad x(t) \in D,\ x(t) \neq 0, \tag{68}$$

(3)

$$\phi(0) = 0, \tag{69}$$

(4)

$$\frac{\partial V}{\partial x} f(x(t), \phi(x(t))) < 0, \quad x(t) \in D,\ x(t) \neq 0, \tag{70}$$

(5)

$$H\left(x(t), \phi(x(t)), \left(\frac{\partial V}{\partial x}\right)^T\right) = 0, \quad x(t) \in D, \tag{71}$$

(6)

$$H\left(x(t), u(t), \left(\frac{\partial V}{\partial x}\right)^T\right) \geq 0, \quad x \in D,\ u \in \Omega. \tag{72}$$

Then with the feedback control $u(\cdot) = \phi(x(\cdot))$, the solution $x(t) = 0, t \geq 0$, of the closed-loop system (64) is locally asymptotically stable, and

$$J(x_0, \phi(x(\cdot))) = V(x_0). \tag{73}$$

Furthermore, the feedback control $u(\cdot) = \phi(x(\cdot))$ minimizes $J(x_0, u(\cdot))$ in the sense that

$$J(x_0, \phi(x(\cdot))) = \min_{u(\cdot) \in S(x_0)} J(x_0, u(\cdot)). \tag{74}$$

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