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Errata Corrige – A Novel Lithium-ion Battery Pack Modeling Framework - Series-Connected Case Study

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This document provides some updates of the governing equations and nomenclature presented in "A Novel Lithium-ion Battery Pack Modeling Framework - Series-Connected Case Study", published in the proceedings of the 2020 American Control Conference (ACC), *IEEE*.

The following color code is used:

- blue: corrections;
- red: to be removed.

I. Governing equations - Table I

Electrochemical dynamics [1]		
Mass conservation in solid phase	$\frac{\partial c_{s,j}}{\partial t} = \frac{D_{s,j}(T)}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial c_{s,j}}{\partial r}\right], \quad j \in [n,p]$ $\frac{\partial c_{s,j}}{\partial r}\Big\|_{r=0} = 0 \quad \frac{\partial c_{s,j}}{\partial r}\Big\|_{r=R_{s,j}} = \frac{\pm I_{app}}{D_{s,j}(T) a_{s,j} A L_j F} + g_{s,j}(c_{s,j}^{surf}, c_{solv}^{surf}, T_c, I_{app}, L_{sei})$	(1)
Mass conservation in electrolyte phase	$\epsilon_{e,j}\frac{\partial c_e}{\partial t} = \frac{\partial}{\partial x}\left(D_{e,j}^{eff}(c_e,T)\frac{\partial c_e}{\partial x}\right) + \left(1-t_0^+\right)\frac{g_{e,j} I_{app}}{A L_j F}, \quad j \in [n,s,p]$ $\frac{\partial c_e}{\partial x}\Big\|_{x=0} = \frac{\partial c_e}{\partial x}\Big\|_{x=L_n+L_s+L_p} = 0$ $D_{e,n}^{eff}(c_{e,n},T)\left(\frac{\partial c_{e,n}}{\partial x}(x,t)\right)\Big\|_{x=L_n} = D_{e,s}^{eff}(c_{e,s},T)\left(\frac{\partial c_{e,s}}{\partial x}(x,t)\right)\Big\|_{x=L_n}$ $D_{e,s}^{eff}(c_{e,s},T)\left(\frac{\partial c_{e,s}}{\partial x}(x,t)\right)\Big\|_{x=L_n+L_s} = D_{e,p}^{eff}(c_{e,p},T)\left(\frac{\partial c_{e,p}}{\partial x}(x,t)\right)\Big\|_{x=L_n+L_s}$	(2)
Charge conservation in electrolyte phase	$\kappa_{e,j}^{eff}(c_e,T)\frac{\partial^2 \Phi_e}{\partial x^2} - \kappa_D^{eff}(c_e,T)\frac{\partial^2 \ln c_e}{\partial x^2} + \frac{I_{app}}{a_{s,j} A L_j} = 0, \quad j \in [n,s,p]$ $\frac{\partial \Phi_e}{\partial x}\Big\|_{x=0} = \frac{\partial \Phi_e}{\partial x}\Big\|_{x=L_n+L_s+L_p} = 0$	(3)
Electrode Overpotential	$\eta_j = \frac{R_g T_c}{0.5F} sinh^{-1}\left(\frac{I_{app}}{2 A a_{s,j} L_j i_{0,j}}\right), \quad j \in [n,p]$	(4)
Exchange Current Density	$i_{0,j} = k_j F\sqrt{c_{e,j}^{avg} c_{s,j}^{surf}\left(c_{s,j}^{max} - c_{s,j}^{surf}\right)}, \quad j \in [n,p]$	(5)
Cell voltage	$V_{cell} = U_p + \eta_p - U_n - \eta_n + \Delta\Phi_e - I_{app}(R_l + R_{el} + R_{sei})$	(6)
Thermal dynamics [2]		
Cell Core Heat Balance	$C_c\frac{dT_c}{dt} = I_{app}(V_{oc} - V_{cell}) + \frac{T_s - T_c}{R_c}$	(7)
Cell Surface Heat Balance	$C_s\frac{dT_s}{dt} = \frac{T_{amb} - T_s}{R_u} - \frac{T_s - T_c}{R_c}$	(8)
Aging dynamics [3], [4]		
Mass conservation in SEI	$\frac{\partial c_{solv}}{\partial t} = D_{solv}(T)\frac{\partial^2 c_{solv}}{\partial r^2} - \frac{dL_{sei}}{dt}\frac{\partial c_{solv}}{\partial r},$ $-D_{solv}(T)\frac{\partial c_{solv}}{\partial r}\Big\|_{r=R_{s,n}} + \frac{dL_{sei}}{dt}c_{solv}^{surf} = \frac{i_s}{F}$ $c_{solv}\Big\|_{r=R_n+L_{sei}} = \epsilon_{sei} c_{solv}^{bulk}$	(9)
SEI layer growth	$\frac{dL_{sei}}{dt} = -\frac{i_s M_{sei}}{2F\rho_{sei}},$	(10)
Side reaction current density	$i_s = -2Fk_f(c_{s,n}^{surf})^2 c_{solv}^{surf}\exp\left[\frac{-\beta F}{R_g T_c}(\Phi_{s,n} - R_{sei}I_{app} - U_s)\right]$	(11)
Cell capacity loss	$\frac{dQ}{dt} = i_s A L_n a_{s,n}$	(12)

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II. Other Equations

- Equation (16):

$$\frac{\partial c_{s,j,i}}{\partial t} = \frac{D_{s,j}}{\Delta r_j^2} \cdot \left[\left(1 + \frac{1}{i}\right) c_{s,j,i+1} - 2c_{s,j,i} + \left(1 - \frac{1}{i}\right) c_{s,j,i-1} \right]$$

- Equation (22):

$$\alpha_{s,j} = \frac{D_{s,j}}{\Delta r_j^2},\ \beta_{s,j} = \begin{cases} \dfrac{-1}{AL_j F a_{s,j} \Delta r_j} & \text{if } j = n \\ \dfrac{1}{AL_j F a_{s,j} \Delta r_j} & \text{if } j = p \end{cases},$$

- Equation (25):

$$\dot{\boldsymbol{c}}_e = \alpha_e A_e \boldsymbol{c}_e + \beta_e B_e \boldsymbol{u}, \text{ where } \beta_e = \frac{(1 - t_0^+)}{AL_j F \epsilon_{e,j}},$$

$$\boldsymbol{c}_e = \begin{bmatrix} \boldsymbol{c_{e,n}} \\ \boldsymbol{c_{e,s}} \\ \boldsymbol{c_{e,p}} \end{bmatrix}_{N_{el} \text{ x } 1} \quad \alpha_e = \frac{1}{\Delta_x^2 \epsilon_{e,j}} \text{ and } N_{el} = \sum_j N_{x,j}$$

III. Nomenclature

$c_{s,j}$	Concentration in solid phase [mol/m^3]	c_e	Concentration in electrolyte phase [mol/m^3]	c_{solv}	Solvent concentration [mol/m^3]
Q	Cell Capacity [Ah]	I_{batt}	Applied current [A]	Φ_e	Electrolyte Potential [V]
$\Phi_{s,n}$	Anode Surface Potential [V]	η_j	Overpotential [V]	$i_{0,j}$	Exchange Current Density [A/m^2]
U_j	Open circuit potential (electrode) [V]	V_{oc}	Open circuit potential (cell) [V]	i_s	Side reaction current density [A/m^2]
$D_{s,j}$	Solid phase diffusion [m^2/s]	$R_{s,j}$	Particle radius [m]	$a_{s,j}$	Specific interfacial surface area [m^{-1}]
A	Cell cross sectional area [m^2]	L_j	Domain thickness [m]	F	Faraday's constant [C/mol]
$\epsilon_{e,j}$	Electrolyte Porosity	$D_{e,j}$	Electrolyte phase diffusion [m^2/s]	t_0^+	Transference number
$c_{s,j}^{max}$	Maximum electrode concentration [mol/m^3]	$\kappa_{e,j}$	Electrolyte conductivity [S/m]	κ_D	Diffusional conductivity [S/m]
k_j	Intercalation rate constant [m$^{2.5}$/s-mol$^{0.5}$]	R_l	Lumped contact resistance [Ω]	R_g	Universal gas constant [J/mol-K]
$\epsilon_{s,j}$	Active volume fraction of solid phase	$\epsilon_{f,j}$	Active volume fraction of filler/binder	D_{solv}	Solvent diffusion coefficient in SEI layer [m^2/s]
ϵ_{sei}	SEI Layer porosity	ρ_{sei}	SEI layer density [kg/m^3]	κ_{sei}	SEI layer ionic conductivity [S/m]
c_{solv}	Solvent concentration [mol/m^3]	M_{sei}	Molar mass of SEI layer [kg/mol]	U_s	Solvent reduction potential [V]
k_f	Solvent Reduction rate constant [mol$^-2$s$^-1$]	β	Side Reaction charge transfer coefficient	C_s	Heat Capacity of cell surface [J/K]
C_c	Heat Capacity of cell core [J/K]	R_c	Conductive resistance - core/surface [K/W]	R_u	Convective resistance - surface/surroundings [K/W]
R_m	Cell-to-cell heat transfer resistance [K/W]	T_{amb}	Ambient temperature [K]	$N_{r,j}$	Number of radial discretization points
$N_{x,j}$	Number of cartesian discretization points	N_{el}	Total number of electrolyte discretization points	N_{sei}	Number of SEI layer discretization points

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