On-Line Power Management Optimization of a Hybrid Electric Vehicle with Non Linear MPC and Battery re-charge Equivalent Cost

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***Abstract*—This paper proposes a power management strategy for an hybrid electrical vehicle based on the *Non-linear Model Predictive Control* (NLMPC) that uses an equivalent consumption cost to control the state of charge of the battery. The predictive control is formulated as a repeated solution of a finite horizon minimum consumption problem which respects input and state constraints and vehicle dynamics also including the efficiency of the gear box. The reference speed and acceleration in the prediction step of NLMPC horizon is obtained by integrating forward the dynamic equations of the vehicle model with a fast semi-analytical method that does not use information about the past behaviour of the vehicle. The corresponding demanded torque is then optimally split between the two energy sources as solution of NLMPC and used in each instant for the definition of the power management strategy. The proposed NLMPC is tested on a simplified driver simulator of the hybrid vehicle and the on-line results are compared both with conventional Adaptive-Equivalent Consumption Minimisation Strategy (A-ECMS) strategies and with the global optimal strategy off-line computed that serves as benchmark.**

I. Introduction

Modern society relies more and more on fossil fuel-based transportation for freely moving goods and people [11]. Shifting towards sustainable transportation could considerably reduce oil consumption and carbon emission. Electrified vehicles come in the form of hybrid electric, plug-in hybrid and all-electric vehicles. Before a mass market production of only-electric vehicles (EVs) takes place, there are still challenges that need to be tackled such as range anxiety, high cost and long battery charging time. Hybrid Electric Vehicles (HEVs), on the other hand, overcome the cost and range issues of a pure EV still providing many advantages compared to traditional IC vehicle including fuel consumption and emmission reduction via regenerative braking and more efficient engine operation combined with better drivability.

HEVs can be classified, based on the configuration of the drivetrain, as series hybrid, parallel hybrid, power-split hybrid [9]. Regardless their drivetrain configuration, any HEV must be equipped with a supervisory controller that optimizes the power split among the vehicle actuators. Usually, the objective of the energy management strategy is to minimize fuel consumption and, possibly, emission over a driving cycle without compromising the vehicle performance. Several strategies have been proposed to solve this problem, both off-line and on-line. The *off-line* methods assume the knowledge of the entire driving cycle. These methods allow to compute a benchmark solution, i.e., the lowest achievable consumption, that can also be analyzed to design a suitable on-line strategy. In the *on-line* methods the power split decision is taken at each time step, during vehicle operation. Several methods have been proposed [16] where some of them are based on heuristic techniques, whereas others on minimisation of an appropriate instantaneous cost function.

In the present, both an off-line and on-line energy management strategy are proposed which are based on *direct optimization approach* via full discretisation (*i.e.* Non Linear Programming, NLP). Section II describes in details the model of the hybrid vehicle and its power-train. Section III defines the minimum consumption problem as an optimal control which is solved off-line over a standardised test using the Direct Method. The off-line solution is used as a benchmark and it is compared with the results of a proposed on-line solution (Section IV) based on Non Linear Model Predictive (NLMPC) scheme and the conventional Adaptive-ECMS strategies with feedback from State of Charge (SOC) [13].

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II. HYBRID VEHICLE MODEL

In this study, the vehicle longitudinal dynamics modeled neglects vehicle suspension dynamics and wheel slip [7] resulting in the following equation

$$m_{\text{eq}}\dot{v}(t) = F_t - (F_a(t) + C_{\text{rr0}}), \tag{1}$$

where $m_{\text{eq}} = (m_v + m_J)$, m_v vehicle mass and $v(t)$ is the vehicle speed, F_a is the aerodynamic friction given by:

$$F_a(v) = c_f\, v^2, \tag{2}$$

where c_f is a constant that takes into account the density of the ambient air, the aerodynamic drag coefficient and the vehicle frontal area. C_{rr0} represents the rolling friction, assumed constant. The traction force F_t is the force generated by the prime mover. Figure 1 shows a schematic representation of this relationship. Finally, the equivalent mass m_J accounts for

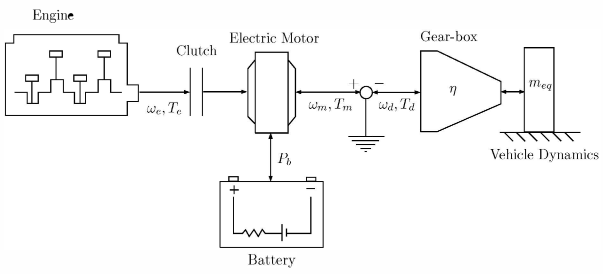

Fig. 1. Schematic of hybrid vehicle powertrain[12]

all rotating masses and is defined as follows:

$$m_J = \sum_{i=1}^{N} m_{J,i} \qquad m_{J,i} = \begin{cases} J_i \frac{\gamma_i^2}{r_w^2}\eta_g & \text{if } \dot{v}(t) > 0 \\ J_i \frac{\gamma_i^2}{r_w^2}\frac{1}{\eta_g} & \text{if } \dot{v}(t) < 0 \end{cases} \tag{3}$$

where J_i, γ_i are respectively the $i-th$ rotating mass moment of inertia and transmission ratio. It is worth noticing that the driveline efficiency η_g affects the equivalent mass in a different way during traction and braking.

A. Internal Combustion Engine Model

Owing to the faster chemical process dynamics of the internal combustion engine, a static map is used to predict fuel consumption based on engine torque and speed. The Willians line model is used to design an analytical engine model that

TABLE I
VEHICLE PARAMETERS

Parameters	Value	Units	Description
m_v	1400	[kg]	vehicle mass
r_w	0.3107	[m]	wheel radius
η_g	0.95	[$-$]	transmission efficiency
c_f	0.46875	[kg/m]	aerodynamic drag coefficient
C_{rr0}	0.02	[N]	rolling friction coefficient
J_{em}	0.0226	[kg m^2]	EM moment of inertia
J_{ice}	0.1598	[kg m^2]	ICE moment of inertia
δ	0.01	–	regularisation parameter

expresses the input power P_{in} as an affine function of the net (output) power P_{out} [16]:

$$P_{\text{in}} = \lambda_0(\omega) + \lambda_1(\omega)\, P_{\text{out}}, \tag{4}$$

where $P_{\text{out}} = T_e\,\omega$ is the mechanical power, while $P_{\text{in}} = \dot{m}_f\, H_l$ is the fuel power, and where λ_0 can be interpreted as friction loss function, and λ_1 the conversion efficiency of the machine; both the parameters are polynomial functions of shaft angular speed ω. Given that the fuel lower heating value H_l is a constant parameter we can write the fuel consumption rate $\dot{m}_f(\omega, T_e)$ as

$$\dot{m}_f = \begin{cases} \sum_{j=0}^{2}\sum_{i=0}^{1} a_{ij}\omega^j\,(\omega\, T_e)^i & \text{if } T_e > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Hence the engine is operating when T_e is greater than zero (*i.e.* the consumption is different than zero). T_e is bounded as follows:

$$T_e^{\min}(\omega) \le T_e \le T_e^{\max}(\omega) \tag{6}$$

$$T_e^{\max}(\omega) = \begin{cases} T_e^{\max}(\omega) & \text{if } \omega_{\text{id}} \le \omega \le \omega_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where ω_{id} represents the idle engine angular speed and $\omega_{\max}$ the engine maximum allowable angular speed. Moreover the clutch is considered open when the engine is operating and closed vice versa.

Parameters $a_{i,j}$ of Eq. 5 are identified with a least-square method using experimental data (see Table II). In [15] it is shown that the approximation of the actual engine map points with the model used here is always less than 5%, and even smaller over high engine efficiency region.

B. Torque split and Inertia

From the driving cycle, the torque requested upstream the gear-box is given by

$$T_d = \frac{1}{\gamma}\begin{cases} F_t\, r_w/\eta_g & \text{if } \dot{v}(t) \ge 0 \\ F_t\, r_w \eta_g & \text{if } \dot{v}(t) < 0 \end{cases}, \qquad \alpha = \frac{\dot{v}(t)\,\gamma}{r_w}. \tag{8}$$

where α is the angular acceleration upstream of the gear-box.

The requested torque must also verify torque split balance equation:

$$T_d = \begin{cases} T_e + T_m + T_b - J_e \frac{\mathrm{d}}{\mathrm{d}t}\omega - T_{\text{br}}, & \text{if } T_e(t) > 0 \\ T_e + T_m + T_b - T_{\text{br}}, & \text{if } T_e(t) = 0 \end{cases} \tag{9}$$

where ω is the engine shaft angular speed and J_e the inertia of the engine rotating masses that is added here since, when the engine is disengaged, its inertia cannot be computed in the total inertia of the vehicle m_J. T_{br} accounts for bearing losses as described in next subsection. $T_b < 0$ is the torque due to the braking systems computed upstream of the gear-box.

TABLE II
PARAMETERS OF ICE MODEL OF EQUATION (5) AND BATTERY MODEL OF EQUATION (16) USED IN SIMULATIONS.

ICE		Battery	
Parameters	**Value**	**Parameters**	**Value**
a_{00}	0.0002928	n_{cells}	60
a_{10}	$-1.646 \cdot 10^{-6}$	e_0	3.4093
a_{01}	$1.205 \cdot 10^{-5}$	e_1	1.4127
a_{20}	$3.462 \cdot 10^{-9}$	e_2	−1.2567
a_{11}	$5.238 \cdot 10^{-8}$	e_3	0.57
a_{02}	$-7.192 \cdot 10^{-23}$	r_0	0.0023
		r_1	−0.0007
		r_2	0.0004
		Q	6.5 [Ah]
		$I_{\text{charge}}^{\max}$	130 [A]

C. Electric Motor

The equation that models the electrical machine, both for motor and generator mode, is

$$P_b = P_m + P_{\text{loss}}, \tag{10}$$

The power losses can be split into mechanical and electrical losses and have been modeled using equation

$$P_{\text{loss}} = k_2\, T_m^2 + (k_1 + k_3)\, \omega, \tag{11}$$

where k_3 is the parameter linked to the ***Iron Loss*** [8] and k_1 is the parameter that describes the contribution of bearings losses [10]. It is assumed that $k_3 \ll k_1$, thus the ***Iron Loss*** contribution is neglected [6]. Therefore, equation (11) becomes

$$P_{\text{loss}} = k_2\, T_m^2 + k_1\, \omega. \tag{12}$$

Then since the mechanical losses do not depend on the motor torque their contribution is always present in the drive-line. Therefore the parameter k_1 can be modelled as a constant torque ($T_{br} = k_1$ if $\omega > 0$ otherwise $T_{br} = 0$) due to the friction force in the bearings which is added to Eq. 9. Thus power losses reduce to $P_{\text{loss}} = k_2\, T_m^2$. The torque T_m exchanged with the prop-shaft is bounded between

$$T_m^{\min}(\omega) \leq T_m \leq T_m^{\max}(\omega). \tag{13}$$

D. Gear shift strategy

The implemented gearbox is a five gear transmission and the total gear ratio γ, considering the *differential* gear ratio $\gamma_f = 4.1$, is $\gamma = \gamma_r\, \gamma_f$.

The gear switching strategy is

- if $\omega(t) > 350$ up-shift
- if $\omega(t) < 150$ down-shift

Given the gear shifting strategy and the driving cycle, angular speed α, acceleration $\dot{v}(t) = a(t)$ and torque demanded T_d are easily computed from Eq. (1) and Eq. (9).

E. Battery

The battery State of Charge (SOC) is a measure of the residual capacity of a battery as the ratio between current I_b and total charge Q

$$\dot{\text{SOC}} = -I_b/Q. \tag{14}$$

The battery can be modelled using a simple controlled voltage source in series with a resistance [17]. The current that flows in the battery is the solution of the equation $P_b = E\, I_b - R\, I_b^2$:

$$I_b = \left(E - \sqrt{E^2 - 4\, R\, P_b}\right) / (2R) \tag{15}$$

where P_b is the battery power and the value of voltage source. E and the internal resistance R are given as polynomial function of SOC, as:

$$E(\text{SOC}) = (e_0 + e_1\, \text{SOC} + e_2\, \text{SOC}^2 + e_3\, \text{SOC}^3)\, n_c, \tag{16}$$

$$R(\text{SOC}) = (r_0 + r_1\, \text{SOC} + r_2\, \text{SOC}^2)\, n_c, \tag{17}$$

where e_0,e_1,e_2,e_3 and r_0,r_1,r_2 are parameters (obtained experimentally) and n_c represents the number of cells in the battery pack (see Tab. II).

The battery power P_b is limited by minimum power $P_b^{\min}(\text{SOC}) = -I_{\text{ch}}^{\max}\,(E + R\, I_{\text{ch}}^{\max})$ which depends on the maximum charge current $I_{\text{ch}}^{\max}$ and the maximum power the battery can supply $P_b^{\max}(\text{SOC}) = E^2/(4\, R)$. Therefore

$$P_b^{\min}(\text{SOC}) \leq P_b \leq P_b^{\max}(\text{SOC}). \tag{18}$$

Starting from the definition of $P_b^{\max}$ we can re-define I_b as

$$I_b = \frac{P_b}{\sqrt{R}(\sqrt{P_b^{\max}} + \sqrt{P_b^{\max} - P_b})}. \tag{19}$$

Combining Eqs. (14) and (19) the differential equation that governs the battery SOC becomes

$$\dot{\text{SOC}} = -\frac{P_b}{\sqrt{R}(\sqrt{P_b^{\max}} + \sqrt{P_b^{\max} - P_b})\, Q}. \tag{20}$$

III. MINIMUM CONSUMPTION PROBLEM: OFF-LINE PROBLEM

The optimal minimum fuel consumption problem consists in finding the optimal selection of vehicle actuators, e.g., engine and electric motor, over specific driving cycle. Several driving cycles are available to test the energy management strategies; the Artemis Urban driving cycle is used in this work. For the sake of simplicity a *quasi-static* approach is used to solve the problem [7]. Under this assumption the number of the states and inputs of the model decreases making the problem easier to solve.

The general optimisation problem is the following:

$$\begin{cases} \min\limits_{u \in \mathcal{U}} \left\{ J(u) := \displaystyle\int_0^T \dot{m}_f(\omega, T_e)\, \mathrm{d}t \right\} \\ \text{subject to :} \\ \dot{x}(t) = f(x(t), u(t), t) \\ \boldsymbol{b}(x(0), x(T)) = \boldsymbol{0} \\ \boldsymbol{c}(x(t), u(t)) \geq \boldsymbol{0} \end{cases} \tag{21}$$

where $x = SOC(t)$. The input variable $u(t)$ represent a measure of the torque split between the two power source (engine and motor) that can be defined in several ways. The SOC dynamics equation f is described by Eq. (20). Vector $\boldsymbol{b}$ lists the initial and final conditions: $SOC(0) = 0.55$ and $0.55 < \mathrm{SOC}(T) < 0.56$ (charge sustaining mode). $\boldsymbol{c}(x(t), u(t))$ represents the state and input constraints (Eqs. (6),(13),(9),(18),(10)). Moreover

$$0.4 < \mathrm{SOC}(t) < 0.8. \tag{22}$$

A. Solution Method: NLP

The optimal control problem Eq. (21) may be solved at least with three different families of methods [2]. In [10] it was solved with an indirect method. Here a direct method is used with full discretisation. Therefore the speed v and the acceleration a of the vehicle are assumed to be constant over each time step h of the test cycle as well as the angle α of the road slope and thus computed with a finite difference approximation (f_k represents the finite difference approximation of function $f(t)$ evaluated at $t_k = kh$):

$$\bar{v}_{k+1/2} = \frac{v_k + v_{k+1}}{2}, \quad \bar{a}_{k+1/2} = \frac{v_{k+1} - v_k}{h}. \tag{23}$$

The force F_t acting on the wheels to drive the vehicle can be directly computed (considering the road slope always zero) inverting the model Eq. (1):

$$\bar{F}_{k+1/2} = m_{eq}\, \bar{a}_{k+1/2} + \frac{1}{2}\,\rho_a\, A_f\, c_d\, \bar{v}^2_{k+1/2} + c_r\, m_v\, g. \tag{24}$$

The angular speed $\bar{\omega}_{k+1/2}$, the angular acceleration $\bar{\alpha}_k$ and the torque $\bar{T}_{d,k+1/2}$ upstream the gear-box can be computed knowing the efficiency η_g and the gear-ratio $\gamma_{k+1/2}$ of the gear box during each interval. The input of the problem becomes the vector $\boldsymbol{u} = [T_e(t), T_m(t), T_b(t)]^T$. Under these assumptions the optimal control problem in Eq. (21) can be written as the Non-Linear Programming (NLP) problem

$$\min J(\mathbf{z}) = \sum_{k=1}^{N} \dot{m}(\omega_{k-1/2}, T_{e,k-1/2}) \tanh\left(\frac{T_{e,k-1/2}}{\epsilon}\right) h, \tag{25}$$

subjected to the following system of non-linear equations $h_k(\mathbf{z}) = 0$:

$$\begin{aligned} h_k(\mathbf{z}) &= T_{e,k+1/2} + T_{m,k+1/2} + T_{b,k+1/2} - T_{d,k+1/2} \\ &\quad - T_{\mathrm{bear},k+1/2} - \alpha_{k+1/2}\, J_e \tanh\left(\frac{T_{e,k+1/2}}{\epsilon}\right), \\ h_{k+N}(\mathbf{z}) &= x_{k+1} - x_k + \frac{I_{\mathrm{b},k+1/2}(T_{m,k+1/2})}{Q}\, h, \end{aligned} \tag{26}$$

which represents the torque split of Eq. (9) and the battery SOC dynamics (represented by the variable x_k) which is discretised using the forward Euler approximation (Eq. (14)). The value of $I_{b,k+1/2}$ is computed as a function of $T_{m,k+1/2}$ using Eqs. (10) and (19). The reader may note that the switch conditions in Eqs. (5) and (9) have been approximated by the hyperbolic tangent function in order to create continuous and differentiable functions. The problem is completed by the following inequalities $g_k(\mathbf{z}) \geq 0$:

$$\begin{aligned} g_k(\mathbf{z}) &= P_{b,k+1/2} + I_{\mathrm{ch}}^{\max}\left(E(x_k) + R(x_k)\, I_{\mathrm{ch}}^{\max}\right) \geq 0, \\ g_{k+N}(\mathbf{z}) &= -P_{b,k+1/2} + \frac{E(x_k)^2}{4\,R(x_k)} \geq 0, \end{aligned} \tag{27}$$

and the following constraints on the state and inputs variables:

$$0 \leq T_{e,k+1/2} \leq T_e^{\max}(\omega_{k+1/2}), \tag{28}$$

$$T_m^{\min}(\omega_{k+1/2}) \leq T_{m,k+1/2} \leq T_m^{\max}(\omega_{k+1/2}), \tag{29}$$

$$T_{b,k+1/2} < 0, \tag{30}$$

$$0.4 < x_k < 0.8, \tag{31}$$

$$0.55 < x_N < 0.56, \tag{32}$$

and initial condition $x_0 = 0.55$. The unknowns of the problem is the vector $\mathbf{z} \in \mathbb{R}^{4N+1}$

$$\begin{aligned} \mathbf{z} = \{&x_0, ..., x_N, T_{e,1/2}, ..., T_{e,N-1/2}, \\ &T_{m,1/2}, ..., T_{m,N-1/2}, T_{b,1/2}, ..., T_{b,N-1/2}\}. \end{aligned} \tag{33}$$

The reader may note that, in principle, T_e could be computed from the algebraic equation of Eq (26) but since the relation is non linear it is better to leave the algebraic equation in the optimisation problem.

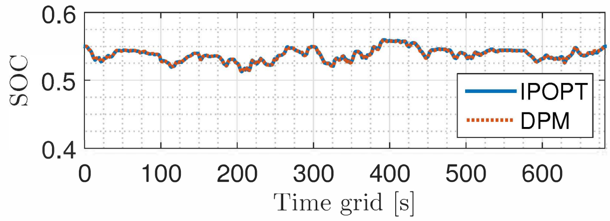

Fig. 2. Evolution of the state of charge during the driving cycle: off-line solution.

B. Numerical results: NLP solution

The NLP problem (Eq. (25) - (33)) is solved in Matlab using IPOPT [18]. The results shown in Fig. 4 show that the ICE is on for 6.3% of the total driving cycle time and gives more than the demanded torque for 88.4% of the time (i.e. when charging the battery). The electric motor/generator torque T_m is different than zero for 100% of the total driving cycle time. Finally braking torque T_b is zero for 99.7% of the total driving cycle time since the generator is able to provide almost always the braking torque necessary to decelerate the vehicle (because of type of test adopted). As expected the engine works at the maximum torque allowed when is on because the efficiency increases with the torque (Figure 3). It is worth pointing out that the electric motor is the main torque source, both in acceleration and in deceleration. The power needed to recharge the battery is mainly provided during braking by the electric motor; the remaining power comes from the engine (that works at the maximum efficiency point) and adsorbed by the electric motor in generator mode. Therefore the key factor in the hybrid electric vehicle is the regenerative braking which allows to recover part of the kinetic energy accumulated in acceleration.

The same problem was also solved using the Dynamic Programming (DP) with the DPM solver (with Boundary Line Method) [4] for comparison. It turned out that DPM is accurate as NLP (a fuel consumption of $66.6[g]$) but at the price of more computational effort (DP took 27s versus 7s for NLP). On the other hand NLP needs a suitable first guess that is difficult to be generated for the whole driving cycle but it is easier for a smaller problem. Moreover the Dynamic Programming function DPM works with limited number of state variables and it does not allow to define an objective function that depends on the final state. This result makes the indirect method interesting for real-time control strategies, in particular for the model predictive control as it is explained in the next section.

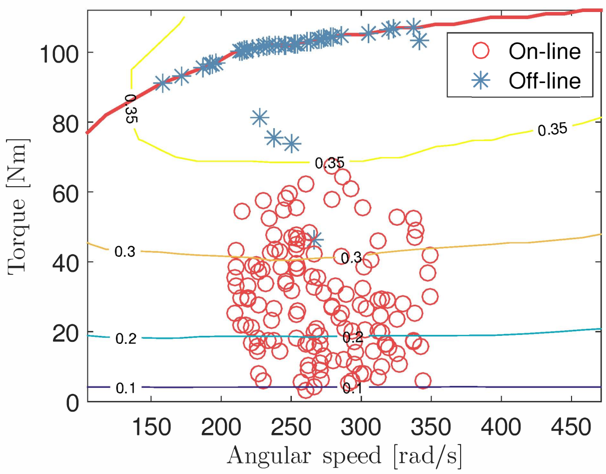

Fig. 3. Distribution of engine torque values during the driving cycle.

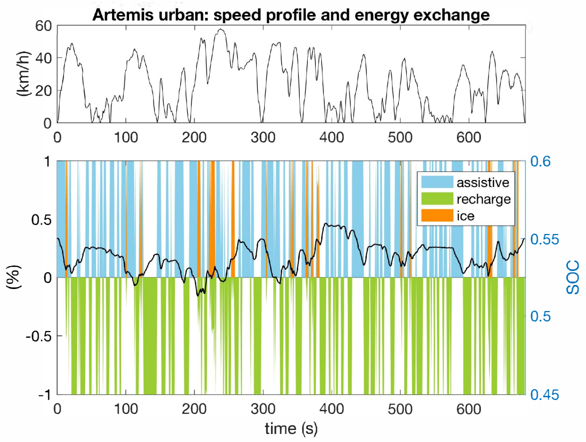

Fig. 4. Top plot shows the speed profile of the Artemis Urban cycle used in the simulation. The lower plot shows the solution of the energy management. Electric motor torque T_m is split between assistive (positive) and recharge (negative) torque. ICE torque T_e is also shown. All the torques have been normalised with respect to the requested torque T_d. Finally, the state of charge has been superimposed.

IV. On-line NLMPC Minimum Consumption Problem

The solution of the minimum fuel consumption problem treated in the section III makes it possible to estimate the optimal energy management strategy to achieve the minimum fuel consumption during a predefined driving cycle. To achieve this results it uses the a-priori knowledge of the driving cycle speed trace which is not available in real time case. Therefore real-time control strategy can rely on instantaneous speed only yielding only a local optimal real-time management.

In the past fifteen years many studies have investigated various methods for the real-time control of hybrid electric vehicle power-train in order to achieve a sub-optimal solution of the minimum fuel consumption problem [14]. Two of these real-time methods have been recognised as suitable for the implementation in commercial vehicles ECUs: *Heuristic methods* and the *Adaptive Equivalent Consumption Minimisation Strategy* (A-ECMS) [16]. The A-ECMS consists in the minimisation at each time step of an appropriately defined instantaneous cost function. Usually the cost function is derived from the Hamiltonian function of the optimal control problem solved with Minimum Principle of Pontryagin:

$$H(x,u,t) = P_{\text{fuel}}(u,t) + s(t)\, P_{\text{ech}}(x,u,t). \qquad (34)$$

$P_{\text{fuel}}(u,t)$ and $P_{\text{ech}}(x,u,t)$ take into account the power provided by the engine and by the electric motor: The equivalence factor $s(t)$ is initialised starting from the off-line results and corrected with a feedback on the battery SOC. We propose to solve on-line the problem Eq. (21) with the same approach used for the off-line problem within a NLMPC scheme on a short horizon where the the future speed profile is estimated for a short horizon. The proposed method is then compared with the A-ECMS one.

A. Model Predictive Control

The on-line control here presented is formulated as a repeated solution of a finite horizon optimal control problem with respect to system dynamics, input and state constraints[3]. For each t_k sampling period of the ECU and for the positive demanded power, the following sequence is repeated over the predicted horizon t_p:

- Generation of the estimated future trajectory (i.e. speed profile and demanded torque).
- Minimisation of the fuel consumption starting with current state of charge.
- Application of the power split strategy considering the first instant of the estimated trajectory.

The generation of the estimated future speed of the vehicle $\hat{v}(\tau)$ is carried out integrating the differential equation that describes the vehicle dynamics from t_k to $t_k + t_p$;

$$\dot{\hat{v}}(\tau) = a - c_1\, \hat{v}(\tau)^2, \quad \hat{v}(0) = v(t_k) \qquad (35)$$

where $v(t_k)$ is the speed of the vehicle in the instant t_k. The solution is computed analytically, as described in [5], assuming that the power-train will provide a constant torque T_k (the torque requested at the instant t_k) throughout the predicted horizon. The parameters a and c_1 are defined, considering a positive value of T_k as

$$a = \left(\frac{(T_k - T_{\text{bear}})\,\gamma(t_k)\,\eta_g}{r_w m_{\text{tot}}} - \frac{C_{\text{rr0}}}{m_{\text{tot}}} \right), \quad c_1 = \frac{c_f}{m_{\text{tot}}}, \quad (36)$$

where $\gamma(t_k)$ represents the gear ratio at the instant t_k, η_g the gear-box efficiency, T_{bear} the motor bearings friction torque, $C_{\text{rr}\bullet}$ the force due to the rolling resistance of tyres and C_f the parameter related to the air friction of the vehicle; m_{tot} represents the total mass of the vehicle considering the inertia of the components connected to the power-train in the instant t_k. The problem is simplified decreasing the number of the input variables to one, i.e. $u = T_e$, since it is solved only if a positive power is demanded. Based on the assumption above the minimum fuel consumption problem Eq. (21) was modified adding a Mayer term $\Theta(\hat{\mathbf{x}}(t_p))$ to push the final state of charge towards a desirable value:

$$\min_u J(u) := \int_0^{t_p} \dot{m}(\hat{x}(\tau), u(\tau), \tau)\,\mathrm{d}\tau + \Theta(\hat{x}(t_p)). \quad (37)$$

$\Theta(\hat{x}(\tau), \tau)$ represents the equivalent consumption as a function of the difference between the final state of charge and the desirable x_{des}. The idea is to define the fuel consumption that brings the state of charge from $\hat{x}(t_p)$ to x_{des} considering the vehicle travelling at constant speed $\hat{v}(t_p)$. This leads to a definition of an equivalent consumption that depends only on the final state of charge $\hat{x}(t_p)$ assuming that the ICE works at the maximum torque, that is when it is more efficient as suggested by off-line results. The equivalent consumption is computed assuming that the battery parameters R and E are constant and equal to $R(t_p)$ and $E(t_p)$; this is possible since the variation of the battery parameters is small when a small variation of SOC occurs. Under these assumptions the expression of the equivalent consumption become

$$\Theta(\hat{x}(t_p)) = \dot{m}(T_e^{\max}, \hat{\omega}(t_p)) \frac{(x_{\text{des}} - \hat{x}(t_p))\,Q}{I_{b,eq}}, \quad (38)$$

where $\dot{m}(T_e^{\max}, \hat{\omega}(t_p))$ represents the fuel flows when the engine works at angular speed $\hat{\omega}(t_p)$ and torque $T_e^{\max}$. $I_{b,eq}$ represents the constant current that flows in the battery when a constant torque $T_e^{\max}$ is adsorbed by the generator. In order to remain close to the desirable state of charge it has been considered the absolute value of $(x_{\text{des}} - \hat{x}(t_p))$ approximated with the following function

$$|x_{\text{des}} - \hat{x}(t_p)| \approx \sqrt{(x_{\text{des}} - \hat{x}(t_p))^2 + \delta^2}, \quad (39)$$

to make it differentiable — the smaller δ the better the approximation. The NLMPC is solved subdividing the predicted horizon t_p in N small time intervals of length $h = t_p/N$, applying the quasi-static approach already discussed in section III-A.

The first guess of the minimisation problem is the key factor of the problem solution and it has been defined in the following way (to avoid in-feasibilities):

$$\begin{cases} T^{\text{first}}_{e,k-1/2} = \min(T_{d,k-1/2}, T^{\max}_{e,k-1/2}), & k = 1, ..., N \\ T^{\text{first}}_{m,k-1/2} = T_{d,k-1/2} - T^{\text{first}}_{e,k-1/2}, & k = 1, ..., N \end{cases} \quad (40)$$

and the state of charge x_k^{first} is computed by integrating the model equation of the battery. Then the first guess becomes $\mathbf{z}_0 = \{x^{\text{first}}, T_e^{\text{first}}\}$.

During braking T_m can be computed as the maximum allowed (considering the minimum motor torque $T_m^{\min}$ and the minimum battery power $P_b^{\min}$) without solving the NLMPC problem.

B. Simulation Environment

The energy management control scheme has been tested using a simulator of the vehicle longitudinal dynamics and designing a driver model that controls the accelerator pedal in order to generate a desired speed as described in [1] (see Figure 5). The driver model generates a pedal position

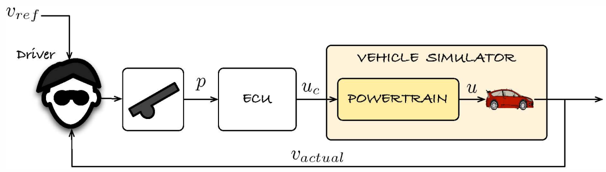

Fig. 5. Longitudinal Dynamics Simulator with driver model in the loop

signal based on the difference between desired (*i.e.* selected speed profile) and actual speed. The pedal position is then transformed into a demanded torque u_c which is then used in Eq. (35) to predict the future speed in the prediction horizon t_p and then to solve the problem Eq. (37).

V. Result Discussion and Comparison with A-ECMS Strategy

The optimal control was computed over a predicted horizon of length $1s$ and subdivided in time steps of length $0.1s$ and it was tested for the Artemis Urban driving cycle. The resulting fuel consumption has been compared with a second simulation, where the vehicle is equipped with only the engine (Table III). The simulation considers zero fuel consumption when the angular speed of the engine shaft is less than the idle speed and when the vehicle speed is zero. Moreover the vehicle mass is assumed to be the same of the hybrid vehicle. From the

TABLE III
Simulation Results.

Power-train	Fuel cons. [l]	Mean fuel cons. [km/l]	SOC end
Only ICE	0.30	14.88	–
Hybrid (with NLMPC)	0.12	36.35	0.55
Hybrid (with A-ECMS)	0.14	32.20	0.54

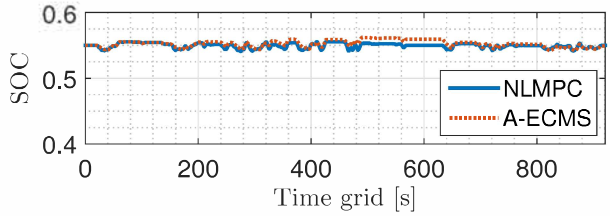

Fig. 6. Evolution of the SOC; comparison between NLMPC and A-ECMS results.

results obtained we observe that the state of charge (Figure 6) remains close to the desirable one and the final one is equal to the initial with a fuel consumption reduction of about $60\,\%$ with respect to a conventional vehicle (Table III). Additionally, unlikely the result of the off-line problem, the engine torque values are distributed all over the engine map (see Figure 3) because the minimum consumption problem over the predicted horizon always tries to recharge the battery when the SOC is different than the desirable one. It is worth noting that the direct method works fine for the real-time control strategy with the implemented first guess. The result is still far from the off-line minimum consumption problem which is unreachable since a prefect knowledge of the future is not possible (except for autonomous vehicle). The key factor of the NLMPC is the Mayer's term which can be defined in many ways depending on the desired behaviour of the battery. Moreover without a model of the past behaviour of the vehicle only a short prediction horizon is computable.

Finally the result of the NLMPC method has been compared with the A-ECMS one. The proposed method gives a better result with respect to the A-ECMS applied with the chosen parameters (see Table III). Since the A-ECMS strategy depends on some parameters that have to be manually tuned, a comparison of the two method from the point of view of the saved fuel is not easy. The interesting result of the comparison is the fact that the NLMPC method gives a result close to the A-ECMS one without the need of user-defined parameters.

VI. Conclusions

The paper has proposed a power management strategy based on the NLMPC paradigm that, off-line, performs better than Dynamic Programming using less computational time. The minimum fuel problem is formulated as a Non Linear Programming problem with a detailed model of the hybrid vehicle power train. The same model and approach has been applied for real time solution of energy management problem using a suitable first guess and adding a mayer term which is an equivalent consumption to control the variation of the battery SOC in the Predicted Horizon. The method can easily account for road slope and information from driver's route and in priciple can produce similar result than off-line calculation for autonomous vehicle (where speed is known in advance).

Several improvements are still possible such as a new definition fo the Mayer term to fully exploit the potential of the battery which still tends to be close to the desired value. By observing Figure 6 we can notice that the SOC tends to remain close to the desired one. Moreover the minimum consumption problem in the NLMPC has been solved only by using a quasi-static approach: a future improvement could be the definition of the problem taking into account the dynamics of the vehicle (dynamic approach).

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