Nonlinear Economic Model Predictive Control for SI Engines Based on Sequential Quadratic Programming

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***Abstract*—This paper proposes a model predictive torque control strategy for Spark Ignition (SI) engines with external Exhaust Gas Recirculation (EGR). The proposed Nonlinear (economic) Model Predictive Controller (NMPC) tries to minimize fuel consumption with given Indicate Mean Effective Pressure (IMEP) tracking reference and abnormal combustion constraints like knock and combustion stability. A Nonlinear Programming (NLP) problem is formulated and solved using Sequential Quadratic Programming (SQP) to obtain the desired control actions. The SQP exploits the Gauss-Newton like structure of the real time NLP problem to simplify computation of Hessian matrix. Simulation results demonstrate that the proposed model predictive IMEP control can track the IMEP reference for engine cycles without active constraints (with a RMS tracking error of 1.1%). When the IMEP reference conflicts with constraints, the SQP MPC can efficiently find close to optimal control actions that are similar to those from off-line feed forward calibration.**

I. Introduction

The control objectives of an IC engine management system should be such that the demanded engine output torque is delivered while minimizing fuel consumption and preventing abnormal combustion phenomena that could damage the mechanical components and interrupt the normal operation of the engine. These control objectives favor the application of model based optimal control strategies. While many articles discussed the possibility of applying Model Predictive Control (MPC) to regulate engine torque output [1]-[4], minimizing fuel consumption and meeting combustion constraints are not well addressed by previous literature. The most important factor causing this dilemma is that the engine models used for MPC torque control are heavily simplified to allow the optimal control action to be found within a reasonable sampling period. However, this simplification process reduces the model accuracy., making the optimality of the obtained control action questionable. This paper proposes an economic model predictive Indicated Mean Effective Pressure (IMEP) control strategy for SI engines with external EGR. This strategy is able to utilize complex high-fidelity engine models to find the optimal control actions while achieving target IMEP, reducing fuel consumption and avoiding abnormal combustion.

Control oriented engine air path and torque generation models are well established [5]. Most of these models are constructed in the time domain making them favorable for controllers with fixed sampling time. The most important drawback of this control strategy is that the IC engine is an inherently discrete event system with cyclic operation characteristics. The SI engine system is modeled and controlled in the engine cycle domain in this research. While this approach agrees with the discrete nature of both MPC and IC engines, it also benefits from the fact that most control oriented combustion models were constructed in the engine cycle domain [6], [7]. It is convenient to impose constraints like knock and combustion stability during the calculation of optimal control actions.

. This research reveals that the real time nonlinear optimization problem solving for the optimal control action is not convex for IMEP control of SI engines with external EGR, leading to multiple local minima issues. Global NLP solvers, like dynamic programming and particle swarm, can be employed to MPC applications [8][9]. Stability of MPC with global optimal solutions, using terminal state penalties, was shown in [10][11]. However, these global NLP solvers require numerous evaluations of the system model, which are not feasible for cyclic engine control applications. Most model predictive engine control researchers have selected sub-optimal strategies to reduce computational demand [12][13][14]. The linear parameter variant (LPV) MPC is a widely adopted sub-optimal predictive controller for nonlinear systems [15][16]. The validity of LPV MPC is based upon the assumption that the system behavior remains linear-like if the system states are not very far from the nominal point of linearization. The LPV MPC was firstly considered in this research work. However, the performance is not satisfactory due to the fact that the investigated engine system is highly nonlinear, nullifying the locally linear assumption. In this case, the optimization cannot converge to a local minimum or guarantee the feasibility of computed control action. The later situation is more undesirable since it could lead to misfire or knock phenomena that can damage the engine and interrupt normal operation.

Sequential Quadratic Programming (SQP) is a continuous NLP algorithm based on Newton's method [17]. Previous research has discussed the possibility of applying SQP to NMPC [14]. The most important advantage of SQP is that it transforms complex NLP into a sequence of sub-level quadratic programming (QP) problems (hence the name). The sub-QP can be solved efficiently with active set algorithms. As a result, the original nonlinear objective and constraint functions are only evaluated before the sub-QP, saving significant computation time compared to other NLP solvers. This characteristic is advantageous for engine control applications since most high-fidelity engine models are complex in nature with multiple calibration maps and ODEs. Conventional SQP algorithms are not favorable for real time MPC applications due to the heavy computational load in computing the Hessian matrix for complex or implicit system models. In this case, numerical differentiation methods are necessary (e.g. algorithmic differentiation and finite difference). In practice, the Hessian is often approximated

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with first order derivatives to reduce computational burden. Quasi-Newton methods are well discussed by [18] and [19]. The Broyden – Fletcher – Goldfarb – Shanno (BFGS) rank-two update method is widely used in SQP applications [20], [21]. *Goldsmith* [22] proposes a disaggregated Hessian approximation approach, which is more computationally effective than BFGS. *Quirynen et al.* [14] proposes using an algorithmic differentiation approach to calculate the exact Hessian. These methods artificially set the approximated Hessian to be Positive Definite (PD) so that a computationally efficient convex active set QP algorithm can be applied. However, the exact Hessian of the original NLP may not be PD. In this case the approximated Hessian reduces the convergence rate making the SQP less favorable for MPC applications. It is noticed that the use of non – convex QP algorithms with non-PD Hessian SQP is discussed by [22]. However, their computational efficiency is yet not high enough for most MPC applications. This paper exploits the Gauss-Newton-like structure of the investigated SI engine IMEP control problem to simplify the Hessian matrix computation. Furthermore, the Hessian of the proposed SQP strategy is inherently PD such that well developed convex QP algorithms can be applied.

This paper is organized as follows. Section II introduces the control oriented high-fidelity engine model. Section III formulates and analyzes the NLP to obtain desired control actions. Section IV discusses the proposed SQP MPC strategy and Section V provides simulation results. Finally, Section VI concludes the contribution of this research work highlighting possible future extensions.

II. CONTROL ORIENTED HIGH-FIDELITY ENGINE MODEL

This research focuses on IMEP control of SI engines with external EGR system. The fuel injection control is assumed to maintaining stoich air-to-fuel ratio (AFR). The manifold temperature is assumed to be constant since the EGR is cooled with a heat exchanger. Finally, the air mass in the air-path system is considered uncompressible. Figure 1 shows the block diagram of the controlled engine system.

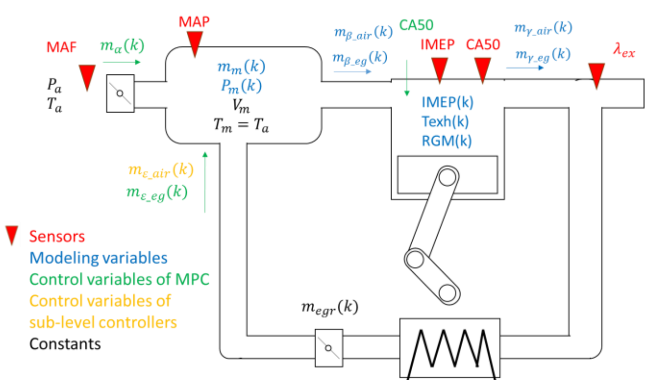

Figure 1. Block diagram of the SI engine system with external EGR.

The proposed MPC is designed in the engine cycle domain. The MPC manipulates throttle (air mass flow per cycle), EGR valve (air mass flow per cycle) and combustion phasing ($u = [m_\alpha, m_\varepsilon, CA50]$. CA50 is the crank angle of 50% mass fraction burned). These variables are sent to lower level controllers with fast update frequency as references. The two-layer supervision control structure exploits the frequency separation of different system dynamics making it favorable for many fast MPC applications (e.g. [23]).

In the engine cycle domain, the air-path system can be modeled according to mass balancing methods (page 161 to 163 in[5]). The exhaust gas and air mass flow into the engine can be modeled with two first order difference equations:

$$
\begin{aligned}
m_{\beta_{air}}(k+1) &= \frac{1}{K+1} m_{\beta_{air}}(k) + \frac{K}{K+1} m_\alpha(k) \\
m_{\beta_{eg}}(k+1) &= \frac{1}{K+1} m_{\beta_{eg}}(k) + \frac{K}{K+1} m_\varepsilon(k)
\end{aligned} \tag{1}
$$

where:

$$K = \frac{\eta_V(k) V_d}{V_m}$$

η_V is volumetric efficiency.
V_d is engine displacement.
V_m is intake manifold volume

It can be observed from (1) that the manifold dynamics are independent of engine speed (regardless of the slowly varying volumetric efficiency η_V), unlike most time domain models. In-cylinder gas composition includes air, exhaust gas, fuel and other minor species that are neglected in this research. The amount of air and fuel can be determined by $m_{\beta_{air}}$ assuming stoich AFR. The amount of in-cylinder exhaust gas is the summation of $m_{\beta_{eg}}$ and RGM. This research adopts the semi-empirical *Fox* model proposed by [24]. The Fox model separates the RGM into two parts: 1) from trapped residual at exhaust valve closing (EVC) due to un-swept cylinder volume and 2) exhaust gas backflow into the cylinder and intake runner during the valve overlap period. After adding terms ΔP_{exh} and ΔP_m to account for wave tuning dynamics to the original Fox model, the residual gas mass for each engine cycle can be calculated according to:

$$
RGM = C_1 \frac{P_{exh}}{RT_{exh}} V_c + C_2 \sqrt{\frac{P_{exh}}{RT_{exh}} \big((P_{exh} + \Delta P_{exh}) - (P_m + \Delta P_m)\big) A_{flow}} \frac{OLV}{\omega_e} \tag{2}
$$

where:

$P_{exh} \approx 110\ kPa$ is the exhaust pressure.
T_{exh} is the exhaust temperature.
R is gas constant.
V_c is the cylinder clearance volume.
A_{flow} is effective flow area during valve overlap period.
OLV is overlap volume which is the cylinder volume difference between EVC and IVO.
C_1, C_2 are calibration factors.
ω_e is engine speed.

Unlike engine models in the time domain, it is not necessary to model the transport delay between induction and torque generation in the engine cycle domain. The gross IMEP is modeled using *Willians* approximation method [5]:

$$IMEP = e\big(\omega_e, CA50, x_{egr}\big) P_f - P_0 \tag{3}$$

where:

e is the 'slop' factor related to engine speed ω_e, CA50 and residual gas fraction x_{egr}.

$P_f = LHV \cdot \frac{m_{\beta_{air}}}{\sigma_0 V_d}$ is fuel effective pressure.

σ_0 is stoich AFR

V_m is intake manifold volume

$P_0 = P_{exh} - P_m$ is pumping effective pressure (PMEP).

$P_m = \frac{m_\beta R T_m}{\eta_V V_d}$ is manifold pressure.

An energy balance method is utilized to calculate exhaust temperature T_{exh} for the residual gas mass model. The IC engine transforms the chemical energy of the injected fuel into mechanical work (can be calculated with gross IMEP) and rejected heat, which is the summation of heat transfer to coolant and exhaust enthalpy. Thus the exhaust gas temperature can be calculated according to:

$$T_{exh} = \frac{V_d\left(P_f - IMEP\right)(1-\vartheta)}{c_p m_{\beta_{air}}(1 + 1/\sigma_0)} \tag{4}$$

where:

c_p is constant pressure gas heat capacity.

ϑ is ratio of transferred heat to coolant (in terms of the total rejected heat). It can be estimated with engine speed and load [26][27]

Figure 2 shows IMEP and T_{exh} validation results for the proposed engine model against experimental results. The error band is less than 5% for more than 98% of the test points.

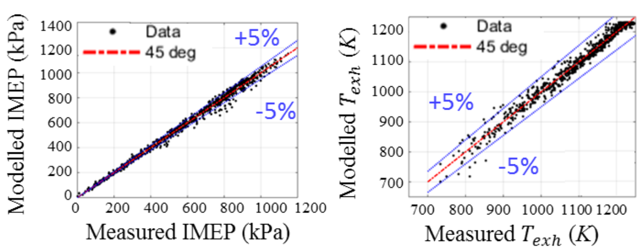

Figure 2. Validation of IMEP and exhaust temperature prediction

Covariance of Indicated Mean Effective Pressure (COV of IMEP) is utilized as an indication of combustion variability. The proposed model predictive IMEP control should maintain the COV of IMEP below a certain value. *Lee et al.* [29] suggested that the COV of IMEP has strong correlation with combustion duration. In this research, the COV of IMEP is correlated to the cylinder air mass flow $m_{\beta_{air}}$ and CA90. CA90 is computed with an Artificial Neural Network (1 hidden layer and 10 neurons) with CA50, RPM and $m_{\beta_{air}}$ as inputs. The knock model is a fully empirical model (n-D lookup table) as a function of RPM, CA50, $m_{\beta_{air}}$ and RGM. The output of this model is the normalized knock intensity KI, which indicates knock is likely if $KI \geq 1$. Both the COV of IMEP and knock models are able to achieve less than 10% RSME with negligible computation time.

In summary, the proposed engine model is a 4th order ODE model:

$$\begin{aligned} x(k+1) &= f_x\big(x(k), u(k)\big) \\ y(k) &= f_y\big(x(k)\big) \\ z(k) &= f_z\big(x(k)\big) \end{aligned} \tag{5}$$

where:

$$x \in \mathbb{R}^4, x = \left[m_{\beta_{air}}, m_{\beta_{eg}}, RGM, x_{CA50}\right]^T$$

$$y \in \mathbb{R}^1, y = IMEP$$

$$z \in \mathbb{R}^3, z = [P_m, COV, KI]^T$$

$$x_{CA50}(k) = CA50(k-1).$$

The CA50 output of the MPC is the target value for the next engine cycle, which induces a unit step delay.

III. Optimization Problem Formulation

The objective of the proposed model predictive IMEP control is to track IMEP reference with minimum fuel consumption. This determines the stage cost of the objective function should penalize the least square error of IMEP tracking and fuel consumption. The fuel consumption can be calculated with engine air mass flow $m_{\beta_{air}}$ (we assume stoich AFR engine operation). A terminal state penalty is included for stability considerations.

$$J\big(x(k), U(k)\big) = x(k+N)^T Q_f x(k+N) + \sum_{i=k}^{k+N-1} \frac{1}{2} q\left(y(i) - y_{ref}(i)\right)^2 + u(i)^T r \tag{7}$$

where:

$$r = \tilde{r}[1,0,0]^T, \tilde{r} \in \mathbb{R}^1_{>0}, q \in \mathbb{R}^1_{>0}, Q_f \in \mathbb{R}^{4\times4}_{>0}$$

$$U(k) = [u(k), u(k+1), \dots u(k+N-1)]^T$$

It is noticed that the fuel consumption penalty makes the MPC an economic optimal controller rather than a conventional tracking MPC. However, the objective function of this specific control application still preserves the general Gauss-Newton structure. Next section discusses exploiting this property to reduce computation load.

The proposed model predictive engine control has constraints on COV of IMEP, knock intensity and manifold pressure (less than the ambient pressure since the engine is naturally aspirated). The air mass flow through the throttle m_α and EGR valve m_ε are non –negative to be physically reasonable. Finally, the following equation shows the complete NLP that needs to be solved per engine cycle to obtain the optimal control sequence for the N steps of the future horizon.

$$\min_{U(k)} J\big(x(k), U(k)\big)$$

$$\text{s.t.} \begin{cases} x(i+1) = f_x\big(x(i), u(i)\big) \\ y(i) = f_y\big(x(i)\big) \\ z(i) = f_z\big(x(i)\big) \\ z(i) - b_z \leq 0 \\ -u(i) - b_u \leq 0 \end{cases} \tag{8}$$

where:

$$i = k+1, k+2, \dots k+N$$

$$b_z = [P_{ambient}, COV_{ub}, KI_{ub}]^T$$

$$b_u = [0, 0, -\infty]^T$$

After transferring the equality constraints to the cost function, the NLP can be written in a more compact form:

$$\min_{U(k)} J\big(x(k), U(k)\big) \quad \text{s.t.} \quad l\big(x(k), U(k)\big) \le \mathbf{0} \tag{9}$$

where:

$$l: \mathbb{R}^{4+3N} \rightarrow \mathbb{R}^{5N}$$

IV. SQP STRATEGY WITH GAUSSIAN HESSIAN APPROXIMATION

SQP is a numerical algorithm searching for the local optimal solution of problem (9). With a given initial guess of U_0, the SQP computes the searching direction ΔU by solving a sub-quadratic programing problem as following:

$$\min_{\Delta U(j)} \frac{1}{2} \Delta U^T HJ_{(x_k, U_0)} \Delta U + \Delta U^T \nabla J_{(x_k, U_0)} \quad \text{s.t.} \quad l(x_k, U_0) + \nabla l_{(x_k, U_0)} \Delta U \le \mathbf{0} \tag{10}$$

The search step size toward direction ΔU is scaled by a factor α which was generated by solving a line search problem of a merit function of the original NLP. For each major iteration j (whereas the iterations for the sub-QP are referred to as minor iterations), the updated solution is calculated as:

$$U_0(j+1) = U_0(j) + \alpha \Delta U(j) \tag{11}$$

The rest of this chapter will discuss the computation of Hessian and Jacobians of the NLP, and the merit function technique.

A. Hessian and Jacobian Calculation

The Hessian $HJ_{(x_k, U_0)}$ can be calculated by taking the second order derivatives of $J\big(x(k), U(k)\big)$ with respect to $U(k)$:

$$HJ_{(x_k, U_0)} = 2 \left(\frac{\partial Y}{\partial U}\bigg|_{x(k), U_0} \right)^T Q \frac{\partial Y}{\partial U}\bigg|_{x(k), U_0} + 2 \frac{\partial^2 Y}{\partial U^2}\bigg|_{x(k), U_0} Q\big(Y_0 - Y_{ref}\big) \tag{12}$$

where:

$$Y = [y(k), y(k+1), \dots y(k+N-1), x(k+N)]^T$$

$$Y_0 = Y(x_k, U_0), Q = \begin{bmatrix} q & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & q & 0 \\ 0 & \cdots & 0 & Q_f \end{bmatrix} \in \mathbb{R}^{(N+4)\times(N+4)}$$

The second term in equation (12) can be neglected if $Y_0 \approx Y_{ref}$. This assumption is reasonable if the initial guess is not far from the optimal solution. This can be achieved with warm start techniques or additional calibration effort to improve the initial guess of the optimal solution. Furthermore, this assumption becomes more reasonable as the SQP converges to the optimal solution. It is noticed that the tracking performance weighting matrix Q is PD. However, the Hessian may be PSD depending on the rank of $\partial Y / \partial U$. A remedy to this situation is to add a quadratic penalty of the search step ΔU to the sub-QP. The weighting on this penalty should be small and PD so that the search step is not overly conservative. The following equation shows the proposed Hessian calculation:

$$HJ_{(x_k, U_0)} = 2 \left(\frac{\partial Y}{\partial U}\bigg|_{x(k), U_0} \right)^T Q \frac{\partial Y}{\partial U}\bigg|_{x(k), U_0} + S \tag{13}$$

where:

$$S = \begin{bmatrix} s & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s \end{bmatrix} \in \mathbb{R}_{>0}^{3N \times 3N}$$

The Jacobian of the objective function J can be computed as:

$$\nabla J_{(x_k, U_0)} = -2 \frac{\partial Y}{\partial U}\bigg|_{x(k), U_0} Q\big(Y_{ref} - Y_0\big) + R \tag{14}$$

where:

$$R = [r, r, \dots r]^T \in \mathbb{R}^{3 \times N}$$

The Jacobian of the constraint function, l, can be generated in the similar fashion. The $\partial Y / \partial U$ can be calculated using a finite difference linearization approach, exploiting the fact that the manifold dynamics are linear and known. For the proposed control strategy, $\partial Y / \partial U$ is fully computed with $4 \times N + 1$ number of engine model evaluations.

B. Merit Function Technique

The full search step ΔU calculated from sub-QP can be very aggressive if the original NLP is highly nonlinear. This situation makes the SQP converge to different local optimal solutions for similar system states and future tracking references, leading to control chattering issues. Let us define a merit function $g: U \rightarrow \mathbb{R}^1$ such that:

$$g(U) = J(x(k), U) + \sigma \sum_{i=1}^{q} \max\big(0, l_i(x(k), U)\big) \tag{17}$$

where:

q is the total number of constraints.

σ is the penalty on the constraints violation.

The scaling of the search step α is obtained by solving the following one-dimensional unconstrained search problem:

$$\alpha(i) = arg \min_{\alpha} g(U + \alpha \Delta U) \tag{18}$$

It is suggested that the scaled step $\alpha \Delta U$ results in the least objective function value along the search direction (with properly selected σ), which is smaller or equal to that of the start point. Therefore, it maintains the stability criterion (16). The following flow chart summaries the entire proposed SQP model predictive IMEP control strategy.

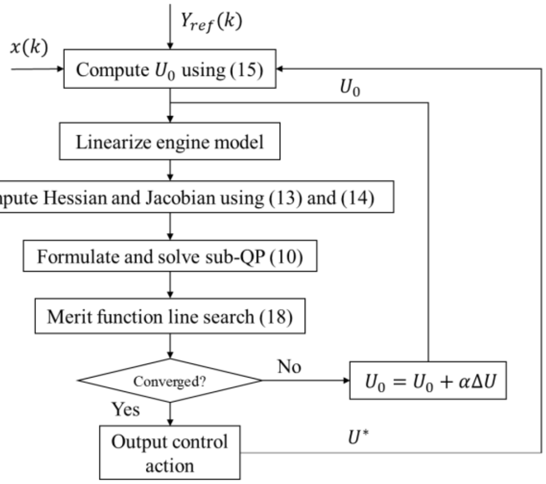

Figure 3. Flow chart of the proposed SQP model predictive IMEP control.

V. Simulation Results

Figure 4 and Figure 5 show the engine performance and control actuation of the proposed SQP model predictive IMEP controller in simulation with comparison to a LTV MPC. The LTV MPC shares the same finite difference real time linearization and warm start techniques as the proposed SQP MPC. The engine speed is fixed at 1500 RPM during this simulation. The COV of IMEP limit is selected as 6%.

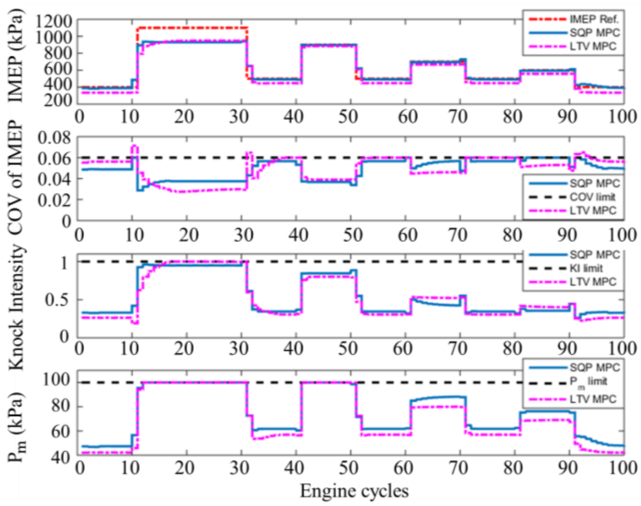

Figure 4. Engine performance comparison between LTV MPC and SQP MPC IMEP control.

It can be observed from Figure 4 that the proposed SQP MPC is able to track the IMEP reference without violating the knock and COV of IMEP constraints. The reason for the large tracking error between 10 ~ 30 engine cycles is due to the conflict of knock constraint. In comparison, the LTV MPC exhibits steady state tracking error and violation of constraints during transient scenarios.

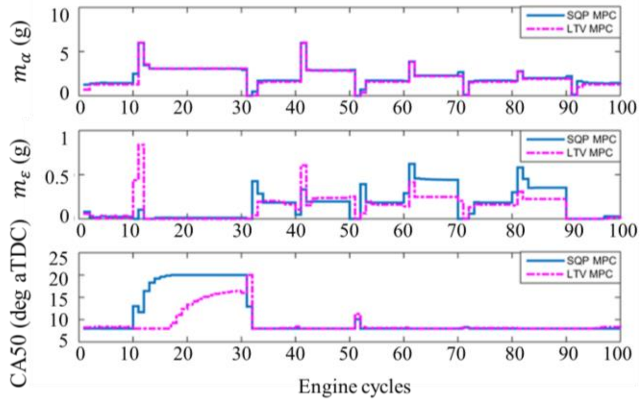

Figure 5. Control actuation comparison between LTV MPC and SQP MPC IMEP control.

The control actuation of the proposed SQP MPC is demonstrated in Figure 5. For the situation of "tip in" (around 10s, 40s and 60s), the throttle air mass flow m_α overshoots during IMEP reference steps. This maneuver is to compensate for the manifold delay and quickly increase the IMEP output. During "tip out" situations (around 30s, 50s, 70s and 90s), the throttle air mass is reduced to zero initially to compensate for manifold delay. Then it converges to steady state value without oscillation. It can be observed that the EGR flow is shut down before the throttle in order to prevent excessive residual gas fraction and meet with the COV of IMEP constraint. When the IMEP demand is not high (30~40s and 60~70s), the MPC asks for MBT combustion phasing and maximum amount of EGR, as permitted by the COV constraint. If the IMEP demand is very high (10~30s and 40~50s), the MPC reduces EGR to maximize engine air mass flow. All these observations are in agreement agree with the calibration rules for traditional map based IMEP controls.

The proposed controller is also evaluated for 10^6 consecutive engine cycles with a random IMEP tracking reference. The RMSE of IMEP tracking is 13 kPa for engine cycles without active constraints. Table I shows that range of operation conditions of this validation.

Table I. Operation Condition Range of SQP MPC Validation.

	Min	Max
RPM	800	6000
Manifold pressure (kPa)	30	100
IMEP reference (kPa)	250	1200

Table II. Statistics of the Proposed SQP Model Predictive IMEP Controller.

	Mean	Max	Min
Number of major iterations	4.2	11	1
Execution time per engine cycle (ms)	2.6	6	0.5
Execution time per major iteration (ms)	0.63	0.76	0.41
Time for model evaluation per iteration (ms)	0.39	0.42	0.38
Time for QP per iteration (ms)	0.23	0.32	0.02

Table II summarizes the execution time statistics of the proposed SQP MPC. The simulation is carried out on a desktop computer with 3.2 GHz 64 bit CPU and 16 GB of RAM.

VI. CONCLUSION

This research proposes a nonlinear model predictive control strategy for SI engine IMEP based on a SQP algorithm. The control objective is to track IMEP reference while minimizing fuel consumption. This economic MPC also respects abnormal combustion constraints during the search for optimal control action. The proposed SQP MPC is designed to work in the engine cycle domain, which reduces the engine speed dependence of air-path dynamics. The SQP algorithm is tailored for this application to improve the computational efficiency. It exploits the Gauss-Newton like structure of the NLP formulated for MPC to simplify computation of the Hessian matrix. The merit function step scaling improves global convergence performance and eliminates steady state control chattering issues. Finally, the finite difference linearization technique makes the SQP strategy adaptive to different types of engine models with complex structures. Simulation results demonstrate that the proposed model predictive IMEP control strategy achieves its design objectives, in terms of tracking torque reference, minimizing fuel consumption and respecting combustion constraints. The computational time analysis of the proposed SQP MPC demonstrates high potential for real time implementation with current prototype controllers or future production ECUs.

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