Pattern Recognition Technique Based Active Set QP Strategy Applied to MPC for a Driving Cycle Test

Qilun Zhu, Simona Onori and Robert Prucka

Abstract*—Application of constrained Model Predictive Control (MPC) to systems with fast dynamics is limited by the time consuming iterative optimization solvers. This paper proposes a fast and reliable Quadratic Programming (QP) strategy to solve MPC problems. While the optimal control action is calculated with a fast online dual QP algorithm, a "warm start" technique is adopted to reduce iterations of the online search process. The warm start solution is calculated from a predicted active constraint set generated by a pattern recognition function (Artificial Neural Network, ANN, is discussed). This function is calibrated with data from Monte Carlo simulation of the MPC controller over finite sampling points of the state-space. The proposed MPC strategy can adapt to applications with long prediction/control horizons, Linear Parameter Varying (LPV) dynamics and time varying constraints with balance between computation time, memory requirement and calibration effort. This MPC approach is applied to control vehicle speed for a HIL driving cycle test on an engine dynamometer. Simulation results demonstrate the speed profile tracking error of the MPC "driver" can be 67% less than a PID "driver". Furthermore, smooth throttle/brake actuations, similar to human drivers are achieved with the MPC controller.

I. INTRODUCTION

Applications of MPC in the automotive industry are being discussed intensively for its ability to improve system transient performance, manage constraints and reduce control effort (e.g. [1][2][3]). However, for systems with fast dynamics, the heavy computation burden of constrained MPC is a big challenge for hardware implementation. Although microprocessors are getting faster, most MPC methods for mechanical systems control are difficult to be implemented into prototype controllers.

Receding horizon constrained linear MPC controllers are the most common and fundamental design applied to systems with fast dynamics. These controllers optimize their control sequence for the future horizon using Quadratic Programming (QP). Since the development of active set QP algorithms in the 1980s, this solver has been the fastest option for online operation [4][5][6][7]. This active set algorithm is based on the fact that QP problems have closed form solutions if the active constraint set of the optimal solution is known. In [4][5], active set algorithm based on the primal QP problem is proposed. The advantage of this method versus dual active set methods is that it keeps the solution feasible during the search for the optimal solution. However, because of the formulation of MPC, the primal QP problem usually contains large number of constraints. It requires a time consuming Phase I optimization to find a feasible initial solution. The dual of the original QP problem with Lagrange multipliers as independent variables, on the other hand, has a much more uniformed constraint set ($\lambda > 0$). The search for the dual feasible initial solution can be done by calculating the optimal solution of the primal problem without any constraints. Dual active set QP methods (e.g. [6][7]) exploit this speed advantage, making them favorable for online operation with fast update frequency. [8] employed the Schur-complement dual active set QP method to the MPC application. Most dual QP methods can be applied to MPC and tested with fast prototype controllers. However, they are still not fast enough to be implemented into the ECUs of a production vehicle or other common industry level microprocessors. In addition to these two types of active set methods, the primal-dual (or interior point) method is another option to solve QP for MPCs [9][10]. It is not widely considered for fast MPCs since it requires more computational effort to complete each iteration. Furthermore, the difficulty of finding a "warm" start point is another reason that makes it not suitable for fast online operation.

Computational effort of QP can be greatly reduced with a reasonable guess of the initial search point [8][11][12][13]. All active set QP methods can benefit from a reasonable guess of which constraints may be active. [11][12][13] proposed the online active set QP based MPC strategy. Based on the assumption that the active set of constraints does not vary a lot between consecutive control steps for most MPC applications, this approach utilized the active set information of previous the control step to formulate the warm start point for the QP problem of current control cycle. Then the QP is solved using a parametric programming method, which generates a suboptimal solution if terminated prematurely. This MPC approach was tested experimentally with the application of diesel engine EGR and VGT control [13]. The disadvantage of this approach is the assumption on which it is based. For some MPC applications, especially with nonlinear system models, time varying dynamics and constraints, the active constraint set can change dramatically between each control update, leading to an increased number of iterations to find the optimal solutions.

It has been discussed in literature that the QP searching for optimal solution of MPC could be completed offline, while the online execution of MPC was transformed into a fast state and reference based control law, termed Piece-Wise Affine (PWA) function [14]. This approach was applied to multiple automotive related MPC research applications [15][16][17][18][19][20]. The fast and straight forward execution process made it possible to validate these results with hardware tests. The calibration and execution process of this MPC approach shares some similarities with that of dynamic programming, including its disadvantages. In the cases with long prediction horizon and a large number of constraints, the calibration time and memory required to store the PWA function gain matrices become less acceptable. On the other hand, the stability of MPC controllers often relies on

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long prediction/control horizon [21] and a high number of constraints [22]. For Linear Time Invariant (LTI) MPC applications, it is possible to examine the Karush – Kuhn – Tucker (KKT) conditions for all constraint combinations to guarantee the validity of the control action calculated with the PWA function [14]. However, for MPC with time varying dynamics and constraints this process may be difficult even impossible to complete. Although the PWA gain matrices can be interpolated or extrapolated from stored values in these cases, the control action may not be optimal or feasible for the control horizon.

In this paper, we propose an MPC-based strategy that can be considered as a combination between online active set methods and a similar concept of the PWA function approach. For the execution process, a fast dual active set QP solver is selected to search for the optimal solution from the warm start point that is calculated according to current system states and future reference. A pattern recognition function is used to estimate the initial guess of the active constraint set with reasonable accuracy. For the offline calibration process, instead of checking the KKT condition of all possible constraint combinations, a pattern function is trained with data from objective oriented Monte Carlo simulation. The memory requirement of this approach is also greatly reduced since it only needs to store a function instead of numerous PWA gain matrices. Compared to using the active constraint set of the previous control step as a warm start, the pattern function can generate a better "guess" of the start point and reduce the number of iterations to find the optimal solution. The proposed MPC strategy is applied to track the vehicle speed profile by manipulating gas and brake pedals during driving cycle tests. In [23][24][25] Hardware – in-the– Loop (HIL) or Engine – in-the– Loop (EIL) setups to complete driving cycle tests on the engine dynamometer are discussed. While the actual engine and control system were installed in the dyno test environment, the rest of the powertrain and vehicle longitudinal dynamics were simulated with in real time models. This testing method allows testing of powertrain component design and control without building the prototype vehicles. Since it is unlikely to employ a human driver for this type of testing, most previous work used classical controllers (e.g. PID) to track the designated speed profile. It is speculated that using MPC in this application could solve many potential tuning issues and provide better speed tracking performance.

The rest of the paper is organized as follows. Section II describes the pattern recognition technique-based QP strategy to solve the MPC problem. Section III gives the details of the HIL driving cycle test setup and modeling. Section IV provides simulation results. Finally, Section V concludes the contribution of this research work and possible future extension.

II. PATTERN RECOGNITION TECHNIQUE-BASED QP STRATEGY

The most common MPC formulations focus on systems whose dynamics can be linearized and discretized into state-space formation. The state-space matrices can be different for each step, describing a LPV system with state vector $x \in \mathbb{R}^n$, control inputs $u \in \mathbb{R}^j$ and outputs $y \in \mathbb{R}^l$.

$$\begin{gathered} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = C(k)x(k) \end{gathered} \tag{1}$$

where:

$$A(k) \in \mathbb{R}^{n\times n}, B(k) \in \mathbb{R}^{n\times j}, C(k) \in \mathbb{R}^{l\times n}$$

With the information of current system states X_0, the sequence of future system outputs Y_N of the prediction horizon N_p can be considered as an affine function of the future control action sequence U_N of control horizon N_c. The gain matrices F_N and G_N are formulated with the LPV state space function (1) (refer to Appendix A).

$$Y_N = F_N X_0 + G_N U_N \tag{2}$$

where:

$$\begin{gathered} Y_N = \left[y(k+1), \cdots y(k+N_p)\right]^T \\ X_0 = x(k) \\ U_N = [u(k), u(k+1), \cdots u(k+N_c)]^T \end{gathered}$$

The optimal control action sequence can be calculated by solving the optimization problem whose cost function penalizes the sum of weighted quadratic norm of both tracking errors and control action. The constraints that are commonly encountered in MPC include control magnitude, control changing rate and state magnitude. These constraints can be integrated and transformed into one linear inequality constraint system that is imposed on the future control action sequence U_N.

$$\begin{gathered} \min_{U_N}\left[(Y_N - R_N)^T Q_y (Y_N - R_N) + U_N^T P_u U_N\right] \\ s.t.\ A_{cons} U_N < b_{cons} \end{gathered} \tag{3}$$

where:

$$\begin{gathered} A_{cons} \in \mathbb{R}^{p\times j}, b_{cons} \in \mathbb{R}^p \\ R_N = \left[r(k+1), \cdots r(k+N_p)\right]^T \end{gathered}$$

r is tracking reference.

Q_y and P_u are symmetrical positive definite weighting matrices of reference tracking error and control effort, respectively.

The above process transfers the equality constraints of system dynamics into the objective function, accelerating the search for an optimal solution. After substituting (2) into (3), the original optimization problem can be transformed into a QP form.

$$\begin{gathered} \min_{U}\left(\frac{1}{2} U_N^T H U + f^T U_N\right) \\ s.t.\ A_{cons} U_N < b_{cons} \end{gathered} \tag{4}$$

where:

$$\begin{gathered} H = 2\left(G_N^T Q_y G_N + P_u\right) \\ f = -2G_N^T Q_y (R_N - F_N X_0) \end{gathered}$$

Equation (4) is referred to as the primal QP problem of the MPC. The primal QP algorithms can be applied at this point to solve (4) and obtain the optimal control action sequence. However, the primal QP has multiple linear inequality constraints, which make it difficult to find a feasible initial solution. A phase one optimization is usually required to identify a feasible start point. Instead of conducting a two phase optimum search for the primal optimization problem, this work employs dual QP algorithms to find the optimal solution. The conversion of the primal QP into a dual QP with Lagrange multipliers λ as independent variables is demonstrated by:

$$\min_{\lambda}\left(\frac{1}{2}\lambda^T M\lambda + N\lambda\right) \quad s.t.\ \lambda \geq 0 \tag{5}$$

where:

$$M = A_{cons}H^{-1}A_{cons}^T$$
$$N = b_{cons} + A_{cons}H^{-1}f$$

The initial solution U_0 of this QP can be easily obtained by solving the primal QP without any constraints (equivalent to letting $\lambda = 0$).

$$U_0 = -H^{-1}f \tag{6}$$

Once the optimal solution (λ^*) of the dual problem is found, the optimal control action U^* can be calculated according to:

$$U^* = U_0 - H^{-1}A_{cons}^T\lambda^* \tag{7}$$

A. Pattern Recognition Based Active Set Identification

The warm start of the dual QP is a semi-positive vector λ_0 that is close to the optimal solution λ^*. If it is known which constraints are active for the optimal solution, the constraint system of the primal and dual QP can be separated as the following:

Primal	Dual	
$A_{act}U_N = b_{act}$	$\lambda_{act} > 0$	(8)
$A_{ina}U_N < b_{act}$	$\lambda_{ina} = 0$	(9)

Equation (8) shows active partition of the constraint system while (9) shows the inactive partition. After dropping the inactive constraints and substituting the primal active constraints partition into the objective function (4), the primal-dual problems have the closed form solutions as:

$$\lambda_{act}^* = -(A_{act}H^{-1}A_{act}^T)^{-1}(b_{act} + A_{act}H^{-1}f) \quad \lambda_{ina}^* = 0 \tag{10}$$

$$U^* = U_0 - H^{-1}A_{act}^T\lambda_{act}^* \tag{11}$$

Substituting f from (4) into (10), λ_{act}^* can be transformed into a PWA function corresponding to X_0 and R_N with three gain matrices. For LTI MPC, these gain matrices are constant.

$$\lambda_{act}^* = K_X X_0 + K_R R_N + K_0 \tag{12}$$

Where:

$$K_R = 2(A_{act}H^{-1}A_{act}^T)^{-1}A_{act}H^{-1}G_N^T Q_y \quad K_X = -K_R F_N \quad K_0 = -(A_{act}H^{-1}A_{act}^T)^{-1}b_{act} \tag{13}$$

For LTI MPCs with single state ($n = 1$) and one step prediction horizon ($N_p = 1$), the active constraint sets can be visualized as 2D polyhedra in the state space (Figure 1 left). This plot is generated using the MPC controller designed for the driving cycle application with 1 step prediction horizon and LTI dynamics model. In this case, R_N is the speed reference for the next step while X_0 is the current vehicle speed. One drawback of the PWA function calibration is that the calibration time grows exponentially with control horizon and number of constraints. Furthermore, almost all matrices in (13) can be time varying for Nonlinear MPC (NMPC) applications. For instance, if the limitation of engine power and quadratic aerodynamic drag are considered, the boundaries of active constraint set polyhedra are curved making the calibration of PWA gain matrices by checking KKT conditions difficult (Figure 1 right).

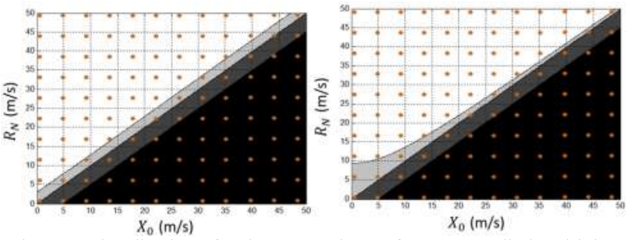

Figure 1: visualization of active constraint set for MPC applied to driving cycle tests with 1 step prediction horizon. Orange dots represent sampling points for Monte Carlo simulation. Left plot is for constant dynamics and constraints while the righ plot shows the constraint set considering engine power limit and quadratic aerodynamic drag.

The identification of active set of constraint in the state-space can be considered as a pattern recognition process. Let $\xi \in \mathbb{R}^p$ be a binary vector that has the same length p as λ. Elements of ξ is one if the corresponding constraint is active:

$$\begin{cases} \xi(i) = 0, & if\ \lambda(i) = 0 \\ \xi(i) = 1, & else \end{cases}, \text{for i} = 1,2, \dots p \tag{14}$$

Therefore, different ξ vectors can uniquely represent which constraints are active. Then the active set of constraints identification problem can be reduced to the fitting of the pattern function:

$$\xi = h(X_0, R_N) \tag{15}$$

The training data to fit the pattern function can be generated using Monte Carlo simulation with sufficient samplings points of the state-space (orange dots in Figure 1). The selection of sampling resolution and range can be practice - oriented to reduce calibration time. Many pattern recognition techniques can be applied to this application, including ANN [26], fuzzy logic [27] and optimal margin classification [28]. For this research work, an ANN with scaled conjugate gradient training algorithm is selected because of its fast execution.

TABLE I: ANN PATTERN FUNCTION WITH DIFFERENT HIDDEN LAYER SIZE.

Hidden layer size	Max ξ diff.	Mean ξ diff.	Memory (KB)
5, 0	10	1.79	7.2
10, 0	8	1.14	11.3
10, 10	5	1.08	13.4
20, 0	5	0.83	19.3
50, 0	5	0.80	33.8

The most obvious advantage for the proposed active set method is the reduction in memory requirements. Storage of the pattern function h is usually negligible compared to hundreds of gain matrices for the traditional PWA methods.

In order to demonstrate this property, the MPC applied to the driving cycle test is evaluated under random step inputs for 10^6 consecutive control cycles with prediction and control horizon expanded to 50 and 20 steps. The traditional PWA cannot be evaluated in this case due to the astronomical memory requirement. Table I compares the performance and memory requirement of the ANN pattern function with different network sizes. It can be observed that a simple double-layer ANN can predict the active sets of constraints with reasonable accuracy and memory demand. The rest of the analysis focuses on the ANN with 10 neurons on the two hidden layers.

TABLE II: COMPARISON BETWEEN COLD START AND DIFFERENT WARM START TECHNIQUES.

		Cold start	Warm start	
			Previous cycle	ANN
Constant system	Max iter. #.	59	11	8
	Mean iter #	41	7	3
Varying system	Max iter. #.	59	23	9
	Mean iter #	43	14	3

The performance of active set prediction methods is measured with ξ difference, which is defined as the number of ξ elements that is different from the ξ^* of the optimal solution. Most optimization algorithms take more iterations to find the optimal solution with larger ξ difference although this relationship may not be exactly linear. Table II shows that the two warm start techniques generate close to optimal initial guesses of active sets. Comparing to the warm start technique that uses active set information of previous control cycle (traditional online active set method), the proposed ANN pattern function can provide more accurate initial guesses of active sets. This significantly reduces the number of online iterations to find the optimal solution. Figure 2 shows that the ξ predicted by ANN pattern function has two elements different from the optimal ξ^* for more than 90% of control updates. Another important characteristic of the new approach is that it does not rely on the boundaries between active set polyhedra to be straight (Figure 1). Therefore, the accuracy of active sets prediction does not degenerate with varying dynamics model and constraints (Table II and Figure 2). However, identifying active sets for LPV MPC can benefit from training data with higher resolution of the state-space.

Depending on the control objectives, the pattern function can include other inputs to capture non-state dependent variation of dynamics model and constraints. For instance, the vehicle mass m can alter the vehicle longitudinal dynamics. The pattern function inputs are augmented by including the vehicle mass to predict the active sets with consideration of vehicle loading conditions:

$$\xi = h(X_0, R_N, m) \tag{16}$$

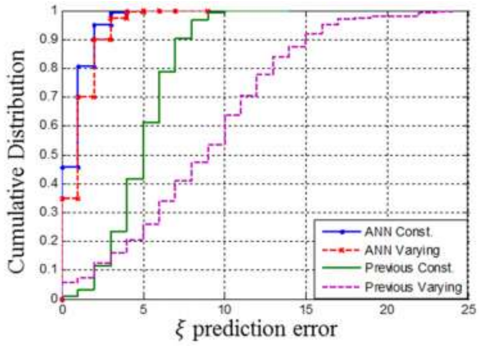

Figure 2: cumulative distribution function as a function of prediction error ξ between different warm start techniques. Both constant and varying system dynamics/constraints are evaluated.

B. Hildreth Search Method for λ^*

Without using the predicted active sets to compute control actions directly, the online optimal search process increases the tolerance to initial guess error. Therefore, it is possible to use training data with coarser resolution to calibrate the pattern function. For applications with long prediction and control horizons, this trait of the proposed MPC strategy allows for faster calibration and more robustness against system variations than PWA methods.

The dual QP methods of Hildreth and D'Espo [29][30] are applied to search for optimal solution λ^* for computational efficiency. Without matrix inversions during each iteration, the algorithm is fast and reliable [31]. Possibilities of applying other online QP algorithm will be considered in the future work of this research.

III. HIL DRIVING CYCLE TEST SETUP AND MODELING

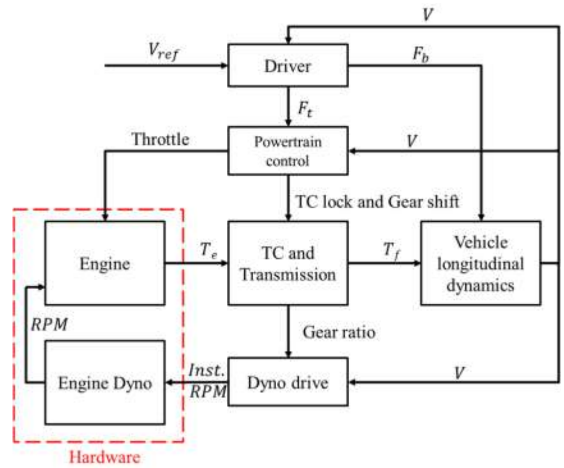

Figure 3: block diagram of HIL driving cycle test setup. V_{ref} is the reference speed. T_e and T_f are torque output from engine and final drive, respectively.

Figure 3 shows a block diagram of the HIL driving cycle test setup. Other than the engine and dynamometer (red dash circle), the rest of the components are simulated with real time models. The engine dyno speed is determined by current vehicle speed and gear ratio. The throttle and engine speed are sent to the dyno through the software interface, while the engine torque measured by the dyno is sent back. Both engine and dyno are currently replaced with mathematical models. The engine model is a high-fidelity

Control Oriented Model (COM) [32]. The dyno is assumed to be able to measure engine torque instantaneously. Its delay to RPM command is considered as a first order delay with time constant of 0.7s.

The proposed MPC strategy is implemented as the "driver". The MPC control objective is to mimic a real driver on actual roads, who foresees the desired vehicle speed several seconds ahead of time. The update frequency of the MPC is set to be 0.5s. Both prediction and control horizons are selected as 10 steps during the simulation. The optimization problem is formulated as described in Section I. Both speed tracking error and control effort are being minimized with respect of weightings. The gas and braking pedal actuations are interpreted as demand of normalized traction and braking force ranging from 0 to 1. While the maximum braking force c_1 is assumed to be constant, the maximum of traction force is limited by the maximum engine power P_{max} at the specific vehicle speed.

$$\mu mg + \frac{1}{2} C_D A_{front} \rho V^2 + m\dot{V} = \frac{P_{max}}{V} F_t - c_1 F_b \qquad (17)$$

Where:

A_{front}	Frontal area
C_D	Drag coefficient
F_t, F_b	Traction, braking force
V	Vehicle velocity
μ	Coefficient of rolling resistance

The longitudinal vehicle dynamics in Equation (17) are linearized at each step according to reference velocity along the prediction horizon. This LPV system model is applied to formulate the MPC controller.

IV. Simulation Results

Simulations are conducted implementing the proposed MPC strategy within the HIL driving cycle test using the following main parameters.

TABLE III: IMPORTANT PARAMETERS FOR THE SIMULATION

Engine	Displacement	3.2 L
	Max torque	287 Nm
	Max power	157 kw
Powertrain	Gear ratio	4.1/2.4/1.6/1.2/0.9/0.7
	Final drive	2.8
Vehicle	Mass	1800 kg
	$C_D \times A$	0.25 × 2.1
FTP Driving Cycle	Max/Average speed	91/34 km/h
	Max accel./decal.	3/-3 m/s^2

The performance of two MPC "drivers" is compared to a PID controller. Figure 4 is a snap shot of vehicle speed and normalized driver actuation from a portion of the drive cycle. Both MPC "drivers" demonstrate the ability to optimize control actions according to future reference and constraints. The first MPC (MPC 1 in the figures) has small weighting on the control effort (P_u). Therefore, it tends to change the control actuation aggressively to match the speed profile. The mean speed tracking error (MSTE) is 3.8%. The second "economical" MPC (MPC 2 in the figures) has a larger P_u resulting smooth pedal actions. Although its speed tracking performance (MSTE 6.5%) is slightly worse than the "aggressive" MPC, the "economical" MPC still follows the speed profile better than the PID "driver", whose MSTE is 11.6%.

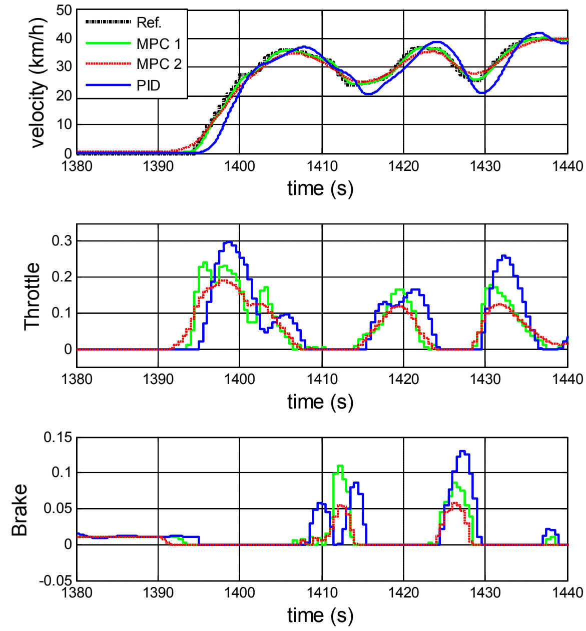

Figure 4: performance comparison between aggressive MPC (MPC 1), economical MPC (MPC 2) and PID "drivers"

V. Conclusion

In this paper, a pattern recognition based active set QP strategy is proposed to solve an MPC problem. This MPC is solved by means of an online QP based on the Hildreth algorithm. Without matrix inversions each iteration, this algorithm is fast and reliable. The initial point of the online search is calculated from current system states and future reference with a pattern function. The recognition of the pattern function is complete with data generated from Monte Carlo simulation. The pattern function can also include non-state dependent variables as input to further improve the adaptive-ness of nonlinearities. Compared to a traditional PWA approach, the pattern function requires less memory space, making it possible to handle problem with long prediction/control horizons and a large number of constraints. The online search process guarantees the robustness against pattern function fitting error. Simulation results indicate that it may be possible to employ a simple ANN to reasonably predict the active constraint sets for MPC with a long control horizon and varying dynamics and constraints. The prediction accuracy is better than using the previous active set directly. As a result, the warm start point generated by the pattern function can significantly reduce iterations when finding the optimal solution.

The proposed MPC strategy is applied to HIL driving cycle by manipulating the gas and brake pedals. The formulation of the MPC involves varying dynamics and constraints, which is efficiently solved by the pattern recognition active set QP method. Simulation results demonstrate that the MPC driver can track the speed profile with relative error less than 7%. The method also produces realistic control actions like a real human driver, who forecasts the desired vehicle speed several seconds ahead. As a byproduct of the smooth pedal action, fuel economy improvement can also be realized.

The actual hardware validation of the entire system will be carried out in the near future. Other possible future expansion of this work includes: 1) comparing different pattern recognition methods for active sets identification; 2) integrating this warm start technique with other online QP solver; 3) applying this MPC strategy to systems with faster dynamics and 4) replacing human driver for chassis dyno cycle tests with the proposed MPC control strategy.

APPENDIX

F_N *and* G_N *in equation* (2) are formulated as (dropping step indicator k)

$$F_N = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}, G_N = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \cdots & CA^{N_p-N_c}B \end{bmatrix}$$

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