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Recasting the HEV Energy management problem into an Infinite-Time Optimization Problem including Stability

Roberto Mura, *Student Member, IEEE,* Vadim Utkin, *Fellow, IEEE,* Simona Onori, *Member, IEEE*

***Abstract*— This paper deals with the problem of finding a closed-form optimal solution for the energy management problem in charge-sustaining hybrid electric vehicles. For the first time, a generalized stability and optimality framework for this type of problem is proposed. In the proposed control design, the energy management problem, that by its very nature is a finite-time optimal control problem, is reformulated as a nonlinear-nonquadratic infinite-time optimization problem, leading to a family of state-feedback based control laws providing optimality with respect to an infinite time horizon performance functional while guarantee asymptotic stability.**

The analytical solution is obtained through a Lyapunov-based dissipative approach. The proposed optimal energy management control law is implemented in a pre-transmission parallel hybrid heavy duty truck and its performance is compared to the benchmark solution provided by the Pontryagin's Minimum Principle (PMP). Reduced computational efforts and lower sensitivity of the control parameter, while maintaining performances within 2% from the optimal value, makes the novel control design a breakthrough in energy management control research.

I. Introduction

Hybrid electric vehicles (HEVs) are equipped with two or more on-board propulsion devices and energy sources [1]. The additional degree of freedom offered from the hybrid architecture is used to find an optimal split of the power demand between the internal combustion engine (ICE) and the electric machines (EMs), while minimizing a performance objective, commonly the fuel consumption, although other objectives such as pollutant emissions, battery aging, drivability or a combination of them ([2], [3]) can also be included. The control layer responsible for this purpose is the Supervisory Controller or Energy Management Module, which represents the extra layer of control needed in hybrid vehicles to generate the optimal actuator set points.

The HEV energy management problem can be cast into an optimal control problem where the objective is to minimize a functional cost defined over the time of a driving mission. Several methods to tackle this problem have been proposed in literature over the past decades (see, for example, [4], [5], [6] and references therein). The simplest way to deal with the problem, by not involving explicit minimization or optimization, is by designing rules to manage the on-board energy of the vehicle. Rule-based control and fuzzy logic strategies are within this category. Their advantage is in a fast computational efficiency [7], at the price of a large calibration parameters set, which require an *ad hoc* tuning that depends on the architecture.

To fully exploit the potential of the hybrid electric architectures model-based optimal control methods have been used, showing the benefits of a more sofisticated control design in terms of improved performences over less computational demanding rule-based strategies. For instance, global optimization methods like dynamic programming (DP) and genetic algorithms (GA) [8] were used to find off-line solutions. Stochastic dynamic programming (SDP) was succesuflly used for on-line control developemnt [9], [10] and model-predictive control (MPC) based methods have been also applied to solve the energy management problem where a short-term optimization horizon in the future is considered during which the driving cycle is predicted and the optimal power split is found [11], [12]. Instantaneous optimization methods have also been proposed. Among them are enclosed Pontryagin's Minimum Principle (PMP) [6], [13], [14] and the Equivalent Consumption Minimization Strategy (ECMS) [2], [15]. Both methods, that have been shown in [13] to be equivalent, consist in the minimization, at each time step, of an instantaneous suitable adjoint cost function, leading to a sub-optimal result close to the global optimum. Both ECMS and PMP require calibration of a tuning parameter, referred to as equivalence factor and co-state, respectively. The optimality is achieved only when future driving information is known, thus making them not suitable candidates for in-vehicle implementation (if the optimality is sought). To this end, on-line adaptation of the tuning parameter as driving dynamics change was proposed (A-ECMS, [16]), where the adaptation of the parameter is performed on-line through PI-like adaptive law, either predicting the driving cycle, or exploiting the correlation between the co-state and the battery state of charge [16], [17].

Both PMP and ECMS require the adjoint cost function (the Hamiltonian or the equivalent fuel consumption function) to be minimized instantaneously. This operation, that needs to be executed on-board at each tick of the clock, despite being computationally expensive, can lead in some cases to unpredictable results, due to the fact that the Hamiltonian is in many instances of the driving cycle not a convex function of the control variable, as one can see in Fig. 1 where the Hamiltonian function is plotted for three different points of the driving cycle. Different control values are thus equally suitable in the minimization process, leading to a not unique solution of the optimal control problem. With this, loss of optimality is accompanied to poor drivability. To overcome these issues, a new research direction has been sought. Inspired by [18] and [19] on theoretical results on optimal nonlinear regulation problem involving non quadratic cost functionals, a first attempt to propose a new framework for the energy management problem was given in [20]. In this work the authors cast the energy management problem into a nonlinear optimal regulation problem where the battery state of charge SOC was optimally regulated to its reference target in the case of zero disturbance. Preliminary results showed the feasibility of the closed-form control law in the simple case of vehicle at standstill. Reduction in computational execution and decreased sensitivity of the control parameter with respect to driving conditions were also showed. Nonetheless two issues were not being addressed properly in [20]: the definition of stability and the extension of the finite-time cost function into an infinite-time functional (needed to fully use results from [18], [19]). Starting

Roberto Mura is with the Department of Electronics and Information, Politecnico di Milano, Milano, 20132 Italy. He was a visiting scholar at the Center for Automotive Research The Ohio State University, Columbus, 43212, USA at the time of this work. [mura@elet.polimi.it]

Vadim Utkin is with the Department of Electrical and Computer Engineering, The Ohio State University, Columbus, 43210, USA. [utkin@ece.osu.edu]

Simona Onori is with Automotive Engineering Department, Clemson University, Greenville, 29607, USA. She was with the Center for Automotive Research The Ohio State University at the time of this research. [sonori@clemson.edu]

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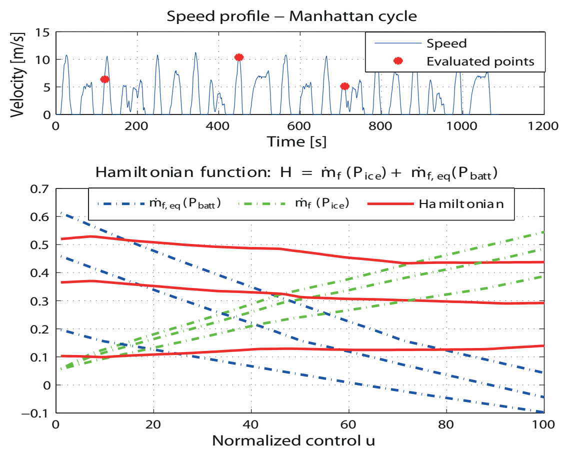

Fig. 1: Hamiltonian function $\mathcal{H}$ (bottom) evaluated for different instances of a Manhattan driving cycle (top).

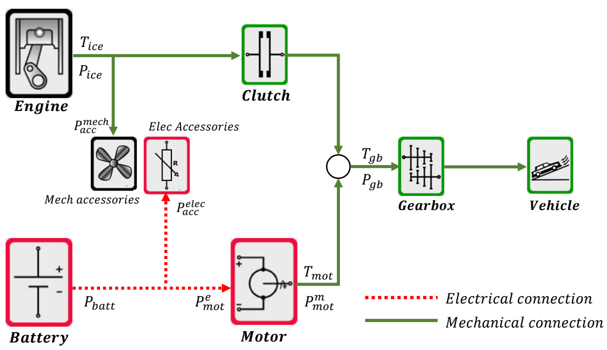

Fig. 2: Power flow diagram of pre-transmission parallel HEV.

from this, this paper proposes an extension and improvement of concepts initially presented in [20]. The objective is to find an analytical, closed-form, energy management strategy that is real-time implementable and that assures optimality and stability. To this end, the problem is cast into a nonlinear infinite-time optimal regulation problem, and a Lyapunov based approach is used to design an analytical control law which produces performances comparable to PMP solutions, used as a proxi for global optimum.

II. Hybrid Heavy-duty truck model

The vehicle model used in this work is a heavy duty pre-transmission parallel HEV, whose schematic is shown in Fig. 2. The engagement or disengagement of the clutch makes the engine to be connected to the transmission, and thus to the rest of the powertrain. The vehicle can operate in three different modes which are described in the following, depending on the status of clutch and the gear position.

1) Electric Mode. In this mode the clutch is open and the vehicle uses only the battery and the electric motor for propulsion; the engine is not connected to the wheels and is switched off. Since there is just one propulsion device in this mode the torque/power requested by the driver at the wheels is totally satisfied using the electric drivetrain and no optimization is needed. The instantaneous torque/power balance equations are:

$$\begin{cases} T_{mot}(t) = T_{gb}(t) \\ P_{batt}(t) = P^{e}_{mot}(t) + P^{elec}_{acc} \\ \omega_{mot}(t) = \omega_{gb}(t) \end{cases} \qquad \forall t \in [0, T] \tag{1}$$

where $T_{gb}(t)$ and $\omega_{gb}(t)$ are the instantaneous gearbox torque and speed; $T_{mot}(t)$, $\omega_{mot}(t)$ and $P^{m}_{mot}(t)$ are the instantaneous electric motor torque, speed and mechanical power; P^{elec}_{acc} represents the electrical accessory power (considered constant) and $P^{e}_{mot}(t)$ represents the instantaneous electrical power of the electric motor:

$$P^{m}_{mot} = \begin{cases} \eta_{mot} \cdot P^{e}_{mot} & P^{e}_{mot} > 0 \\ \dfrac{1}{\eta_{mot}} \cdot P^{e}_{mot} & P^{e}_{mot} < 0 \end{cases} \tag{2}$$

where η_{mot} represents the efficiency of the electric motor.

2) Parallel mode with neutral gear. This mode of operation occurs when the vehicle is at standstill with the clutch closed and the gearbox in neutral position. The engine is still connected to the transmission, but its speed is free to vary as the gearbox is in neutral position. The instantaneous torque, power and speed balance equations to be satisfied are:

$$\begin{cases} T_{ice}(t) - T^{mech}_{acc}(t) = -T_{mot}(t) \\ P_{batt}(t) = P^{e}_{mot}(t) + P^{elec}_{acc} \\ \omega_{mot}(t) = \omega_{ice}(t) = \omega^{*}_{ice}(t) \end{cases} \qquad \forall t \in [0, T] \tag{3}$$

where $T_{ice}(t)$ and $\omega_{ice}(t)$ represent the instantaneous engine torque and speed; $T^{mech}_{acc}(t)$ is the instantaneous mechanical accessory torque; and $\omega^{*}_{ice}(t)$ represents the instantaneous optimal engine speed obtained by selecting the maximum efficiency operating line of the engine. Being the total power requested at the wheels zero, the power balance equations can be written, assuming a constant efficiency for the electric motor (η_{mot}), as:

$$P_{gb}(t) = 0 = P_{ice}(t) - P^{mech}_{acc}(t) + \frac{1}{\eta_{mot}} \cdot \left(P_{batt}(t) - P^{elec}_{acc} \right) \tag{4}$$

3) Parallel. In this mode both devices are used for propulsion, with clutch closed and the engine connected to the wheels. The speed at the wheels determines, through the transmission, the speed of both the electric machine and the engine. The powertrain equations are:

$$\begin{cases} T_{ice}(t) - T^{mech}_{acc} + T_{mot}(t) = T_{gb}(t) \\ P_{batt}(t) = P^{e}_{mot}(t) + P^{elec}_{acc} \\ \omega_{mot}(t) = \omega_{ice}(t) = \omega_{gb}(t) \end{cases} \qquad \forall t \in [0, T] \tag{5}$$

$$P_{gb} = \begin{cases} P_{ice} - P^{mech}_{acc} + \eta_{mot} \cdot \left(P_{batt} - P^{elec}_{acc} \right) & P^{e}_{mot} > 0 \\ P_{ice} - P^{mech}_{acc} + \dfrac{1}{\eta_{mot}} \cdot \left(P_{batt} - P^{elec}_{acc} \right) & P^{e}_{mot} < 0 \end{cases} \tag{6}$$

The vehicle model described above has been implemented in a PSAT (Powertrain Simulation Analysis Toolkit) detailed forward model simulator. PSAT is a state-of-the-art flexible powertrain simulation software developed by Argonne National Laboratory, running in MATLAB/Simulink environment, which provides access to dynamic models of different mechanical and electrical components of several hybrid vehicle configurations [21].

III. Energy management problem - Classical formulation

The objective of the energy management strategy in a HEV is to find the optimal power split between the primary and secondary energy sources that minimizes a given objective function over an entire driving cycle. In particular the aim is to minimize the total mass of fuel m_f [g] during a driving mission of length T, starting from $t = 0$, thus is equivalent to minimize the cost J_T:

$$J_T = \int_0^T \dot{m}_f(u(t))dt \tag{7}$$

where $\dot{m}_f$ is the instantaneous fuel consumption rate expressed in [g/s] and $u(t)$ is the control variable.

A. System dynamics and constraints

When solving the energy management problem, the only state variable is the battery state of charge $SOC(\cdot)$, whose dynamics are defined as:

$$\dot{SOC}(t) = -\alpha \frac{I(t)}{Q_{max}} \tag{8}$$

where α represents the Coulombic efficiency [1], $I(t)$ [A] the actual battery current (positive in discharging, negative in charging) and Q_{max} [Ah] the maximum battery capacity. In a charge-sustaining HEV, the net energy variation in the battery over a given driving cycle should be zero. This condition is guaranteed by imposing that the SOC is the same at the beginning and at the end of the cycle. Equation (9) represents the global constraints of the optimal problem:

$$SOC(0) = SOC(T) = SOC_{ref} \tag{9}$$

Local constraints are imposed on state and control variables as well. These constraints mostly concern physical operation limits, such as maximum engine torque and speed, maximum motor power, and the battery SOC limits. For pre-transmission parallel HEV powertrain local constraints are expressed as:

$$\begin{cases} P_{batt,min} \leq P_{batt}(t) \leq P_{batt,max} \\ SOC_{min} \leq SOC(t) \leq SOC_{max} \\ T_{x,min} \leq T_x(t) \leq T_{x,max} & \forall t \in [0,T] \\ P_{x,min} \leq P_x(t) \leq P_{x,max} \\ \omega_{x,min} \leq \omega_x(t) \leq \omega_{x,max} & x = ice, mot \end{cases} \tag{10}$$

where the last three inequalities represent limitations on the instantaneous engine and motor torque, and speed respectively; $(\cdot)_{min}$; $(\cdot)_{max}$ are the minimum and maximum value of power/SOC/torque/speed at each instant. In particular battery power limits are not constant but depend on internal parameters like V_{oc} [V], the battery open circuit voltage, and R_s [Ω], the battery internal resistance. Moreover, powertrain constraints are also enforced at each instant to ensure that the total power demand at the wheels is satisfied, in accordance to the specific mode of operation. Within this formulation the energy management problem can be defined as follows:

Problem 1: The energy management problem is a constrained optimal control problem where the cost function (7) is minimized under system dynamics (8), global and local constraints ((9) and (10), respectively).

We refer to Problem 1 as the standard HEV energy management problem. Typical SOC behavior resulting from solving Problem 1 is shown in Fig. 3.

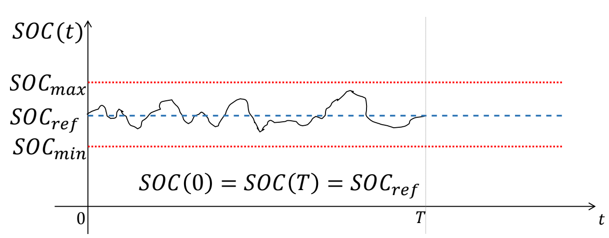

Fig. 3: SOC profile obtained solving the HEV energy management problem as formulated in Problem 1.

B. Fuel flow rate consumption - Engine Model

In the new control design proposed in this paper we use an analytical model of the engine fuel consumption rather than a map-based model which is generally used to solve this type of problems [1]. In particular, the engine chemical power (P_{chem}) is given as a function of the engine power (P_{ice}) and speed (ω_{ice}) using a Willans line model:

$$P_{chem}(t) = e_0(\omega_{ice}(t)) + e_1(\omega_{ice}(t)) \cdot P_{ice}(t) \tag{11}$$

where $e_0(\omega)$ represents the engine friction losses and $e_1(\omega)$ the conversion efficiency of the machine. A good approximation of the friction losses and conversion efficiency coefficients is given by expressing e_0 and e_1 as a quadratic fitting with respect to engine speed, as:

$$\begin{cases} e_0(\omega_{ice}(t)) = e_{00} + e_{01} \cdot \omega_{ice}(t) + e_{02} \cdot \omega_{ice}^2(t) \\ e_1(\omega_{ice}(t)) = e_{10} + e_{11} \cdot \omega_{ice}(t) + e_{12} \cdot \omega_{ice}^2(t) \end{cases} \tag{12}$$

where $e_{ij} > 0$, $i, j = 0, 1, 2$ are the constant Willans line coefficients, $P_{chem} = \dot{m}_f \cdot Q_{LHV}$ (Q_{LHV} is the lower heating calorific value of diesel in [kJ/kg]) is the chemical power input to the engine and $P_{ice} = T_{ice}\omega_{ice}$ is the engine power output. Hence, the fuel consumption rate can be written as:

$$\dot{m}_f(t) = \frac{1}{Q_{LHV}} \left[e_0(\omega_{ice}(t)) + e_1(\omega_{ice}(t)) \cdot P_{ice}(t) \right] \tag{13}$$

otherwise written by means of the coefficients p_0 and p_1:

$$\dot{m}_f(t) = p_0(\omega_{ice}) + p_1(\omega_{ice}) P_{ice}(t) \tag{14}$$

that can be written also as an explicit function of P_{batt} by means of (4):

$$\dot{m}_f = n_0(\omega_{mot}) + n_1(\omega_{mot}) P_{batt}(t) \tag{15}$$

where the relation between coefficients p_0, p_1 and n_0, n_1 is:

$$\begin{cases} n_0 = p_0 + p_1 \left(P_{acc}^{mech} + \dfrac{1}{\eta_{mot}} P_{acc}^{elec} \right) \\ n_1 = -\dfrac{p_1}{\eta_{mot}} \end{cases} \tag{16}$$

The Willans line fuel consumption rate model, together with a suitable description of the battery model, is used in the next section to reformulate the energy management problem as an infinite-time horizon optimal problem including stability.

IV. Infinite-Time Nonlinear Optimization Problem including Stability

The novel approach proposed in this paper consists in re-thinking the standard finite-time optimal control problem in HEV (Problem 1) as a nonlinear non-quadratic optimization problem over an infinite time horizon. In particular, a Lyapunov based approach is used to design the controller, leading to a family of feedback closed-form control laws that guarantee stability and optimality with respect to an adjoint cost function. This adjoint cost (defined later in this section) guarantees a bound on the worst-case value of the nonquadratic cost functional over a prescribed set of bounded input disturbances. To ensure optimality of vehicle operation when $t > T$, the $[0 \;\; T]$ optimization horizon is extended into the infinite horizon $[0 \;\; \infty]$, so as to lead to a new cost function defined over $[0 \;\; \infty]$:

$$J_\infty = \int_0^\infty \dot{m}_f(u(t)) \; g(t) \; dt \tag{17}$$

by means of the scalar positive function $g(t)$,

$$g(t) = \frac{1 + \alpha \left(\dfrac{t}{T}\right)^q}{1 + \left(\dfrac{t}{T}\right)^q} \qquad 0 < \alpha < 1, \quad k > 0. \tag{18}$$

The role of the function $g(t)$, shown in Fig. 4, is to penalize the

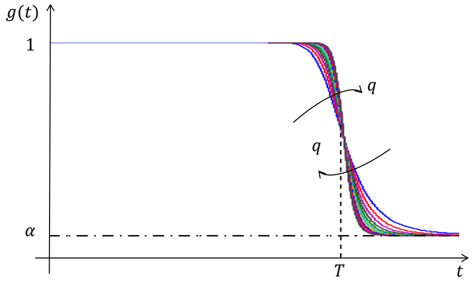

Fig. 4: Behavior of $g(t)$ for different values of q. Note that for $q \to \infty$ is obtained that $J_\infty = J_T + J_\alpha$ where J_T is defined in (7) and $J_\alpha = \int_T^\infty \dot{m}_f \cdot \alpha \, dt$.

action of the control $u(t)$ for $t > T$ in order to approximate the finite-time cost J defined in (7) with the infinite-time functional defined above.

Before presenting the new control design, a set of mathematical preliminaries from [19] is included in the next sub-section for ease of discussion, consisting in a set of sufficient conditions requiring, among other things, the nonlinear system to be dissipative with respect to a supply rate function.

A. General Framework

Let be $\mathcal{D} \subset \mathbb{R}^n$ an open set and let $\mathcal{U} \subset \mathbb{R}^m$, with $0 \in \mathcal{D}$ and $0 \in \mathcal{U}$. Moreover, let $\mathcal{W} \subset \mathbb{R}^d$. Consider now the controlled dynamical system:

$$\begin{array}{c} \dot{x}(t) = F(x(t), u(t)) + G(x(t))w(t) \\ x(0) = x_0, \quad w(\cdot) \in \mathcal{L}_2, \quad t \geq 0 \end{array} \tag{19}$$

with performance output vector:

$$z(t) = h(x(t), u(t)) \tag{20}$$

where $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ satisfies $F(0,0) = 0$, $G : \mathbb{R}^n \to \mathbb{R}^d$, $h : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ satisfies $h(0,0) = 0$ and the control $u(\cdot)$ is restricted to the class of admissible controls such that $u(t) \in \mathcal{U}$, $\forall t \geq 0$. Given a feedback control law $u(t) = \phi(x(t))$, the closed-loop system has the form:

$$\begin{array}{l} \dot{x}(t) = F(x(t), \phi(x(t))) + J(x(t))w(t) \quad x(0) = x_0, \quad t \geq 0 \\ z(t) = h(x(t), \phi(x(t))) \end{array} \tag{21}$$

With respect to the open loop system (19) and the closed one (21), the following assumptions and definitions are given.

Assumption 1: The mapping $\phi : \mathcal{D} \to \mathcal{U}$ satisfies sufficient regularity conditions such that the resulting closed-loop system (21) has a unique solution forward in time.

Let $L : \mathcal{D} \times \mathcal{U} \to \mathbb{R}$ and $\mathcal{S}$ the set of regulation controllers for the nonlinear system with $w(t) \equiv 0$.

Definition 1: $\mathcal{S}(x_0)$ = {$u(\cdot)$: $u(\cdot)$ is admissible and such that $\dot{x}(\cdot)$ given by (19), starting from initial state condition x_0, satisfies $x(t) \to 0$ as $t \to \infty$ with $w(t) \equiv 0$ }.

Definition 2: γ is the system $\mathcal{L}_2$ gain from w to z, representing the maximum energy amplification of the input signal $w \in \mathcal{L}_2$ on the performance variable z:

$$\|z(\cdot)\|_2 \leq \gamma \|w(\cdot)\|_2 \tag{22}$$

Definition 3: Given the system (19), the performance output (20) and a Lyapunov function $V : \mathcal{D} \to \mathbb{R}$, a positive storage function $\Gamma : \mathcal{D} \times \mathcal{U} \to \mathbb{R}$, and the given supply rate function $r : \mathbb{R}^p \times \mathbb{R}^d \to \mathbb{R}$ are introduced as follows:

$$\begin{cases} \Gamma(x, u) = \dfrac{1}{\gamma^2} \left(\dfrac{\partial V}{\partial x} \right)^2 f(u) \\ r(x, w) = \gamma^2 w^2 - z^2 \end{cases} \tag{23}$$

where $f(u)$ is a generic scalar nonlinear function of the control variable u.

Theorem 1: (from [19]) Consider the nonlinear dynamical system (19) and (20) with performance functional:

$$J(x_0, u(\cdot)) = \int_0^\infty L(x(t), u(t)) \, dt \tag{24}$$

where $u(\cdot)$ is an admissible control. Assume that there exist a Lyapunov function $V : \mathcal{D} \to \mathbb{R}$, a positive storage function $\Gamma : \mathcal{D} \times \mathcal{U} \to \mathbb{R}$, the given supply rate function $r : \mathbb{R}^p \times \mathbb{R}^d \to \mathbb{R}$, and a control law $\phi : \mathcal{D} \to \mathcal{U}$ such that:

$$\begin{array}{ll} 1. & V(0) = 0 \\ 2. & V(x) > 0, \quad x \in \mathcal{D}, \quad x \neq 0 \\ 3. & \phi(0) = 0 \\ 4. & \dfrac{\partial V(x)}{\partial x} F(x, \phi(x)) < 0, \quad x \in \mathcal{D}, \quad x \neq 0 \\ 5. & \dfrac{\partial V(x)}{\partial x} G(x) w \leq r(z, w) + L(x, \phi(x)) + \Gamma(x, \phi(x)) \\ & x \in \mathcal{D}, \quad w \in \mathcal{W} \\ 6. & \begin{cases} \mathcal{H}(x, \phi(x)) = 0, \quad x \in \mathcal{D} \\ \mathcal{H}(x, u) \geq 0, \quad x \in \mathcal{D}, \quad u \in \mathcal{U} \end{cases} \end{array} \tag{25}$$

where

$$\mathcal{H}(x, u) = \frac{\partial V(x)}{\partial x} F(x, u) + L(x, u) + \Gamma(x, u) \tag{26}$$

Then:

- there exists a neighborhood $\mathcal{D}_0 \subset \mathcal{D}$ of the origin such that the zero solution $x(t) \equiv 0$ of the undisturbed ($w(t) \equiv 0$) system is locally asymptotically stable;
- if $x_0 \in \mathcal{D}_0$ then the feedback control $u(\cdot) = \phi(x(\cdot))$ minimizes $\mathcal{J}(x_0, u(\cdot))$ in the sense that:

$$\mathcal{J}(x_0, \phi(x(\cdot))) = \min_{u(\cdot) \in \mathcal{S}(x_0)} \mathcal{J}(x_0, u(\cdot)) \tag{27}$$

  where $\mathcal{J}(x_0, \phi(x(\cdot)))$ is the adjoint functional defined as:

$$\mathcal{J}(x_0, u(\cdot)) = \int_0^\infty [L(x(t), u(t)) + \Gamma(x(t), u(t))] \, dt \tag{28}$$

  and in addition:

$$J_\infty(x_0, \phi(x(\cdot))) \leq \mathcal{J}(x_0, \phi(x(\cdot))) \tag{29}$$

- if $\mathcal{D} = \mathbb{R}^n$, $\mathcal{U} = \mathbb{R}^m$, $w(t) \equiv 0$, and $V(x) \to \infty$ as $\|x\| \to \infty$, then the zero solution $x(t) \equiv 0$ of the closed loop system is globally asymptotically stable.

Proof: The proof of Theorem 1 is given in Chapter 10 of [19]. ■

B. Application to the energy management problem in HEVs

Following the lines of Theorem 1, a Lyapunov-based approach is used to obtain a state-feedback control law to find the optimal torque/power split between the engine and the electric motor. In the new control framework, the power requested (P_{req}) is regarded as a $\mathcal{L}_2$ disturbance.

Without loss of generality, the state of energy (SOE), defined as the amount of battery energy stored at the present time ($E(t)$) to the maximum battery energy capacity (E_{max}) , is used as state variable in this discussion. SOE is related to SOC by the following relationship:

$$SOE(t) = SOC(t)\frac{V_L(t)}{V_{oc}^{max}} = \frac{E(t)}{E_{max}} \tag{30}$$

where V_L is the battery terminal voltage and V_{oc}^{max} the maximum open circuit voltage. From here on the explicit dependence on time will be omitted. The SOE dynamics are:

$$\begin{cases} S\dot{O}E = -\alpha \frac{P_{batt}}{E_{max}} \\ E_{max} = Q_{max} \cdot V_{oc}^{max} \end{cases} \tag{31}$$

Assuming $k = \frac{\alpha}{E_{max}\eta_{mot}}$, the battery SOE error $\zeta = SOE_{ref} - SOE$ is introduced, whose dynamics are described as a function of the control input (P_{ice}) and the disturbance (P_{req}) (as shown later in (34)). Note that in parallel mode the power requested is the sum of accessory powers ($P_{acc}^{elec} + P_{acc}^{mecc}$) and the gearbox power (P_{gb}). When the vehicle is not moving ($v = 0$), instead, the power requested P_{req} only accounts for the accessory loads power. The disturbance power P_{req} is thus following:

$$P_{req} = \begin{cases} P_{gb} + \eta_{mot}P_{acc}^{elec} + P_{acc}^{mecc} & v > 0 \quad \forall t \in [0 \quad T] \\ \eta_{mot}P_{acc}^{elec} + P_{acc}^{mecc} & v = 0 \quad \forall t \in [T \quad \infty] \end{cases} \tag{32}$$

Consider a compact set $\mathcal{Z} \subset \mathbb{R}$ such that $\zeta \in \mathcal{Z}$, a set $\mathcal{U} \subset \mathbb{R}$ such that $P_{ice} \in \mathcal{U}$, and a set $\mathcal{W} \subset \mathbb{R}$ such that $P_{req} \in \mathcal{W}$ and P_{req} in $\mathcal{L}_2$. The control, state and disturbance domains are:

$$\begin{cases} \mathcal{Z} = [SOE_{ref} - SOE_{max}, SOE_{ref} - SOE_{min}] \\ \mathcal{U} = [0, P_{ice}^{max}] \\ \mathcal{W} = \{P_{req} : P_{req} \in \mathcal{L}_2\} \end{cases} \tag{33}$$

Then consider the control system

$$\begin{cases} \dot{\zeta} = -kP_{ice} + kP_{req}, \quad \zeta(0) = \zeta_0 \\ z = \zeta \end{cases} \tag{34}$$

where $\zeta = 0$ is an equilibrium point and z is the performance variable, and the functional cost defined as:

$$J_\infty(\zeta_0, P_{ice}(\cdot)) = \int_0^\infty \frac{p_0(\omega_{ice}) + p_1(\omega_{ice}) \cdot P_{ice}(t)}{Q_{LHV}} g(t)\, dt \tag{35}$$

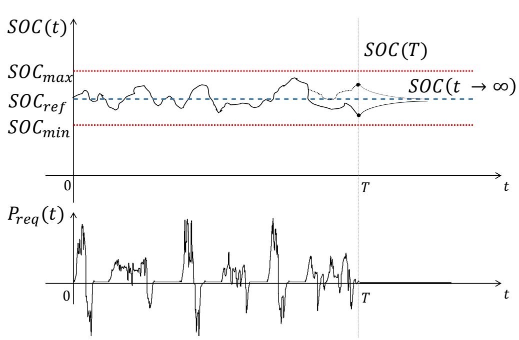

Fig. 5: Infinite time optimization and stability in HEV energy management problem.

Problem 2: The infinite-time optimal problem consists in minimizing the cost function (35) under system dynamics (34), with state and control variables defined in the sets $\mathcal{Z}$ and $\mathcal{U}$ respectively, and $P_{req} \in \mathcal{W}$.

Definition 4: Consider the Problem 2 with $P_{req} \equiv 0$ and $\phi(\zeta(t))$ an optimal solution for the problem. Then the origin $\zeta(t) = 0$ of the closed-loop system under $\phi(\zeta(t))$ is asymptotically stable if $\zeta(t) \to 0$ for $t \to \infty$.

When solving Problem 2 in presence of P_{req} given by (32), a typical SOC behavior is shown in Fig. 5. It can be noticed that the global constraint (9), given in the standard control problem, requiring $SOC(T)$ to be equal to the reference value SOC_{ref} is not met. In other words, different SOC values can be taken $t = T$, but the convergence to SOC_{ref} is guaranteed only as $t \to \infty$ (see Fig. 5).

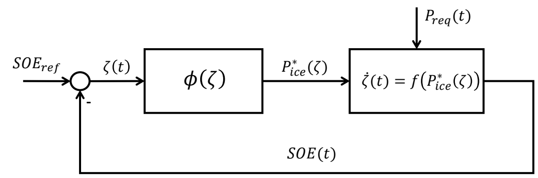

Fig. 6: Closed-loop energy management scheme.

V. Non linear optimization control strategy (NL-OCS) design

With respect to the system (34) the Hamiltonian function $\mathcal{H}$ (26) takes on the following form:

$$\mathcal{H}(\zeta, P_{ice}, \lambda) = \dot{m}_f(P_{ice}) + \lambda \cdot (kP_{ice}) + \Gamma(\zeta, P_{ice}) \tag{36}$$

where $\dot{m}_f$ is the instantaneous cost function and $\Gamma(\zeta, P_{ice})$ is a positive definite function of ζ and the control variable, and λ is the co-state variable. In order to have the Hamiltonian function zero at the minimum value, a shifting of the $\mathcal{H}$ is operated as follows:

$$\bar{\mathcal{H}}(\zeta, P_{ice}, \lambda) = \mathcal{H}(\zeta, P_{ice}, \lambda) - p_0 \tag{37}$$

Theorem 2: Consider the system (34) with functional cost (35). Then, the feedback control law $P_{ice}^*(\zeta)$ defined as:

$$P_{ice}^* = \phi(\zeta) = \begin{cases} \frac{2k^2\left(\mu^4\zeta^3\right)^2}{\left(k\mu^4\zeta^3 - p_1(\omega_{ice})g(t)\right)\gamma^2} & \zeta > \bar{\zeta}_1 \\ \zeta^2 & 0 < \zeta \le \bar{\zeta}_1 \end{cases} \tag{38}$$

with $\bar{\zeta}_1 = \left(\frac{p_1}{k\mu^4}\right)^{\frac{1}{3}}$, is such that:

1. the solution $\zeta(t) = 0$, $t \ge 0$ of the closed-loop system is locally asymptotically stable in accordance to Definition 4.
2. the adjoint performance functional $\mathcal{J}(\zeta_0, P_{ice}(\zeta))$ (28) is minimized.

The proof of Theorem 2 follows the steps of the proof of Theorem 1 (Chapter 10 of [19]). The sufficient conditions from Theorem 1 are satisfied with the proposed optimal controller (38), leading the conditions 1. and 2. in Theorem 2 to hold true. Hence, the origin $\zeta = 0$ of the closed-loop system is optimally locally asymptotically stable when $P_{req} = 0$. Moreover, P_{ice}^* is optimal with respect to the adjoint functional $\mathcal{J}(\zeta_0, P_{ice}(\cdot))$, which represents an upper bound for $J(\zeta_0, P_{ice}(\cdot))$. The optimal control law obtained in Theorem 2 is implemented via the closed-loop system shown in Fig. 6.

VI. Simulation results

In this section simulation results obtained with the proposed control law NL-OCS (Eq. 38) are presented and compared to the optimal global solution obtained from PMP (which is a proxi for the optimal global solution). The characteristics of the vehicle used in this study are shown in Table I. The only parameter the

TABLE I: Vehicle characteristics

Vehicle mass	19878 kg
Engine power	194 kW
Motor power	200 kW
Battery energy capacity	7.5 kWh
Electrical accessory	7 kW
Mechanical accessory	4 kW

analytical control law (Eq. 38) depends on is μ. In particular, μ has an impact on the difference between $SOC(T)$, and the reference value SOC_{ref}, therefore on the amount of fuel consumed. The calibration of μ is carried out to obtain minimum fuel consumed while ensuring charge sustainability ($SOC(T) - SOC_{ref} = 0$). In this section simulation results for a concatenation of four different driving cycles are shown. Table II summarizes the overall fuel consumption obtained with the new porposed design versus the PMP. The NL-OCS law consumes 2.3% more fuel when compared to PMP, but shows a better capacity in keeping the $SOC(t)$ close to the reference SOC_{ref}, both urban and highway examples of the driving cycle. For the simulation scenario used, the overall execution time needed to run the NL-OCS is almost 6 times lower than the time used by the PMP solution. Another advantage of the proposed strategy is in the rather low sensitivity of optimality of the control parameter μ with respect to driving missions (when compared to the sensitivity of the optimal co-state in PMP).

TABLE II: Efficiency and fuel consumption comparison between the PMP and NL-OCS solutions in the simulated scenario

Strategy	Fuel consumption m_f [kg]	ICE efficiency
PMP	12.93	0.319
NL-OCS	13.24 (+2.3%)	0.310 (-3.1%)

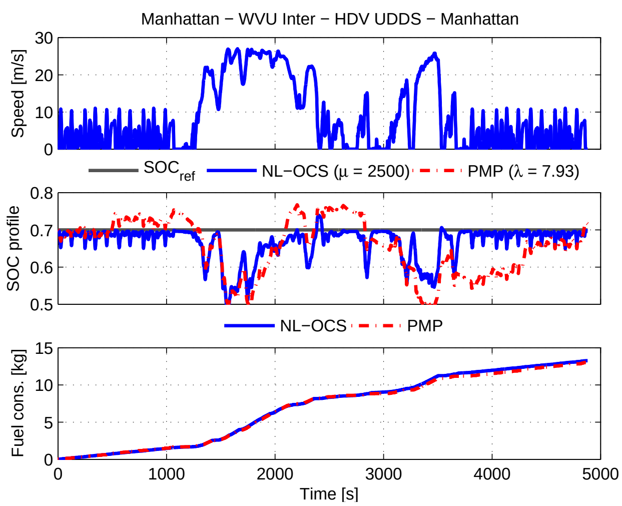

Fig. 7: Comparison of fuel consumption and SOC profile between PMP and NL-OCS.

VII. Conclusions

The novelty of this paper consists in a re-thinking of the energy management problem in charge-sustaining HEVs as a nonlinear-nonquadratic infinite-time optimization problem. In doing that, a new stability framework for HEVs has also been proposed. It has been shown that the new framework allows to design a family of state-feedback based control laws that provide optimality with respect to an infinite time horizon performance functional while guarantee asymptotic stability. Simulations results e have shown that while maintaining the performances near to the optimal (within 2%) a reduction of computational effort (about 6 times lower) is obtained. Moreover, a law sensitivity of the control parameter with respect to optimality is obtained which makes the newly proposed strategy a great candidate for on-board implementation.

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