

A prognostic methodology for interconnected systems: preliminary results

Simona Onori^{*,#}, Giorgio Rizzoni^{**}, Andrea Cordoba-Arenas^{**}

^{*}Center for Automotive Research, The Ohio State University, Columbus, OH USA

^{**} Center for Automotive Research and Dept of Mechanical and Aerospace Engineering,
The Ohio State University, Columbus, OH USA

[#] corresponding author (e-mail: onori.1@osu.edu)

Abstract: This paper introduces a systematic methodology for life cycle management of complex engineering systems, based on the interaction between dynamic system models and dynamic models of damage propagation. The approach is to model and predict the degradation propagation among subsystems that are highly interconnected and tightly integrated within a system, to predict overall system remaining useful life (RUL) based on knowledge of subsystem RUL. The outcome is a new approach to health management in complex systems that establishes a single model-based framework for the prognosis, and life cycle management of a complex system, based on dynamic system models.

Keywords: Prognostic, interconnected systems, aging, battery.

1. INTRODUCTION

Prognosis and Health Management (PHM) aims to understand the State Of Health (SOH) and Remaining Useful Life (RUL) of a system, helping to make informed and timely life cycle management decisions, reducing warranty and maintenance costs while improving serviceability, availability and safety. Life cycle management decisions include: 1) life-extending management; 2) scheduling maintenance, including minimizing unscheduled maintenance, extending maintaining cycles, and maintaining effectiveness through timely repair actions; and 3) end-of-life decision-making to help determine whether product life can be extended, whether any component can be reused, and what subsystems should be disposed of to minimize system costs.

The present paper proposes a methodology for life cycle management of complex engineered products. Real world systems are inherently subject to faults and aging. These faults must be detected and isolated at an early stage, and damage evolution tracked for safety, availability and serviceability. Fault detection recognizes abnormal behaviour of components and processes through information contained in measured signals. Model-based prognostics and health monitoring determines the system or process SOH and estimates its RUL by tracking damage variables through damage evolution models. The field of model based diagnosis, or Fault Detection and Identification (FDI), has seen significant progress with respect to model-based algorithmic approaches to residual generation, and can be considered relatively mature (Gertler, 1998; Isermann, 2006; Patton, 1989, 2000). However, the problem of incorporating model based prognostics and state of health monitoring functions into system fault diagnosis and reconfiguration strategies to provide life-cycle supervision of complex systems and processes remains to date an open research problem. Unlike FDI, the field of model-based PHM is a relatively new area of study (Orchard, 2009; Goebel, 2008; Vachtsevanos, 2006). The structural engineering community provides a number of developments of interest in this paper, motivated by the fact

that failures due to structural fatigue can lead to catastrophic consequences (e.g., in aeronautical and marine applications), (Chelidze, 2004; Kozlowski, 2001; Todinov, 2001). In general, monitoring SOH has reached a more advanced stage of development than prognosis, mainly because SOH monitoring assesses the condition of a system or process in the present time and does not involve the challenge of predicting its behaviour in the future. Figure 1 depicts a high-level view of the relationship between fault diagnosis and prognosis. Fault diagnosis relies on system models that may be static, but are often dynamic, and typically results in a binary decision. One of the outcomes of a well-implemented fault diagnosis strategy is the opportunity to reconfigure the control strategy.

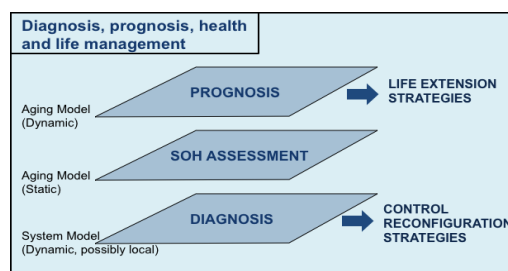


Figure 1. Relationship between diagnosis, prognosis, health management and life cycle supervision.

SOH assessment, on the other hand, represents a continuous range of conditions. Often, a static model is used to correlate measured variables to an aging variable, to obtain an estimate of SOH. If a dynamic aging model is available, then prognosis becomes possible, in that a dynamic model permits estimation into the future. The ability to conduct model-based prognosis makes it possible to incorporate life extension strategies in the supervisory controller.

Key challenges in developing model-based prognostics algorithms are:

- 1) It is necessary to identify and experimentally validate damage variables – not always an easy and generalizable process, as it usually involves very lengthy experiments

- under controlled conditions, which do not necessarily reflect actual aging in real life.
- 2) Once a damage variable is identified, there remains the challenge of reliably extracting features or estimating parameters from experimental data that closely correlate with the damage variable; and
 - 3) Damage evolution is invariably a nonlinear phenomenon, making the modelling of it more difficult, and is also dependent on initial conditions (e.g. structural or material defect distribution) (Chelidze et al. 2004).

The aim of this paper is to propose a systematic methodology to overcome these challenges and to solve the problem of incorporating PHM functions into FDI strategies. The problem is approached in a systematic and integrated fashion so as to grow the field of system/process supervision from a collection of ad hoc PHM approaches to a design methodology comparable to the advances made in control system theory in the past several decades. To this end, the foundation of this work is to study the behaviour of systems that can be modelled using the well-known *singular perturbation* model paradigm. This approach can be generalized to a broad class of systems, and can lend itself to a general approach to the problem of SOH and RUL estimation.

A second research topic that is addressed in this research is the interconnection of components and subsystems into complex systems. Such complex interconnections present serious challenges in extrapolating the aging behavior of individual components and subsystems to the entire system, as the relationship between the SOH of an individual component or subsystem and that of the overall system is not necessarily obvious – in other words, the composite effect of degradation in individual components and subsystems that may result in an acceptable SOH when taken one at a time, could be an unacceptable SOH of the system. Intuitively, it should be apparent that it is possible for each component of a system to have a certain residual useful life left, and to have a significantly lower life expectation of the overall system, as the combined effect of individual component aging on the overall system is likely to be nonlinear. Today, there is little or no literature that addresses the relationship between subsystem- or component-level prognosis and system-level prognosis (Abbas et al. 2009; Saxena et al. 2008). In this paper we aim to develop a formal approach to understanding the propagation of degradation in a complex system.

2. MODEL-BASED PROGNOSIS FRAMEWORK

The model-based prognostics methodology for interconnected systems developed in this paper builds upon the model-based prognostic framework of single systems of (Serrao et al. 2009) and depicted which has the following characteristics. It is designed to be implemented in real time and without any change in the system operation, i.e. only with the sensors available in the system (no intrusive measurements are needed). It uses a dynamic system model in state space form. The model links the states expressed variables of the

"fast" dynamics with the states of the "slow" damage evolution.

The damage evolution is assumed to be very slow such that the coupled system ("fast" and "slow" dynamics) can be modelled according to singular perturbation theory. Damage tracking is accomplished by a multi-time-scale approach.

With reference to Figure 2, damage tracking is accomplished by a multi-time-scale modelling strategy based on measurements of fast-varying states. The "slow" dynamics, i.e. the damage evolution, uses the measurements of the system input and output, combined with a model of the damage evolution. The resulting overall multi-rate scheme is such that the fast dynamics are evaluated every t seconds, while the slow varying variables, i.e. the aging parameters, are computed every $N \times t$, where N represents the duration of one "aging period". The prediction of the remaining useful life depends on the current damage state, as well as the future operation of the system.

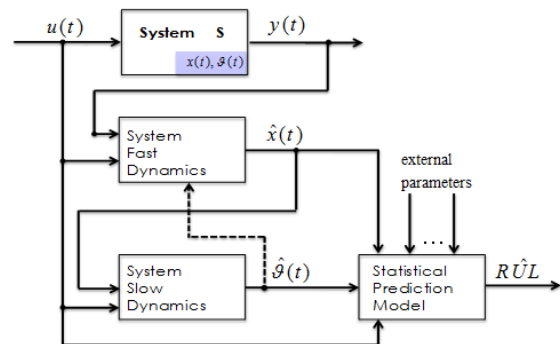


Figure 2. General scheme for model-based prognostics.

The general scheme can be divided in two stages:

- (a) Damage evolution tracking (and therefore, SOH estimation).
 - (b) Remaining useful time (RUL) estimation.
- In the following subsections the framework is described in detail.

2.1 Modeling dynamic systems subject to aging

A generic dynamic system subject to aging can be described by:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, \vartheta_1, u_1) \\ \dot{\vartheta}_1 &= \varepsilon \cdot g_1(x_1, \vartheta_1, u_1) \\ y_1 &= C_1 x_1 + D_1 u_1 \end{aligned} \quad (1)$$

where:

- $x_1 \in R^n$ is the set of state variables associated with the fast dynamic behavior of the system;
- $\vartheta_1 \in R^n$ is the set of damage variables, i.e. the system parameters that change with the age of the system;
- ε_1 is a positive scalar ($\varepsilon_1 \ll 1$) that represents the fact that the dynamics of the damage variables are much slower than the system dynamics
- $u_1 \in R^l$ are the external inputs acting on the system;

- $p_1 \in R^k$ are the aging factors which have an effect on the aging of the system. The vector p_1 can be composed of states, inputs and/or external parameters;
- $y_1 \in R^j$ are the system output vector.

Since the parameter ε is very small, Eq. (1) represents a standard singular perturbation model with slowly drifting parameters (Khalil, 2002). The dynamic variation of ϑ_1 is assumed to be much slower than the main system dynamics, so that the value of the parameters ϑ_1 can be considered constant when dealing with the system dynamic equation $\dot{x}_1 = f_1(x_1, \vartheta_1, u_1)$.

To facilitate the analysis of the system aging modelling, we define a mapping function ξ , named damage measure, which maps the domain of the system parameters or damage variables into the scalar domain of damage measure ranging in the interval $[0, 1]$. For scalar damage variable, the normalized damage measure can be used to express the progression of the aging process (Serrao et al. 2009) as:

$$\xi_1 = \frac{\vartheta_0 - \vartheta_1}{\vartheta_0 - \vartheta_f} \quad (2)$$

where, ϑ_0 is the damage variable value of the system when no aging had taken place and ϑ_f is the damage variable value of the system at the end of life.

The aging equation $\dot{\vartheta}_1 = \varepsilon \cdot g_1(x_1, \vartheta_1, u_1)$ that appears in (1) can be written in terms of ξ and the number of cycles n rather than ϑ and time, with a simple rescaling of the variables:

$$\frac{d\vartheta_1}{dt} = \frac{1}{t_c} \frac{d\vartheta_1}{dn}; \quad (3)$$

where, t_c is the duration of each cycle (hence $dn = dt/t_c$) and hence, using (2), $d\vartheta_1 \rightarrow d\xi_1$. Thus, the aging model (1) can be represented in terms of damage measure as:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, \vartheta_1, u_1) \\ \frac{d\xi_1}{dn} &= \eta \cdot \phi_1(\xi_1, p_1) \end{aligned} \quad (4)$$

$$y_1 = C_1 x_1 + D_1 u_1$$

where η is a positive scalar much smaller than 1. Much of the relevant work in the field of prognostics comes from the structural engineering community, where failure due to structural fatigue can lead to catastrophic consequences (e.g., in aeronautical applications). In the study of mechanical fatigue the most common approach to modelling the aging of a mechanical component is the use of the Palmgren-Miner rule, illustrated in Figure 3. The rule states that the life of a component under a sequence of variable loads is reduced each time by a finite fraction. This reduction corresponds to the ratio of the number of cycles spent under the given load condition and the number of cycles that the component would last if subjected to that same load condition for its entire life.

In other words, if n_i is the number of cycles spent under the load condition p_i and N_i is the number of cycles that the new component would last if it were cycled under condition p_i until failure, the end-of-life due to a sequence of variable loads $p_i, i = 1, \dots, W$, corresponds to the condition:

$$\frac{\Delta n_1}{N_1} + \frac{\Delta n_2}{N_2} + \dots + \frac{\Delta n_w}{N_w} = 1 \quad (5)$$

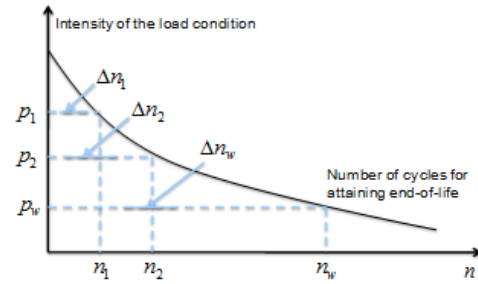


Figure 3. Exposure for fractions of life Δn_i at different intensity levels p_i of the loading conditions (Todinov, 2001).

that is, the end-of-life is reached when the cumulative of the fractions of life reduction reaches the unit value. The total life is a function of the loading conditions, and is obtained from experimental data for a wide variety of loads, components, and materials. Experimental results collected over the course of the years for basic mechanical components are readily available as tabulated data. This empirical rule has found wide usage in mechanical fatigue analysis because it is followed with fairly good approximation in many cases of practical relevance.

This rule implies that the order in which the loads are applied is not important, but that their effect is accumulated and that the progression of damage is linear. An important result tying the Palmgren-Miner rule to the damage accumulation was formulated by Todinov (Todinov, 2001). He proved that the additive law coming from Palmgren-Miner rule is equivalent to the equation $\frac{d\xi_1}{dn} = \eta \cdot \phi_1(\xi_1, p_1)$ that appears in (4), if and only if the damage evolution rate $\frac{d\xi_1}{dn}$ under a constant load p_1 can be factored as a product of a function $\phi_1(\xi_1)$ of the current amount of damage and a function $\sigma_1(p_1)$ of the load p_1 i.e.:

$$\frac{d\xi_1}{dn} = \eta \cdot \phi_1(\xi_1) \sigma_1(p_1) \quad (6)$$

where, $\eta \cdot \phi_1(\xi_1) \sigma_1(p_1)$ is a non-negative function. Equation (6) allows to track the progression of aging if the functions $\phi_1(\xi_1)$ and $\sigma_1(p_1)$ are known. The two functions are defined as the age factor function and the severity factor function, respectively (Onori et al. 2012). Damage growth is directly influenced by aging factors or stresses. In general, aging factors are a subset of the control inputs, $u_1(t)$, and internal states, $x_1(t)$. In the system aging modeling framework, we defined the vector of aging factors as $p_1 = [x_1, u_1]$. Hence, the severity factor function in Eq. (6) is a nonlinear function of elements of the vector p_1 , e.g., $\sigma_1(p_1) = \sigma_1(x_1, u_1)$.

The aging model just described applies to a non interconnected system. Next, we present a general formulation of the model-based prognostic approach for the case of interconnected systems.

3. AGING MODELING IN INTERCONNECTED SYSTEMS: A NOVEL FRAMEWORK

A complex system is composed of several subsystems that interact with each other dynamically. These constituent subsystems and their interactions make up the whole system. When a fault condition or aging event arises in one of the subsystems, not only does the subsystems behaviour change, but the interaction of that subsystem with the other subsystems may also change. The aim of this work is to formally understand the propagation of degradation in complex systems and to develop a unified framework for complex system prognosis. As an example, let us consider two isolated systems S_1 and S_2 subject to aging. The set of dynamic equations describing system evolution and aging dynamics are given in Fig. 4.

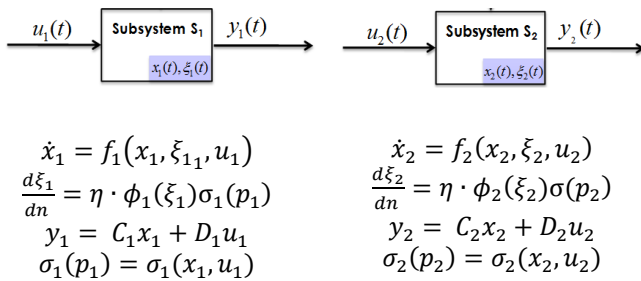


Figure 4. Two isolated systems S_1 and S_2 and their set of dynamic equations

The two systems S_1 and S_2 can be connected either in series or in parallel. Let us now consider the case in which a series interconnections is formed between S_1 and S_2 , as shown in Fig. 5. In this configuration, the input to system S_2 is the output of system S_1 , i.e. $y_1(t) = u_2(t)$. If system S_1 is subject to aging, then we can define the *aging residual* as the difference between the output of S_1 with no aging, $y_1(t)$ and the output of S_1 with aging, $y_{1,a}(t)$, i.e.:

$$\Delta y_1(t) = y_1(t) - y_{1,a}(t) \quad (7)$$

Moreover, the aging residual (7) can be expressed as a function of the damage variable ξ_1 of system S_1 when no other faults are acting on the system (sensor or actuator faults), as follows:

$$\Delta y_1(t) = y_1(t) - y_{1,a}(t) = C_1 (x_1(t) - x_{1,a}(t)) = \Psi(\xi_1) \quad (8)$$

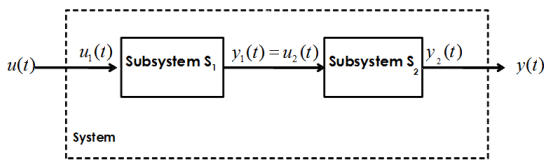


Figure 5. Series interconnection of subsystems S_1 and S_2 .

In Eq. 8 the aging residual is a function of the damage variable ξ_1 only. Note that if system S_1 is not subject to aging then $\Delta y_1(t) = 0$. When S_1 and S_2 are interconnected and S_1 is subject to aging the damage in S_1 propagates to S_2 through the interconnection:

$$u_2(t) = y_{1,a}(t) = \Delta y_1(t) + y_1(t) = \Psi(\xi_1) + y_1(t) \quad (9)$$

Because of the interconnection, S_2 is “affected by the damage” occurring in S_1 . Thus, the severity factor function, $\sigma_2(p_2)$, becomes:

$$\sigma_2(p_2) = \sigma_2(x_2, \Psi(\xi_1) + y_1(t))$$

Finally, the downstream system is:

$$\begin{aligned} \dot{x}_2 &= f_1(x_2, \xi_2, \Psi(\xi_1) + y_1(t)) \\ \frac{d\xi_2}{dn} &= \eta \cdot \phi_2(\xi_2) \sigma(x_2, \Psi(\xi_1) + y_1(t)) \\ y_2 &= C_2 x_2 + D_2 (\Psi(\xi_1) + y_1(t)) \\ \sigma_2(p_2) &= \sigma_2(x_2, u_2) \end{aligned} \quad (10)$$

Equation (10) shows that any aging that takes place in system S_1 will clearly affect the state dynamics, the aging dynamics, and the output of system of S_2 in this cascade interconnection.

The damage in the downstream system S_2 is not only a function of the actual damage of system S_2 itself and of the external operating factors, but also of the actual damage of the upstream system S_1 . Moreover, the aging residual for the downstream system S_2 , defined as $\Delta y_2(t) = y_2(t) - y_{2,a}(t)$, is in general a nonlinear function of the damage variables of the two subsystems S_1, S_2 , i.e. $\Delta y_2(t) = \Psi_2(\xi_1, \xi_2)$, indicating that the aging of the upstream system affects the aging of the downstream system. Hence, the prognosis and SOH assessment of the downstream system cannot be carried out as if system S_2 were isolated: the interconnection between the two subsystems needs to be considered.

Depending on the signal directionality, the role of the upstream and downstream system can be reversed (for example, a battery can be discharged to provide motive power, or can be recharged to recover kinetic energy during vehicle braking). This means that, in general, the rate of progression of the aging of a subsystem depends on the actual aging of the subsystem itself, on the external operating factor, and on the actual aging of the subsystems to which it is interconnected.

This fact poses some serious challenges to the study of prognosis and SOH estimation of complex systems, which cannot be addressed using or relying on modularity based principles. This example, however simple, generates a number of research questions:

1. How does the system topology affect the functional relationship among aging variables in interconnected subsystems? We will consider cascade, parallel, and feedback configurations, and combinations of these.
2. How does one analytically and quantitatively assess the effect of the damage propagation among subsystems? For example, we may be interested in determining whether damage propagation coupling between subsystems is negligible or must be explicitly considered.
3. If damage propagation among subsystems is not negligible, is it possible to control one or more system inputs or operating conditions such that this coupling can be attenuated? One can think of this as a form of damage mitigation or life extension. Alternatively, can one find

limits or bounds on states and inputs for which such propagation is attenuated?

4. CASE STUDY: BATTERY STRING

An especially rich example is the case of a battery pack module. A battery pack in a hybrid vehicle is a collection of modules, which are in turn made up of series/parallel combinations of individual cells. Each module is typically managed by a *Battery Management System* or BMS. Figure 6 depicts a sketch of two possible configurations for a battery module containing the same number of cells. The one on the left is termed “3P8S” because consists of 3 parallel strings of 8 cells in series; the one on the right is termed “8S3P” because consists of 8 elements in series, each consisting of 3 cells in parallel. Fig. 7 shows the state of charge (SOC) divergence that occurs when one of the cells in one of the strings of the 3P8S configuration is damaged and has reduced capacity and increased resistance.

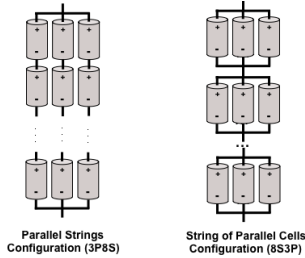


Figure 6. Battery module topologies (Yurkovich, 2010)

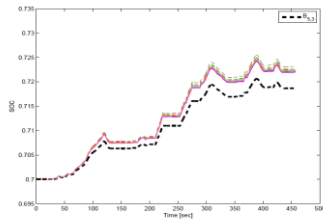


Figure 5.32: SoC plot of severely perturbed pack

Figure 7. SOC divergence of three cells (8S3P case) (Yurkovich, 2010)

In this preliminary work, we show the proposed methodology is shown on a simple case of battery string consisting of two cells in series, as drawn in Fig. 8. To evaluate and demonstrate the proposed methodology we use aging models derived from aging experiments on individual cells (Todeschini et al. 2012; Onori et al 2011). Each cell is modelled in terms of its fast and aging dynamics.

In particular, we consider the scenario that a given amount of power is requested to the battery string of Fig. 8.

Aging experiments conducted at OSU CAR on battery cells have shown that the resistance increases almost linearly as the battery ages (Spagnol et al. 2010). The slope of growth, though, depends on the given aging conditions, i.e. T , SOC , C_{rate} . For the purpose of demonstrating the mechanics of proposed approach we consider aging data of a Li-ion cell aged at 55 °C, from 0-30% SOC and at 2 C_{rate} (Todeschini et al. 2012).

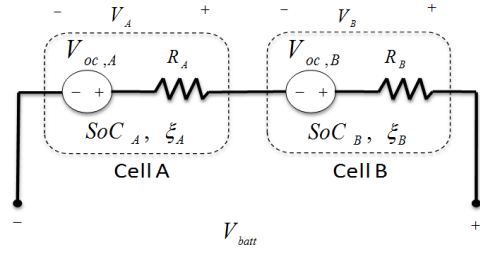


Figure 8. String of two cells in series.

The resistance growth of this cell is shown in Fig. 9. For both Cell A and Cell B the fast dynamics, slow dynamics and output equations are described below (the subscript x is used in the equations as to indicate that the same equations hold true for both Cell A and Cell B):

Fast Dynamics:

$$S\dot{O}C_x = -\frac{I}{Q_x} \quad (11)$$

Slow Dynamics:

The resistance growth can be modelled as:

$$R_x = R_{0,x} + m_x(SOC_x, T_x, C_{rate}) \cdot Ah \quad (12)$$

The damage measure, ξ_x , for the resistance is given as:

$$\xi_x = \frac{R_x - R_{0,x}}{R_f - R_{0,x}}$$

Hence,

$$R_x = \left(\frac{R_f - R_{0,x}}{\lambda_x} \right) \cdot \xi_x + R_{0,x} = \lambda_x \xi_x + R_{0,x} \quad (13)$$

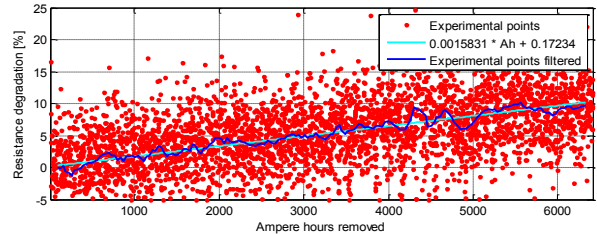


Fig. 9. Resistance growth of a Li-ion A123 ANR 26650 battery cell. The slope of growth depends on aging factors such as: SOC of operation, Temperature and C_{rate} .

Thus, the aging dynamics are:

$$\frac{d\xi_x}{dAh} = \frac{m_x(SOC_x, T_x, C_{rate})}{R_f - R_{0,x}} \quad (14)$$

Output Equation:

$$P_{string} = V_{batt} \cdot I \quad (15)$$

The voltage drop across each cell is:

$$V_A = V_{oc,A} - I_A R_A \quad (16)$$

$$V_B = V_{oc,B} - I_B R_B \quad (17)$$

At the interconnection, current outputted from Cell A is entering Cell B, i.e., $I_A = I_B = I$. Thus, substituting current from (16) into (17), results in:

$$\begin{aligned} V_B &= V_{oc,B} - IR_B = V_{oc,B} - \frac{R_B}{R_A}(V_{oc,A} - V_A) \\ &= V_{oc} - \frac{R_B}{R_A}(V_{oc} - V_A) \end{aligned} \quad (18)$$

Equation 18, obtained under the reasonable assumption that the two cells have the same open circuit voltage, shows that, if the two cell resistances were perfectly equal, i.e. $\frac{R_B}{R_A} = 1$, then $V_B = V_A$. In general, Eq. 18 indicates that the changes in the electrical characteristics of Cell A will have an effect on the output voltage of Cell B. In particular, substituting the resistance aging model of Eq. 13 into Eq. 18, we observe that the output voltage Cell B, in series with Cell A, is indeed a function of the aging of Cell A. In fact,

$$V_B = V_{oc} - \frac{\lambda_B \xi_B + R_{0,B}}{\lambda_A \xi_A + R_{0,A}}(V_{oc} - V_A) \quad (19)$$

If we assume equal initial resistance for each cell, $R_{0,A} = R_{0,B} = R_0$, and $\lambda_A = \lambda_B = \lambda$, then Eq. 19 reduces to:

$$V_B = V_{oc} - \frac{\lambda \xi_B + R_0}{\lambda \xi_A + R_0}(V_{oc} - V_A)$$

If the cell upstream in the interconnection is aging faster than the cell downstream, then $\frac{\lambda \xi_B + R_0}{\lambda \xi_A + R_0} < 1$ and V_B is no longer equal to V_A , as it should be if the two cell were aging at the same rate. As a consequence of the faster aging of the upstream cell, higher current will flow into the circuit to satisfy the given power request. Higher current in the circuit will affect the aging dynamics of the cell downstream as well.

5. CONCLUSIONS

In this paper, a new approach to model-based prognosis and life cycle management of complex engineering systems has been introduced. In particular, a mathematical framework to deal with the modelling of degradation propagation among interconnected subsystems, to predict overall system remaining useful life (RUL) is proposed. Using the well-known singular perturbation model paradigm and in the case of series interconnected subsystems, we have shown that the rate of damage progression of a subsystem depends not only on the actual aging of the subsystem itself and the external operating factors, but also on the aging of the interconnected subsystems. New challenges in this arena have been pointed out which will be investigated by the authors in future studies.

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