

An Optimal Regulation Strategy for Energy Management of Hybrid Electric Vehicles

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Abstract—The issue of designing an analytical optimal solution to the problem of energy management for charge-sustaining hybrid electric vehicles is addressed. In particular, it is shown that, by suitably casting the energy management problem into a nonlinear optimal regulation problem and using an appropriate control Lyapunov function candidate, it can be proved that the state-feedback based optimal control law (with respect to minimum fuel consumption) produces a charge-sustaining behavior. We provide sufficient conditions for state feedback based control law to guarantee asymptotic stability and optimality with respect to an infinite horizon performance functional. The optimal control law is implemented in a series hybrid electric vehicle and the performance of the proposed energy management strategy is shown in simulation for a specific driving case.

I. INTRODUCTION

Hybrid electric vehicles (HEVs) combine two or more energy sources for their propulsion. Usually, one or more electric machines and an energy storage system are connected to a conventional internal combustion engine (ICE) [1]. The energy management problem in HEVs consists of finding the optimal power split between the ICE and the electric machines while minimizing a performance objective. The minimization can be performed with respect to several objectives like fuel consumption, emissions, battery aging, etc., or a combination of the above [2], [3]. A possible classification of the energy management strategies proposed in the literature over the past decade can be done based on the feasibility of implementation of the strategy in a real vehicle. In particular, two main categories can be identified. The first category involves the use of classical optimal control techniques to assure global optimality of the solution. Dynamic programming (DP) and Pontryagin's minimum principle (PMP) belong to this category. DP assumes *a-priori* knowledge of the driving cycle and solves the problem backwards in time, considering all the possible power split choices at each instant. This approach gives the global optimal solution and it has been used as a benchmark solution [4], [5]. PMP, on the other hand, formulates and minimizes a Hamiltonian function (a function of the instantaneous cost and the state constraint) at each instant to obtain the extremal solution [6], [7]. PMP gives only the necessary conditions

(not sufficient) that must be satisfied by the optimal solution (known as *extremal* solutions). In the second category we find strategies that can be implemented in the real-vehicle, which do not necessarily guarantee optimality. For example, equivalent consumption minimization strategy (ECMS), adaptive energy management strategies and rule based control strategies belong to this category. The basic idea of ECMS is to reformulate the global optimization problem into a local optimization problem. The method can give very good results, but the equivalence factors (tuning parameters) must be determined with optimization techniques, and are dependent on driving cycles [8], [9]. An initial version of adaptive equivalent consumption minimization strategy (A-ECMS) was proposed where the tuning parameters of ECMS were calculated by either predicting the driving cycle, [10], or using a look up table for the correlation between the factors and the driving cycle, [11]. A different version of A-ECMS as proposed in [12], [13], [14] adapts the tuning parameter using the correlation between equivalence factor and state of charge (SOC) of battery.

It is clear that if the strategies guarantee global optimality using classical optimal control techniques, and make use of future driving information they cannot be implemented in a real vehicle. The second category of strategies which can be implemented in a real vehicle, though can give performance close to optimal solution (by fine tuning), is clearly sub-optimal. For example, the optimality of ECMS depends on the optimal equivalence factor for a given driving cycle [15], [12]. Thus, different driving conditions will result in different optimal equivalence factors. Because some of strategies in the second category involve minimization at each instant, this requires significant computational power on board, which sometimes is not available.

Hence the need for optimal energy management strategies which can be easily implemented in a real vehicle still ensuring optimality is evident. The optimal control problem under study is the minimization of a non quadratic cost functional subject to nonlinear system dynamics. Inspired by the tutorial exposition of a simplified framework for optimal nonlinear regulation in feedback control problems involving non quadratic cost functionals shown in [16], we propose a new framework to cast and solve the energy management problem in HEVs in this paper.

This paper aims at finding an energy management strategy that can ensure optimality and stability and be easily implemented in a real vehicle. In particular, it is shown that, by suitably casting the energy management problem into a nonlinear optimal regulation problem and using an

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appropriate control Lyapunov function candidate, it can be proved that the state-feedback based optimal control law (with respect to minimum fuel consumption) produces a charge-sustaining behavior. This paper proposes and proves sufficient conditions on stability and optimality of the state feedback control law designed for a series medium duty truck hybrid electric vehicle.

The paper is organized as follows: Section II describes the energy management problem in a series HEV, the engine fuel consumption rate model and the battery model; Section III lists the mathematical preliminaries required to state and prove the main results of the paper found in Section IV where sufficient conditions for asymptotic stability of the origin and optimality of the control law are proved. In Section V the proposed control law is implemented and simulation results show the effectiveness of the solution proposed. Finally, conclusions and future work are discussed in Section VI.

II. ENERGY MANAGEMENT PROBLEM IN HEVS

The objective of the energy management strategy in a HEV is to find the optimal power split between the primary and secondary energy sources that minimizes a given objective function over an entire driving cycle. In this paper, we consider the problem of minimizing the total mass of fuel, m_f [g] during a driving mission, or equivalently, minimizing the following cost J :

$$J = \int_{t_0}^{t_f} \dot{m}_f(u(t)) dt \quad (1)$$

where \dot{m}_f [g/s] is the instantaneous fuel consumption rate, $u(t)$ is the control action, and $t_f - t_0$ is the optimization horizon.

The energy management problem, by its very nature, is a constrained optimal control problem, where the objective function (1) is minimized under a set of both local and global constraints on the state and control variables, as outlined in the following.

System Dynamics. The system dynamics are given in terms of state-of-charge (SOC) variation with respect to time according to:

$$S\dot{O}C(t) = -\alpha \frac{I(t)}{Q_{max}} \quad (2)$$

where α represents the Coulombic efficiency [1]; $I(t)$ [A] is the current flowing in (positive) and out (negative) of the battery and Q_{max} [Ah] is the maximum battery charge capacity. The battery is modeled through the zero-th order equivalent circuit mode, shown in Fig. 1, whose parameters are: the equivalent resistance, R_{eq} and the open circuit voltage, V_{oc} . These parameters depend on several factors, the most important being, the SOC and the temperature. Because in a charge-sustaining HEV, the battery is used only over a limited range of SOC (typically between 0.5-0.8 SOC), the model parameters do not vary significantly as a function of SOC, and they are considered constant in this work. Moreover, in this study we neglect the effect of the temperature on the battery parameters, leaving the

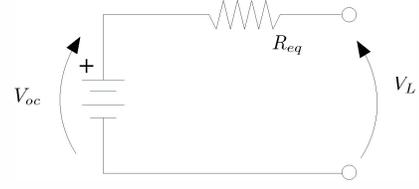


Fig. 1. Zero-th order electrical circuit model of the battery

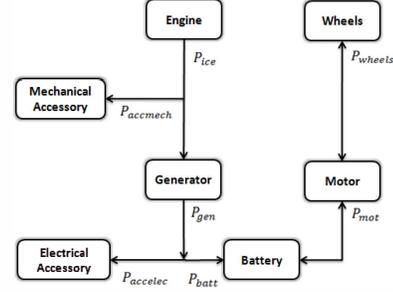


Fig. 2. Power flow diagram of series HEV

investigation in the case of temperature dependent parameters to future studies.

With reference the equivalent circuit model of Fig. 1, the SOC variation can be expressed as a function of battery power. In fact, the voltage at the battery pack terminals, $V_L(t)$, is given by:

$$V_L(t) = V_{oc} - I(t)R_{eq}, \quad (3)$$

Multiplying (3) by current $I(t)$ on both sides, the battery power, P_{batt} , is obtained as:

$$P_{batt}(t) = V_L(t)I(t) = V_{oc}I(t) - I^2(t)R_{eq}. \quad (4)$$

Solving the quadratic equation (4), the battery current $I(t)$ is obtained as:

$$I(t) = \frac{V_{oc} - \sqrt{(V_{oc})^2 - 4R_{eq}P_{batt}(t)}}{2R_{eq}}. \quad (5)$$

This result, substituted into (2) generates the nonlinear mapping:

$$S\dot{O}C(t) = -\alpha \frac{V_{oc} - \sqrt{(V_{oc})^2 - 4R_{eq}P_{batt}(t)}}{2R_{eq}Q_{max}}. \quad (6)$$

Global Constraints. In a charge sustaining HEV, the net energy from the battery is zero over a given driving mission, meaning that the SOC at the end of the driving cycle should be the same as that in the beginning of the driving cycle, i.e.,

$$SOC(t_f) = SOC(t_0). \quad (7)$$

where $SOC(t_0)$, $SOC(t_f)$ represent the battery SOC at the beginning and end of the driving cycle.

Local Constraints. Local constraints are imposed on the state and control variables. These constraints mostly concern physical operation limits, such as the maximum engine torque and speed, the motor power, or the battery SOC. For

a series HEV powertrain, shown in Fig. 2, local constraints are expressed as:

$$\begin{cases} P_{batt,min} \leq P_{batt}(t) \leq P_{batt,max}, \\ SOC_{min} \leq SOC(t) \leq SOC_{max} \\ T_{x,min} \leq T_x(t) \leq T_{x,max}, \\ \omega_{x,min} \leq \omega_x(t) \leq \omega_{x,max}, \quad x = ice, gen, mot. \end{cases} \quad (8)$$

where the last two inequalities in (8) represent limitations on the instantaneous engine, motor and generator torque and speed respectively; $(\cdot)_{min}, (\cdot)_{max}$ is the minimum and maximum value of power/SOC/torque/speed at each instant.

Moreover, drivability constraints are also enforced at each instant to ensure that the total power demand at the wheels is satisfied. In a series HEV, the engine and generator are connected in series with the battery pack and can be operated independent of the vehicle speed. The vehicle uses an all electric propulsion with an electric motor connected directly to the wheels. Because the electric motor (*mot*) propels the vehicle, its speed and torque are directly determined by the driver's torque request and the only degree of freedom is the torque of the generator (T_{gen}). The torque/power balance equations that must be satisfied are:

$$\begin{cases} T_{ice}(t) - T_{accmech}(t) = -T_{gen}(t), \\ T_{mot}(t) = T_{gb}(t), \\ P_{batt}(t) = P_{mot,e}(t) + P_{gen,e}(t) + P_{accelec}, \\ \omega_{mot}(t) = \omega_{gb}(t), \\ \omega_{ice}(t) = \omega_{gen}(t), \end{cases} \quad (9)$$

where T_{gb} is the instantaneous gearbox torque and ω_{gb} is the instantaneous gearbox speed; $T_{accmech}$ and $P_{accelec}$ represent the instantaneous mechanical accessory torque and electrical accessory power; and, $P_{mot,e}, P_{gen,e}$ represent the instantaneous electrical power at input/output terminals of the electric machines.

A. Engine Fuel Consumption Rate Model

The fuel consumption to be minimized over a driving cycle, is generally modeled as a map for every possible combination of engine speed and torque. The engine fuel consumption rate, \dot{m}_f , can be expressed as a closed-form expression of the engine power and speed using an appropriate Willan's line model [17], [15]. In general, for any energy conversion device, the efficiency of the device can be modeled by representing the input power as an affine function of the output power and losses. In the case of internal combustion engine, the input power ($P_{in} = P_{chem}$) is written as an affine function of the output power ($P_{out} = P_{ice}$). The slope a_1 and intercept a_0 of the equation are polynomial functions of engine speed, represented by:

$$\begin{cases} P_{in}(t) = a_0(\omega_{ice}(t)) + a_1(\omega_{ice}(t))P_{out}(t), \\ a_0(\omega_{ice}(t)) = a_{00} + a_{01} \cdot \omega_{ice}(t) + a_{02} \cdot \omega_{ice}^2(t), \\ a_1(\omega_{ice}(t)) = a_{10} + a_{11} \cdot \omega_{ice}(t) + a_{12} \cdot \omega_{ice}^2(t), \end{cases} \quad (10)$$

where $a_{ij}, i, j = 0, 1, 2$ are Willans line coefficients, $P_{in} = P_{chem} = \dot{m}_f \cdot Q_{LHV}$ is the chemical power input to the

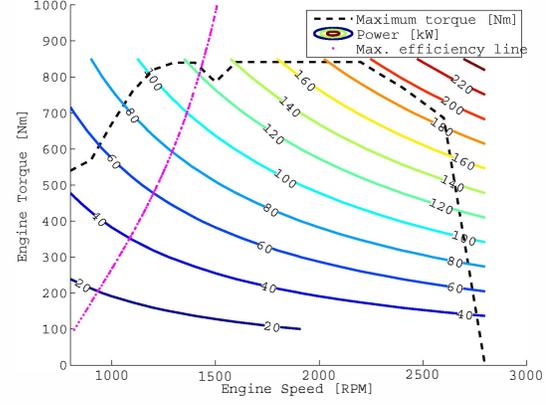


Fig. 3. Maximum efficiency operating line

engine and $P_{out} = P_{ice} = T_{ice} \cdot \omega_{ice}$ is the engine power output. Given the engine power P_{ice} and speed ω_{ice} , the fuel consumption rate can be written as

$$\dot{m}_f(t) = \frac{1}{Q_{LHV}} [a_0(\omega_{ice}(t)) + a_1(\omega_{ice}(t))P_{ice}(t)] \quad (11)$$

In a series HEV, the engine speed ω_{ice} is independent of the vehicle speed and can be chosen to operate the engine at the most efficient operating point given the engine power request. Thus, the optimal engine speed is known for any given P_{ice} can be calculated separately by minimizing the chemical power P_{chem} in the manner

$$\frac{\partial P_{chem}}{\partial \omega_{ice}} = 0 \Rightarrow \omega_{ice,opt} = -\frac{1}{2} \frac{a_{01} + a_{11}P_{ice}}{a_{02} + a_{12}P_{ice}}. \quad (12)$$

For a given power requested from the engine (P_{ice}), the maximum efficiency operating line, shown in Fig. 3, is decided using the optimal engine speed $\omega_{ice,opt}$. The fuel consumption rate (consumed by operating the engine in the most efficient speed for a given power) can now be expressed as an affine function of P_{ice} alone, as:

$$\dot{m}_f(t) = m_0 + m_1 P_{ice}(t) \quad (13)$$

where m_0 and m_1 are known constants obtained from (11) and (12). Moreover, since P_{ice} is a function of the control input P_{batt} ,

$$\begin{aligned} P_{ice}(t) &= P_{accmech} + \frac{1}{\eta_{gen}} [\eta_{mot} P_{gb}(t) + P_{accelec}] + \\ &\quad - \frac{1}{\eta_{gen}} P_{batt}(t) \end{aligned} \quad (14)$$

where η_{mot}, η_{gen} are efficiencies of the generator and electric motor and $P_{accmech}$ is the mechanical accessory power. Ultimately the fuel consumption rate $\dot{m}_f(t)$ is a direct function of the control input, P_{batt} , i.e.,

$$\dot{m}_f(t) = s_0 + s_1 P_{batt}(t) + s_2 P_{gb}(t), \quad (15)$$

through coefficients s_0, s_1, s_2 which are known constants,

obtained from (14) and (15) expressed as follows:

$$\begin{cases} s_0 = m_0 + m_1 \left(P_{accmech} + \frac{1}{\eta_{gen}} P_{accelec} \right), \\ s_1 = -\frac{m_1}{\eta_{gen}}, \\ s_2 = \frac{m_1}{\eta_{gen} \eta_{mot}}. \end{cases} \quad (16)$$

The engine fuel consumption rate model (15) and the battery SOC dynamics (6) described in this section will be used in the remainder of the paper to cast the energy management problem as a nonlinear optimal regulation problem. The closed-form expression for the fuel consumption rate using Willans line model will be used in finding an analytical optimal control law, which is a significant contribution in the HEV literature. The next section aims at presenting the mathematical background needed for the results proved in Section IV.

III. MATHEMATICAL PRELIMINARIES

We formulate the optimal control problem for energy management in HEVs as a nonlinear optimal regulation problem by studying the deviation of the battery SOC from a constant reference value SOC_{ref} . For the remainder of the paper, all the variables are implicitly assumed to be functions of time i.e., $x \Rightarrow x(t)$ and the constants are explicitly described. The battery SOC error and the battery SOC dynamics are defined as:

$$\begin{cases} e = SOC_{ref} - SOC, \\ \dot{e} = \alpha \frac{V_{oc} - \sqrt{(V_{oc})^2 - 4R_{eq} P_{batt}}}{2R_{eq} Q_{max}} = f_e(P_{batt}). \end{cases} \quad (17)$$

In this section, we introduce some mathematical preliminaries for the scalar system (17) with single control input, which are instrumental to the discussion presented in the next section.

Consider an open set $\mathcal{D} \subset \mathbb{R}$ such that $e \in \mathcal{D}$ an arbitrary set $\mathcal{U} \subset \mathbb{R}$ such that $P_{batt} \in \mathcal{U}$ and $0 \in \mathcal{D}, 0 \in \mathcal{U}$. In the HEV problem, the state domain and control domain can be defined as

$$\begin{cases} e \in \mathcal{D} = [SOC_{ref} - SOC_{max}, SOC_{ref} - SOC_{min}], \\ P_{batt} \in \mathcal{U} = [P_{batt,min}, P_{batt,max}] \end{cases}$$

Furthermore, let $f_e : \mathcal{U} \rightarrow \mathbb{R}$ satisfy $f_e(0) = 0$. Now consider the controlled system

$$\dot{e} = f_e(P_{batt}), \quad e(0) = e_0, \quad t \geq 0. \quad (18)$$

where the control input $P_{batt}(\cdot)$ is restricted to the class of *admissible* controls consisting of measurable functions $P_{batt}(\cdot)$ such that

$$P_{batt} \in \Omega, \quad t \geq 0, \quad (19)$$

where the control constraint set $\Omega \subset \mathcal{U}$ is compact and $0 \in \Omega$. Let an *optimal control law* P_{batt}^* be a measurable mapping $P_{batt}^* : \mathcal{D} \rightarrow \Omega$ satisfying $P_{batt}^*(0) = 0$. Now the system (17) with feedback control $P_{batt}(t) = P_{batt}^*(e(t))$, has the form

$$\dot{e} = f_e(P_{batt}^*(e)), \quad e(0) = e_0, \quad t \geq 0. \quad (20)$$

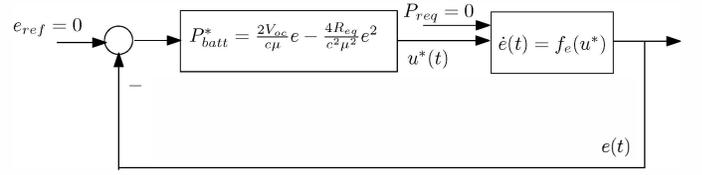


Fig. 4. Energy management strategy as a nonlinear optimal regulation problem

In order to address the problem of characterizing feedback controllers that minimize a performance functional, let $\mathcal{H} : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, $\dot{m}_f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ and $p \in \mathbb{R}$ such that,

$$\mathcal{H}(e, P_{batt}, P_{gb}, p) \triangleq \dot{m}_f(P_{batt}, P_{gb}) + p \cdot f_e(P_{batt}). \quad (21)$$

where $\mathcal{H}(\cdot, \cdot, \cdot, \cdot)$ is the Hamiltonian function, $\dot{m}_f(\cdot, \cdot)$ is the instantaneous cost function expressed in (15) and p is the co-state variable.

Because the Hamiltonian function has to take the minimum value when the optimal control input is applied, a new augmented Hamiltonian function $\bar{\mathcal{H}}$ is defined as:

$$\bar{\mathcal{H}}(e, P_{batt}, P_{gb}, p) \triangleq \mathcal{H}(e, P_{batt}, P_{gb}, p) - s_0. \quad (22)$$

where s_0 is the Willans line coefficient defined in (16).

IV. NONLINEAR OPTIMAL REGULATION OF HEV

In this section, we solve the energy management problem formulated in Section II according to a nonlinear optimal regulation framework (Fig. 4) which guarantees that the battery SOC error dynamics is regulated to zero. The result that follows gives sufficient conditions under which the origin $e = 0$ can be locally asymptotically stabilized under nonlinear state feedback control which also guarantee optimality with respect to the fuel consumption over an infinite time horizon. In this paper, sufficient conditions for stability and optimality are given in the case where no external inputs or disturbances enter the system (18). This scenario corresponds to $P_{gb} = 0, \forall t \geq 0$ in the engine fuel consumption rate model (15) and the system initial condition being different from zero, i.e. $e(0) = e_0 \neq 0$. In the context of charge-sustaining HEVs, the considered scenario corresponds to having the vehicle switched on without any tractive force at the wheels ($v(t) = 0$) with the actual battery SOC being at not at the reference value, i.e., $SOC_{ref} \neq SOC(0)$. What follows is the first of a series of original results obtained by the authors on stability and optimality in the context of energy management problem in HEVs, that builds upon the main results of [16].

Theorem 1: Consider the system (18) with performance functional

$$J(e_0, P_{batt}(\cdot)) \triangleq \int_0^\infty \dot{m}_f(P_{batt}) dt. \quad (23)$$

Then with the feedback control $P_{batt} = P_{batt}^*(e)$, where P_{batt}^* satisfies:

$$\begin{cases} P_{batt}^* = \frac{2V_{oc}}{c} e - \frac{4R_{eq}}{c^2} e^2, \\ c = \frac{2R_{eq} Q_{max} s_1}{\alpha}. \end{cases} \quad (24)$$

the solution $e(t) = 0$, $t \geq 0$ of the closed-loop system (20) is locally asymptotically stable and the optimal feedback control law $P_{batt}^*(e(\cdot))$ minimizes $J(e_0, P_{batt}(\cdot))$.

Proof: Considering the candidate Lyapunov function $V(e) = \frac{1}{2}e^2$, local asymptotic stability of the origin $e(t) = 0$ and optimality of P_{batt}^* with respect to $J(e_0, P_{batt}(\cdot))$ are proven by showing that conditions listed in [16] are met for the candidate Lyapunov function.

- 1) The Lyapunov function $V(e)$ has a minimum value of 0 at the origin

$$V(0) = 0; \quad (25)$$

- 2) The candidate Lyapunov function $V(e)$ is a positive definite function. In fact, $V(e)$ is a quadratic function of e

$$V(e) > 0 \quad \forall e \in \mathcal{D}, e \neq 0; \quad (26)$$

- 3) The optimal feedback control law is zero at the origin:

$$P_{batt}^*(0) = 0; \quad (27)$$

- 4) Asymptotic stability of the origin is achieved when the optimal control law is applied, i.e. $\dot{V}(P_{batt}^*) < 0$.

$$\begin{cases} \frac{\partial V}{\partial x} f_e(P_{batt}^*(e)) < 0 \quad \forall e \in \mathcal{D}, e \neq 0, \\ = e(t) f_e(P_{batt}^*(e)) < 0, \\ \Rightarrow \begin{cases} f_e(P_{batt}^*(e)) < 0 \Rightarrow P_{batt}^*(e) < 0 \quad \forall e > 0, \\ f_e(P_{batt}^*(e)) > 0 \Rightarrow P_{batt}^*(e) > 0 \quad \forall e < 0 \end{cases} \end{cases} \quad (28)$$

This analysis provides conditions on the sign of state feedback control law $P_{batt}^*(e)$ as:

$$\begin{cases} P_{batt}^*(e) < 0, \quad \forall e > 0, \\ P_{batt}^*(0) = 0, \\ P_{batt}^*(e) > 0, \quad \forall e < 0. \end{cases} \quad (29)$$

- 5) The Hamiltonian function $\bar{\mathcal{H}}$ takes on the minimum value of zero when the optimal control law ($P_{batt} = P_{batt}^*(e)$) is applied:

$$\begin{cases} \bar{\mathcal{H}}(e, P_{batt}^*(e), (\frac{\partial V}{\partial e})^T) = 0, \\ \Rightarrow \dot{m}_f(P_{batt}^*(e)) = -\frac{\partial V}{\partial e} f_e(P_{batt}^*(e)) \quad \forall e \in \mathcal{D}. \end{cases} \quad (30)$$

From (30), substituting the expression of fuel consumption (15), the optimal control law $P_{batt}^*(e)$ from nonlinear state feedback is:

$$\begin{cases} P_{batt}^* = \frac{2V_{e,u}}{c} e - \frac{4R_{e,g}}{c^2} e^2, \\ c = \frac{2R_{e,g} Q_{m,e,x} s_1}{\alpha}. \end{cases} \quad (31)$$

- 6) The Hamiltonian function $\bar{\mathcal{H}}$ takes on a value greater than zero when a control law (P_{batt}) other than the optimal control law (P_{batt}^*) is applied:

$$\begin{cases} \bar{\mathcal{H}}(e, P_{batt}, (\frac{\partial V}{\partial e})^T) \geq 0, \\ \Rightarrow \dot{m}_f(e, P_{batt}) \geq \frac{\partial V}{\partial e} f_e(P_{batt}) \quad \forall e \in \mathcal{D}, u \in \Omega. \end{cases} \quad (32)$$

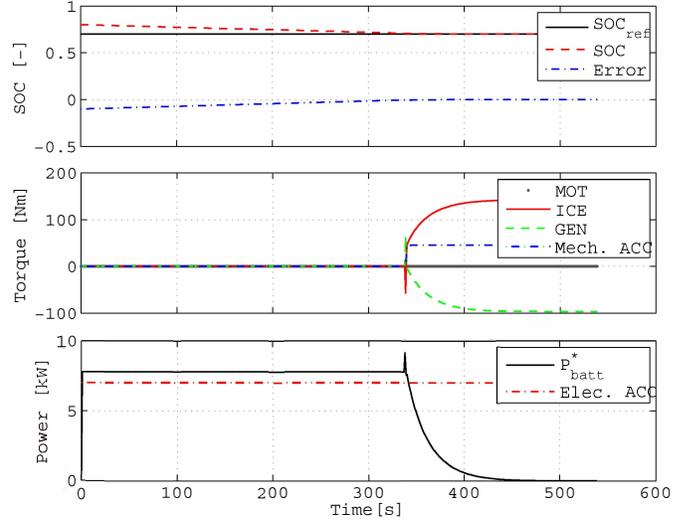


Fig. 5. Optimal battery SOC, SOC error, ICE, GEN, MOT torques and optimal battery power ($SOC_0 = 0.8$)

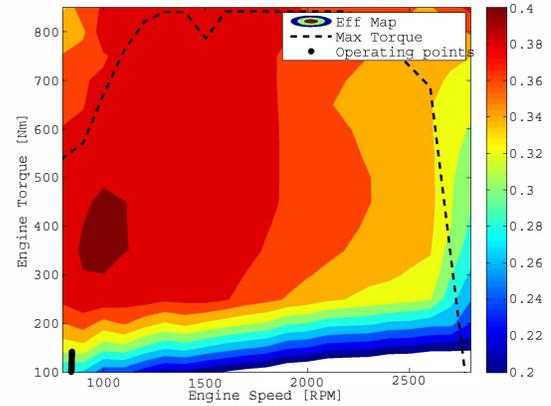


Fig. 6. Optimal engine operating points ($SOC_0 = 0.8$)

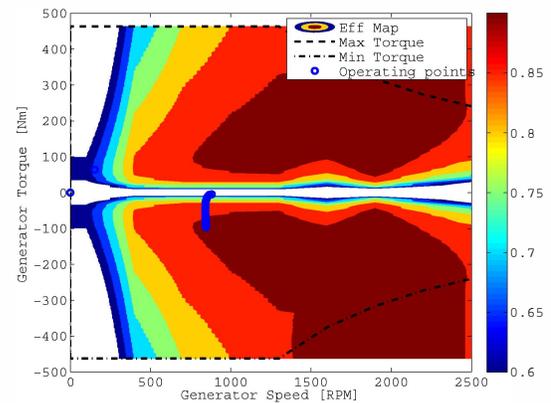


Fig. 7. Optimal generator operating points ($SOC_0 = 0.8$)

TABLE I
VEHICLE CHARACTERISTICS

Vehicle mass	18000 kg
Engine capacity	6.7 L Diesel
Engine power	194 kW
Motor power	200 kW
Generator power	45 kW
Battery energy capacity	7.5 kWh (27 MJ)
Electrical Accessory	7 kW
Mechanical Accessory	4 kW

V. SIMULATION RESULTS

The series heavy-duty HEV is simulated using a longitudinal and quasi-static forward vehicle simulator [18]. All the vehicle components are modeled using quasi-static map-based models and the characteristics of the vehicle modeled in this paper are shown in Table I. Because the mechanical accessory represents the accessory load due to the operation of the engine, it is supplied as torque using the engine throughout the simulation. All the remaining secondary accessory loads in the vehicle are lumped into the electrical accessory power supplied using electrical power.

The scenario investigated in this work consists in the vehicle being switched on without any tractive force at the wheels (the vehicle is at standstill, *i.e.* $v_{veh} = 0$). In this situation, the optimal control law (31) is implemented and shows that the origin of the *SOC* error dynamics, $e = 0$ is stabilized asymptotically while minimizing the fuel consumed by the engine over an infinite time horizon. As no tractive force is requested at the wheels, the system (Fig. 4) is excited by an initial value of battery *SOC* different from SOC_{ref} ($SOC_{ref} = 0.65$). When $SOC_0 > SOC_{ref}$, the control law depletes the battery initially to supply the accessory power and then uses the engine and generator to maintain the battery *SOC* at the reference value as shown in Fig. 5. After supplying the electrical accessory power directly from the battery, the engine and the generator are used together to maintain the battery *SOC* at SOC_{ref} . In the absence of external tractive force, the engine and generator are operated as shown in Figures 6 and 7 to supply the mechanical accessory torque and the electrical accessory power. Given the small power request from the accessories, the engine is operated at a low speed and low torque region as shown in Fig. 6 and the generator is operated in the negative torque region to generate electrical power to supply electrical accessory power (Fig. 7).

VI. CONCLUSION AND FUTURE WORK

This paper has aimed at casting the energy management problem in a charge sustaining series HEV as a nonlinear optimal regulation problem. The paper uses a zero *th* order model for the battery and an appropriate Willans line model of the engine fuel consumption rate to design the optimal control law based on the results from [16]. The solution proposed in this paper is the first of a series of results on proving asymptotic stability of the closed loop system using a state-feedback based optimal control law while

minimizing fuel consumption. The analytical control law has been implemented in a forward simulator and the results are shown for a specific scenario (vehicle at stand still). The main contribution of the work lies in the development of stability and optimality framework to design and analyze energy management strategies in the absence of external disturbances. The objective of the future work is to extend the stability and optimality framework developed here to include external disturbances (vehicle in traction mode).

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