Preprints of the 18th IFAC World Congress
Milano (Italy) August 28 - September 2, 2011

A New Modeling Approach to Predict 'Peukert Effect' for Lead Acid Batteries

Antonio Manenti*[,1], Simona Onori**
Yann Guezennec****

**Dipartimento di Elettronica e Informazione Politecnico di Milano,*
Milan, 20133 Italy (e-mail: manenti@elet.polimi.it)
***Ohio State University, Center for Automotive Research, OH 43212*
USA (e-mail:onori.1@osu.edu, corresponding author)
**** Ohio State University, Department of Mechanical Engineering, OH 43210*
USA (e-mail:guezennec.1@osu.edu)

Abstract: This paper presents a new modeling structure to capture the 'Peukert effect' in lead-acid (PbA) batteries. Despite the fact that new battery chemistries are available nowadays for use in electric and hybrid vehicles, PbA batteries still play an important role in automotive and stationary applications. In particular, this papers focuses on PbA batteries used in deep cycle applications such as military ground vehicles, all-electric vehicles and power backup systems for which a novel battery model is proposed in order to capture, and therefore predict, the apparent capacity reduction phenomenon ('Peukert effect') as a consequence of the current at which the battery is discharged. The novel battery model structure, based on an equivalent electrical circuit representation of the battery, is shown to reproduce this phenomenon under different operating conditions. The model is verified in simulations. An experimental test plan is then designed to calibrate and validate the model against real data.

1. INTRODUCTION

The development of battery models for automotive applications has been a research topic for many years. The complexity of designing such models comes from the intrinsic nature of these systems that involve both electrical and electrochemical aspects. A suitable battery model representative of the battery behavior under certain operating conditions, allows to predict the system output, which in turns is important when a model-based diagnosis/prognosis framework is to be built. This is motivated by safety and reliability issues.

Depending on the particular application, each model has to meet different requirements. Often, in vehicle applications, it is desirable to deal with low order models which do not require high computational effort. The goal of this type of models might be limited to track specific variables, for instance state of charge (*SoC*), terminal voltage, body temperature etc. On the other hand, in some other application fields, the accuracy of the results and the understanding of the behavior of the entire battery have the priority over simulation time and computational requirements.

These can be related to the battery itself (aging, sulfation, self-discharge, etc.), to the environment (temperature, humidity, etc.) or to the operating conditions (current load, resting time, etc.).

Over the years, the input-output behavior of batteries has been modelled mainly through electrochemical and equivalent electrical circuit models (Pascoe, 2003). The former are usually adopted by battery manufacturers and researchers to understand the influence on battery performance of the use of new materials, change in physical dimensions and tolerance on abuses (Rincon-Mora, 2006). Complexities of electrochemical models generally prohibit them from being used effectively in embedded systems where computational power is usually limited. The latter are more intuitive and easy to handle when it comes to deal with battery-powered systems, and they do not require a deep knowledge of cell chemistry principles. Equivalent circuit models represent a simplification of electrochemical models by using electrical circuit elements to describe the battery behavior. Being based on electrical components, such models can be handled by the majority of standard circuit simulators increasing their versatility and usefulness. These are more frequently adopted in on-vehicle applications where the battery type is well defined and a simple model is used by the battery management system (BMS) to prevent dangerous situations or, for example, predict the SoC and state of health (SoH) of battery pack (Codeca, 2008).

[1] Antonio Manenti has conducted this work while he was a visiting scholar at Center for Automotive Research at Ohio State University.

While not the most sophisticated model structure possible, the equivalent circuit model is selected in this paper as the model structure to represent the Peukert effect, because of its simplicity and universality.

Despite the fact that new battery chemistries are available nowadays for use in EVs, HEVs and PHEVs, PbA batteries still play an important role in automotive and stationary applications thanks to their low cost and their ability to supply high power. They are extensively used in deep cycle applications such as military ground vehicles, all-electric vehicles such as fork lift and tow motors, renewable energy storage and power backup systems as well as electric wheelchair and indoor passenger vehicles. Moreover, this chemistry is extensively adopted in all stationary applications, such as uninterruptible power systems (UPS) and telecommunication back-up systems and storage unit for solar panel applications (Kattakayam, 2004).

Although this battery chemistry has been used for more than a century, and still to be used, some interesting phenomena have not been completely understood and therefore correctly managed by most of the battery models. Among them, there is the so called 'Peukert effect'. This is a phenomenological effect which describes the apparent capacity reduction of a battery as the current discharge rate increases. Another phenomenon is the so-called capacity recovery effect.

The battery capacity represents the maximum amount of energy that can be extracted out of the battery and deep-cycle PbA battery applications are such that the battery is designed to be regularly discharged to most of its capacity. Thus, predicting the remaining battery capacity is an essential requirement for the successful of such applications. One of the main factors, though, which influence the battery capacity, is the discharge current.

This work presents a new PbA battery model structure based on equivalent electrical circuit able to rationally capture the aforementioned 'Peukert phenomenon' and capacity recovery effect under all operating conditions. The paper has the following structure: in Section 2, a review of models based on equivalent electrical circuit is given. Section 3 describes the 'Peukert effect' and Section 4 proposes a new model structure to suitable capture this apparent capacity reduction effect and the capacity recovery phenomenon. In Section 5, the evaluation of model parameters using data provided from battery manufacturer is provided. Finally, in Section 6 conclusions and guide lines for future work are given.

2. EQUIVALENT ELECTRICAL CIRCUIT FOR PbA BATTERIES

Since the appearance of the first PbA batteries, models have been developed to capture their behavior. Equivalent circuit models turn out to be a very flexible and intuitive way to describe the battery behavior even without a deeply knowledge of the battery chemistry (Tebbutt, 1999). Due to their simplicity, they result useful in model-based algorithms for SoC and SoH estimation (Plett, 2004; Kim, 2006) which, in turn, become attractive for their efficiency, robustness and low tuning requirements (Hu, 2011).

Although, very accurate equivalent circuit models exist, they typically require distributed or nonlinear elements such as transmission lines and Warburg impedances which make them almost impossible to be used in embedded applications. Hu (Hu, 2009) has shown that the battery response to currents profiles typical of automotive applications can be approximated by linear equivalent circuits containing only resistors and capacitors elements, but that requires the model to use scheduled parameters over the entire operating domain of *SoC* and temperature.

One of the problems to tackle when using an equivalent electrical circuit model is the identification and calibration of the parameters. Depending on the accuracy required to the model to mimic the battery behavior, each parameter can be function of many independent variables such as temperature, load/charge current, *SoC* and aging. In order to find out the relation between all these parameters, the first step consists of performing a suitable number of well defined experiments designed specifically to span the range operating conditions over which the model is intended to be valid. Then, the least squares (Bamieh, 2002) or subspace identification (Verdult, 2002) approaches can be used in case, for example, of linear parameter varying (LPV) discrete systems. Otherwise, for systems described by continuous time nonlinear differential equations, a procedure aimed at minimizing the error between the measured battery voltage and the model terminal voltage can be adopted using multi-parameter optimization algorithms.

PbA batteries present an interesting phenomenon often not explicitly modeled by battery models, namely the 'Peukert effect'. This effect manifests itself as an apparent reduction in battery capacity as the load current increases. This effect is very pronounced for PbA battery and can lead to very large reduction in apparent capacity (measured in Amp-hours). Battery manufacturers provide their batteries with a measure, i.e. "rated capacity", which is the capacity a battery is able to deliver under a constant current discharge at ambient temperature. The rated capacity for PbA batteries is usually specified at C/8, C/10 or C/20[2]. It is observed that the higher the rate the less the charge capacity a battery can deliver. This effect is heuristically captured by the so-called Peukert equation (described in Section 3 by Equation 1). Despite the practical importance of this effect and its implication for design and operation of battery systems under deep discharge scenarios, to the best of our knowledge, there are no results related to representation of the this phenomenological effect through electric circuit models. Although, the Peukert law (Equation 1) to de-rate the capacity according to the load applied, is mostly used. This simplification is based on the fact that, in many applications, the battery pack is mainly used under well defined (constant) load conditions in which the Peukert equation provides a fair approximation of the available capacity. This, in turn, means that the average load current is constant and around a certain range of values and

[2] A battery rated at 120Ah provides 120A for one hour if discharged at 1C. A discharge of 1C draws a current equal to the rated capacity. The same battery discharged at C/20 would discharge within (or no more than) 20 hours when a current discharge profile of 6A is used.

so the battery capacity can be assumed constant (calculated using the mean value). Unfortunately, this assumption does not hold when the current loads strongly vary over time. A different approach consists assuming the battery capacity as function of the load current. However, capacity values obtained using this method would show inconsistent results since the challenging problem to accurately relate the battery capacity to a variable current. In (Ceraolo, 2000), for example, a low pass filtering process is used to extract the "average current". Moreover, as described in the next section, the reduction of the battery capacity is not simply related to the magnitude of load current but also to the presence of resting times. In fact, besides the Peukert effect, there is the so-called recovery effect: during periods of no or very low discharge, the battery can recover the capacity "lost" during periods of high discharge to a certain extent. In this way the effective capacity is increased and the battery lifetime is lengthened. For all types of batteries these effects occur. However, the extent to which they are exhibited depends on the battery type. The novel model structure proposed in this paper is able to capture both the Peukert effect and the capacity recovery effect as discussed in Section 3 and 4 for PbA batteries.

3.PEUKERT EFFECT

Peukert effect, first introduced by the German scientist W. Peukert in 1897 (Peukert, 1897), expresses the capacity of a PbA battery in terms of the rate at which it is discharged. As the rate increases, the battery available capacity decreases. Usually, the discharge rate for a battery is not directly given in terms of discharge current in [A] but in terms of C-rate. A value of *C/N* indicates a current magnitude such that is able to completely discharge the battery in *N* hours. For instance, a battery with a rated capacity of C_R=120Ah can be discharged at a discharge rate of C/20 (I_R=6A) for nominally 20 hours.

Due to the Peukert effect, for example, discharging the same battery of rated capacity of 120Ah at C/10 (12A) and C/5 (24A) leads to an actual amount of charge that can be depleted of 110Ah and 95Ah, respectively (under the same temperature).

In the following, C_1 is referred to the capacity that the battery can provide when discharged with a constant current of 1A. Similarly, C_R is the capacity obtained discharging the battery at C-rate of *C/R*, finally, C_{av}, is the available capacity when the battery is discharged at a discharge current *I*.

The Peukert equation (Peukert, 1897) is given by:

$$I^n t_f = const = C_1 \text{ [Ah]} \tag{1}$$

where t_f is the time (in hours) the battery takes to go from fully charged to fully discharged and *n* is Peukert exponent, experimentally determined. Equation (1) indicates that, if the battery is discharged at constant current *I* for a period of duration t_f it can provide a capacity of C_1.

By very definition, the total available capacity of a battery discharged with a constant current *I* from time $t_0=0$ when fully charged at capacity C_{t0} to time $t = t_f$ until its capacity C_T is zero (the voltage cut-off is reached) is:

$$C_{t0} - C_T = C_{av} = \int_0^{t_f} I(t)dt \xrightarrow{I=const.} I * t_f \tag{2}$$

By solving Equation(1) with respect to t_f and using Equation (2) one can express the available capacity as a function of C_1, the discharge current *I* and the Peukert exponent *n*, as:

$$C_{av} = C_1 I^{(1-n)} \tag{3}$$

Since most battery manufacturers specify the nominal battery capacity as calculated at C/20 rather than the capacity available when discharging at 1A, Equation (1) is replaced with Equation (4) to obtain the battery time to run:

$$t_f = \frac{R\left(\frac{C_R}{R}\right)^n}{I^n} \tag{4}$$

If the manufacturer claims a rated capacity of C_R=120Ah (as measured at C/20, *R*=20) and the Peukert exponent for that particular battery is *n*=1.1, then by means of Equation (4) the battery will "last" 20h at *I*=6A and only 9.3h at *I*=12A.

The main disadvantage of Equation (1) and (4) is that, in correspondence of very low magnitude currents they predict infinite discharge time. To overcome this inconsistency, the following formula is often adopted (Jackey, 2007):

$$C_{av}(I) = \frac{K_c C_0}{1+(K_c-1)\left(\frac{I}{I^*}\right)^\delta} \tag{5}$$

In Equation(5), C_0 represents the available capacity obtained for a reference current I^* whereas the term $K_c C_o$ is the maximum capacity obtainable at very low (theoretically zero) current and δ is a tunable parameter with the type of battery.

None of the formulas studied so far in this section account for the temperature dependence. To address this issue, i.e. the dependence of the capacity on electrolyte temperature t_f (expressed in °C and supposed constant), Equation (6) can be used (Jackey, 2007):

$$C_{av}(I,\theta) = C_{av}(I)\left(1 + \frac{T}{-T_f}\right)^\varepsilon \tag{6}$$

Where T_f is the electrolyte freezing temperature that depends mainly on the electrolyte specific gravity (about -70°C for typical electrolyte composition) and $C_{av}(I)$ is the value returned by Equation (5) when calculated at 0°C.

Despite all the aforementioned methods can describe quite effectively the effect of a reduction in (apparent) battery capacity under different current loads, they fail when the battery capacity recovery phenomenon is to be modeled. In fact, during battery rest (after a discharge run), a recovery can occur (Linden, 2002; Aylor, 1982). This phenomenon has been treated in depth by (Sharkh, 2006) where a lead-acid battery with a nominal capacity of 70Ah was initially discharged with a constant current of 5A providing a total capacity slightly less than the nominal value (point C in Figure 1). After a complete charge, the same battery was discharged at 50A until its terminal voltage reached the minimum cut-off value of 10V, providing a whole capacity of 44Ah (point A in Figure 1). After the battery was taken at rested for 6 hours, it was discharged again at 5A. Under these conditions the battery further provided other 20Ah for a total charge delivered by the battery of 64Ah (point B in Figure 1).

This phenomenon has been explained as the reformation of the hydrated gel zones in the electrode active centres during the waiting period (Pavlov, 2002). The lesson learned here is that the total amount of charge that can be depleted from the battery does not depend only on the current magnitude but also on the overall current profile, which account for resting times.

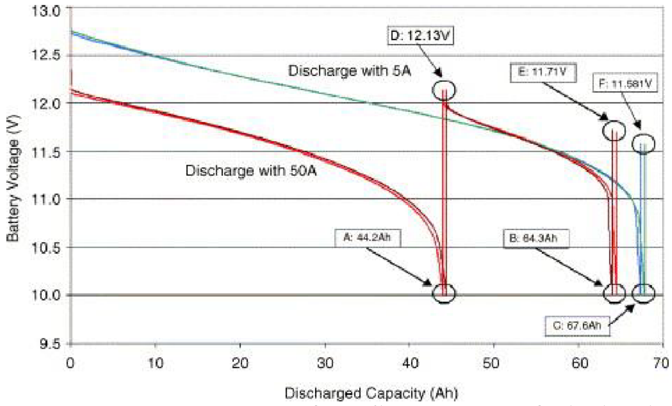

Figure 1 - Capacity recovery after a 6hrs resting time for lead acid batteries, (Sharkh, 2006).

In practice, Peukert equation should be interpreted with care. Equation (1) does not mean that when a battery is discharged 'fully' (*i.e.*, down to a minimum voltage) at a high discharge current, it is completely empty. In fact, a seemingly empty battery discharged at high current will still have some available capacity at a lower discharge current after some resting time.

4. PEUKERT EFFECT MODEL FOR PBA BATTERIES

A simple battery model structure able to capture the 'Peukert phenomenon' is presented in this section. This model is structured specifically to be able to accurately reproduce the Peukert and capacity recovery effects. It is built upon the rich literature on simple equivalent electrical circuit models, but rationally extends them to explicitly capture the phenomenological behavior described by the Peukert effect. The new model structure proposed is based on the equivalent electric circuit shown in Figure 2.

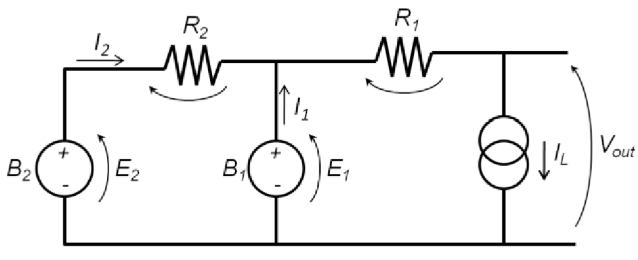

Figure 2 - Battery equivalent circuit to model 'Peukert effect'.

The main idea behind the model design is to split the total capacity C_n of the battery into two energy buffers. A first energy buffer B_1 (directly observable at the battery terminals) is only a (large) fraction of the total capacity. Its voltage E_1 depends on its state of charge (SoC_1). The resistance R_1 represents approximately the internal resistance of the battery. The additional buffer B_2 (internally connected to the first one and invisible at the terminals) is added. In the limit of vanishingly small R_2 (or small currents), this models behaves like a single energy buffer. In the limit of large currents, the first buffer gets emptied before the second (inner) one and the battery appears to be empty (but recovers after some time, as the two buffers equilibrate). This is a very coarse two term approximation of diffusion-driven phenomena (sometimes modeled as transmission lines from an electrical stand point).

Accordingly, the total capacity is split into the two buffers through the parameter β (0≤β≤1):

$$C_{b1}(0) = C_{b10} = \beta C_n \quad (7)$$

$$C_{b2}(0) = C_{b20} = (1-\beta) C_n \quad (8)$$

where C_{b10} and C_{b20} are the initial capacities ($t=0$) of the buffer B_1 and B_2, respectively .The amount of capacity stored in each buffer at $t > 0$, depends on the current flowing through each buffer. In particular:

$$C_{b1}(t) = C_{b10} - \int_0^t I_1(\tau)\, d\tau \quad (9)$$

$$C_{b2}(t) = C_{b20} - \int_0^t I_2(\tau)\, d\tau \quad (10)$$

where positive current indicates a current that flows out of the buffer (as in Figure 2). Differentiating (9) and (10) gives:

$$\dot{C}_{b1}(t) = -I_1(t) \quad (11)$$

$$\dot{C}_{b2}(t) = -I_2(t) \quad (12)$$

Moreover, combining:

$$I_2 = \frac{E_2 - E_1}{R_2} \quad (13)$$

with Kirchhoff's current law at the node gives the following expression for I_1:

$$I_1 = I_L - I_2 = I_L - \frac{E_2 - E_1}{R_2} \quad (14)$$

In Equation (14), I_L represents the load current (positive flowing into the load). For every β, the input-output behavior of the battery is guaranteed if the voltage across each buffer is an increasing monotonic function of their state of charge, that is:

$$E_1 = f(SoC_1) = f\left(\frac{C_{b1}(t)}{Cb_{10}}\right) \quad (15)$$

$$E_2 = f(SoC_2) = f\left(\frac{C_{b2}(t)}{C_{b20}}\right) \quad (16)$$

Typically, the open circuit voltage as a function of the *SoC* (for 12 V PbA batteries) follows the trend shown in Figure 3.

Thus, the model will output a total battery capacity as found by integrating the input load current starting from the initial time ($t=0$) until the moment at which the outer buffer (B_2) gets emptied (at $t = t_f$), i.e.:

$$C_{av,model} = \int_0^{t_f} I_L(t) dt \quad (17)$$

The fundamental parameters of the model are β and R_2 which, while independent of load current and *SoC*, are in general non linear functions of the temperature, *T*, and aging or SoH:

$$\beta = f_\beta(\theta, aging) \quad (18)$$

$$R_2 = f_{R_2}(\theta, aging) \quad (19)$$

In the following, the model parameters are calibrated at constant temperature and for a new battery with data available from the battery manufacturer.

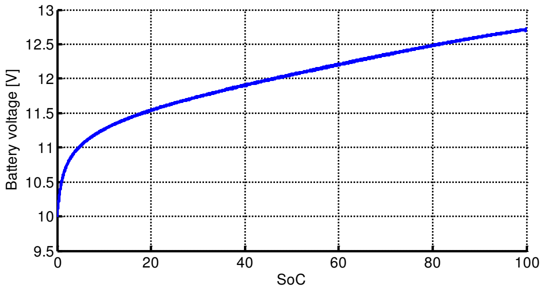

Figure 3 – Experimental data showing the open circuit voltage vs. SoC for a 12 V 120Ah PbA ArmaSafe+ battery manufactured by Hawker (Enersys Inc.)

5. MODEL EVALUATION

The model presented in Section 4 has been evaluated in simulation using a Simulink® based model. We consider a 120Ah ArmaSafe+ battery manufactured by Enersys Inc. Hawker (Enersys Inc.) and data provided by the manufacturer are being used in this initial model assessment. Testing experiments are being designed to experimentally validated the novel model structure against different load scenarios. Table 1 lists the battery rated capacity under different constant load currents. The same information is plotted in Figure 4.

Rated Capacity [Ah]	**Time [hours]**	**C-rate**	**Current [A]**
120	20	C/20	6
110	10	C/11	11
100	5	C/6	20
80	1	2C/3	80

Table 1 - Capacity data for ArmaSafe+ battery (Enersys Inc.)

We refer to the vector of capacities represented by green points with respect to different load current in Figure 4 as $C_{manufacturer}$ (used later on in the identification process described by Equation (23)). An exponential curve fitting is provided also in Figure 4 from which the Peukert exponent is easily derived.

As a result, by means of Equation (3) and the exponential fitting law used to interpolate the manufacturer data of Figure 4, the following values are obtained:

$$C_1 = 159Ah \tag{20}$$

$$n = 1.1565 \tag{21}$$

A parameter identification procedure is used in order to identify values of the parameters β and R_2 of the model. An optimization process aimed at minimizing the root mean square error ε given by:

$$\varepsilon = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(C_{manuf} - C_{av,model}(\beta, R_2)\right)^2\right]} \tag{22}$$

is used which give:

$$R_2 = 0.38\Omega \tag{23}$$

$$\beta = 0.53 \tag{24}$$

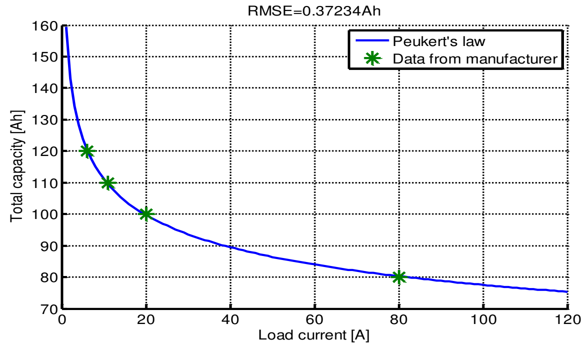

Figure 4 – The green dots represent data provided by the manufacturer while the blue fitting line represents the Peukert trend. The overall rmse is below 1Ah (0.3%).

The model output (in terms of available capacity) and the data provided by the battery manufacturer are plotted in Figure 5.

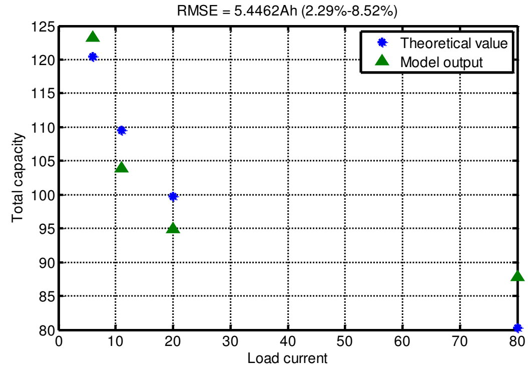

Figure 5 - Comparison between data provided by the manufacturer and model outputs (rmse<5% or <6Ah).

Figure 6 shows the model response when a load current I_L of 20A (C/6) is requested; moreover, it also shows how this current is split in two currents flowing out of two buffers (positive values indicate discharge), I_1 and I_2. While during the initial phase of discharge the contribution from the buffer B_1 is more significant, towards the end of the discharge buffer B_2 takes on progressively (Equation (14) is satisfied at each instant of time).

The model proposed is also able to reproduce the charge recovery process. In fact, after the first discharge at high current, the buffer B_1 gets emptied and since no more charge can be provided by the battery, the load is being disconnected forcing I_L=0. Under this condition, due to the voltage difference between the two buffers, a "negative" current I_1 (flowing into the buffer) will recharge the empty buffer (thus, making the buffers effectively share their charge). This process is shown in Figure 7.

When the initial conditions describe by in Equation (7) and (8) are restored (after an adequate resting time), this process (charge replenish) is stopped. At this point, the 'recovered' capacity is given by:

$$C_{rem} = C_0 - C_{av,model}$$

The battery capacity recovery effect is shown in Figure 8 on the voltage response curve as a result of the simulated model. After the first high current discharge at 20A the battery provides 95Ah. Then, after a resting period, 42Ah are 'recovered' when a low current (6A) is given to the battery, accounting for a total of 137Ah the battery is able to provide.

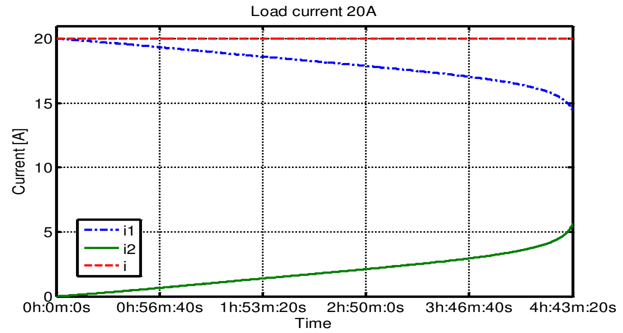

Figure 6 – Battery current 'split' at the node (see Figure 2) under a 20A load. Both buffers contribute to the total current request $(I_1 + I_2 = I)$

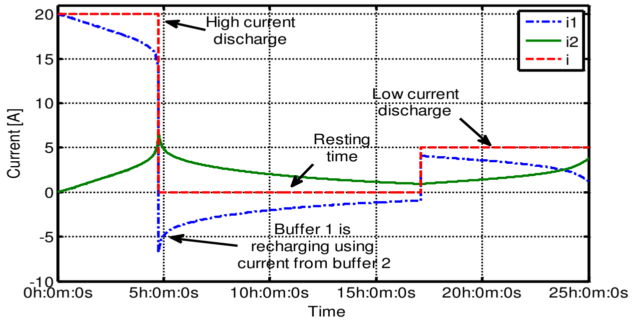

Figure 7 – Capacity recovery effect as simulated by the proposed two-buffer model. After the initial current discharge (20A) the battery is first rested $(I = 0)$ and then followed by a low current (5A) discharge.

7. CONCLUSIONS AND FUTURE WORK

In this paper, a novel battery model able to capture the Peukert and the capacity recover effects for PbA batteries has been presented. Simulation results show the effectiveness of the model proposed against battery manufacturer data. The model has been shown to effectively capture both phenomena for different load currents. Due to its simplicity, the model proposed can easily be incorporated into real time applications to predict *SoC* and time to run, essential for mission-critical applications (military, telecom, hospitals). A testing plan has been designed at the Center for Automotive Research The Ohio State University to calibrate and validate the model proposed against experimental data.

8. REFERENCES

Aylor J. H., A. Thieme and B. W. Johnso (1982). A battery state-of-charge indicator for electric wheelchairs. In: *IEEE Transactions on Industrial Electronics*, 39(5), pp.398-409.

Bamieh B. and L. Giarre'. (2002). Identification of linear parameter varying models. In: *International Journal of Robust and Nonlinear Control*, (12), pp.841-53.

Ceraolo, M. (2000). New Dynamical Models of Lead–Acid Batteries. In: *IEEE Transaction on Power Systems*, 15(4), pp.1184-1190.

Chen M. and G. A. Rincon-Mora (2006). Accurate Electrical Battery Model Capable of Predicting Runtime and I–V Performance. In: *IEEE Transaction on energy conversion*, June. pp.504-511.

Codeca' F., S. M. Savaresi and G. Rizzoni (2008). On battery State of Charge estimation: a new mixed algorithm. In: *17th IEEE International conference on Control Applications, Part of 2008 IEEE Multi-conference on systems and control*. San Antonio, Texas, USA.

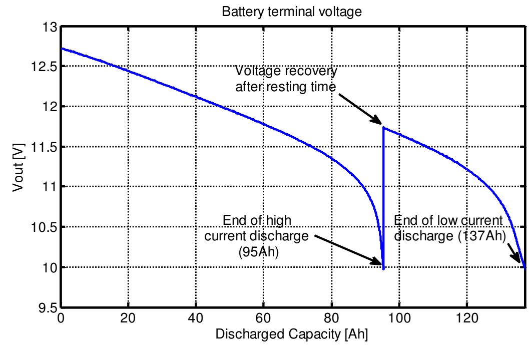

Figure 8 - Battery voltage response from the model. After a resting time from the first discharge the battery experiences a voltage and capacity recovery, providing further energy.

Doerffel D., S. A. Sharkh (2006). A critical reviewnext term of using the Peukert equation for determining the remaining capacity of lead-acid and lithium-ion batteries. In: *Journal of Power Sources*, 155(2), pp.395-400.

Enersys Inc. Hawker, n.d. 6140-01-485-1472 *Armasafe Plus Battery Maintenance Guide*. Technical manual.

Hu Y., S. Y. Yurkovich, Y. Guezennec and B. J. Yurkovich (2009). A technique for dynamic battery model identification in automotive applications using linear parameter varying structures. *Journal of Power Sources*, 17(10), pp.1190-1201.

Hu Y., S. Y. Yurkovich, Y. Guezennec and B. J. Yurkovich (2011). Electro-thermal battery model identification for automotive applications. *Journal of Power Sources*, 196(1), pp.449-457.

Jackey R. A. (2007). A simple, effective lead acid battery modeling process for electrical system component selection. In: *SAE World Congress & Exihibition*. Detroit (MI) USA.

Kattakayam T. A. and K. Srinivasan (2004). Lead acid batteries in solar refrigeration systems. In: *Renewable Energy*, 29(8), pp.1243-1250.

Kim I.I. (2006). The novel state of charge estimation method for lithium battery using sliding mode observer. *Journal of Power Sources*, 163(1), pp.584-590.

Linden, D. (2002). *Handbook of Batteries*. 3rd ed. Mc Graw-Hill.

Pavlov D. and G.Petkova (2002). Phenomena That Limit the Capacity of the Positive Lead Acid Battery Plates. In: *Journal of the Electrochemical Society*, 149(5), pp.A644-A653.

Peukert W (1897). *Elektrotechnische Zeitschrift*.

Plett G. L. (2004). Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs: Part 2. Modeling and identification. In: *Journal of Power Sources*, 134(2), pp.262-276.

Pascoe P. E. and A. H. Anbuky (2003). A VRLA battery simulation model. In: *Energy conversion and management*, Aug. pp.1015 - 1041.

Salameh Z. M., M. A. Casacca and W. A. Lynch (1992). A mathematical model for lead acid batteries. *IEEE Transactions on Energy Conversion*, 7(1), pp.93-98.

Tebbutt M. A. (1999). A breakthrough in "on line" VRLA battery condition analysis. In: *International Telecommunicatin Energy Conference*. Copenhagen (Denmark).

Verdult V., M. Verhaegen (2002). Subspace identification of multivariable linear parameter-varying systems. In: *Automatica*, 38(5), pp.805-814.